# Essays on Macroeconomic Stabilization

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Rohan Kekre

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*Dissertation Advisors:*
**Professor Emmanuel Farhi**
**Professor Gita Gopinath**

*Author:*
**Rohan Kekre**

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# Abstract

Motivated by policy debates emerging from the U.S. Great Recession and Eurozone crisis, I study the stabilization role of monetary, fiscal, and macroprudential policies in response to short-run fluctuations. In the first essay on "*Unemployment Insurance in Macroeconomic Stabilization*", I characterize the role of unemployment insurance (UI) generosity as a particular instrument of fiscal policy, and use my framework to quantitatively evaluate the employment and welfare effects of UI extensions in the U.S. over 2008-13. In the second essay on "*Labor Market Frictions in a Monetary Union*", I study stabilization trade-offs and optimal monetary policy in a monetary union where labor markets are frictional and heterogeneous across member states, with implications for the sustainability of the Euro and policy of the ECB. In the third essay on "*Firm vs. Bank Leverage over the Business Cycle*", I develop a general equilibrium model explaining the contrasting cyclical behavior of non-financial corporate and bank leverage in U.S. data, and study its implications for macroprudential regulation in banking. Methodologically, these essays share a focus on building theoretical models of closed and open economies to address policy-relevant questions in macroeconomics, drawing on additional ideas from related fields such as public economics and finance.

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To my parents

# Introduction

The global downturn in advanced economies beginning in 2008 raised fundamental questions regarding the appropriate conduct of macroeconomic stabilization. Going forward, how should monetary policy cope with constraints posed by a zero lower bound on the nominal interest rate, or a fixed exchange rate such as that (implicitly) within the Eurozone? Given these constraints, what is the positive and normative role of fiscal policy? Alongside these conventional instruments, is there a role for macroprudential regulation over the cycle?

In this dissertation I make progress on these questions in three essays studying monetary, fiscal, and macroprudential policy in response to short-run fluctuations. In the first essay on "*Unemployment Insurance in Macroeconomic Stabilization*", I theoretically examine the role of unemployment insurance (UI) generosity as a particular instrument of fiscal policy when monetary policy is constrained, and apply these insights quantitatively in an evaluation of the macroeconomic effects of UI extensions in the U.S. over 2008-13. In the second essay on "*Labor Market Frictions in a Monetary Union*", I ask how frictional labor markets affect stabilization trade-offs and optimal monetary policy in a monetary union, drawing out implications for the sustainability of the Euro and policy of the ECB. In the third essay on "*Firm vs. Bank Leverage over the Business Cycle*", I develop a general equilibrium model explaining the contrasting cyclical behavior of firm and bank leverage in U.S. data, and study the implications of this framework for macroprudential regulation in banking.

In the first and longest essay, I argue that the interaction between UI and aggregate demand motivates a natural role for expanded UI generosity as a particular fiscal component of stabilization policy during recessions. In contrast to standard analyses in public economics which have focused on UI in partial equilibrium, I embed UI in a macroeconomic framework with search frictions and incomplete markets, and where inefficient fluctuations arise from nominal rigidities and constraints on monetary policy. In a two-period version of this framework capturing the short-run and long-run, I provide the first analytical characterization of optimal UI alongside monetary policy in general equilibrium. Due to an aggregate demand externality arising from differences in the marginal propensities to consume between the unemployed and employed, and an effect of low aggregate demand on the social cost of disincentivizing labor supply, optimal UI naturally exceeds the classic level in public finance when the economy is slack. Quantitatively, an infinite horizon extension of this framework calibrated to the U.S. economy suggests that the 2008-13 benefit extensions indeed had large, positive effects on employment and welfare through aggregate demand.

In the second essay, I argue that while labor market frictions may not change the criteria for optimal currency areas, they do matter for optimal monetary policy in a monetary union. I begin by embedding search and matching frictions in an otherwise standard model of a two-country monetary union with sticky prices and TFP shocks. Under a benchmark set of preferences and technologies, I first establish that, despite arbitrary frictions across countries, the standard Mundellian result survives: the constrained efficient allocation is attainable if and only if productivity shocks are symmetric. I then characterize optimal monetary policy in the presence of asymmetric shocks, finding that it features relative accommodation of the more sclerotic member of the union by targeting smaller output and inflation distortions in that country, all else equal. These results suggest that while labor market frictions may not directly bear on the optimality of the single currency in the Eurozone, they should be incorporated in the design of stabilization policy by the ECB.

In the third essay, I argue that the procyclicality of bank leverage may be a constrained efficient feature of business cycles, and thus not require offsetting macroprudential regulation. In a general equilibrium setting with asymmetric information between issuers and investors, non-financial firm leverage is countercyclical because the lemons discount in equity issuance falls in a boom, whereas bank leverage is procyclical because as diversified intermediaries banks see their capacity to issue safe debt, the cheapest form of external finance, increase in a boom. This theory explains additional cross-sectional leverage patterns in U.S. data, such as between broker/dealers and commercial banks, and between more and less bank-dependent firms. In normative analysis, I then find that while the level of bank leverage may be constrained inefficient, its procyclicality is not, since safe debt issuance is an efficient response to informational and technological frictions in the economy.

Across these essays, I build theoretical general equilibrium models of closed and open economies, and draw on tools and ideas from related fields such as public economics and finance, to speak to important policy-relevant debates in macroeconomics. Together, these essays advance our understanding of the appropriate use of monetary, fiscal, and macroprudential instruments in macroeconomic stabilization.

# Chapter 1

# Unemployment Insurance in Macroeconomic Stabilization

## 1.1 Introduction

Economists have long viewed unemployment insurance (UI) as an important automatic stabilizer — but should it also serve as a discretionary tool in the stabilization of short-run fluctuations? Since the 1950s, policymakers in the United States have treated UI generosity as precisely such an instrument, routinely extending benefits in recessions. This practice was expanded in unprecedented and controversial fashion during the Great Recession, when benefit durations were raised almost four-fold at the depth of the downturn. Supporters of increased generosity have pointed to the stimulus benefits of transfers to the unemployed, many of whom face binding liquidity constraints. Critics have emphasized the counterproductive supply-side effects of more generous UI.[1]

The standard analysis of UI in the economics literature is largely silent on this debate because it has been developed in a partial equilibrium setting which assumes away interac-

[1] See Summers (2010), Congressional Budget Office (2012), and Blanchard *et al.* (2013) for support of more generous UI. See Barro (2010) and Mulligan (2012) for more critical commentary on its effects.

tions between UI and macroeconomic slackness.[2] This paper embeds UI in a macroeconomic framework with search frictions and incomplete markets, and where inefficient fluctuations arise from nominal rigidities and constraints on monetary policy. In this setting, I theoretically characterize the effects of UI generosity on equilibrium output, employment, and welfare. I study these effects quantitatively in a dynamic simulation of the 2008-13 benefit extensions in the U.S. I conclude that expanded UI generosity can, and should, play an important role in the policy response to economic slackness.

Theoretically, the interaction between UI and aggregate demand naturally motivates a role for higher generosity when the economy is slack. Nominal rigidities imply that an increase in UI generosity affects output through a redistribution effect on aggregate demand, governed by the difference in marginal propensities to consume (MPCs) between the unemployed and employed. In terms of social welfare, this generates an aggregate demand externality from transfers when the economy is inefficiently slack, as when monetary policy is constrained. In addition, low aggregate demand in such a recession itself changes the social cost of disincentivizing labor supply, the sign of which depends on whether a reduction in employment changes goods supply or demand by more in the short run. When the unemployed have a higher MPC than the employed and are net debtors, these channels imply a positive effect of redistribution on output, and optimal generosity which is higher than the classic partial equilibrium formula would imply.

