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## D–branes and Spinning Black Holes

J.C. Breckenridge♣, R.C. Myers◇,  
*Physics Department, McGill University,  
Montreal, PQ, H3A 2T8, Canada*

A.W. Peet♠,  
*Joseph Henry Laboratories, Princeton University  
Princeton, NJ 08544, USA*

and

C. Vafa♡  
*Lyman Laboratory of Physics, Harvard University,  
Cambridge, MA 02138, USA*

### Abstract

We obtain a new class of spinning charged extremal black holes in five dimensions, considered both as classical configurations and in the Dirichlet(D)–brane representation. The degeneracy of states is computed from the D–brane side and the entropy agrees perfectly with that obtained from the black hole side.

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♣ e-mail: [jake@haydn.physics.mcgill.ca](mailto:jake@haydn.physics.mcgill.ca)

◇ e-mail: [rcm@hep.physics.mcgill.ca](mailto:rcm@hep.physics.mcgill.ca)

♠ e-mail: [peet@puhep1.princeton.edu](mailto:peet@puhep1.princeton.edu)

♡ e-mail: [vafa@string.harvard.edu](mailto:vafa@string.harvard.edu)

## 1. Introduction

Recently, significant progress has been made in understanding the degrees of freedom giving rise to the entropy of certain black holes in string theory [1] (and references therein)<sup>1</sup>. This was achieved by using the beautiful representation of solitons carrying Ramond–Ramond charge as D–branes [4].

In this work we report additional progress in this direction, by investigating spinning black holes in five spacetime dimensions, in theories with  $N = 4$  supersymmetry<sup>2</sup>. The black holes which we will consider carry electric charge  $Q_F$  and an antisymmetric tensor charge  $Q_H$ , and are spinning generalizations of the solutions of [1]. We concentrate on BPS–saturated states, i.e. extremal black holes, so that we may rely on adiabatic arguments for the invariance of the expression for the entropy under changes in the string coupling.

We construct black hole solutions which have equal–magnitude angular momenta in the two independent planes, i.e.  $|J_1| = |J_2| \equiv J$ . The spinning black hole entropy is computed and is found to be

$$S_{BH} = 2\pi \sqrt{\frac{Q_H Q_F^2}{2} - J^2}$$

The entropy obtained from counting spinning D–brane states is found, for large charges, to be

$$S_{micro} = 2\pi \sqrt{Q_H \left(\frac{1}{2} Q_F^2 + 1\right) - \frac{1}{4} (|J_1| + |J_2|)^2}$$

This is in exact agreement with the black hole answer in the case of interest, namely for large charges  $Q_{H,F}$  and spins, as well as  $|J_1| \simeq |J_2| = J$ . We find it remarkable that the agreement is precise, including the crucial numerical factors.

This paper is organized as follows. In Section 2 we review actions of heterotic string theory on  $T^4$  and Type II theory on  $K3$  and subsequent compactifications to five dimensions on an  $S^1$ , and outline our method for generating the solution. Section 3 contains a summary of the black hole solutions. In Section 4, we explain the counting of states from the D–brane side, and we end with some comments in Section 5.

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<sup>1</sup> See also the recent works [2], [3]

<sup>2</sup> This easily extends to the case of Type II compactified on  $T^5$ , which has  $N = 8$  supersymmetry.

## 2. Actions and solution-generating

We will begin with a five dimensional black hole which spins in a single plane, and is a solution of the five-dimensional Einstein equations. We add a trivial flat dimension with coordinate  $y$ , and the metric is thus [5]

$$\begin{aligned}
 ds_6^2 &= G_{6\mu\nu} dx^\mu dx^\nu \\
 &= -dt^2 + (r^2 + a^2) \sin^2 \theta d\varphi^2 + \frac{m}{\rho^2} (dt + a \sin^2 \theta d\varphi)^2 \\
 &\quad + \frac{\rho^2}{r^2 + a^2 - m} dr^2 + \rho^2 d\theta^2 + r^2 \cos^2 \theta d\psi^2 + dy^2
 \end{aligned} \tag{2.1}$$

where  $\rho^2 = r^2 + a^2 \cos^2 \theta$ . This black hole can be thought of as a solution of the six dimensional low energy action of heterotic string theory. It is a solution which has only the metric excited but no gauge fields, antisymmetric tensor, dilaton, or moduli fields turned on. From it, we will eventually obtain via string/string duality a charged spinning black hole solution of the Type II theory in five dimensions. This black hole will be a spinning generalization of the solution in [1].

