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HYPER VERSION

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Vacuum states for AdS_2 black holes

Marcus Spradlin and Andrew Strominger

Department of Physics, Harvard University
Cambridge, MA 02138
E-mail: `spradlin@feynman.harvard.edu`, `andy@planck.harvard.edu`

Abstract: An AdS_2 black hole spacetime is an AdS_2 spacetime together with a preferred choice of time. The Boulware, Hartle-Hawking and SL$(2,\mathbb{R})$ invariant vacua are constructed, together with their Green functions and stress tensors, for both massive and massless scalars in an AdS_2 black hole. The classical Bekenstein-Hawking entropy is found to be independent of the temperature, but at one loop a non-zero entanglement entropy arises. This represents a logarithmic violation of finite-temperature decoupling for AdS_2 black holes which arise in the near-horizon limit of an asymptotically flat black hole. Correlation functions of the SL$(2,\mathbb{R})$ invariant boundary quantum mechanics are computed as functions of the choice of AdS_2 vacuum.

Keywords: Black Holes, 2D Gravity.

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Contents

1. Introduction

Two-dimensional anti-de Sitter space (AdS_2) has arisen in at least three distinct but related contexts within string/black hole physics. The first is as the near-horizon geometry (together with an S^2 factor) of the extremal Reissner-Nordstrom solution [1]. AdS_2 is a stable attractor solution of the equations which govern how the geometry changes as the horizon is approached [2], and as such is expected to play a central role in the physics of black holes. AdS_2 made a second appearance in studies of two-dimensional quantum gravity, where it provides an $\mathrm{SL}(2,\mathbb{R})$ invariant ground state for Liouville gravity [3, 4], and a rich arena for the study of two-dimensional black holes [5, 6, 7, 8, 9]. Most recently it is the black sheep in the family of AdS/CFT dualities [10], having so far resisted a fully satisfactory realization of the duality [11, 12, 13, 14, 15, 16]. One hopes that this can be remedied and that in the process a clearer relation between the different aspects of AdS_2 physics will emerge.

In this paper we investigate properties of both massive and massless quantum field theory on an AdS_2 background. In section 2 we review the appearance of AdS_2 in near-horizon black hole geometries. This motivates the definition of an AdS_2 black hole as an AdS_2 spacetime together with a preferred choice of time. In section 3 it is shown in the quantum theory that the choice of time affects the vacuum state. We discuss the Hartle-Hawking, Boulware and $\mathrm{SL}(2,\mathbb{R})$ invariant AdS_2 black hole vacua and the Hawking temperature measured by various families of observers. It is shown that the vacua defined with respect to Poincaré or global time are equivalent to one another and to the Hartle-Hawking vacuum. The Boulware vacuum, which is associated to the preferred choice of time, is not in general equivalent. Section 4 concerns the entropy of an AdS_2 black hole. The classical Bekenstein-Hawking entropy is temperature-independent. At one loop there is an entanglement entropy which depends logarithmically on the Hawking temperature. This represents a violation of low-energy decoupling between the asymptotically flat and near-horizon regions of the black hole at finite temperature. In section 5 we analyze processes in which the temperature is changed by sending matter into the black hole. In section 6 we turn to massive fields, and give explicit expressions for the Green functions in the Boulware and Hartle-Hawking vacua. The stress-energy expectation values in these vacua are computed in section 7. In section 8, motivated by the AdS/CFT duality, we compute correlation functions of the $\mathrm{SL}(2,\mathbb{R})$ invariant boundary quantum mechanics in the various AdS_2 vacua.

2. AdS_2 black holes in the near-horizon limit

In three dimensions, all negative curvature spaces are locally equivalent to AdS_3. Because of this, for many years it was believed that black holes did not exist for pure gravity in three dimensions. However, BTZ showed that black holes do exist which differ from AdS_3 only by global identifications [17]. The local geometry at the black hole horizon is the same as everywhere else, but it is globally characterized as the surface from behind which nothing can communicate with infinity. This differs from higher dimensional examples in which the geometry has special features at the horizon.

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In two dimensions, all negative curvature spaces are locally AdS_2. We will argue that, much as in three dimensions, AdS_2 black holes nevertheless exist in pure gravity (without dilatons). Similar discussions have appeared in [5, 18]. One way to describe this is that an AdS_2 black hole is AdS_2 together with a choice of (Killing) time t at infinity for which the full region $-\infty < t < \infty$ does not cover all of the boundary of AdS_2. The black hole horizon is then the surface from behind which nothing can escape to the region $-\infty < t < \infty$. We will see that the black holes so defined have characteristic thermodynamic properties.

AdS_2 black holes naturally arise in the near-horizon limits of Reissner-Nordstrom black holes. Following the discussion of [12], the full magnetically-charged solution is

$$\begin{aligned} ds^2 &= -\frac{(r-r_+)(r-r_-)}{r^2}dt^2 + \frac{r^2}{(r-r_+)(r-r_-)}dr^2 + r^2 d\Omega_2^2\,, \\ F &= Q\epsilon_2\,, \end{aligned} \tag{2.1}$$

where ϵ_2 is the volume element on the unit S^2. The locations of the inner and outer horizons are related to the Hawking temperature T_H and charge via

$$\begin{aligned} Q^2 &= \frac{r_+ r_-}{L_p^2}\,, \\ T_H &= \frac{r_+ - r_-}{4\pi r_+^2}\,, \end{aligned} \tag{2.2}$$

where L_p is the Planck length.

We now consider, as in [12], the near-horizon limit

$$L_p \to 0\,, \tag{2.3}$$

with

$$U = \frac{r - r_+}{L_p^2}\,, \qquad Q, T_H \text{ fixed}\,. \tag{2.4}$$

The metric then reduces to

$$\frac{ds^2}{Q^2 L_p^2} = -\frac{U(U + 4\pi Q^2 T_H)}{Q^4}dt^2 + \frac{1}{U(U+4\pi Q^2 T_H)}dU^2 + d\Omega_2^2\,. \tag{2.5}$$

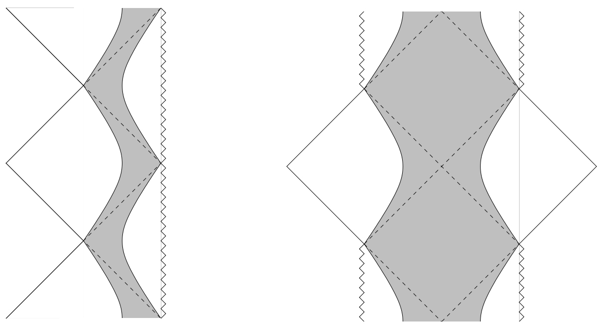
We note that both the ADM energy $2M = r^+ + r^-$ and the entropy $S_{BH} = \pi r_+^2/L_p^2$ go to T_H-independent constants ($M = Q$ and $S_{BH} = \pi Q^2$) in this limit.

The T_H dependence of the metric can be eliminated by a coordinate transformation. Defining

$$t' \pm \frac{Q^2}{U'} = \tanh\left[\pi T_H \left(t \pm \frac{1}{4\pi T_H}\ln\frac{U}{U + 4\pi Q^2 T_H}\right)\right]\,, \tag{2.6}$$

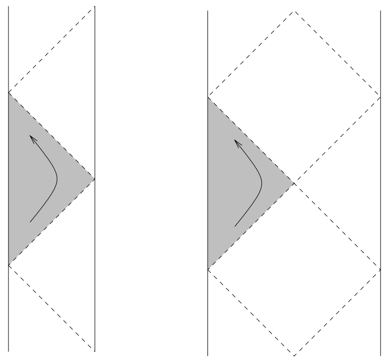
the metric reduces to

$$\frac{ds^2}{Q^2 L_p^2} = -\frac{U'^2}{Q^4}dt'^2 + \frac{1}{U'^2}dU'^2 + d\Omega_2^2\,. \tag{2.7}$$



(a) (b)

Figure 1: (a) Penrose diagram corresponding to the extremal Reissner-Nordstrom black hole. The dashed line is the black hole horizon, and the shaded strip is the near-horizon AdS_2 region. (b) Near-extremal Reissner-Nordstrom black hole and corresponding near-horizon AdS_2 region.



(a) (b)

Figure 2: Penrose diagram for AdS_2. The dashed lines are the horizons inherited from the embedding in extremal (a) or near-extremal (b) Reissner-Nordstrom (compare with figure 1). The arrows indicate the flow of asymptotic time "t".

In terms of

$$\tau \pm \sigma \pm \frac{\pi}{2} = 2 \tan^{-1} \left(t' \pm \frac{Q^2}{U'} \right) , \tag{2.8}$$

the metric becomes

$$\frac{ds^2}{Q^2 L_p^2} = \frac{-d\tau^2 + d\sigma^2}{\cos^2 \sigma} + d\Omega_2^2 \, . \tag{2.9}$$

This is known as the Robinson-Bertotti geometry on $AdS_2 \times S^2$. As illustrated in figures 1a and 1b for the extremal and near extremal cases, the $AdS_2 \times S^2$ region of the full Reissner-Nordstrom geometry is a ribbon which zigzags its way up through the infinite chain of universes.

Since T_H can be eliminated by a coordinate transformation, the classical near horizon theory is independent of T_H. We shall see in the next section that this is not the case in the quantum theory, because the definition of a vacuum state in general depends on a choice of time, or equivalently a preferred family of observers. An AdS_2 spacetime which arises as the near horizon geometry of Reissner-Nordstrom is indeed endowed with a preferred choice of time "t", namely, the one associated to the Killing vector which generates unit time translations in the asymptotically flat spatial infinity of the Reissner-Nordstrom geometry. As is evident from figure 2, as this preferred time coordinate t runs over the full range $-\infty < t < +\infty$, only part of the timelike boundaries of AdS_2 is covered. We shall refer to this boundary region as spatial infinity. The future black hole horizon can then be defined as the boundary of the region from which nothing can escape to spatial infinity. The past horizon is then the boundary of the region which cannot be accessed from spatial infinity. These horizons coincide with the Killing horizon of the preferred Killing vector.

In the extremal case $T_H = 0$ depicted in figure 2a, the exterior of the black hole is a wedge, the corner of which extend to the far boundary of AdS_2. For $T_H \neq 0$ (figure 2b), the exterior of the black hole is still a wedge, but it extends only halfway across AdS_2.

3. AdS_2 black hole thermodynamics

In this section we discuss the thermal properties of AdS_2 black holes. We consider mainly the case of a free massless scalar field, deferring the massive case to section 6.