Quantitatively, I find that the 2008–13 benefit extensions in the U.S. indeed had large, positive effects on employment and welfare operating through these channels. I extend the framework used in the theoretical analysis to simulate an infinite-horizon model with finite-duration UI. Relative to a counterfactual path of benefits capped at 9 months of duration in the calibrated model, I find that the observed extensions to 22 months prevent a further rise in the unemployment rate of 2–5 percentage points, depending on the value of calibrated parameters. Across calibrations, I further find that the observed extensions

[2] There is a growing literature developed since the Great Recession which studies UI over the business cycle, which my paper joins and which I discuss later in this section. My analysis is distinguished by accounting for interactions between UI and aggregate demand, a key part of the policy debate.

generate a strict Pareto improvement relative to the counterfactual. Remarkably, even the employed gain from the policy change: while they expect to finance incremental transfers to the unemployed, the stronger labor market induced by transfers more than compensates by raising their continuation utility should they lose their jobs.

My paper is the first to analytically characterize the role of UI generosity alongside monetary policy in stabilizing an economy with nominal rigidities. Three frictions on the real side of the economy, and my characterization of optimal policy absent nominal rigidities, set the stage for this stabilization problem. First, search and matching frictions in the tradition of Diamond (1981), Mortensen (1982), and Pissarides (1984) (hereafter, DMP) give rise to involuntary unemployment. Second, market incompleteness with respect to unemployment risk generates an efficient role for the public provision of UI. Third, unobservable worker search intensity in the matching process leads to a moral hazard cost in the provision of UI. In this environment, I obtain closed form formulas by specializing to the case of a two-period economy with a "short run" and "long run". Absent nominal rigidities, the efficient level of UI generosity is characterized by a general equilibrium version of the benchmark Baily (1978)-Chetty (2006) formula from public finance. In terms of sufficient statistics, this formula demonstrates that optimal UI balances the welfare gain from consumption insurance with the disincentive cost from moral hazard.

My first main result is a *generalized Baily-Chetty* formula characterizing optimal UI generosity in the presence of macroeconomic slackness. Slackness arises from the combination of nominal rigidity and an inability of monetary policy to perfectly stabilize the economy on its own, owing to a zero lower bound on the nominal interest rate.[3] There are two distinct channels through which slackness affects the optimal generosity of UI. First, an aggregate demand externality in the class identified by Farhi and Werning (2015) generates

[3] A constraint on monetary policy is necessary to motivate a second-best stabilization problem when prices or wages are fully rigid. Another constraint I study is one faced by a small open economy with a fixed exchange rate, where all of my results carry over (see section 1.2.6). A more distinct setting of interest is one where prices can partially adjust, and the economy is buffeted by "cost-push" shocks. While the stimulative effects of UI extensions through aggregate demand would carry over, the supply-side effects of UI may be more costly than they are in the present setting. The optimal policy prescription may thus change. I leave the analysis of this case to future research.

a distinction between the private and social value of transfers. If unemployed workers' MPC out of income exceeds that of employed workers in the short run, the social value of transfers exceeds the private gains from consumption insurance because UI can raise aggregate demand and thus economic activity. Second, low aggregate demand itself changes the social cost of disincentivizing labor supply. The disincentive effect of UI is welfare-relevant because lower employment affects economy-wide resources available for consumption.[4] At each date, the resulting social cost depends on the shadow price of aggregate resources and the effect of lower employment on the net supply of goods, keeping in mind that lower employment not only reduces production but also changes the composition of demand. In the short-run, if unemployed workers are net debtors, lower employment unambiguously reduces net supply. In a recession caused by low aggregate demand, a low shadow price on resources makes this less costly.

My second main result is a formula for the *UI multiplier* characterizing the effect of an increase in unemployed workers' income on equilibrium output. A marginal increase in their income affects output through a redistribution effect on aggregate demand, governed by the difference in MPCs between the unemployed and employed. This effect underlies the aggregate demand externality in the normative analysis, and is absent in prior analyses of the positive effects of UI in partial equilibrium or flexible price settings. It only relies on nominal rigidities, though constraints on monetary policy motivate why there may be no endogenous policy response to the stance in fiscal policy. There is a subtle difference from the standard Keynesian intuition, however: supply-side elasticities still matter even when production adjusts to meet desired demand. In particular, when vacancies adjust to offset the reduction in search effort induced by higher UI, this raises recruiting intensity, which is costly in a frictional labor market. On the margin, this effect may be small relative to the aggregate demand effects of transfers. But it underscores why, at the extreme, complete insurance need not maximize output even in a demand-determined world. It also explains

[4] In standard analyses of UI in public finance, there is a fiscal externality of UI on the government's budget constraint. In general equilibrium, the analogous externality is on the economy's resource constraints at each date.

why disincentives still matter in determining the optimal generosity of UI.

In an enriched quantitative model, I then assess the effects of the unprecedented expansion of UI generosity in the U.S. during the Great Recession. The federal-state Extended Benefits program and the federal government's Emergency Unemployment Compensation Act of 2008 together raised benefit durations from an average of 26 weeks across states to 99 weeks at the depth of the recession. To study these policies, I generalize the model to an infinite horizon setting combining the dynamics of the DMP labor market with rich consumption and savings decisions in the tradition of Bewley (1983), Huggett (1993), and Aiyagari (1994) (hereafter, BHA). This allows me to capture important dynamic issues introduced by policy affecting benefit durations in a long-lasting recession, such as effects on precautionary saving. It also allows me to accommodate the endogenously changing employment and wealth distribution over the Great Recession in shaping the effects of UI.

In the steady-state of the calibrated model, the MPCs of unemployed agents rise sharply with duration of unemployment — an endogenous outcome, not a calibrated target. In my benchmark calibration, the long-term unemployed (those unemployed for at least 6 months, and who thus exhaust benefits in steady-state) have a monthly MPC which exceeds that of the employed by 0.24. This arises from the combination of two forces operating as an agent proceeds through an unemployment spell: (i) precautionary behavior which tends to raise the MPC at any given level of wealth, and (ii) the endogenous decision to draw down on assets, pushing the agent closer to the borrowing constraint. To use the language of public finance, it means that in a BHA framework with DMP labor market dynamics calibrated to match the incidence and duration of unemployment in the U.S. economy, the long-term unemployed are an extremely promising "tag" for high MPCs.[5]

In response to a shock which pushes the economy with fully sticky prices against the zero lower bound, I find that the 2008-13 benefit extensions have large, positive effects on employment and welfare through aggregate demand. At date 1 of the simulation, which

[5] I use "tag" as defined by Akerlof (1978). It means that long-term unemployment is an observable characteristic which correlates strongly with the MPC, a characteristic which is less easily observable but is what fundamentally matters for stabilization policy according to my theoretical formulas.

corresponds to July 2008, there is an unanticipated shock to preferences which raises desired saving, and thus reduces the natural rate of interest. The path of fundamentals and the assumed monetary policy rule imply a binding zero lower bound on the interest rate for 90 months (consistent with "lift-off" from the zero lower bound in December 2015). In my baseline experiment, I compare outcomes under the observed extensions to counterfactual benefits capped at 9 months of duration, consistent with the greatest average degree of generosity across U.S. states at any point prior to the Great Recession. Relative to this counterfactual, the observed extensions prevent a further rise in the unemployment rate of 2–5 percentage points and generate a strict Pareto improvement. Even employed agents gain because of the stronger labor market induced by transfers to the long-term unemployed. Notably, these results are reversed when prices are flexible, where disincentives dominate and motivate a reduction in utilitarian social welfare.