Before we explain our method for generating the final solution, let us review some salient features of low-energy actions for the heterotic and Type II theories in six and five dimensions. We will work with the six dimensional actions in the sector with just one abelian gauge field  $F = d \wedge A$  and no moduli. This gauge field is taken to be a right-handed<sup>3</sup> internal gauge field on the heterotic side, and a field of Ramond-Ramond origin on the Type II side. Our notation is such that heterotic fields are denoted by primes, six dimensional fields have a subscript 6 so as to distinguish them from five dimensional fields, and we use the conventions of [1]. We have on the heterotic side, to lowest order in  $\alpha'$  [6]

$$S_{het}(T^4) = \int d^6x \sqrt{-g'_6} e^{-2\phi'_6} \left[ R'_6 + 4(\partial_\mu \phi'_6)^2 - \frac{1}{12} H'^2_{6\mu\nu\lambda} - \frac{1}{4} F'^2_{6\mu\nu} \right]$$

with  $\mu = 0, \dots, 5$  and

$$H'_{6\mu\nu\lambda} = \partial_\mu B'_{6\nu\lambda} - \frac{1}{2} A'_{6\mu} F'_{6\nu\lambda} + (\text{cyclic})$$

Note that the Chern-Simons terms come from the internal gauge fields. For Type IIA

$$\begin{aligned}
 S_{IIA}(K3) &= \int d^6x \left[ \sqrt{-g_6} \left[ e^{-2\phi_6} \left( R_6 + 4(\partial_\mu \phi_6)^2 - \frac{1}{12} H^2_{6\mu\nu\lambda} \right) - \frac{1}{4} F^2_{6\mu\nu} \right] \right. \\
 &\quad \left. - \frac{1}{4} \epsilon^{\mu\nu\lambda\rho\alpha\beta} B_{6\mu\nu} F_{6\lambda\rho} F_{6\alpha\beta} \right]
 \end{aligned}$$

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<sup>3</sup> We take the field to be right-handed so that the extremal configuration is supersymmetric

where

$$H_{6\mu\nu\lambda} = \partial_\mu B_{6\nu\lambda} + (\text{cyclic})$$

These two actions are related by string/string duality

$$\begin{aligned}\phi_6 &= -\phi'_6 \\ g_{6\mu\nu} &= e^{-2\phi'_6} g'_{6\mu\nu} \\ A_{6\mu} &= A'_{6\mu} \\ H_{6\mu\nu\lambda} &= \frac{1}{6} \epsilon'_{\mu\nu\lambda\rho\alpha\beta} \sqrt{-g'_6} e^{-2\phi'_6} H'^{\rho\alpha\beta}_6\end{aligned}$$

Using the standard Kaluza–Klein reduction on the  $S^1$  with coordinate  $y = x^5$ ,

$$\begin{aligned}ds_6^2 &= \tilde{g}_{\mu\nu} dx^\mu dx^\nu + e^{2\sigma} (dy + V_\mu dx^\mu)^2 \\ \phi_6 &= \phi + \frac{1}{2}\sigma \\ B_6 &= \frac{1}{2} [B_{\mu\nu} - \frac{1}{2} (V_\mu \tilde{B}_\nu - \tilde{B}_\mu V_\nu)] dx^\mu \wedge dx^\nu + \tilde{B}_\mu dx^\mu \wedge dy\end{aligned}$$

(in the sector with  $A_y = 0$ ) one finds that

$$\begin{aligned}S_{IIA}(K3 \times S^1) &= \int d^5x \left[ \sqrt{-\tilde{g}} \left[ e^{-2\phi} \left( \tilde{R} + 4(\partial_\mu \phi)^2 - \frac{1}{12} H_{\mu\nu\lambda}^2 \right. \right. \right. \\ &\quad \left. \left. - (\partial_\mu \sigma)^2 - \frac{1}{4} e^{2\sigma} V_{\mu\nu}^2 - \frac{1}{4} e^{-2\sigma} \tilde{H}_{\mu\nu}^2 \right) \right. \\ &\quad \left. \left. - \frac{1}{4} e^\sigma F_{\mu\nu}^2 \right] + \frac{1}{4} \epsilon^{\mu\nu\lambda\alpha\beta} \tilde{B}_\mu F_{\nu\lambda} F_{\alpha\beta} \right]\end{aligned}$$

where  $\tilde{H} = d \wedge \tilde{B}$  and

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} - \frac{1}{2} V_\mu \tilde{H}_{\nu\lambda} - \frac{1}{2} \tilde{B}_\mu V_{\nu\lambda} + (\text{cyclic})$$