3.1 The quantum state

In order to define a vacuum state we need a metric with a timelike Killing vector. The vacuum is then defined as the state annihilated by positive frequency modes of the field operator. Observers at a fixed spatial coordinate x, in a coordinate system in which the metric is time-independent, then detect no particles.

For AdS_2 there are inequivalent choices of time coordinates or equivalently conformal gauge coordinates. For one such coordinate choice the metric takes the form

$$\frac{ds^2}{Q^2 L_p^2} = \frac{-d\tau^2 + d\sigma^2}{\cos^2 \sigma} \,. \tag{3.1}$$

The coordinates (τ, σ) are referred to as global coordinates because they cover all of (the universal cover of) AdS_2 for $-\pi/2 \leq \sigma \leq \pi/2$ and $-\infty < \tau < \infty$. Spatial infinity is at $\sigma = \pm\pi/2$, and the horizons are at $\tau \pm \sigma = 0$. The corresponding vacuum $|0_{\text{Global}}\rangle$, annihilated by modes which are positive frequency with respect to τ, is the familiar $\text{SL}(2, \mathbb{R})$ invariant vacuum for a free scalar field on the strip. We shall see shortly that this is equivalent to the Hartle-Hawking black hole vacuum as well as the Poincaré vacuum.

A second coordinate system is the "Schwarzschild" coordinates, which uses the time t appearing in (2.5). t coincides with the time coordinate inherited from the decoupled asymptotically flat region and, as discussed above, defines the black hole horizon. Eq. (2.5) can be transformed to conformal gauge by

$$x = \frac{1}{4\pi T_H} \ln \frac{U}{U + 4\pi Q^2 T_H} \,, \tag{3.2}$$

in which

$$\frac{ds^2}{Q^2 L_p^2} = \left[\frac{2\pi T_H}{\sinh(2\pi T_H x)} \right]^2 (-dt^2 + dx^2) \,. \tag{3.3}$$

Since the coordinate transformation (3.2) involves only the spatial coordinate and does not change the choice of time, it does not affect the associated vacuum $|0_{\text{Schwarzschild}}\rangle$.

The Schwarzschild coordinates (t, x) and global coordinates (τ, σ) are related by the coordinate transformation

$$\tan \frac{1}{2}(\tau \pm \sigma) = \mp e^{\mp 2\pi T_H (t \pm x)} . \tag{3.4}$$

A natural family of observers are those moving along worldlines of fixed U. This corresponds to trajectories which remain a fixed distance from the black hole horizon. Since the proper time along such worldlines equals Schwarzschild time (up to a constant), such observers will not detect any particles in the state $|0_{\text{Schwarzschild}}\rangle$. The vacuum with this property is known as the Boulware vacuum. Hence we conclude that

$$|0_{\text{Schwarzschild}}\rangle = |0_{\text{Boulware}}\rangle . \tag{3.5}$$

We will see in section 7 that this vacuum has the property that the expectation value of the stress tensor diverges on the horizon.

Since Schwarzschild and global time do not agree, constant-U observers will detect particles in the global vacuum. The transition probabilities for a detector on a constant U-worldline are determined from the Green functions in the global (τ, σ) vacuum $|0_{\text{Global}}\rangle$. It follows from (3.4) that with respect to the proper time τ_D along the detector worldline these are thermal Green functions, simply because the (τ, σ) coordinates are invariant under imaginary shifts $t \to t + i/T_H$. Accounting for the difference between t and proper time τ_D, the detector sees a thermal bath of particles at temperature $\sqrt{g^{00}} T_H = \frac{1}{2\pi Q} \sinh(2\pi T_H x)$. The vacuum with this property is known as the Hartle-Hawking vacuum. Hence we conclude that

$$|0_{\text{Global}}\rangle = |0_{\text{Hartle-Hawking}}\rangle . \tag{3.6}$$

Yet another way to define a vacuum is as the state annihilated by modes which are positive frequency in the Poincaré metric

$$\frac{ds^2}{Q^2 L_p^2} = \frac{-dT^2 + dy^2}{y^2} . \tag{3.7}$$

We use capital T to distinguish the Poincaré time T from the Schwarzschild time t. For $-\infty < T < \infty$ and $0 < y < \infty$ these coordinates cover only the patch defined by $\tau + \sigma < \pi/2$ and $\tau - \sigma > -\pi/2$, and hence only the boundary at $\sigma = -\pi/2$ (the various coordinate systems are illustrated in figure 3). These coordinates are related to the global coordinates by the transformation

$$T \pm y = \tan \frac{1}{2}\left(\tau \pm \sigma \pm \frac{\pi}{2}\right) . \tag{3.8}$$

The (Klein-Gordon) overlap between a positive frequency mode in Poincaré coordinates $\phi^P_{+\omega} = \frac{1}{\sqrt{\pi\omega}} e^{-i\omega T} \sin(\omega y)$ and a mode $\phi^G_n = \frac{1}{\sqrt{\pi|n|}} e^{-in\tau} \sin(n(\sigma + \pi/2))$ with positive $(n = 1, 2, \ldots)$ or negative $(n = -1, -2, \ldots)$ frequency in global coordinates is

$$\langle \phi^P_{+\omega} | \phi^G_n \rangle = i \int_0^\infty dy \left[\phi^P_{-\omega} (\partial_T \phi^G_n) - \phi^G_n (\partial_T \phi^P_{-\omega}) \right]_{T=0} , \tag{3.9}$$

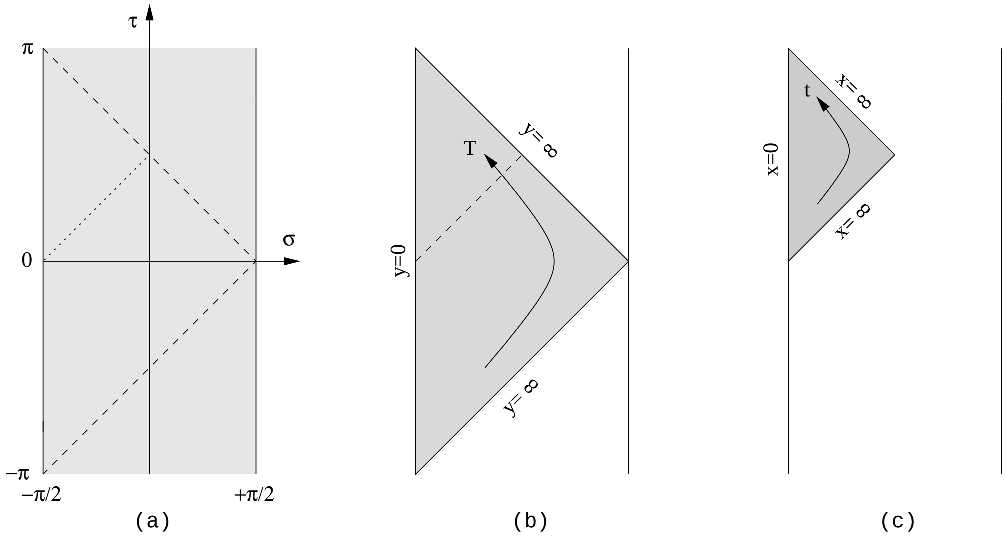


Figure 3: Three coordinate systems on AdS_2. (a) Global coordinates, $-\pi/2 \le \sigma \le \pi/2$ and $-\infty < \tau < \infty$. (b) Poincaré coordinates, $-\infty < T < \infty$, $0 < y < \infty$. (c) Schwarzschild coordinates, $-\infty < t < \infty$, $0 < x < \infty$.

where $\phi^P_{-\omega} = (\phi^P_{+\omega})^*$. On the slice $T = 0$ one has $\sigma + \pi/2 = 2\tan^{-1} y$ and $\partial_T = \frac{2}{y^2+1}\partial_\tau$. Using these facts and $\tan^{-1} y = \frac{1}{2i}\log(\frac{1+iy}{1-iy})$ one can put (3.9) into the form

$$\langle \phi^P_{+\omega} | \phi^G_n \rangle = \frac{1}{\pi}\sqrt{\frac{|n|}{\omega}} \int_{-\infty}^{\infty} dy\, e^{i\omega y}(1+iy)^{-n-1}(1-iy)^{n-1} . \tag{3.10}$$

The contour must be closed in the upper half plane. When n is negative there is no pole in the upper half plane and so the integral vanishes. When n is positive there is a pole at $y = i$, and the result of the integration is

$$\begin{aligned} \langle \phi^P_{+\omega} | \phi^G_{+n} \rangle &= (-1)^n \sqrt{\frac{n}{\omega}} e^{-\omega} L_n^{-1}(2\omega) , \\ \langle \phi^P_{+\omega} | \phi^G_{-n} \rangle &= 0 , \end{aligned} \tag{3.11}$$

where L_n^α is the associated Laguerre polynomial. We conclude that the Bogoliubov transformation is block diagonal, and it follows that the Poincaré annihilation operators are linear combinations of the global annihilation operators and have no overlap with the global creation operators, and hence

$$|0_{\text{Global}}\rangle = |0_{\text{Poincaré}}\rangle . \tag{3.12}$$

This result will be confirmed by the computation of the Green functions for massive scalars in section 6. The equivalence of the global and Poincaré vacua in AdS_n has been discussed in [19].

We note that in the limit $T_H \to 0$, the Schwarzschild metric (3.3) reduces to the Poincaré form

$$\frac{ds^2}{Q^2 L_p^2} = \frac{-dt^2 + dx^2}{x^2} . \tag{3.13}$$

Hence the vacuum associated to the coordinates (3.3) reduces to the $SL(2,\mathbb{R})$ invariant Poincaré vacuum associated to the coordinates (3.13) in the limit $T_H \to 0$. The Hawking temperature T_H can be thought of as a measure of the non-$SL(2,\mathbb{R})$ invariance of the vacuum state associated to (3.3).

4. Entanglement entropy

The presence of a thermal bath of particles around an AdS_2 black hole would normally imply an associated temperature-dependent entropy. However in the near-horizon limit (2.3), (2.4) one finds that

$$S_{BH} \to \pi Q^2 \,, \tag{4.1}$$

independently of T_H. This means that there is no classical temperature-dependent entropy. However at the one loop level there is a quantum correction to the entropy from the entanglement of the near-horizon AdS_2 Hilbert space with the Hilbert space of the decoupled asymptotically flat region. (Strictly speaking when this entropy is nonzero the asymptotically flat region is not fully decoupled.)