The effects of the benefit extensions on employment and welfare appear driven by two forces: (i) the high MPCs of the long-term unemployed and (ii) dynamic amplification arising from the interaction with precautionary saving. Consistent with the analytical results of the first half of the paper, I find in sensitivity analysis that the pattern of MPCs by employment status is the micro-level moment which drives the stimulus from more generous UI.[6] In an infinite horizon environment with incomplete markets, this stimulus is amplified by its interaction with precautionary savings, evidenced by the time-path of equilibrium unemployment under the observed extensions relative to less generous UI. UI extensions at date $t$, by redistributing to high-MPC agents and thus stimulating the economy, lead to higher job-finding rates. Expecting this at $t-1$, agents engage in less precautionary saving, further stimulating the economy at $t-1$, and so on. As a result, in the simulation the greatest stimulus from UI extensions is realized in the initial period of announcement.

As such, my results motivate two important directions for further work to build upon

[6] A broader set of counterfactual policy experiments further suggests the importance of this moment. For instance, I find that the observed extensions deliver greater employment and welfare than any of the counterfactuals I explore with durations of 6 months, but more generous replacement rates. Even though these replacement rate policies increase generosity for any agent who becomes unemployed, the high MPCs of the long-term unemployed render the observed extensions more powerful in stimulating aggregate demand.

the framework developed here. First, future applied work should aim to estimate the profile of MPCs by duration of unemployment, which remains a poorly understood moment despite an active literature in macroeconomics on the level and heterogeneity of MPCs across various groups.[7] Targeting such an estimate could help narrow down the range of effects obtained in this paper, which accounts for the substantial uncertainty there currently is regarding the MPC profile. Second, future work should attempt to better quantify, and assess the robustness of, the dynamic amplification operating through precautionary saving. In ongoing analytical work, I develop a multi-period extension of my short-run/long-run framework to study the effects of UI generosity on precautionary saving and isolate sufficient statistics governing this amplification channel. In ongoing quantitative work, I study the robustness of my results to the assumption of perfect foresight, which likely contributes to the large magnitude of amplification in the simulation experiments presented here.

**Relation to literature.** My paper is the first to analytically characterize the role of UI generosity as a second-best policy instrument when monetary policy is constrained. It joins an emerging literature inspired by the Great Recession focused on other second-best instruments such as conventional government spending (Woodford (2010), Eggertsson and Krugman (2012), Werning (2012)), macroprudential regulation (Farhi and Werning (2015), Korinek and Simsek (forthcoming)), tax policy (Correia *et al.* (2013), Farhi *et al.* (2013)), capital controls (Schmitt-Grohe and Uribe (2012), Farhi and Werning (2013)), and cross-border transfers (Farhi and Werning (2014)).

In characterizing the optimal use of UI generosity in stabilization, I integrate Baily (1978) and Chetty (2006)'s classic analysis of optimal UI with Farhi and Werning (2015)'s recent framework for second-best macroeconomic stabilization. The latter authors build a general theory of macroprudential policies in the presence of nominal rigidities and constraints on monetary policy. My analysis demonstrates that the aggregate demand externalities

[7] To my knowledge, Japelli and Pistaferri (2014) is the only study speaking to MPC heterogeneity by employment status, in a survey of Italian households. Other studies have focused on MPC heterogeneity by income or wealth.

which motivate their theory of macroprudential policy similarly motivate a role for social insurance in macroeconomic stabilization, manifest in optimal UI which departs from the Baily-Chetty benchmark.

Relative to other recent efforts studying the Baily-Chetty formula over the business cycle, mine is uniquely focused on nominal rigidities, generating a key role for the aggregate demand effects of UI. Jung and Kuester (2015) instead study departures from the Baily-Chetty benchmark as part of a broader analysis of optimal labor market policy in a real business cycle model. Landais *et al.* (2015b,a) study departures from the Baily-Chetty benchmark using a sufficient statistic approach which does not need to take a stand on the source of inefficiency, but does rule out any aggregate demand effects of UI. These authors' "rat race effect" is, however, similar to the intuition behind low aggregate demand changing the social cost of disincentives in my optimal UI formula.[8] Kroft and Notowidigdo (2015) explore how the elasticities in the standard Baily-Chetty formula vary over the business cycle, and Schmieder *et al.* (2012) do the same in an analogous formula developed for benefit extensions. The present analysis identifies an important setting when these elasticities are no longer sufficient; the difference in MPCs enters in a distinct, important role.

Quantitatively, my paper is the first to study the effects of UI extensions in a calibrated model with search frictions, incomplete markets, and nominal rigidities. Krusell *et al.* (2010), Nakajima (2012), and Mitman and Rabinovich (2015) analyze UI in calibrated models merging the DMP and BHA traditions, but with business cycle dynamics in the real business cycle tradition. As I show in my analysis, moving from flexible to sticky prices reverses the employment and welfare effects of UI. Krueger *et al.* (2016) find expansionary effects of UI when output responds to changes in aggregate consumption, but model this through productivity and do not include search frictions in the labor market. Christiano *et al.* (2015) and den Haan *et al.* (2015) include nominal rigidities and search frictions in their

[8]In Landais *et al.* (2015b,a), the "rat race effect" reduces the cost of disincentivizing labor supply when tightness is inefficiently low, naturally motivating optimally countercyclical UI generosity. My analysis formalizes why the economy can find itself in such a situation of inefficient tightness, owing to nominal rigidity, a binding zero lower bound, and thus low aggregate demand.

simulations, but the former do not model incomplete markets, and the latter focus on changes in replacement rate generosity in a model with infinite duration UI. den Haan *et al.* (2015) further emphasize the demand for real money balances and its impact on deflation, which is shut down here as I study the cashless limit with fully sticky prices. My framework instead emphasizes MPC heterogeneity, which looms large because I study policies on the duration margin.

Relative to a growing literature exploring the policy consequences of MPC heterogeneity in incomplete markets New Keynesian economies, my paper adds DMP dynamics to the labor market. Oh and Reis (2012) and Guerrieri and Lorenzoni (2015) combined the incomplete markets and New Keynesian traditions, and in such an environment researchers have studied general transfers (Bilbiie *et al.* (2013), Mehrotra (2014), Giambattista and Pennings (2015)), automatic stabilizers (McKay and Reis (2015)), and monetary policy itself (Auclert (2015), Kaplan *et al.* (2016)). All of these have featured a neoclassical labor market. In my setting, DMP dynamics interact with the stimulus from redistribution to generate an endogenous rise in equilibrium job-finding rates. The resulting rise in job-finding rates reduces incentives to precautionary save, amplifying the initial stimulus. This mechanism builds on recent work linking precautionary saving and aggregate demand in incomplete market environments (Ravn and Sterk (2014), Challe *et al.* (2014), Werning (2015)).