In five dimensional Einstein frame, defined by  $g_{\mu\nu} = e^{-4\phi/3} \tilde{g}_{\mu\nu}$ , we obtain

$$\begin{aligned}S_{IIA}(K3 \times S^1) &= \int d^5x \left[ \sqrt{-g} \left( R - \frac{4}{3} (\partial_\mu \phi)^2 - (\partial_\mu \sigma)^2 \right. \right. \\ &\quad \left. \left. - \frac{1}{4} e^{2\sigma-4\phi/3} V_{\mu\nu}^2 - \frac{1}{4} e^{-2\sigma-4\phi/3} \tilde{H}_{\mu\nu}^2 - \frac{1}{4} e^{8\phi/3} X_{\mu\nu}^2 - \frac{1}{4} e^{\sigma+2\phi/3} F_{\mu\nu}^2 \right) \right. \\ &\quad \left. + \frac{1}{4} \epsilon^{\sigma\rho\mu\nu\lambda} (X_\sigma V_{\rho\mu} \tilde{H}_{\nu\lambda} + \tilde{B}_\sigma F_{\rho\mu} F_{\nu\lambda}) \right]\end{aligned}$$

where in this action we have Hodge–dualized the three–form  $H$  via

$$\begin{aligned} H_{\mu\nu\lambda} &= \frac{1}{2} e^{8\phi/3} \sqrt{-g} \epsilon_{\sigma\rho\mu\nu\lambda} X^{\sigma\rho} \\ &= \partial_\mu B_{\nu\lambda} - \frac{1}{2} V_\mu \tilde{H}_{\nu\lambda} - \frac{1}{2} \tilde{B}_\mu V_{\nu\lambda} + \text{cyclic} \end{aligned}$$

Our method for generating the desired black hole is to use a series of transformations, namely boosts involving the time  $t$  and the circle coordinate  $y$ , and string/string duality, as follows. We begin with the metric (2.1) as a heterotic solution in six dimensions. We apply  $O(6,6)$  boosts in the  $(t, y)$  directions, following the five dimensional black hole construction of [7]. In their notations, we use boost parameters  $x = \cosh \alpha$ , and  $\beta = -\alpha$ . The resulting six–dimensional solution has no  $G_{6y\mu}$  for  $\mu < 5$ , but has a  $B_6$  and a  $\phi_6$ . Next, we notice that since this boosted solution has no gauge ( $A_6$ ) fields, the field configuration is a solution for the Type II theory as well. We then apply string/string duality to get back a heterotic solution. Using the  $O(6,6)$  symmetry we can thus apply a final boost, mixing  $t$  and the internal direction involving  $A'_6$ , with parameter  $z = \cosh \gamma$ . We then apply string/string duality to convert the heterotic solution to a Type II solution, and lastly we perform the standard Kaluza–Klein reduction to five dimensions. The above boost parameter  $z$  is carefully chosen to satisfy  $z = 2x^2 - 1$ , i.e.  $\gamma = 2\alpha$ ; this choice reduces the five dimensional dilaton to a constant. The resulting configuration is a charged spinning five dimensional black hole with constant dilaton and constant moduli.

### 3. The extremal black holes

Here we will exhibit the extremal limit of the black holes obtained via the procedure outlined above. To do this, we take the limit  $x \rightarrow \infty$ ,  $a \rightarrow 0$ ,  $m \rightarrow 0$ , such that the quantities  $\mu \equiv m x^2$  and  $\omega \equiv a x$  remain finite, where  $m, a$  are the quantities appearing in the metric (2.1). After doing a coordinate transformation to match with [1],  $r^2 \rightarrow r^2 + \mu$ , we obtain for the extremal metric