In order to compute this entropy one needs to be more precise about how the near-horizon AdS_2 region of the Reissner-Nordstrom black hole is separated from the asymptotically flat region in the $L_p \to 0$ limit. Before taking L_p all the way to zero let us choose a fixed value of radial coordinate $U = U_{\rm max}$ which divides the spacetime so that the AdS_2 region is $0 < U < U_{\rm max}$ while the flat region is $U_{\rm max} < U < \infty$. $U_{\rm max}$ should be in the mouth region where the geometry changes from AdS_2 to flat, so we take $U_{\rm max} = c_0 \frac{Q}{L_p}$. The arbitrary constant c_0 can be taken to be very small so that the boundary is deep in the AdS_2 region, but is held fixed as $L_p \to 0$, so that $U_{\rm max} \to \infty$. We then erect the Hilbert space of, e.g., a scalar field on both regions, with bases denoted $|\psi^i_{\rm AdS}\rangle$ and $|\psi^J_{\rm Flat}\rangle$. A generic state of the quantum field on the Reissner-Nordstrom spacetime – including the vacuum state – is a sum of product states of the form

$$|\psi\rangle = \sum_{iJ} c_{iJ} |\psi^i_{\rm AdS}\rangle |\psi^J_{\rm Flat}\rangle \,. \tag{4.2}$$

The state on the AdS_2 region is then a density matrix

$$\rho_{\rm AdS} = {\rm Tr}_{\rm Flat} |\psi\rangle\langle\psi| = \sum_{ijK} c_{iK} c^*_{jK} |\psi^i_{\rm AdS}\rangle\langle\psi^j_{\rm AdS}| \,. \tag{4.3}$$

Alternately the state on the flat region is

$$\rho_{\rm Flat} = {\rm Tr}_{\rm AdS} |\psi\rangle\langle\psi| = \sum_{IJk} c_{kI} c^*_{kJ} |\psi^I_{\rm Flat}\rangle\langle\psi^J_{\rm Flat}| \,. \tag{4.4}$$

The entanglement entropy is then defined by

$$S_{\rm ent} = -\,{\rm Tr} \rho \ln \rho \,, \tag{4.5}$$

and takes the same value for either $\rho_{\rm Flat}$ or $\rho_{\rm AdS}$. $S_{\rm ent}$ is a measure of the correlation between the portions of the quantum state on the two regions.

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Entanglement entropy for black holes has been discussed in [20]–[26]. In general there are divergences arising from the entanglement of arbitrarily short wavelength modes which overlap the dividing line $U_{\rm max}$. We are interested in finite, temperature-dependent contributions to $S_{\rm ent}$ for the vacuum state on the Reissner-Nordstrom geometry. Such a term arises from the S-wave modes of scalar fields, which reduces to a conformal field on AdS_2. The vacuum entanglement entropy for a conformal field theory of central charge charge c in curved space was derived in [24, 27] as

$$S_{\rm ent} = \frac{c}{6}\rho(\sigma_{\rm max}) - \frac{c}{6}\ln\Delta\,. \tag{4.6}$$

In this expression, $\rho(\sigma_{\rm max})$ is the metric conformal factor in the coordinate system used to define the vacuum evaluated at the dividing line between the two regions, and Δ is a non-universal short-distance cutoff.

The Hartle-Hawking vacuum for an AdS_2 black hole is defined with respect to the global coordinates (3.1), in which

$$\rho = -\ln\cos\sigma\,. \tag{4.7}$$

For small L_p, $U_{\rm max}$ is large and from (2.6) and (2.8) we have

$$\sigma_{\rm max} + \frac{\pi}{2} \sim \frac{2\pi Q^2 T_H}{U_{\rm max}}\,. \tag{4.8}$$

It follows that

$$S_{\rm ent} = -\frac{c}{6}\ln(QT_H) + \text{non-universal}\,. \tag{4.9}$$

Related (although not obviously equivalent) results were obtained with euclidean methods in [9].

Expression (4.9) represents a logarithmic violation of decoupling in the near horizon limit at finite temperature between the flat region and the AdS_2 region. Additional contributions to the entanglement entropy could arise from massive fields as well as higher angular modes of massless fields. However it is not clear if these contributions will survive the near horizon limit since the modes of such fields vanish rapidly near the boundary of AdS_2. It would be interesting to compute $S_{\rm ent}$ in string theory examples and to investigate its origin in the D-brane picture.

5. Making an AdS_2 black hole

In this section we consider simple processes which change the temperature of the black hole. A general spherically symmetric solution of Einstein-Maxwell gravity corresponding to null matter falling in to a Reissner-Nordstrom black hole is

$$\begin{aligned} ds^2 &= -\frac{(r-r_+(v))(r-r_-(v))}{r^2}dv^2 + 2drdv + r^2 d\Omega_2^2\,, \\ F &= Q\epsilon_2\,, \end{aligned} \tag{5.1}$$

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with $r_+ r_- = L_p^2 Q^2$. The null matter has only one nonzero component of its stress tensor:

$$T_{vv} = \frac{\partial_v r_+(v) + \partial_v r_-(v)}{4\pi L_p^2 r^2} \,. \tag{5.2}$$

Let us start with an extreme Reissner-Nordstrom black hole ($r_- = r_+$) and send in a null shockwave of the form

$$T_{vv} = \frac{\pi Q^3 T_0^2 L_p \delta(v)}{r^2} \,, \tag{5.3}$$

where T_0 is a constant with units of temperature. The meaning of this particular form will shortly be clear. From (5.2) we see that this leads to

$$\begin{aligned} r_+ + r_- &= 2QL_p \qquad v < 0 \,, \\ r_+ + r_- &= 2QL_p + 4\pi^2 Q^3 T_0^2 L_p^3 \qquad v > 0 \,. \end{aligned} \tag{5.4}$$

Using $r_+ r_- = Q^2 L_p^2$ one can solve to find

$$\begin{aligned} r_\pm &= QL_p \qquad v < 0 \,, \\ r_\pm &= QL_p \left[1 \pm 2\pi Q T_0 L_p\right] + O(L_p^3) \qquad v > 0 \,, \end{aligned} \tag{5.5}$$

where the higher-order corrections in L_p will not be important. We see then that the Hawking temperature $T_H = (r_+ - r_-)/4\pi r_+^2$ is

$$\begin{aligned} T_H &= 0 \qquad v < 0 \,, \\ T_H &= T_0 + O(L_p) \qquad v > 0 \,. \end{aligned} \tag{5.6}$$

The shockwave (5.3) increases the Hawking temperature of the black hole from zero to T_H, at least in the $L_p \to 0$ limit.

Now we consider a near horizon limit

$$L_p \to 0 \,, \tag{5.7}$$

with

$$U = \frac{r - r_+}{L_p^2} \,, \qquad Q, T_0 \text{ fixed} \,. \tag{5.8}$$

The two-dimensional metric then reduces to

$$\frac{ds^2}{L_p^2} = -\frac{U(U + 4\pi Q^2 T_0 \Theta(v))}{Q^2} dv^2 + 2dU\, dv \,. \tag{5.9}$$

We note that in this limit the energy density (5.2) vanishes. In terms of the coordinates $s^\pm$ defined by

$$\begin{aligned} s^- &= v \,, & s^+ &= v + \frac{2}{U} \,, & &\text{for } v < 0 \,, \\ s^- &= \frac{1}{2\pi Q^2 T_0}(e^{2\pi T_0 v} - 1) \,, & s^+ &= s^- + \frac{2}{U} e^{2\pi T_0 v} \,, & &\text{for } v > 0 \,, \end{aligned} \tag{5.10}$$

the metric (5.9) takes the Poincaré form

$$\frac{ds^2}{Q^2 L_p^2} = -\frac{4ds^+ds^-}{(s^+ - s^-)^2}\,. \tag{5.11}$$

A detector at fixed $U = U_0$ hence has a worldline

$$\begin{aligned} s^+ &= s^- + \frac{2}{U_0}\,, \qquad s^- < 0\,, \\ s^+ &= s^-\left(1 + \frac{4\pi Q^2 T_0}{U_0}\right) + \frac{2}{U_0}\,, \qquad s^- > 0\,. \end{aligned} \tag{5.12}$$

The proper time τ_D along the detector worldline is

$$\begin{aligned} \tau_D &= QU_0 s^-\,, \qquad s^- < 0\,, \\ \tau_D &= \frac{1}{2\pi Q T_0}\sqrt{U_0(U_0 + 4\pi Q^2 T_0)}\,\ln(1 + 2\pi Q^2 T_0 s^-)\,, \qquad s^- > 0\,. \end{aligned} \tag{5.13}$$

Since Poincaré time and worldline time are proportional prior to the shock wave, there will be no particle detection in this region. However, after the shock wave, it follows from (5.10) that s^- is periodic under imaginary shifts of detector proper time. This implies that the detector will detect a thermal bath of radiation at temperature

$$T = T_0 \frac{Q}{\sqrt{U_0(U_0 + 4\pi Q^2 T_0)}}\,. \tag{5.14}$$

The first factor of T_0 is the black hole temperature, while the second is the Tolman factor representing the usual position-dependent temperature for thermal equilibrium in a gravitational field.

In conclusion, (5.9) represents an AdS_2 black hole whose temperature grows as a function of the null coordinate v because matter is being thrown in. A detector stationed at fixed U outside the black hole detects a thermal bath of radiation whose temperature grows as the matter is thrown in.

6. Massive fields and vacua

In this section we extend the previous discussion to the case of massive fields. For the remainder of this paper we set $QL_p = 1$. The proper dependence may be restored using dimensional analysis.