Finally, my calibration approach complements a separate strand of the literature using quasi-experimental research designs to estimate the stabilization effects of UI generosity.[9] The conclusions from this literature are mixed. di Maggio and Kermani (2015) find that more generous UI tends to have a stabilizing impact on business cycle fluctuations through its effect on aggregate demand. Hagedorn *et al.* (2015a) and Hagedorn *et al.* (2015b) instead estimate negative effects of the 2008-13 UI benefit extensions on employment, while

[9] More broadly, my paper complements a growing literature on fiscal policy seeking to estimate the effects of transfers, rather than traditional government spending, on macroeconomic outcomes. Chodorow-Reich *et al.* (2012) estimate large, positive effects on local employment from transfers to states as part of the American Recovery and Reinvestment Act. Romer and Romer (2015) estimate only short-lived aggregate consumption increases from Social Security benefit increases over 1952 to 1991, but find that this may be caused by the response of monetary policy.

Chodorow-Reich and Karabarbounis (2016) estimate negligible effects and Coglianese (2015) estimates positive effects of the same extensions. Such quasi-experimental studies face the challenge of appropriately controlling for the endogeneity of labor market policy in response to fluctuations in economic activity, which may explain the differing results. My approach does not resolve this debate. Instead, I ask what micro-level behavioral moments imply regarding the effects of UI extensions in the laboratory of a rational expectations, general equilibrium model of the U.S. economy at the zero lower bound.

**Outline.** I begin in section 1.2 by characterizing the theoretical role of UI in a short-run/long-run economy featuring constrained efficient UI, nominal rigidities, and potential constraints on monetary policy. I extend this framework in section 1.3 to conduct a quantitative evaluation of the benefit extensions in the U.S. during the Great Recession. Finally, in section 1.4 I conclude.

## 1.2 Theory

In this section I embed UI in a macroeconomic model with inefficient fluctuations to study the role of UI in stabilization. I begin by characterizing an economy with search frictions, incomplete markets, and moral hazard in which the government provision of UI is constrained efficient. I then examine the normative and positive consequences of changes in UI generosity in the presence of short-run fluctuations arising from macroeconomic shocks and nominal rigidities.

I find that general equilibrium interactions between UI and aggregate demand motivate a role for UI in stabilization when monetary policy is constrained. Normatively, optimal UI generosity departs from the classic partial equilibrium trade-off between insurance and incentives due to an aggregate demand externality and an effect of low aggregate demand on the social cost of disincentives. Positively, a marginal increase in UI generosity affects output and employment through a redistribution effect on aggregate demand. The aggregate demand externality summarizes the welfare impact of the redistribution effect when the economy is slack. Both are governed by the difference in MPCs between the unemployed and employed.

### 1.2.1 Economic environment

Consider a closed economy, where the treatment of dynamics is simplified by considering a “short run” with endogenous production at date 1 and “long run” with traded endowments at date 2. Three departures from the neoclassical benchmark are sufficient to motivate a constrained efficient role for UI in the short run, as characterized in the next subsection: a static version of search and matching, an absence of private markets to insure against idiosyncratic unemployment risk, and worker search intensity which is unobserved by the government.

**Technologies, tastes, and endowments.** There are two classes of firms in this economy: measure one producers of a homogenous intermediate good, and measure one retailers who purchase this input and use a pure pass-through technology to sell a differentiated variety to consumers. Producers are perfectly competitive, while retailers are monopolistically competitive.[10]

Producers engage in two classes of activities: recruiting and production. Recruiting proceeds according to a static version of Diamond-Mortensen-Pissarides search and matching frictions, in which workers actively search for jobs and firms post vacancies to hire them. Given aggregate worker search effort $\bar{s}$ and firm vacancies $\bar{v}$, the economy sees an aggregate number of matches

$$m(\bar{s}, \bar{v}) = \bar{m}\bar{s}^{1-\eta}\bar{v}^{\eta}$$

in the short run. Defining labor market tightness $\theta \equiv \frac{\bar{v}}{\bar{s}}$, this implies a vacancy-filling

[10]The distinction between these two classes of firms, common in the New Keynesian literature, is simply made for tractability in separating the problems of production and recruiting on the one hand, and price-setting on the other.

probability $q(\theta)$ and job-finding probability per unit effort $p(\theta)$ defined as

$$q(\theta) \equiv \bar{m}\theta^{\eta-1} = \frac{m(\bar{s},\bar{v})}{\bar{v}},\; p(\theta) \equiv \bar{m}\theta^{\eta} = \frac{m(\bar{s},\bar{v})}{\bar{s}}.$$

A given producer thus rationally expects that by posting $v$ vacancies it will yield $q(\theta)v$ workers. Recruiting has a cost $k$ per vacancy, which I express in units of foregone worker time spent producing.[11] As such, this producer has $q(\theta)v - kv$ units of labor spent in production, which yields

$$f(q(\theta)v - kv) \equiv a(q(\theta)v - kv)^{\alpha}$$

units of output given productivity $a$ and returns to scale $\alpha \in (0,1]$ common to all firms.

On the worker side, there are measure one agents who are ex-ante identical. In the short-run, these agents are endowed with one indivisible unit of labor. Putting in search effort $s$, a particular agent becomes employed with probability $p(\theta)s$, generating ex-ante utility

$$U = (p(\theta)s)\, u^{e}(c_1^{e}, c_2^{e}) + (1 - p(\theta)s)\, u^{u}(c_1^{u}, c_2^{u}) - \psi(s). \tag{1.1}$$

I allow arbitrary non-separabilities between short-run and long-run consumption $\{c_1, c_2\}$ in each employment state, though of course time-separable consumption with discount factor $\beta$ is a special case.[12] I further assume arbitrarily different subutility functions in each employment state $\{u^{e}(\cdot), u^{u}(\cdot)\}$, which can accommodate non-pecuniary costs of employment and/or unemployment. Short-run consumption is itself a CES aggregator over differentiated varieties

$$c_1^{i} = \left[\int_0^1 c_1^{i}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},\; i \in \{e,u\}. \tag{1.2}$$

The cost of search effort $\psi(s)$ is assumed to be separable from consumption for expositional

[11] I follow Shimer (2010) here. Without incumbent workers, it is an abstraction; in the infinite-horizon framework of section 1.3, it more directly maps to the time spent by incumbent workers on recruiting new hires in practice.

[12] Time separable preferences are standard in dynamic macroeconomic models. I am more general here not just because the results do not require them, but also because non-separability affords another useful interpretation of my results as pertaining to a small open economy with non-traded and traded consumption (as I discuss in section 1.2.6).

simplicity, and satisfies standard regularity conditions.[13] Besides their short-run labor endowment, I finally assume that workers are endowed with equal equity shares in the economy's firms, and receive a long-run goods endowment $y_2^e$ or $y_2^u$ which depends on their short-run employment status, capturing permanent income differences between the employed and unemployed.

**Policy, markets, and equilibrium.** I focus on five instruments available to policymakers: three instruments to intervene in the labor market, and two instruments to conduct monetary policy. In the labor market, they can assess a lump-sum tax $t$ and ad-valorem payroll tax $\tau$ on employed workers, and provide a lump-sum payment $b$ to unemployed workers. To conduct monetary policy, they can set the nominal interest rate $i$ between the short- and long-run, as well as directly choose the long-run price level $P_2$.[14,15] Beyond these instruments, I assume for convenience that policymakers can assess an ad-valorem tax on retailers $\tau^r$ and lump-sum tax on all agents $T^r$ which will only be used to offset the monopolistic competition distortions introduced by retailers. Throughout the paper, I will assume that $\tau^r = -\frac{1}{\varepsilon}$ and that $T^r$ is set to finance these subsidies, so that they play no stabilization role in my analysis.[16]

We can now characterize the structure of particular markets in this economy and, given the policy instruments above, the implied optimization problems faced by firms and workers. I first discuss wage determination in the labor market. As is well known, the presence of search frictions generates a continuum of wages which are bilaterally efficient. To resolve this indeterminacy, three wage determination processes are widely used in the

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[13] It is positive ($\psi(s) \geq 0$, strictly for $s > 0$), increasing ($\psi'(s) \geq 0$, strictly for $s > 0$) and convex ($\psi''(s) > 0$).