$$\begin{aligned} ds_5^2(\text{ext}) &= - \left(1 - \frac{\mu}{r^2}\right)^2 \left[ dt - \frac{\mu\omega \sin^2 \theta}{(r^2 - \mu)} d\varphi + \frac{\mu\omega \cos^2 \theta}{(r^2 - \mu)} d\psi \right]^2 \\ &\quad + \left(1 - \frac{\mu}{r^2}\right)^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2 + \cos^2 \theta d\psi^2) \end{aligned}$$

while for the Ramond–Ramond gauge field the nonvanishing components are

$$\begin{aligned} A_t^{(ext)} &= \frac{\sqrt{2}}{\lambda} \frac{\mu}{r^2} \\ A_\varphi^{(ext)} &= \frac{\sqrt{2}}{\lambda} \frac{\omega \mu \sin^2 \theta}{r^2} \\ A_\psi^{(ext)} &= -\frac{\sqrt{2}}{\lambda} \frac{\omega \mu \cos^2 \theta}{r^2} \end{aligned}$$

and for the winding gauge field we have

$$\tilde{B}^{(ext)} = \frac{\lambda^3}{\sqrt{2}} A^{(ext)}$$

While the result of our solution generating procedure yields  $\phi = 0 = \sigma$ , we have shifted these scalars by a constant to

$$e^{\sigma+2\phi/3} = \lambda^2$$

which introduces the scaling of the gauge fields by  $\lambda$  given above [1]. The above fields are the only ones excited in this black hole background. Notice that when we take  $\omega \rightarrow 0$ , we recover the solution of [1].

From the asymptotic metric we obtain for the angular momentum, in the independent planes defined by  $\varphi, \psi$ ,

$$\begin{aligned} J_1 \equiv J_\varphi &= +\frac{\pi}{4} \mu \omega \\ J_2 \equiv J_\psi &= -\frac{\pi}{4} \mu \omega \end{aligned}$$

and for the mass we find

$$M_{ADM} = \frac{3\pi\mu}{4}$$

while the charges are<sup>4</sup>

$$\begin{aligned} Q_H &\equiv \frac{1}{4\pi^2} \int_{S^3} {}^* e^{-2\sigma-4\phi/3} \tilde{H} = \mu/\lambda^2 \\ Q_F &\equiv \frac{1}{16\pi} \int_{S^3} {}^* e^{\sigma+2\phi/3} F = -\frac{\pi}{2\sqrt{2}} \mu \lambda \end{aligned}$$

Note that this black hole, although a solution of the low–energy string theory equations, is not a solution of the Einstein–Maxwell equations in five dimensions. In the spinning configuration, the magnetic dipole field combines with the electric monopole field so that

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<sup>4</sup> The sphere  $S^3$  is at infinity, so we can ignore the effects of the Chern–Simons terms

the Chern–Simons contributions to the equations of motion are nontrivial. (The terms we are referring to are of course distinct from the usual  $\mathcal{O}(\alpha')$  Chern–Simons terms already appearing in the ten dimensional heterotic string theory; we are not including those terms here, as we are doing our analysis to lowest order in  $\alpha'$ .)

Let us now obtain the entropy of this extremal spinning black hole. In the above coordinates, the horizon is at  $r = r_0 = \sqrt{\mu}$ , and its entropy is found to be ( $|J_1| = |J_2| \equiv J$ )

$$\begin{aligned} S_{BH} &= \frac{1}{2}\pi^2\mu\sqrt{\mu - \omega^2} \\ &= 2\pi\sqrt{\frac{Q_H Q_F^2}{2} - J^2} \end{aligned} \tag{3.1}$$

Note that both of these expressions are independent of  $\lambda$ .

We find it satisfying that these extremal rotating charged black holes have a finite–area horizon, and also that the angular momentum is bounded above:  $J_{max}^2 = Q_H Q_F^2/2$  (in going beyond this limit, closed timelike curves develop).

#### 4. Spins of the BPS D–branes

Let us recall the nature of the D–brane states that are responsible for the degeneracy of the extremal black holes that we are considering [1]. We will consider compactification of type IIB on  $K3 \times S^1$  down to five dimensions<sup>5</sup>. Consider those D–brane states which are wrapped around  $S^1$  and partially wrapped around  $K3$ . Let  $Q_F$  denote the charge of the D–brane on the  $K3$  part.  $Q_F$  can be viewed as an element of the  $K3$  homology  $H_*(K3, \mathbf{Z})$  which is identified with how the internal part of the D–brane wraps around  $K3$ . Note that the dot product  $Q_F \cdot Q_F$  is the same as the intersection of cycles in the  $K3$  homology. In the presence of D–branes, we get an effective field theory which lives on the D–brane worldvolume. If we take the size of  $S^1$  to be much larger than that of the  $K3$ , then the D–brane effective field theory will be a theory on  $\mathbf{R} \times S^1$ , where the  $\mathbf{R}$  is the time coordinate.