6.1 Green functions

We consider a massive scalar field ϕ with action

$$S = -\frac{1}{2}\int d^2x\sqrt{-g}\,\left[(\nabla\phi)^2 + m^2\phi^2\right]. \tag{6.1}$$

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The vacuum $|0\rangle$ is completely specified by the two-point function $G(\mathbf{x},\mathbf{y}) = \langle 0|\phi(\mathbf{x})\phi(\mathbf{y})|0\rangle$. In lorentzian spacetimes there are many Green functions.[1] We focus on the Hadamard function

$$G^{(1)}(\mathbf{x},\mathbf{y}) = \langle 0|\{\phi(\mathbf{x}),\phi(\mathbf{y})\}|0\rangle\,, \tag{6.2}$$

which is related to the familiar Feynman propagator $G_F(\mathbf{x},\mathbf{y}) = i\langle 0|T\phi(\mathbf{x})\phi(\mathbf{y})|0\rangle$ by $G^{(1)} = 2\,\mathrm{Im}\,G_F$. To construct the Hadamard function explicitly for a given vacuum one first finds a complete set of positive frequency solutions (i.e. $\phi_\omega \sim e^{-i\omega t}$ where t is the chosen time variable) of the massive wave equation

$$\nabla^2\phi_\omega = m^2\phi_\omega\,, \tag{6.3}$$

normalized with respect to the Klein-Gordon inner product, which in conformal gauge takes the form

$$\langle\phi_\omega|\phi'_\omega\rangle = i\int_\Sigma [\phi^*_\omega(\partial_t\phi_{\omega'}) - (\partial_t\phi^*_\omega)\phi_{\omega'}]\,, \tag{6.4}$$

where the integral is taken over a constant-time slice Σ. We encounter bases $\{\phi_\omega(y)\}$ defined on the half-plane $y \geq 0$ which oscillate as $y \to \infty$ and hence are not integrable. These modes are normalized by requiring that

$$\phi_\omega(y) \to \frac{1}{\sqrt{\pi\omega}}\sin(\omega y + \delta_\omega)\,, \qquad \text{as } y \to \infty\,. \tag{6.5}$$

This gives the correct relativistic delta-function normalization

$$\langle\phi_\omega|\phi_{\omega'}\rangle = 2\omega\int_0^\infty dy\;\phi^*_\omega\phi_{\omega'} = \delta(\omega - \omega')\,. \tag{6.6}$$

Once the modes are known, the Hadamard function is given by

$$G^{(1)}(\mathbf{x},\mathbf{y}) = 2\,\mathrm{Re}\int d\omega\,\phi^*_\omega(\mathbf{x})\phi_\omega(\mathbf{y})\,. \tag{6.7}$$

If the spectrum of ω is discrete then the integrals should be replaced by sums.

6.2 The global vacuum

In this subsection we construct the Green function associated with the global vacuum. The wave equation for a massive scalar in global coordinates is

$$\left[\cos^2\sigma\,(-\partial_\tau^2 + \partial_\sigma^2) - h(h-1)\right]\phi = 0\,, \tag{6.8}$$

where we write $m^2 = h(h-1)$. The normalized positive-frequency solutions are [13]

$$\phi_n = \Gamma(h)2^{h-1}\sqrt{\frac{n!}{\pi\Gamma(n+2h)}}e^{-i(n+h)\tau}(\cos\sigma)^h C^h_n(\sin\sigma)\,, \qquad n = 0,1,\ldots \tag{6.9}$$

[1]A discussion can be found in [28, chapter 4].

where C_n^h is the Gegenbauer polynomial [29, 8.930]. The Hadamard function (6.7) for the global vacuum is therefore

$$G^{(1)}_{\rm Global}(\tau_1,\sigma_1;\tau_2,\sigma_2) = \frac{\Gamma(h)^2 2^{2h-1}}{\pi}(\cos\sigma_1\cos\sigma_2)^h \times \times \sum_{n=0}^{\infty}\frac{n!}{\Gamma(n+2h)}\cos\left[(n+h)(\tau_1-\tau_2)\right] C_n^h(\sin\sigma_1)C_n^h(\sin\sigma_2)\,. \tag{6.10}$$

This sum appears in [30] (the mode sum for AdS_n is calculated in [31]) and gives

$$G^{(1)}_{\rm Global}(\tau_1,\sigma_1;\tau_2,\sigma_2) = \frac{\Gamma(h)^2}{2\pi\Gamma(2h)}{\rm Re}\left[\left(\frac{2}{d_{\rm Global}}\right)^h F\left(h,h;2h;-\frac{2}{d_{\rm Global}}\right)\right], \tag{6.11}$$

where

$$d_{\rm Global}(\tau_1,\sigma_1;\tau_2,\sigma_2) = \frac{\cos(\tau_1-\tau_2)-\cos(\sigma_1-\sigma_2)}{\cos\sigma_1\cos\sigma_2} \tag{6.12}$$

is the $SL(2,\mathbb{R})$ invariant distance function on AdS_2, in global coordinates. This is the known result [32] for the $SL(2,\mathbb{R})$ invariant Green function of a massive scalar on AdS_2. This function has the desired properties: it satisfies the massive wave equation (6.8), has the correct short-distance singularity, $G^{(1)}_{\rm Global} \sim -\frac{1}{\pi}\ln|\epsilon|$ for two points separated by a distance ϵ, and $G^{(1)}_{\rm Global} \sim (\cos\sigma)^h$ as $\cos\sigma \to 0$. For a massless scalar $h=1$ we recover

$$G^{(1)}_{\rm Global} = \frac{1}{2\pi}\ln\left|1+\frac{2}{d_{\rm Global}}\right| = -\frac{1}{2\pi}\ln\left|\frac{\cos(\tau_1-\tau_2)-\cos(\sigma_1-\sigma_2)}{\cos(\tau_1-\tau_2)+\cos(\sigma_1+\sigma_2)}\right|, \tag{6.13}$$

which is the correct massless Green function on the strip, as may be seen by summing the massless Green function on the plane over a collection of image field sources. This is required by the conformal invariance of a massless scalar.

One may explicitly check that the the same Green function is obtained in Poincaré coordinates (3.7), as expected from the equivalence of the corresponding vacua. The massive wave equation in Poincaré coordinates for a positive-frequency mode $\phi = e^{-i\omega T}\chi(y)$ is

$$\left[\frac{\partial^2}{\partial y^2}+\omega^2-\frac{h(h-1)}{y^2}\right]\chi(y) = 0\,. \tag{6.14}$$

The normalized positive-frequency modes (which vanish at the boundary $y=0$) are

$$\phi_\omega(T,y) = e^{-i\omega T}\sqrt{\frac{y}{2}}J_{h-1/2}(\omega y)\,, \tag{6.15}$$

so that the Hadamard function for the Poincaré vacuum is

$$\begin{aligned} G^{(1)}_{\rm Poincaré}(T_1,y_1;T_2,y_2) &= \sqrt{y_1y_2}\int_0^\infty d\omega\cos\left[\omega(T_1-T_2)\right] J_{h-1/2}(\omega y_1)J_{h-1/2}(\omega y_2) \\ &= \frac{\Gamma(h)^2}{2\pi\Gamma(2h)}{\rm Re}\left[\left(\frac{2}{d_{\rm Poincaré}}\right)^h F\left(h,h;2h;-\frac{2}{d_{\rm Poincaré}}\right)\right], \end{aligned} \tag{6.16}$$

(the integral appears in [30]) where

$$d_{\text{Poincaré}}(T_1, y_1; T_2, y_2) = \frac{-(T_1 - T_2)^2 + (y_1 - y_2)^2}{2y_1 y_2} \tag{6.17}$$

is the $\mathrm{SL}(2, \mathbb{R})$ invariant distance function in Poincaré coordinates. Equation (6.16) agrees precisely with (6.11) as anticipated. For a massless scalar ($h = 1$) we recover

$$G^{(1)}_{\text{Poincaré}} = -\frac{1}{2\pi} \ln \left| \frac{-(T_1 - T_2)^2 + (y_1 - y_2)^2}{-(T_1 - T_2)^2 + (y_1 + y_2)^2} \right| , \tag{6.18}$$

which is the usual massless Green function on the half plane, as required by conformal invariance. The term in the denominator can be thought of as coming from an image field source at $y_2' = -y_2$.

6.3 The Boulware vacuum

In this subsection we construct the Boulware Green function. For convenience we temporarily set $2\pi T_H = 1$. One can restore T_H simply by taking $(t, x) \to 2\pi T_H(t, x)$. The massive wave equation for a positive frequency solution $\phi_\omega = e^{-i\omega t}\phi(x)$ reads

$$\left[\frac{\partial^2}{\partial x^2} + \omega^2 - \frac{h(h-1)}{\sinh^2 x} \right] \phi_\omega(x) = 0 \, . \tag{6.19}$$

The solution which vanishes at $x = 0$ is

$$\phi_\omega(t, x) = \sqrt{\frac{\omega}{2}} \frac{\Gamma(h + i\omega)}{\Gamma(1 + i\omega)} e^{-i\omega t} (\sinh x)^{1/2} P^{\frac{1}{2} - h}_{-\frac{1}{2} - i\omega}(\cosh x) \, , \tag{6.20}$$

where P is the associated Legendre function and we have normalized according to (6.5). This gives the Hadamard function

$$G^{(1)}_{\text{Boulware}}(t_1, x_1; t_2, x_2) = (\sinh x_1 \sinh x_2)^{1/2} \times \tag{6.21}$$
$$\times \int_0^\infty \omega \, d\omega \left| \frac{\Gamma(h + i\omega)}{\Gamma(1 + i\omega)} \right|^2 \cos\left[\omega(t_1 - t_2)\right] P^{\frac{1}{2} - h}_{-\frac{1}{2} - i\omega}(\cosh x_1) P^{\frac{1}{2} - h}_{-\frac{1}{2} + i\omega}(\cosh x_2) \, .$$

This integral cannot be evaluated in terms of elementary functions. For the massless case $h = 1$ we have

$$(\sinh x)^{1/2} P^{-\frac{1}{2}}_{-\frac{1}{2} \pm i\omega}(\cosh x) = \sqrt{\frac{2}{\pi}} \frac{\sin \omega x}{\omega} \, , \tag{6.22}$$

and hence

$$\begin{aligned} G^{(1)}_{\text{Boulware}}(t_1, x_1; t_2, x_2) &= \frac{2}{\pi} \int_0^\infty \frac{d\omega}{\omega} \cos\left[\omega(t_1 - t_2)\right] \sin \omega x_1 \sin \omega x_2 \\ &= -\frac{1}{2\pi} \ln \left| \frac{-(t_1 - t_2)^2 + (x_1 - x_2)^2}{-(t_1 - t_2)^2 + (x_1 + x_2)^2} \right| , \end{aligned} \tag{6.23}$$

which again is the correct massless Green function on the half plane. Since it is impossible to rewrite (6.23) as a function of the $SL(2,\mathbb{R})$ invariant distance

$$d_{\text{Boulware}}(t_1,x_1;t_2,x_2) = \frac{-\cosh(2\pi T_H(t_1-t_2)) + \cosh(2\pi T_H(x_1-x_2))}{\sinh(2\pi T_H x_1)\sinh(2\pi T_H x_2)}, \qquad (6.24)$$

we discover that the Boulware vacuum is not $SL(2,\mathbb{R})$ invariant. In particular, it is distinct from the global vacuum.