[14] Throughout the paper, I will denote all nominal objects in upper case, and real objects in lower case.

[15] Following Woodford (2003), I model the economy at the "cashless limit" where money serves as only a unit of account. The assumption that policymakers can directly control the long-run price level along with the nominal interest rate is needed to ensure price level determinacy in this finite horizon environment.

[16] Given the other tax instruments I have assumed, these do not change the set of implementable allocations. I assume them only to eliminate monopolistic competition in a transparent way consistent with the literature.

search and matching literature: Nash bargaining, wage posting, and rigid real wages.[17] My framework can accommodate all three cases, with no changes to the main results on UI in macroeconomic stabilization.[18]

For concreteness, I present here the case where the labor market is characterized by wage posting by producers.[19] Following Acemoglu and Shimer (1999), all producers anticipate a tightness schedule $\theta(W)$ defined over submarkets indexed by their prevailing wage. Since there is a representative producer and worker ex-ante, there will only be one prevailing wage rate in equilibrium; the schedule thus reflects a form of subgame perfection, in which all producers act optimally given shared off-path beliefs. In equilibrium, the schedule $\theta(W)$ will be defined to be that which makes workers indifferent to applying to alternative submarkets offering lower or higher wages.

Given this labor market structure, the representative producer faces

$$\Pi = \max_{v,W} P^I f(q(\theta(W))v - kv) - Wq(\theta(W))v \tag{1.3}$$

given a price of intermediate goods $P^I$.

I turn now to asset markets, where I assume agents cannot buy private insurance against their idiosyncratic risk of unemployment. Beyond that, markets are complete. In particular, uncertainty over macroeconomic aggregates — such as the structure of preferences or the value of long-run endowments $\{y_2^e, y_2^u\}$ — is resolved at date 1. In principle, agents can trade securities indexed to such aggregate sources of risk, but we can ignore this since there is a representative worker ex-ante. Ex-post, agents can trade a bond paying the riskless nominal rate $1+i$ at date 2.

[17]Hall and Krueger (2012) provide evidence on the prevalence of Nash bargaining and wage posting in the U.S. Shimer (2005) and Hall (2005) argue that real wage rigidity is needed to explain observed employment fluctuations.

[18]I discuss my results under these alternative assumptions on wages in more depth in Appendix A.3.

[19]In part, I lead with this case because it applies to arbitrary returns to scale in the production function $\alpha$. Nash bargaining is well-defined when there are constant returns to scale in production, but not when decreasing returns to scale generate a distinction between the marginal product of the "first" and "last" worker. Alternative bargaining protocols such as the one proposed by Stole and Zwiebel (1996) could be considered in that case.

Given this asset market structure, ex-post, worker $i \in \{e, u\}$ faces

$$v^i = \max_{\{c_1^i(j)\}, c_2^i, z_1^i} u^i(c_1^i, c_2^i) \text{ s.t.}$$
$$(RC)_1^i : \int_0^1 P_1(j) c_1^i(j) dj + P_1 z_1^i \leq Y_1^i, \tag{1.4}$$
$$(RC)_2^i : P_2 c_2^i \leq P_2 y_2^i + (1+i) P_1 z_1^i,$$

where $z_1^i$ is $i$'s net asset position in the short-run, and short-run disposable incomes are

$$Y_1^e = (1-\tau)W - P_1 t + (\Pi + \Pi^r - T^r),$$
$$Y_1^u = P_1 b + (\Pi + \Pi^r - T^r),$$

given aggregate retailer profits $\Pi^r \equiv \int_0^1 \Pi^r(j) dj$ defined below.[20] Ex-ante, the representative worker faces

$$v = \max_s (p(\theta)s) v^e + (1 - p(\theta)s) v^u - \psi(s) \tag{1.5}$$

given the specification of ex-ante utility in (1.1).

I now turn to the problem of retailers, which is more standard. Exploiting the standard solution to workers' lower-stage optimization problem in (1.4), retailer $j$ faces

$$\Pi^r(j) = \max_{P_1(j), y_1(j), x(j)} P_1(j) y_1(j) - (1+\tau^r) P^I x(j) \text{ s.t.}$$
$$(Tech)(j) : x(j) = y_1(j), \tag{1.6}$$
$$(Demand)(j) : y_1(j) = \left(\frac{P_1(j)}{P_1}\right)^{-\varepsilon} ((p(\theta)s) c_1^e + (1 - p(\theta)s) c_1^u).$$

where $y_1(j)$ and $x_1(j)$ are final goods produced and intermediate goods purchased by $j$, respectively.

I finally summarize market clearing and the government's budget constraint in this

[20] I have not yet defined the price index $P_1$; in equilibrium this will take on the standard CES form, as given in (1.13). Anticipating this, I use it here so I can directly work with the real net asset positions $z_1^i$ and level of UI $b$.

economy. Final goods market clearing at each date is given by

$$(p(\theta)s)c_1^e(j) + (1 - p(\theta)s)c_1^u(j) = y_1(j)\ \forall j, \tag{1.7}$$

$$(p(\theta)s)c_2^e + (1 - p(\theta)s)c_2^u = (p(\theta)s)y_2^e + (1 - p(\theta)s)y_2^u, \tag{1.8}$$

while in the short-run intermediate goods market clearing is given by

$$\int_0^1 x_1(j)dj = f(q(\theta)\nu - k\nu) \tag{1.9}$$

and bond market clearing is given by

$$(p(\theta)s)z_1^e + (1 - p(\theta)s)z_1^u = 0. \tag{1.10}$$

Assuming that the government separately balances its budgets for policy in the labor market and policy targeted at retailers, we lastly have

$$p(\theta)s\,[P_1 t + \tau W] = (1 - p(\theta)s)P_1 b, \tag{1.11}$$

$$T^r + \tau^r P^I \int_0^1 x(j)dj = 0. \tag{1.12}$$

We are now ready to define an equilibrium in the present environment.

**Definition 1.1.** *A* flexible price and wage equilibrium *is an allocation* $\{\{c_1^e(j)\}, z_1^e, c_2^e, \{c_1^u(j)\}, z_1^u, c_2^u, s, \nu, \{x(j)\}, \{y_1(j)\}\}$*, nominal prices, wages, and profits* $\{P^I, \{P_1(j)\}, P_2, W, \Pi, \{\Pi^r(j)\}\}$*, and tightness schedule* $\theta(W)$ *such that, given policy* $\{b, t, \tau; i, P_2; T^r, \tau^r\}$*:*

1. *producers solve (1.3);*

2. *workers solve (1.4) and (1.5);*

3. *retailers solve (1.6);*

4. $\theta(W)$ *is consistent with worker indifference across submarkets;*

5. *equilibrium tightness is consistent with worker and firm behavior (*$\theta = \frac{\nu}{s}$*);*

6. *goods and bond markets clear at each date according to (1.7)-(1.10); and*

7. *the government's budget is balanced according to (1.11) and (1.12).*

Assuming that the allocation is interior and that the first order conditions of agents' problems are necessary to characterize an optimum, Appendix A.2 characterizes optimality and worker indifference across submarkets in the flexible price and wage equilibrium.