Based on string dualities and observations in [8] it was conjectured [9] that this theory is a sigma model on  $(\frac{1}{2}Q_F \cdot Q_F + 1)$  symmetric product of  $K3$ 's, i.e. on

$$M = \text{Sym}^{\frac{1}{2}Q_F^2 + 1}(K3)$$

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<sup>5</sup> The arguments for compactification on  $T^4 \times S^1$  are (essentially) identical with the replacement of  $T^4$  for  $K3$  in the following discussions. Only the dimension of the manifold enters in the asymptotic growth below.



This conjecture has been checked in essentially all cases, at least up to  $T$ -duality, and found to be true [10], [11]. Actually, as noted in [8], [9], the light-cone helicity of the six dimensional theory, which we identify with the spatial  $O(4)$  holonomy, can also be read off, as follows. Let  $J_1$  and  $J_2$  be the two holonomies in  $O(4)$ . Note that the sigma model on  $M$  is conformal (as it is hyperkahler) and it will give rise to two  $U(1)$ 's from the  $N = 2$  superconformal algebras: one left- and one right-moving. In fact, there will be an  $N = 4$  superconformal algebra in our case, with the  $SU(2)_L \times SU(2)_R$  action to be identified with our  $O(4)$ , but we will only need the  $U(1)_L \times U(1)_R$  subgroup of it here. Let us denote the  $U(1)_L \times U(1)_R$  charges of states by  $(F_L, F_R)$ . Then

$$J_1 = \frac{1}{2}(F_L + F_R)$$

$$J_2 = \frac{1}{2}(F_L - F_R)$$

Consider, for example, the case  $Q_F = 0$ . Noting that the ground states of the sigma model are identified with the  $K3$  cohomology, and that  $F_L$  and  $F_R$  for the ground states run over the values  $\{-1, 0, 1\}$ , we learn that we have the  $(J_1, J_2)$  spectrum consisting of 20 states with  $(0,0)$ , two states with  $(\pm 1, 0)$ , and two states with  $(0, \pm 1)$ . These we recognize as the light-cone oscillator quantum numbers of bosonic strings in 6 dimensions.

The D-brane BPS states considered in [1] correspond to Ramond-Ramond states of this sigma model which are right-moving ground states, and left-moving states of level  $n = Q_H$ . Recall that there is a bound for the  $F_L$  and  $F_R$  with respect to  $L_0$  and  $\overline{L}_0$  [12]. This is easily seen by bosonizing the  $U(1)$  currents: let  $J_L = \sqrt{\hat{c}}\partial\phi$ . A state with charge  $F_L$  will then be represented by an operator

$$\exp\left(\frac{iF_L\phi}{\sqrt{\hat{c}}}\right) \cdot \Phi$$

where  $\Phi$  is an operator from the rest of the conformal field theory which can be made of the oscillator factors of the  $U(1)$  current, but not the momentum modes, plus any other state in the theory. The same story repeats for  $F_R$ . In particular, note that since the dimensions of  $\Phi$  are positive, the dimensions of the operators are restricted by

$$L_0 \geq \frac{F_L^2}{2\hat{c}} \quad \overline{L}_0 \geq \frac{F_R^2}{2\hat{c}}$$

where  $\hat{c}$  is the complex dimension of the manifold. In our case,  $\hat{c} = Q_F^2 + 2$ .