Using a recursion relation satisfied by the Legendre functions one can write down a (very complicated) expression which gives the value of the integral (6.23) for any positive integer h in terms of sums of logarithms and exponential-integral functions $\text{Ei}(z)$ [29, 8.211]. The formulas involved are lengthy and not illuminating. For example, for $h = 2$ one finds

$$G^{(1)}_{\text{Boulware}} = (\coth x_1 \coth x_2) G^{(1)}_{B,(h=1)} - \frac{1}{4\pi} \sum_{a,b,c=\pm 1} \frac{\text{Ei}(a(t_1-t_2)+bx_1+cx_2)}{e^{a(t_1-t_2)}\sinh bx_1 \sinh cx_2}. \qquad (6.25)$$

One can check that $G^{(1)}_{\text{Boulware}}$ constructed in this way satisfies the massive wave equation (6.19), has the correct short-distance singularity $G^{(1)}_{\text{Boulware}} \sim -\frac{1}{\pi}\ln|\epsilon|$, and vanishes as x^h when $x \to 0$. These properties ensure that the Boulware vacuum is a 'good' vacuum, although it is singular along the horizon at $x = \infty$.

Furthermore, by restoring $(t,x) \to 2\pi T_H(t,x)$ one can verify that in the limit $T_H \to 0$, the Hadamard function for the Boulware vacuum reduces to that of the global vacuum (6.16) (with (T,r) replaced by (t,x)), in agreement with the fact that the coordinate systems coincide for $T_H = 0$ (3.13). Thus the Hawking temperature T_H is a measure of the non-$SL(2,\mathbb{R})$ invariance of the Boulware vacuum.

7. The stress tensor

The various vacua in AdS_2 are characterized by differing stress tensor expectation values. In this section we compute these expectation values for both the massless and the massive case.

7.1 Two-dimensional Rindler and Minkowski space

We begin with a review of some well-known features of the thermodynamics of two-dimensional Rindler space. This will clarify the meaning of the various AdS_2 expressions. Readers familiar with this topic should skip to the next subsection.

The Rindler metric

$$ds^2 = -e^{\kappa(U^+-U^-)}dU^+dU^- \qquad (7.1)$$

is related to the Minkowski metric $ds^2 = -du^+du^-$ by the coordinate transformation

$$U^\pm = \pm\frac{1}{\kappa}\ln(\pm\kappa u^\pm), \qquad (7.2)$$

where κ is a constant. Lines of constant $U^+ - U^-$ correspond to the worldlines of observers undergoing constant proper acceleration κ (see figure 4).

Consider a massless scalar field in Minkowski space. We may construct the stress tensor operator $T_{\mu\nu}$ normal-ordered with respect to Minkowski coordinates, $u^\pm$, or with respect to Rindler coordinates $U^\pm$. These two operators are related by the well-known formula

$$T_{++}(U^+) = \left(\frac{\partial u^+}{\partial U^+}\right)^2 T_{++}(u^+) + \frac{1}{12\pi}\sqrt{\frac{\partial u^+}{\partial U^+}}\frac{\partial^2}{\partial U^{+2}}\sqrt{\frac{\partial U^+}{\partial u^+}}\,. \tag{7.3}$$

Here and henceforth the stress tensor in a given coordinate system is always normal-ordered with respect to that coordinate system. The difference in the two stress tensors reflects the fact that observers which are stationary with respect to different coordinate systems detect different particle densities. Plugging in (7.2) gives

$$T_{++}(U^+) = e^{2\kappa U^+} T_{++}(u^+) + \frac{\kappa^2}{48\pi}\,. \tag{7.4}$$

Taking the expectation value of (7.4) in the Minkowski vacuum gives

$$\langle T_{++}(U^+)\rangle_M = \frac{\kappa^2}{48\pi}\,, \tag{7.5}$$

which is the stress-energy density of a thermal bath of particles at temperature $T = \kappa/2\pi$.[2] This may be interpreted as radiation coming from the Rindler horizon. On the other hand, taking the expectation value of (7.4) in the Rindler vacuum gives

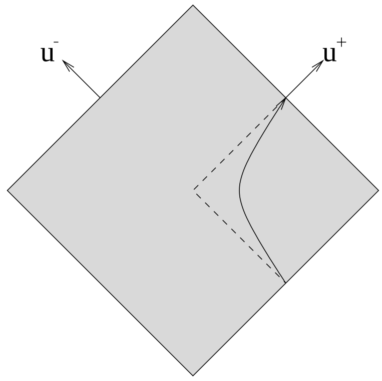

Figure 4: Rindler spacetime. The "right Rindler wedge" ($u^- < 0$ and $u^+ > 0$) is accessible to a Rindler observer that accelerates uniformly to the right. The dashed lines show the past and future horizon (the "Rindler horizon") seen by such an observer.

$$\langle T_{++}(u^+)\rangle_R = -\frac{1}{48\pi (u^+)^2}\,, \tag{7.6}$$

which can be viewed as a divergent Casimir energy arising from the presence of a boundary at the Rindler horizon $u^+ = 0$.

So far we have ignored the other independent component of $\langle T_{\mu\nu}\rangle$, which is determined by the trace anomaly formula

$$\langle T_{+-}\rangle = \frac{1}{2} g_{+-}\langle T\rangle = \frac{R}{48\pi} g_{+-}\,. \tag{7.7}$$

This vanishes for Rindler/Minkowski space but plays a role in AdS_2, where $R = -2$.

The stress tensor for massive scalars in Rindler space has been constructed in [33].

[2]This temperature is related to the fact that the coordinate transformation (7.2) is periodic in imaginary Rindler time with periodicity $\beta = 2\pi/\kappa$, so that any Green function constructed in Rindler coordinates would also be periodic in imaginary Rindler time and would therefore correspond to a thermal Green function at temperature β^{-1}.

7.2 Massless scalar in AdS_2

We now calculate the stress energy for a massless scalar in AdS_2. The results are essentially identical to those we obtained in the previous subsection. It is convenient to work in null coordinates in which the Poincaré and Schwarzschild coordinate systems take the form

$$ds^2 = -\frac{4du^+du^-}{(u^+ - u^-)^2} = -\frac{(2\pi T_H)^2 dU^+ dU^-}{\sinh^2\left[\pi T_H(U^+ - U^-)\right]}, \tag{7.8}$$

where the null coordinates are defined by

$$2\pi T_H U^\pm = 2\pi T_H(t \pm x) = \ln(T \pm y) = \ln u^\pm . \tag{7.9}$$

From the coordinate transformation (7.9) we find

$$T_{++}(U^+) = (2\pi T_H u^+)^2 T_{++}(u^+) + \frac{\pi T_H^2}{12}, \tag{7.10}$$

where $T_{++}(U^+)$ is the stress tensor normal-ordered in the Schwarzschild coordinates and $T_{++}(u^+)$ is the stress tensor normal-ordered in the Poincaré coordinates. Taking the expectation of this equation in the global vacuum gives

$$\langle T_{++}(U^+)\rangle_{\text{Global}} = \frac{\pi T_H^2}{12}, \tag{7.11}$$

which is the stress-energy density of a thermal bath of particles at temperature T_H (again this is to be expected by virtue of the periodicity of the coordinate transformation (7.9) in imaginary Schwarzschild time). On the other hand, taking the expectation value of (7.10) in the Boulware vacuum gives

$$\langle T_{++}(u^+)\rangle_{\text{Boulware}} = -\frac{1}{48\pi(u^+)^2}, \tag{7.12}$$

which may be viewed as Casimir energy arising from a boundary at the black hole horizon.

So far we have discussed the stress tensor in Schwarzschild and Poincaré coordinates. In null global coordinates

$$\frac{1}{2}\left(\tau \pm \sigma \pm \frac{\pi}{2}\right) = \tau^\pm = \tan^{-1} u^\pm , \tag{7.13}$$

the stress tensor picks up a term

$$T_{++}(\tau^+) = \left(\frac{\partial u^+}{\partial \tau^+}\right)^2 T_{++}(u^+) - \frac{1}{12\pi}, \tag{7.14}$$

so that

$$\langle T_{++}(\tau^+)\rangle_{\text{Global}} = -\frac{1}{12\pi}, \tag{7.15}$$

which is the familiar zero-point shift of a $c = 1$ theory on the strip. Curiously, normal-ordering in Poincaré and global coordinates lead to different shifts even though the associated vacua are identical. This is possible because in the former case one uses a continuous set of modes, while in the latter one uses a discrete set, and so the infinite zero-point energy sums are regulated differently. The fact that the expectation value of the stress tensor in the global vacuum vanishes in Poincaré coordinates but not in global coordinates also follows from $\mathrm{SL}(2,\mathbb{R})$ invariance, together with the observation that the inhomogenous term in (7.3) vanishes for $\mathrm{SL}(2,\mathbb{R})$ transformations in Poincaré coordinates but not in global coordinates.

7.3 Point-splitting regularization of massive scalars

The calculation of $\langle T_{\mu\nu}\rangle$ for a massive scalar is significantly more difficult as there is no simple formula such as (7.3). The calculation is complicated by the fact that the expectation value of an operator such as $T_{\mu\nu}$ which is quadratic in the field ϕ is formally divergent and must be regularized and renormalized. We implement the regularization by using the point-splitting technique[3], reviewed briefly below.