### 1.2.2 Optimal policy in the flexible price and wage benchmark

With the economic environment in place, I now seek to characterize optimal government policy given flexible prices and wages. The key result is that optimal UI remains characterized by the classic partial equilibrium trade-off between insurance and incentives from public finance: a Baily (1978)-Chetty (2006) formula. This provides a useful benchmark against which I can study the role of UI in stabilization in the following subsections.

The approach I use to characterize optimal UI is a methodological contribution which can be used in the analysis of other social insurance programs in general equilibrium. Most prior work on optimal UI is in partial equilibrium, which allows the researcher to ignore the effects of UI on macroeconomic aggregates. In the present setting, we do not have this luxury: were we to optimize among the set of competitive equilibria with respect to $b$ and other instruments, the analysis would immediately become complex as wages $W$, profits $\Pi + \Pi^r$, and tightness $\theta$ respond to these changes.

Inspired by Farhi and Werning (2015)'s approach to the study of macroprudential regulation, the innovation is to solve an equivalent problem in which the planner's controls are instead the distribution of wealth across agents and tightness itself. This remains a dual approach to the Ramsey policy problem, but amounts to a reparameterization of the controls. It is distinct from the primal approach to Ramsey policy analysis, more common in macroeconomics, in which the researcher directly optimizes over the set of implementable allocations in terms of consumption and labor supply (see, e.g., Chari and Kehoe (1999)).

This has two advantages. First, it allows us to use the tools of price theory to relate optimal policy prescriptions to behavioral elasticities which are easily interpretable and measurable, despite the complexities of working in general equilibrium with many frictions.

This is particularly powerful once nominal rigidities and constraints on monetary policy are added to the environment, complicating the policy problem. Second, like the primal approach to Ramsey policy analysis, the resulting characterization of optimal policy can be implemented with a broader set of instruments than I have assumed so far, since these leave the set of implementable allocations unchanged. I discuss these implementation issues in more depth in Appendix A.3.

Formally, we make progress as follows. Standard two-stage budgeting given CES preferences over varieties implies a short-run price index

$$P_1 = \left[ \int_0^1 P_1(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \tag{1.13}$$

It follows that $\{c_1^i, c_2^i\}$ which solves the ex-post problem (1.4) for worker $i \in \{e, u\}$ solves the equivalent problem

$$\begin{aligned} v^i &= \max_{c_1^i, c_2^i} u^i(c_1^i, c_2^i) \text{ s.t.} \\ (RC)^i &: c_1^i + p_2 c_2^i = w_1^i \end{aligned} \tag{1.14}$$

where real wealth levels are given by

$$w_1^e = \left[ \frac{1}{P_1} \left( (1-\tau)W - P_1 t + (\Pi + \Pi^r - T^r) \right) \right] + p_2 \left[ y_2^e \right], \tag{1.15}$$

$$w_1^u = \left[ \frac{1}{P_1} \left( P_1 b + (\Pi + \Pi^r - T^r) \right) \right] + p_2 \left[ y_2^u \right], \tag{1.16}$$

and $p_2$, the relative price of long-run consumption in terms of short-run consumption, is simply the inverse of the real interest rate $1 + r$ defined using the standard Fisher equation:

$$p_2 \equiv \frac{1}{1+r}, \text{ where } 1 + r = \left( \frac{P_1}{P_2} \right) (1 + i). \tag{1.17}$$

With this reformulation, (1.14) generates standard Marshallian demand functions $c_1^i(w_1^i, p_2)$ and $c_2^i(w_1^i, p_2)$, and indirect utility $v^i(w_1^i, p_2)$. Then, the representative worker's ex-ante problem (1.5) can be written

$$v = \max_s (p(\theta)s) v^e(w_1^e, p_2) + (1 - p(\theta)s) v^u(w_1^u, p_2) - \psi(s), \tag{1.18}$$

generating a labor supply function $s(w_1^e, w_1^u, p_2, \theta)$.

With these reformulations of workers' problems in hand, the following result greatly simplifies the characterization of the Ramsey optimal allocation.

**Proposition 1.1.** *An allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$ ($\Leftrightarrow$ real interest rate $1 + r$) form part of a flexible price and wage equilibrium if and only if there exist wealth levels $\{w_1^e, w_1^u\}$ satisfying the economy-wide resource constraints*

$$(p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s), \tag{1.19}$$

$$(p(\theta)s)c_2^e + (1 - p(\theta)s)c_2^u = (p(\theta)s)y_2^e + (1 - p(\theta)s)y_2^u, \tag{1.20}$$

*given implementability constraints $c_1^e = c_1^e(w_1^e, p_2)$, $c_1^u = c_1^u(w_1^u, p_2)$, $c_2^e = c_2^e(w_1^e, p_2)$, $c_2^u = c_2^u(w_1^u, p_2)$, and $s = s(w_1^e, w_1^u, p_2, \theta)$ as defined in (1.14) and (1.18).*

Given a utilitarian social welfare function, the Ramsey planning problem can be now stated as

$$\begin{aligned} &\max_{w_1^e, w_1^u, \theta, p_2} (p(\theta)s(\cdot))v^e(w_1^e, p_2) + (1 - p(\theta)s(\cdot))v^u(w_1^u, p_2) - \psi(s(\cdot)) \text{ s.t.} \\ &(RC)_1 : (p(\theta)s(\cdot))c_1^e(\cdot) + (1 - p(\theta)s(\cdot))c_1^u(\cdot) = f(p(\theta)s(\cdot) - k\theta s(\cdot)), \\ &(RC)_2 : (p(\theta)s(\cdot))c_2^e(\cdot) + (1 - p(\theta)s(\cdot))c_2^u(\cdot) = (p(\theta)s(\cdot))y_2^e + (1 - p(\theta)s(\cdot))y_2^u, \end{aligned} \tag{1.21}$$

where I integrate the implementability constraints into the resource constraints, and suppress the dependence of the Marshallian demand and labor supply functions on controls, for brevity.

As is evident from this formulation of the problem, we can immediately use standard results from price theory to characterize agents' behavioral responses given changes in the controls. Using this approach, we can manipulate the first order conditions of (1.21) with respect to $w_1^e$, $w_1^u$, and $p_2$ to obtain the following key result:

**Proposition 1.2.** *Ramsey optimal risk-sharing is characterized by*

$$\frac{\frac{\partial v^e}{\partial w_1^e}}{\underbrace{1 - \frac{1}{p(\theta)s}\left(\frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}}_{\text{GE fiscal externality}}} = \frac{\frac{\partial v^u}{\partial w_1^u}}{\underbrace{1 - \frac{1}{1 - p(\theta)s}\left(\frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}}_{\text{GE fiscal externality}}} \tag{1.22}$$

*where the* production-inclusive excess supply functions $x_1$ *and* $x_2$ *are defined as functions of* $s$ *and* $\theta$, *holding consumption levels fixed:*

$$x_1(s,\theta) \equiv f(p(\theta)s - k\theta s) - [(p(\theta)s)c_1^e + (1-p(\theta)s)c_1^u]\Big|_{c_1^e, c_1^u}, \tag{1.23}$$

$$x_2(s,\theta) \equiv [(p(\theta)s)y_2^e + (1-p(\theta)s)y_2^u] - [(p(\theta)s)c_2^e + (1-p(\theta)s)c_2^u]\Big|_{c_2^e, c_2^u}. \tag{1.24}$$

In (1.22), the Ramsey planner would like to allocate wealth levels to equate agents' private marginal utilities of income $\frac{\partial v^e}{\partial w_1^e}$ and $\frac{\partial v^u}{\partial w_1^u}$, but is limited by moral hazard manifesting itself in a general equilibrium analog of a fiscal externality. In public finance, the fiscal externality reflects the impact of agents' behavioral responses to a policy change on the government's budget constraint. Here, the general equilibrium analog is the impact of agents' behavioral responses on the economy's resource constraints — more precisely, the production-inclusive excess supply functions $\{x_1, x_2\}$. If $\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s} > 0$, lower search reduces the net present value of resources in the economy, which is socially costly. Since transfers to the unemployed reduce incentives to search (that is, $\frac{\partial s}{\partial w_1^u} < 0$ and $\frac{\partial s}{\partial w_1^e} > 0$), the Ramsey planner provides incomplete insurance at the optimum.