We are interested in doing a count in the regime where  $Q_F$  is large but held fixed. Moreover, we take  $Q_H$  to be arbitrarily large. We are also interested in a region with  $|J_1|, |J_2| \gg 1$ . Let us consider the case where the system is the right-moving ground state with fixed  $F_R$ . Then we can consider arbitrarily large values of  $F_L$  to make both  $J_1$  and  $J_2$  large with the same sign<sup>6</sup>. Since the entropy comes from the left-moving Hilbert space, we have to estimate how many left-mover states are still available if we fix  $F_L$ . Considering a regime<sup>7</sup> where  $(Q_H - F_L^2/2\hat{c}) \gg 1$  as well as  $Q_H/Q_F^2 \gg 1$ , the answer is supplied by the bosonization discussed above. Since the total eigenvalue is  $L_0 = n = Q_H$ , and we have used up  $\frac{F_L^2}{2\hat{c}} = \frac{F_L^2}{2Q_F^2+4}$  for the states we are interested in, the  $L_0$  eigenvalue of the extra operator  $\Phi$  is given by

$$L_0(\Phi) = \tilde{n} = n - \frac{F_L^2}{2\hat{c}} = Q_H - \frac{F_L^2}{2Q_F^2 + 4}$$

Since the oscillatory states make the maximum contribution to degeneracy of string states, we learn that effectively we can take  $\tilde{n}$  as the available oscillator number. Therefore, we get a degeneracy growth of ( $\hat{c} = c/3$ ,  $F_L = J_1 + J_2$ )

$$\begin{aligned} d &\sim \exp\left(2\pi\sqrt{\frac{\tilde{n}\hat{c}}{2}}\right) = \exp\left(2\pi\sqrt{\left(Q_H - \frac{F_L^2/4}{\frac{1}{2}Q_F^2 + 1}\right)\left(\frac{1}{2}Q_F^2 + 1\right)}\right) \\ &\sim \exp\left(2\pi\sqrt{Q_H\left(\frac{1}{2}Q_F^2 + 1\right) - \frac{1}{4}(|J_1| + |J_2|)^2}\right) \end{aligned}$$

We have used absolute value signs for  $J_i$  in order to write the final answer in its most general form, independently of whether or not  $J_1$  and  $J_2$  have the same sign. Then the entropy is

$$S_{micro} \sim 2\pi\sqrt{Q_H\left(\frac{1}{2}Q_F^2 + 1\right) - \frac{1}{4}(|J_1| + |J_2|)^2} \quad (4.1)$$

Taking  $|J_1| = |J_2| = J$ , we see that this formula agrees with what we found for the entropy (3.1) of the spinning black hole. Note that this computation also sharpens the computation in [1] where, in principle, one should have counted only the spin-zero D-branes to make the comparison with the non-spinning black hole.

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<sup>6</sup> To consider  $J_1$  and  $J_2$  with the opposite sign, the entropy would have come from the right-movers and we would be considering large values of  $F_R$ .

<sup>7</sup> It may be that our final results are valid beyond this regime of charges. Further note that the given regime does not exclude the possibility that the ratio of  $F_L^2/2\hat{c}$  to  $Q_H$  is only slightly less than one.

## 5. Comments and Acknowledgements

In this work we have found spinning black hole solutions in five dimensions, which we believe are new, whose entropy agrees precisely with the result obtained by using D-brane technology. It is satisfying that the two methods give the same result, and we regard this as additional evidence for the D-brane picture of [4].

One would also like to consider the case where the two angular momenta,  $J_1$  and  $J_2$ , are not equal. With respect to our D-brane calculation, we note that in the right-moving ground state  $|F_R\rangle$  is bounded as [12]

$$|F_R| = |J_1 - J_2| \leq \frac{\hat{c}}{2} = \frac{1}{2}Q_F^2 + 1$$

Hence the difference between the spins can not be arbitrarily large. By combining this bound with the previously noted relation  $Q_H/Q_F^2 \gg 1$ , one can demonstrate that our calculations are valid for  $|F_R|/|F_L| = |J_1 - J_2|/|J_1 + J_2| \ll 1$ . Hence one should not expect to see a difference in the spins at the macroscopic level of the black hole computations. In fact, one can construct a nonextremal black hole analogous to that presented here in which the two angular momenta are independent. However, one finds demanding that the extremal or supersymmetric limit be nonsingular requires setting  $|J_1| = |J_2|$  [13]. Hence the D-brane and black hole results are in perfect agreement on this aspect of the calculation, as well.

Lastly, we would like to point out that in the degeneracy formula there are power corrections which give rise to logarithmic corrections to the entropy (4.1). These should correspond to logarithmic corrections to the entropy on the black hole side, and it would be interesting to investigate whether these do in fact arise as one-loop corrections in the black hole geometry.

Work on the nonextremal versions of the black holes exhibited here is in progress. It appears that the results of [2], [3] can be extended to the nonextremal spinning case [13].

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