The stress tensor for a massive scalar field ϕ is

$$T_{\mu\nu}(\mathbf{x}) = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + m^2\phi^2\right). \tag{7.16}$$

In conformal gauge

$$ds^2 = -e^{2\rho}dw^+dw^-, \tag{7.17}$$

one has

$$\begin{aligned} T_{++} &= \partial_+\phi\partial_+\phi\,, \\ T_{--} &= \partial_-\phi\partial_-\phi\,, \\ T_{+-} &= T_{-+} = -\frac{1}{2}g_{+-}m^2\phi^2\,. \end{aligned} \tag{7.18}$$

Following [35], we define the point-split stress tensor operator as follows. Consider any non-null geodesic through $\mathbf{x}$, and let $x^\mu(\epsilon) = (w^+(\epsilon), w^-(\epsilon))$ be the point on the geodesic at a proper distance $\epsilon > 0$ from $\mathbf{x}$. The geodesic may be characterized by its normalized tangent vector at $\mathbf{x}$, $\tau_0^\mu \equiv \tau^\mu(0)$, where

$$\frac{dx^\mu(\epsilon)}{d\epsilon} = \tau^\mu(\epsilon)\,, \qquad \tau_\mu\tau^\mu = -e^{2\rho}\tau^+\tau^- \equiv \Sigma = \pm 1\,. \tag{7.19}$$

The geodesic equations may be solved for w^+ in a power series in ϵ, giving

$$\begin{aligned} w^+(\epsilon) &= w_0^+ + \epsilon\tau_0^+ - \epsilon^2(\partial_+\rho)(\tau_0^+)^2 + \\ &\quad + \frac{1}{3}\epsilon^3\left[(4(\partial_+\rho)^2 - \partial_+^2\rho)(\tau_0^+)^3 - \partial_-\partial_+\rho\tau_0^-(\tau_0^+)^2\right] + O(\epsilon^4)\,, \end{aligned} \tag{7.20}$$

[3]See [34] for a detailed discussion.

where ρ on the right-hand side is always evaluated at w_0. Switching $+$ and $-$ in this expression yields the solution for $w^-(\epsilon)$. We define the point-split stress tensor operator by

$$T_{++}(\mathbf{x};\epsilon,\tau_0^\mu) = U_\epsilon U_{-\epsilon}\frac{1}{2}\left\{\partial_+\phi(\mathbf{x}(\epsilon)),\partial_+\phi(\mathbf{x}(-\epsilon))\right\},$$
$$T_{+-}(\mathbf{x};\epsilon,\tau_0^\mu) = -\frac{1}{2}m^2 g_{+-}\left\{\phi(\mathbf{x}(\epsilon)),\phi(\mathbf{x}(-\epsilon))\right\}, \tag{7.21}$$

and similarly for T_{--}. In this expression

$$U_\epsilon \equiv \left(\frac{dw^+(0)}{d\epsilon}\right)^{-1}\frac{dw^+(\epsilon)}{d\epsilon}. \tag{7.22}$$

These factors arises because $\partial_+\phi(\mathbf{x}(\pm\epsilon))$ must be parallel transported back to $\mathbf{x}(0)$ in order to obtain a quantity which transforms as a tensor [34]. Upon taking the expectation value of both sides in some vacuum V, we find that

$$\langle T_{++}(\mathbf{x};\epsilon,\tau_0^\mu)\rangle_V = \left[U_\epsilon U_{-\epsilon}\frac{\partial}{\partial w_1^+}\frac{\partial}{\partial w_2^+}\frac{1}{2}G_V^{(1)}(\mathbf{x}_1,\mathbf{x}_2)\right]_{\substack{\mathbf{x}_1 = \mathbf{x}(\epsilon)\\ \mathbf{x}_2 = \mathbf{x}(-\epsilon)}},$$
$$\langle T_{+-}(\mathbf{x};\epsilon,\tau_0^\mu)\rangle_V = \left[-\frac{m^2}{2}g_{+-}\frac{1}{2}G_V^{(1)}(\mathbf{x}_1,\mathbf{x}_2)\right]_{\substack{\mathbf{x}_1 = \mathbf{x}(\epsilon)\\ \mathbf{x}_2 = \mathbf{x}(-\epsilon)}}, \tag{7.23}$$

and similarly for $\langle T_{--}\rangle$.

7.4 Application of the point splitting procedure

In all of the cases we consider the Hadamard function has the usual short-distance behavior

$$G^{(1)}(w_1^+,w_1^-;w_2^+,w_2^-) = -\frac{1}{2\pi}\ln\left|(w_1^+-w_2^+)(w_1^--w_2^-)\right| + \cdots, \tag{7.24}$$

where the dots denote terms which are finite as $\mathbf{x}_2$ approaches $\mathbf{x}_1$, and the point-split stress tensors have the general form

$$\langle T_{++}(\mathbf{x};\epsilon,\tau^\mu)\rangle = -\frac{1}{4\pi}\left[\frac{1}{\epsilon^2} - 16\Sigma\pi f_2(\mathbf{x})\right]\tau_+\tau_+ + f_1(\mathbf{x}) + O(\epsilon\ln\epsilon),$$
$$\langle T_{+-}(\mathbf{x};\epsilon,\tau^\mu)\rangle = \frac{m^2}{4\pi}g_{+-}\left[\ln\epsilon + f_3(\mathbf{x})\right] + O(\epsilon\ln\epsilon), \tag{7.25}$$

where the three functions f_1, f_2, and f_3, which depend only on the point $\mathbf{x}$ and not on ϵ or $\tau^\pm$, encode all of the physical information in the point-split stress tensor. To simplify the notation we here and henceforth drop the subscript 0 on τ^μ. Finally, making use of the fact that $g_{++}=0$ and

$$-\frac{1}{2}e^{2\rho} = g_{+-} = 2\Sigma\tau_+\tau_-, \tag{7.26}$$

we can combine both expressions in (7.25) into a single covariant expression for the point-split stress tensor,

$$\langle T_{\mu\nu}(\mathbf{x};\epsilon;\tau^\mu)\rangle = \frac{1}{8\pi}\left[\frac{\Sigma}{\epsilon^2} - 16\pi f_2(\mathbf{x})\right](g_{\mu\nu} - 2\Sigma\tau_\mu\tau_\nu) + \theta_{\mu\nu}(\mathbf{x}) + \\ + \frac{m^2}{4\pi} g_{\mu\nu}\left[\ln\epsilon + f_3(\mathbf{x})\right] + O(\epsilon\ln\epsilon)\,, \tag{7.27}$$

where $\theta_{\mu\nu}$ is the traceless tensor whose components in the $w^\pm$ coordinate system are

$$\theta_{++} = \theta_{--} = f_1(\mathbf{x})\,, \\ \theta_{+-} = \theta_{-+} = 0\,. \tag{7.28}$$

The regularized stress tensor $\langle T_{\mu\nu}(\mathbf{x};\epsilon,\tau^\mu)\rangle$ diverges in the limit $\epsilon \to 0$, and furthermore the precise behavior of the divergence depends on the direction of approach τ^μ. The renormalized stress tensor is obtained [35] by discarding all of the terms in (7.27) which depend explicitly on either ϵ or τ^μ,

$$\langle T_{\mu\nu}(\mathbf{x})\rangle = g_{\mu\nu}\left[\frac{m^2}{4\pi} f_3(\mathbf{x}) - 2f_2(\mathbf{x})\right] + \theta_{\mu\nu}(\mathbf{x})\,. \tag{7.29}$$

From (7.27) we see that the terms which diverge as $\epsilon \to 0$ are universal and do not depend upon the particular state under investigation (i.e. they do not depend on the f_i). Therefore the divergent terms always cancel out when we calculate the differences between stress tensors in different vacua.

7.5 Energy of the global vacuum

We begin by calculating $\langle T_{\mu\nu}(u^+,u^-)\rangle_{\text{Global}}$ for the $\text{SL}(2,\mathbb{R})$ invariant global vacuum in Poincaré coordinates. The only rank 2 symmetric, conserved, $\text{SL}(2,\mathbb{R})$ invariant tensor is $g_{\mu\nu}$, so we expect that $\langle T_{\mu\nu}\rangle_{\text{Global}} = c g_{\mu\nu}$ for some constant c. In the notation of the previous subsection we find

$$f_1 = 0\,, \qquad f_2 = \frac{1 + 3h(h-1)}{48\pi}\,, \qquad f_3 = \psi(h) + \gamma. \tag{7.30}$$

where $\psi(z) = \partial \ln\Gamma(z)/\partial z$ and $\gamma = -\psi(1)$ is Euler's constant. Hence the renormalized stress tensor (7.29) is

$$\langle T_{\mu\nu}\rangle_{\text{Global}} = \frac{g_{\mu\nu}}{2\pi}\left[-\frac{1}{12} - \frac{h(h-1)}{2}\left(\frac{1}{2} - \psi(h) - \gamma\right)\right]. \tag{7.31}$$

We have obtained the same result by applying Pauli-Villars regularization. Note that when $h = 1$ we recover

$$\langle T_{\mu\nu}\rangle = -\frac{g_{\mu\nu}}{24\pi}\,, \tag{7.32}$$

which is the massless Weyl anomaly $\langle T_{\mu\nu}\rangle = \frac{R}{48\pi} g_{\mu\nu}$, with $R = -2$ for AdS_2.

7.6 Energy of the Boulware vacuum

This calculation is significantly more complicated. In particular, we cannot use $SL(2,\mathbb{R})$-invariance to argue that $\langle T_{\mu\nu}\rangle_{\text{Boulware}}$ is proportional to $g_{\mu\nu}$, and indeed we find that this is not the case. To simplify the resulting expressions slightly we introduce

$$\langle T_{\mu\nu}\rangle' = \langle T_{\mu\nu}\rangle_{\text{Global}} - \langle T_{\mu\nu}\rangle_{\text{Boulware}}, \tag{7.33}$$

with $\langle T_{\mu\nu}\rangle_{\text{Global}}$ given by (7.31), which is the energy difference between the global and Boulware vacua. (Note that $\langle T_{++}\rangle' = -\langle T_{++}\rangle_{\text{Boulware}}$ since $\langle T_{++}\rangle_{\text{Global}} = 0$.) Using the Hadamard function (6.21) constructed above, we find for $h = 1, 2, 3$ the result

$$\begin{aligned}
\langle T_{++}\rangle'_{h=1} &= \frac{\pi T_H^2}{12}\,,\\
\langle T_{+-}\rangle'_{h=1} &= 0\,,\\
\langle T_{++}\rangle'_{h=2} &= \frac{\pi T_H^2}{12}\left[1 - 6\,\text{csch}^2 z + 12 F(z)\,\text{csch}^4 z\right],\\
\langle T_{+-}\rangle'_{h=2} &= \frac{g_{+-}}{2\pi}\left[1 - \ln\left|\frac{\sinh z}{z}\right| - 2F(z)\,\text{csch}^2 z\right],\\
\langle T_{++}\rangle'_{h=3} &= \frac{\pi T_H^2}{12}\left[1 - 18\,\text{csch}^2 z - 36F(2z)\,\text{csch}^6 z + 18F(z)(3\cosh 2z + 5)\,\text{csch}^6 z\right],\\
\langle T_{+-}\rangle'_{h=3} &= \frac{3g_{+-}}{2\pi}\left[\frac{3}{2} - \ln\left|\frac{\sinh z}{z}\right| + \frac{3}{2}F(2z)\,\text{csch}^4 z - 6F(z)\coth^2 z\,\text{csch}^2 z\right],
\end{aligned} \tag{7.34}$$

where we write $z = 2\pi T_H x$ for simplicity. We have introduced the function

$$F(w) = \int_0^w \frac{du}{u}\sinh^2 u\,. \tag{7.35}$$

A conjectured expression for a general value of h is

$$\begin{aligned}
\frac{\langle T_{++}\rangle'}{\pi T_H^2} &= \frac{1}{12} - \frac{h(h-1)}{4\sinh^2 z}\left[1 - h(h-1)\int_0^z \frac{du}{u}\frac{\sinh^2 u}{\sinh^2 z} F\left(h+1, 2-h, 3, \frac{\sinh^2 u}{\sinh^2 z}\right)\right],\\
\langle T_{+-}\rangle' &= \frac{h(h-1)g_{+-}}{4\pi}\left\{\psi(h) + \gamma - \int_0^z du\left[\coth u - \frac{1}{u}F\left(h, 1-h, 1, \frac{\sinh^2 u}{\sinh^2 z}\right)\right]\right\},
\end{aligned} \tag{7.36}$$

where again $z = 2\pi T_H x$. Note that when h is an integer the hypergeometric series terminates, giving a polynomial which can be explicitly integrated with relative ease (although the result is not be expressible in terms of elementary functions but again involves the exponential-integral function $\text{Ei}(z)$). One can also check that both components (7.36) vanish as $T_H \to 0$, as should be expected. Figures 5 and 6 show $\langle T_{++}(x)\rangle'$ and $\langle T_{+-}(x)\rangle'$ for some values of h.