As suggested by the connection to the standard fiscal externality in public finance, we can manipulate (1.22) to obtain a characterization of optimal UI consistent with the classic Baily (1978)-Chetty (2006) formula. A key step is to recognize that (1.23) and (1.24) in fact imply

$$\begin{aligned}\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s} &= p(\theta)\Bigg[\underbrace{f'(p(\theta)s - k\theta s)\left(1 - \frac{k}{q(\theta)}\right) + p_2(y_2^e - y_2^u)}_{\text{e-u diff in lifetime marginal product}} \\ &\qquad - \underbrace{[(c_1^e + p_2c_2^e) - (c_1^u + p_2c_2^u)]}_{\text{e-u diff in lifetime consumption}}\Bigg] \\ &\equiv p(\theta)\Big[\omega\Big],\end{aligned} \tag{1.25}$$

where I refer to $\omega$, defined at a particular allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$, as the *size of transfers*. The connection between $\omega$ and transfers arises because, in a competitive equilibrium, employed agents will be paid their marginal product. If the difference between employed and unemployed consumption is less than their difference in marginal product, this must be induced by a wealth transfer between these agents in equilibrium. This is formalized by the following lemma.

**Lemma 1.1.** *In a flexible price and wage equilibrium, a particular size of transfers $\omega$ (as defined in (1.25)) is implemented by UI benefits $b$ satisfying*

$$\omega = \frac{1}{p(\theta)s} b.$$

Combining Proposition 1.2 with Lemma 1.1, we obtain the following implementation result:

**Proposition 1.3.** *Ramsey optimal risk-sharing is implemented by UI benefits $b$ satisfying a general equilibrium version of the Baily-Chetty formula*

$$\underbrace{\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon_b^{P(unemp)}}_{Incentives} = \underbrace{\frac{\frac{\partial u^u}{\partial c_1^u} - \frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}}}_{Insurance}, \tag{1.26}$$

*where $\varepsilon_b^{P(unemp)}$, the micro elasticity of the probability of being unemployed at date 1 with respect to an increase in $b$, is given by*

$$\varepsilon_b^{P(unemp)} = -\left(\frac{p(\theta)s}{1-p(\theta)s}\right) \frac{\partial s}{\partial w_1^u} \frac{b}{s} > 0. \tag{1.27}$$

Hence, as in the classic partial equilibrium analysis, optimal UI trades off the private gains from consumption insurance against the (micro) disincentive effect of UI. Why does this partial equilibrium trade-off carry over to the general equilibrium case? Intuitively, we have another supply-side instrument — the payroll tax $\tau$ — which can be used to offset any general equilibrium effects of UI on firms' labor demand. Consistent with the intuition behind the Diamond and Mirrlees (1971) result on production efficiency, UI is then left to solve the partial equilibrium problem.

Two additional results on the Ramsey optimum are outlined in Appendix A.3. First, I characterize the Ramsey optimality condition with respect to tightness $\theta$, and show that the optimal payroll tax $\tau$ implements a *generalized Hosios condition* in the labor market. Hosios

(1990) demonstrates that a particular surplus-sharing rule among firms and workers will induce efficiency in models with search and matching frictions, as it induces both sides to internalize the search externalities they impose on others in the labor market. In the present setting, there are additional general equilibrium fiscal externalities from search behavior. I demonstrate that the resulting optimal surplus-sharing rule requires a worker surplus share less than that implied by the Hosios condition. In a competitive equilibrium characterized by wage posting, this is implemented by a positive payroll tax, or more precisely, a progressive income tax schedule.[21]

Second, I demonstrate that the optimal set of monetary policy instruments $\{i, P_2\}$ is indeterminate. This arises from a *real/nominal dichotomy* with flexible prices and wages, consistent with more standard models in monetary economics. The choice of the nominal rate and long-run price level serves to pin down the level of prices $P_1$ by the Fisher equation in (1.17), but this has no effect on the real allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$.

Two other sets of authors, Landais *et al.* (2015b,a) and Jung and Kuester (2015), have previously derived a Baily-Chetty formula for optimal UI in general equilibrium. The dual approach I have developed here differs from their solution approaches.[22] The difference matters in allowing me to transparently study the case with both nominal rigidities and heterogeneity in MPCs, which they do not consider — and to which I now turn.

### 1.2.3 Introducing nominal rigidities and inefficient fluctuations

I now motivate the study of UI in stabilization by adding two ingredients familiar from the monetary economics tradition: nominal rigidity and potential constraints on monetary policy, such as a zero lower bound on the nominal interest rate.

In particular, suppose that all retailers have posted prices in advance of date 1 at some

[21] To my knowledge, this result is novel. Lehmann and van der Linden (2007) demonstrate that a payroll tax may be needed alongside a UI benefit to implement the constrained efficient allocation, but do not sign it. Landais *et al.* (2015b) note that their "efficiency term" is related to the Hosios condition in ensuring efficiency in the labor market, but do not specify how surplus shares should precisely differ from the Hosios condition.

[22] Landais *et al.* (2015b,a) use a dual approach, but with a different parameterization of the controls. Jung and Kuester (2015) use the primal approach.

level:

$$P_1(j) = \bar{P}_1 \ \forall j \tag{1.28}$$

With identical pre-set prices, each retailer $j$ facing problem (1.6) will simply produce and accommodate desired demand, provided it can earn non-negative profits:

$$x(j) = y_1(j) = \begin{cases} (p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u \text{ provided } \bar{P}_1 \geq (1+\tau^r)P^I, \\ 0 \text{ otherwise.} \end{cases} \tag{1.29}$$

We thus obtain the following definition of equilibrium.

**Definition 1.2.** *A* fully sticky price equilibrium *is an allocation, set of nominal prices, wages, and profits, and tightness schedule such that, given policy, conditions 1-2 and 4-7 of Definition 1.1 are satisfied, and condition 3 is replaced by (1.28) and (1.29).*

In the usual way, nominal rigidities break the real/nominal dichotomy characterized in the last subsection and enable monetary policy to be used in macroeconomic stabilization. In particular, revisiting (1.17), the real interest rate can now directly be controlled by policymakers through their choice of $\{i, P_2\}$:

$$1 + r = \left(\frac{\bar{P}_1}{P_2}\right)(1+i). \tag{1.30}$$

But monetary policy may face constraints. I consider two constraints observed in practice:

$$\begin{cases} i \geq 0 \\ P_2 \leq \bar{P}_2 \end{cases} \tag{1.31}$$

First, the nominal interest rate is bounded below by zero, the assumed nominal rate of return on cash.[23] Second, the long-run price level is constrained above by $\bar{P}_2$, equivalent to a constraint on expected inflation in view of sticky short-run prices $\bar{P}_1$. This can be motivated as a political economy constraint on policymakers, or the result of (unmodeled) costs of inflation between the short- and long-run. It can also reflect the stickiness of inflation

[23] As argued by Eggertsson and Woodford (2004), this constraint holds even in an economy at the "cashless limit" such as the one under present study, provided that agents have the *option* of holding currency.

expectations among the public.