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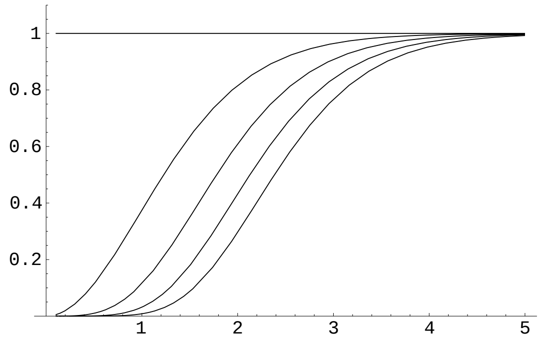

Figure 5: Energy of a massive scalar in the Boulware vacuum. This plot shows $\frac{12}{\pi T_H^2}\langle T_{++}(x)\rangle'$, as defined in (7.33), as a function of $z = 2\pi T_H x$, for scalar fields of mass $h = 1, 2, 3, 4, 5$ (from top to bottom).

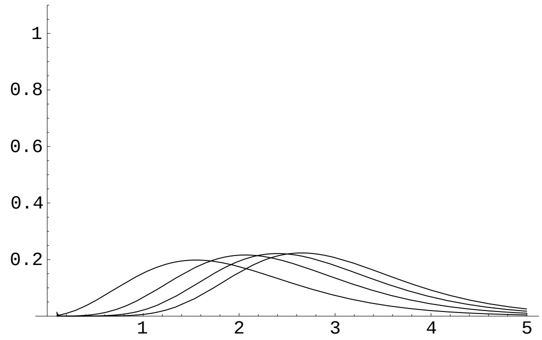

Figure 6: This plot shows $\frac{12}{\pi T_H^2}\langle T_{+-}(x)\rangle'$ as a function of $z = 2\pi T_H x$ for scalar fields of mass $h = 2, 3, 4, 5$ (from left to right). It vanishes identically for $h = 1$. The scales are the same as in figure 5.

Evidence that (7.36) is the correct expression for all values of h is

a) Special cases. It correctly reduces to (7.34) for $h = 1, 2, 3$.

b) Conservation. The stress tensor should satisfy $\nabla^\mu T_{\mu\nu} = 0$, which in Schwarzschild coordinates gives one nontrivial equation,[4]

$$\frac{\pi^2 T_H^2}{\sinh^2(2\pi T_H x)} \frac{\partial}{\partial x}\langle T\rangle + \frac{\partial}{\partial x}\langle T_{++}\rangle = 0\,, \tag{7.37}$$

where $\langle T\rangle = 2g^{+-}\langle T_{+-}\rangle$, which is indeed obeyed by (7.36).

c) Behavior near the boundary. We saw earlier that the Schwarzschild modes behave like $\phi \sim x^h$ near the boundary $x = 0$. Therefore the stress tensor, which is quadratic in $\partial\phi$, should vanish as $\langle T_{\mu\nu}\rangle \sim x^{2(h-1)}$ as $x \to 0$. Again, this can be checked explicitly for (7.36). In particular, the physical requirement that $\langle T_{\mu\nu}\rangle$ vanishes at the boundary fixes any overall additive constant, and the fact that $g_{\mu\nu}$ diverges as x^{-2} precludes us from adding any constant multiple of the metric to $\langle T_{\mu\nu}\rangle$.

d) Behavior near the horizon. Finally we can consider the behavior near the horizon at $x \to \infty$. Everything becomes massless sufficiently close to the horizon. To see this, note that $g_{\mu\nu} \propto (\sinh x)^{-2} \to 0$, in which case the lagrangean density becomes

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}\left[(\nabla\phi)^2 + m^2\phi^2\right] \sim -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\,, \tag{7.38}$$

[4]The other equation essentially says that $\langle T_{\mu\nu}\rangle$ should be time-independent.

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where the inverse metric in the kinetic term $(\nabla\phi)^2$ cancels the zero coming from $\sqrt{-g}$. Thus one should expect, and we indeed find, that for $x \gg 1$ and any h the expressions (7.36) tend to the massless values

$$\langle T_{++}\rangle' \to \frac{\pi T_H^2}{12}\,, \qquad \langle T_{+-}\rangle' \to 0\,, \qquad x \gg 1\,. \tag{7.39}$$

This fixes the overall normalization of $\langle T_{++}\rangle$, which in turn fixes the normalization of $\langle T_{+-}\rangle$ through the conservation equation (7.37).

8. Boundary correlation functions

It is expected [10] that string theory on AdS_2 can be described as conformally invariant quantum mechanics on the boundary of AdS_2. The conformal invariance of the 1-dimensional boundary theory is a consequence of the $SO(2,1)$ isometry group of AdS_2. Boundary correlation functions evaluated in any vacuum other than the natural $SO(2,1)$ invariant vacuum, such as the Boulware vacuum, will therefore not be conformally invariant. However, we have seen that the parameter T_H is a measure of the non-SL$(2,\mathbb{R})$ invariance of the Boulware vacuum, so we expect the nonconformal corrections to boundary correlation functions in the Boulware vacuum to vanish as $T_H \to 0$. In this section we derive these boundary correlators and verify that this is the case.

8.1 Brief review

In order to fix our conventions and notation we begin with a very quick overview of the calculation of boundary correlation functions using the bulk propagator. The AdS/CFT duality [10] states that for every bulk field ϕ there is a corresponding local operator $\mathcal{O}$ on the boundary $\mathcal{B}$, with

$$Z_{\text{eff}}(\phi) = e^{iS_{\text{eff}}(\phi)} = \langle T e^{i\int_{\mathcal{B}} \phi_{\text{b}}\mathcal{O}}\rangle\,, \tag{8.1}$$

where S_{eff} is the effective action in the bulk and ϕ_{b} is the field ϕ restricted to the boundary [36, 37]. Let $\mathcal{O}_h$ be the boundary operator of conformal weight h which couples to the bulk scalar ϕ of mass $m^2 = h(h-1)$, and let $G_V(y,z;y',z')$ be the bulk two-point function of ϕ in coordinates where the boundary lies at $y=0$ and is parametrized by z. This could be Poincaré coordinates with $(y,z)=(y,T)$, Schwarzschild coordinates with $(y,z)=(x,t)$, or global coordinates with $(y,z) = (\cos\sigma,\tau)$. The subscript V is a reminder that the two-point function G_V expresses a choice of vacuum. Boundary correlation functions will depend on the choice of vacuum in the AdS_2 bulk [38, 39].

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The two-point function should vanish as y^h as either point approaches the boundary, and we define the bulk-boundary propagator for the corresponding vacuum state by [40]

$$K_V(y,z;z') = \lim_{y'\to 0}\left[(y')^{-h}G_V(y,z;y',z')\right]. \tag{8.2}$$

(We ignore overall constants throughout this section). If we are given some boundary data $\phi_0(z')$ for the field ϕ, then we can use (8.2) to extend ϕ_0 into the bulk by writing

$$\phi(y,z) = \int dz' K_V(y,z;z')\phi_0(z'). \tag{8.3}$$

Then $\phi(y,z)$ satisfies the equation of motion in the bulk because K satisfies the equation of motion in the variables (y,z). Next we plug the solution (8.3) into the action (6.1). Upon integrating by parts, the action can be expressed as the boundary term

$$S = \lim_{y\to 0}\left[\frac{1}{2}\int dz\ \phi(y,z)\partial_y\phi(y,z)\right]. \tag{8.4}$$

In the limit as we take the cutoff $y \to 0$ the bulk-boundary propagator should approach a delta-function

$$K_V(y,z;z') \to y^{-h+1}\delta(z-z') \tag{8.5}$$

so we can replace

$$\phi(y,z) \to y^{-h+1}\phi_0(z). \tag{8.6}$$

Then (8.4) becomes

$$S = \frac{1}{2}\int dz\, dz'\ \phi_0(z)\phi_0(z')\left[\lim_{y\to 0} y^{-h+1}\partial_y K_V(y,z;z')\right]. \tag{8.7}$$

The generating function for correlation functions of $\mathcal{O}_h(z)$ in the boundary theory coupled to the source $\phi_0(z)$ is given by the exponential of i times (8.7), so recalling (8.2) we find that (again, up to constants) [41]

$$\langle\mathcal{O}_h(z)\mathcal{O}_h(z')\rangle_V = \lim_{y,y'\to 0}\left[(y')^{-h}y^{-h+1}\partial_y G_V(y,z;y',z')\right]. \tag{8.8}$$

8.2 Correlation functions in the global vacuum

Substituting the global vacuum two-point function (in Poincaré coordinates) (6.16) into (8.2) gives the familiar bulk-boundary propagator

$$K(y,T_1;T_2) = \frac{y^h}{(y^2-(T_1-T_2)^2)^h}, \tag{8.9}$$

which leads to the conformally invariant boundary correlation function

$$\langle\mathcal{O}_h(T)\mathcal{O}_h(0)\rangle_{\text{Global}} = \frac{1}{T^{2h}}. \tag{8.10}$$

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For purposes of comparison, it will be convenient to write (8.10) in Schwarzschild coordinates. Recalling that the relation between the Poincaré time T and the Schwarzschild time t on the boundary is $2\pi T_H t = \ln T$ and using the conformal transformation law

$$\mathcal{O}'(z') = (\partial_z z')^{-h} \mathcal{O}(z) \,, \tag{8.11}$$

we can write (8.10) in the form

$$\langle \mathcal{O}_h(t) \mathcal{O}_h(0) \rangle_{\text{Global}} = \left[\frac{T_H}{\sinh{(\pi T_H t)}} \right]^{2h} , \tag{8.12}$$

As expected, (8.10) is periodic in imaginary Schwarzschild time with periodicity T_H^{-1} and therefore represents a thermal state at temperature T_H. For small separations (8.10) has the universal UV limit $\langle \mathcal{O}_h(t) \mathcal{O}_h(0) \rangle \sim 1/t^{2h}$, while in the IR limit the two-point function is exponentially suppressed due to the thermal background, $\langle \mathcal{O}_h(t) \mathcal{O}_h(0) \rangle_{\text{Global}} \sim e^{-2\pi T_H h t}$.

8.3 Correlation functions in the Boulware vacuum

Now we apply (8.8) directly to the Boulware vacuum without first constructing the Boulware bulk-boundary propagator K_{Boulware} from (8.2). However, one can check that K_{Boulware} is given by the Poincaré bulk-boundary propagator (8.9) plus correction terms which are subleading in $z - z'$, so that (8.5) is still satisfied, and proportional to positive powers of T_H, so that K_{Boulware} reduces to (8.9) as $T_H \to 0$.

Using

$$(\sinh x)^{1/2} P^{\frac{1}{2}-h}_{-\frac{1}{2} \pm i\omega}(\cosh x) = \frac{2^{1/2-h}}{\Gamma(h+1/2)} x^h + O(x^{h+2}) \tag{8.13}$$

and the Boulware vacuum Green function (6.21), we find from (8.8) that

$$\langle \mathcal{O}_h(t) \mathcal{O}_h(0) \rangle_{\text{Boulware}} = \int_0^\infty \omega d\omega \left| \frac{\Gamma(h+i\omega)}{\Gamma(1+i\omega)} \right|^2 \cos \omega t \,, \tag{8.14}$$

where we have dropped all overall numerical constants. This integral is not convergent but may be defined by analytic continuation. The problem is that the limit (8.8) does not commute with integration over ω. We present a quick way of getting the answer, which gives perfect agreement with a more careful analysis where one computes the integral first and then takes the limits.[5]

[5] Alternatively, one may insert a factor of $e^{-\epsilon\omega}$ into the integral (8.14). At least when h is an integer, the integral may be done explicitly, and the result is finite in the limit $\epsilon \to 0$ and agrees precisely with the result we present.

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Define $F_h(t)$ to be the quantity in (8.14). Then

$$F_{h+1}(t) = \int_0^\infty \omega\, d\omega \left|\frac{\Gamma(h+1+i\omega)}{\Gamma(1+i\omega)}\right|^2 \cos\omega t$$
$$= \int_0^\infty \omega\, d\omega \left|\frac{\Gamma(h+i\omega)}{\Gamma(1+i\omega)}\right|^2 (h^2+\omega^2)\cos\omega t$$
$$= (h^2 - \partial_t^2)F_h(t)\,. \tag{8.15}$$

This should be valid for all h. To start the recursion we evaluate

$$F_1(t) = \left[\int_0^\infty d\omega\ \omega^n \cos\omega t\right]_{n=1} = \left[-n!\, t^{-n-1} \sin\left(\frac{n\pi}{2}\right)\right]_{n=1} = -\frac{1}{t^2}\,, \tag{8.16}$$

where the quantity in brackets, which is strictly valid only for $-1 < \mathrm{Re}(n) < 0$, is analytically continued to $n = 1$. The solution to (8.16) and (8.15), up to (h-dependent!) constants, may be summarized by the suggestive expression

$$\langle \mathcal{O}_h(t)\mathcal{O}_h(0)\rangle_{\mathrm{Boulware}} = \left[\frac{T_H}{\sinh(\pi T_H t)}\right]^{2h}_{\mathrm{singular}}, \tag{8.17}$$

where we have restored the proper T_H-dependence. The subscript 'singular' indicates that only the singular terms in the expansion of the right-hand side of (8.17) around $t = 0$ are to be kept. For example, for $h = 3$ we find

$$\langle \mathcal{O}_3(t)\mathcal{O}_3(0)\rangle_{\mathrm{Boulware}} \propto \frac{1}{t^6} - \frac{\pi^2 T_H^2}{t^4} + \frac{8\pi^4 T_H^4}{15t^2}\,. \tag{8.18}$$

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References

[1] B. Carter, in *Black Holes*, C. de Witt and B.S. de Witt eds., Gordon and Breach, New York 1973.

[2] S. Ferrara, R. Kallosh and A. Strominger, $N = 2$ *extremal black holes*, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072].

[3] E. D'Hoker and R. Jackiw, *Classical and quantal Liouville field theory*, *Phys. Rev.* **D 26** (1982) 3517.

[4] E. D'Hoker, D.Z. Freedman and R. Jackiw, SO(2,1)-*invariant quantization of the Liouville theory*, *Phys. Rev.* **D 28** (1983) 2583.

[5] R.B. Mann, *Lower dimensional black holes*, *Gen. Rel. Grav.* **24** (1992) 433.

[6] S.P. Trivedi, *Semiclassical extremal black holes*, *Phys. Rev.* **D 47** (1993) 4233 [hep-th/9211011].

[7] A. Strominger and S. P. Trivedi, *Information consumption by Reissner-Nordstrom black holes*, *Phys. Rev.* **D 48** (1993) 5778 [hep-th/9302080].

[8] J.P.S. Lemos, *Thermodynamics of the two-dimensional black hole in the Teitelboim-Jackiw theory*, *Phys. Rev.* **D 54** (1996) 6206 [gr-qc/9608016].

[9] V. Frolov, D. Fursaev, J. Gegenberg and G. Kunstatter, *Thermodynamics and statistical mechanics of induced Liouville gravity*, *Phys. Rev.* **D 60** (1999) 024016, [hep-th/9901087].

[10] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[11] A. Strominger, *AdS_2 quantum gravity and string theory*, *J. High Energy Phys.* **01** (1999) 007 [hep-th/9809027].

[12] J. Maldacena, J. Michelson and A. Strominger, *Anti-de Sitter fragmentation*, *J. High Energy Phys.* **02** (1999) 011 [hep-th/9812073].

[13] T. Nakatsu and N. Yokoi, *Comments on hamiltonian formalism of AdS/CFT correspondence*, *Mod. Phys. Lett.* **A 14** (1999) 147 [hep-th/9812047].

[14] M. Cadoni and S. Mignemi, *Asymptotic symmetries of AdS_2 and conformal group in $d = 1$*, hep-th/9902040.

[15] G.W. Gibbons and P.K. Townsend, *Black holes and Calogero models*, *Phys. Lett.* **B 454** (1999) 187 [hep-th/9812034].

[16] P.K. Townsend, *The M(atrix) model/AdS_2 correspondence*, hep-th/9903043.

[17] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional spacetime*, *Phys. Rev. Lett.* **69** (1992) 1849 [hep-th/9204099].

[18] W.T. Kim, *AdS_2 and quantum stability in the cghs model*, *Phys. Rev.* **D 60** (1999) 024011 [hep-th/9810055].

[19] U.H. Danielsson, E. Keski-Vakkuri and M. Kruczenski, *Vacua, propagators, and holographic probes in AdS/CFT*, *J. High Energy Phys.* **01** (1999) 002 [hep-th/9812007].

[20] L. Bombelli, R.K. Koul, J. Lee and R.D. Sorkin, *A quantum source of information for black holes*, *Phys. Rev.* **D 34** (1986) 373.

[21] M. Srednicki, *Entropy and area*, *Phys. Rev. Lett.* **71** (1993) 666 [hep-th/9303048].

[22] L. Susskind and J. Uglum, *Black hole entropy in canonical quantum gravity and superstring theory*, *Phys. Rev.* **D 50** (1994) 2700 [hep-th/9401070].

[23] C. Callan and F. Wilczek, *On geometric entropy*, *Phys. Lett.* **B 333** (1994) 55 [hep-th/9401072].

[24] T.M. Fiola, J. Preskill, A. Strominger and S.P. Trivedi, *Black hole thermodynamics and information loss in two-dimensions*, *Phys. Rev.* **D 50** (1994) 3987 [hep-th/9403137].

[25] D. Kabat, *Black hole entropy and entropy of entanglement*, *Nucl. Phys.* **B 453** (1995) 281 [hep-th/9503016].

[26] F. Larsen and F. Wilczek, *Renormalization of black hole entropy and of the gravitational coupling constant*, *Nucl. Phys.* **B 458** (1996) 249 [hep-th/9506066].

[27] C. Holzhey, Ph.D. Thesis, Princeton University, 1993.

[28] S. A. Fulling, *Aspects of quantum field theory in curved space-time*, Cambridge University Press, 1989.

[29] I.S. Gradshteyn and I.M Ryzhik, *Tables of integrals, series and products*, Academic Press, San Diego 1994.

[30] A.P. Prudnikov, Yu.A. Brychkov and O.I. Marichev, *Integrals and series, volume 2: special functions*, Gordon and Breach, 1986.

[31] C.P. Burgess and C.A. Lütken, *Propagators and effective potentials in anti-de Sitter space*, *Phys. Lett.* **B 153** (1985) 137.

[32] W.A. Bardeen and D.Z. Freedman, *On the energy crisis in anti-de Sitter supersymmetry*, *Nucl. Phys.* **B 253** (1985) 635.

[33] R. B. Mann and T.G. Steele, *Thermodynamics and quantum aspects of black holes in* $(1+1)$ *dimensions*, *Class. and Quant. Grav.* **9** (1998) 475.

[34] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, 1982.

[35] P.C.W. Davies and S.A. Fulling, *Quantum vacuum energy in two dimensional space-time*, Proc. R. Soc. Lond. **A354** (1977) 59.

[36] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[37] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[38] V. Balasubramanian, P. Kraus and A. Lawrence, *Bulk vs. boundary dynamics in anti-de Sitter space-time*, *Phys. Rev.* **D 59** (1999) 046003 [hep-th/9805171].

[39] V. Balasubramanian, P. Kraus, A. Lawrence and S.P. Trivedi, *Holographic probes of anti-de Sitter space-times*, *Phys. Rev.* **D 59** (1999) 104021 [hep-th/9808017].

[40] T. Banks, M.R. Douglas, G.T. Horowitz and E. Martinec, *AdS dynamics from conformal field theory*, hep-th/9808016.

[41] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the* CFT_d/AdS_{d+1} *correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058].