The constraints in (1.31) can be combined with the Fisher equation in (1.30) to generate a lower bound on the economy's real interest rate:

$$1 + r \geq \left(\frac{\bar{P}_1}{\bar{P}_2}\right). \tag{1.32}$$

This can become an important source of inefficiency, the consequences of which I trace out in the next subsection for the determination of optimal UI. It also motivates why the real interest rate may remain unchanged in response to a marginal change in UI generosity, generating results for the positive effects of UI in the following subsection which reverse the conclusions of prior work.

### 1.2.4 Normative role of UI: a *generalized Baily-Chetty* formula

I begin by revisiting the Ramsey optimal generosity of UI. The core result is a generalization of the Baily-Chetty formula when monetary policy is constrained, providing a precise characterization of the channels through which optimal UI should vary for macroeconomic stabilization purposes.

Characterizing optimal UI in this even more complex environment is again made tractable using the following equivalence result on the set of implementable allocations:

**Proposition 1.4.** *An allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$ ($\Leftrightarrow$ real interest rate $1+r$) form part of a fully sticky price equilibrium if and only if there exist wealth levels $\{w_1^e, w_1^u\}$ satisfying the economy-wide resource constraints*

$$(p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s),$$
$$(p(\theta)s)c_2^e + (1 - p(\theta)s)c_2^u = (p(\theta)s)y_2^e + (1 - p(\theta)s)y_2^u,$$

*and the ZLB implementability constraint*

$$p_2 \leq \bar{p}_2 \equiv \frac{\bar{P}_2}{\bar{P}_1}, \tag{1.33}$$

*given implementability constraints $c_1^e = c_1^e(w_1^e, p_2)$, $c_1^u = c_1^u(w_1^u, p_2)$, $c_2^e = c_2^e(w_1^e, p_2)$, $c_2^u =$*

*$c_2^u(w_1^u, p_2)$, and $s = s(w_1^e, w_1^u, p_2, \theta)$ as defined in (1.14) and (1.18).*

Intuitively, the ZLB implementability constraint reflects the fact that a lower bound on the real interest rate amounts to an upper bound on the relative price of long-run consumption in terms of short-run consumption. When the real interest rate is up against the bound, long-run consumption is cheap relative to short-run consumption, reducing aggregate demand and thus output in the short-run — in a sense I make more precise below.

Comparing Proposition 1.4 to Proposition 1.1, it follows that the planning problem is identical to that under flexible prices and wages, except with (1.33) as an additional constraint:

$$\begin{aligned} \max_{w_1^e, w_1^u, \theta, p_2} (p(\theta)s(\cdot))v^e(w_1^e, p_2) + (1 - p(\theta)s(\cdot))v^u(w_1^u, p_2) - \psi(s(\cdot)) \text{ s.t.} \\ (RC)_1 : (p(\theta)s(\cdot))c_1^e(\cdot) + (1 - p(\theta)s(\cdot))c_1^u(\cdot) = f(p(\theta)s(\cdot) - k\theta s(\cdot)), \\ (RC)_2 : (p(\theta)s(\cdot))c_2^e(\cdot) + (1 - p(\theta)s(\cdot))c_2^u(\cdot) = (p(\theta)s(\cdot))y_2^e + (1 - p(\theta)s(\cdot))y_2^u, \\ (ZLB) : p_2 \leq \bar{p}_2 \end{aligned} \tag{1.34}$$

The close relationship between (1.34) and (1.21) made apparent with this reformulation of the problem will allow us to tightly compare optimal UI to its benchmark level under flexible prices and wages.

I now characterize the solution to this planning problem separately for the cases where the zero lower bound does not, and does, bind at the optimum.

**Slack zero lower bound.** When the zero lower bound does not bind, planning problems (1.34) and (1.21) are identical. We immediately obtain the following result:

**Proposition 1.5.** *If the zero lower bound is slack, the Ramsey optimal allocation is identical to that under flexible prices and wages.*

The key implication of this result is that Ramsey optimal risk-sharing characterized in Proposition 1.2 does not change despite the presence of nominal rigidities. The rationale is that, while there is a new distortion in nominal rigidity, risk-sharing across employed and unemployed agents need not be affected, for policymakers have the right tool to offset the price distortion using monetary policy. Indeed, defining the *natural rate of interest* $1 + r^n$ to be the real interest rate in the Ramsey optimal allocation under flexible prices and wages, monetary policy can be used to target precisely this natural rate via the Fisher equation in (1.30).

The implementation of Ramsey optimal risk-sharing in the labor market is more subtle, however, as implied by the following lemma regarding the size of transfers $\omega$ as defined in (1.25).

**Lemma 1.2.** *In a fully sticky price equilibrium, a particular size of transfers $\omega$ is implemented by UI benefits $b$ satisfying*

$$\omega = \frac{1}{p(\theta)s} b + \underbrace{(\mu - 1)\, w}_{\substack{\text{Add'l transfer} \\ \text{from markup} \\ \text{variation}}},$$

*where $\mu \equiv \frac{\bar{P}_1}{P^I}$ is the gross retailer mark-up and $w \equiv \frac{W}{\bar{P}_1}$ is the real wage.*

Comparing this to the implementation of transfers $\omega$ in Lemma 1.1, we see that markup variation under sticky prices can generate an additional wealth transfer across employed and unemployed agents beyond that induced by government UI policy. The sign and direction of this transfer depends on the allocation of equity shares across agents. In the present environment, employed and unemployed agents have identical equity shares since they are ex-ante identical. Since only the employed have a claim on labor income, an increase in markups amounts to a reallocation of wealth from labor to profit income, and thus a transfer to the unemployed.

The behavior of mark-ups in the Ramsey optimum is indeterminate, however. As is standard in models with sticky prices, and as is discussed further in Appendix A.3, implementing the Ramsey optimal level of production in the economy pins down the after-tax real wage $(1 - \tau)\frac{W}{\bar{P}_1}$ which induces the necessary worker labor supply. However it is that the payroll tax $\tau$ is set, the pre-tax real wage $\frac{W}{\bar{P}_1}$ and thus the inverse gross mark-up $\frac{P^I}{\bar{P}_1}$ will adjust to induce the necessary labor supply to satisfy Ramsey optimal consumption demand.[24]

Given this indeterminacy, I assume that the payroll tax is used to keep markups unchanged from the flexible price and wage allocation (i.e., at $\mu = 1$). This allows me to focus attention on redistribution through UI alone, particularly since the redistributive role of mark-ups is likely small in practice.[25] Furthermore, any other choice would lead to costly price dispersion if some retailers could adjust their prices. We thus obtain the following implementation of the Ramsey optimum.

**Proposition 1.6.** *The Ramsey optimum is implemented by monetary policy* $\{i, P_2\}$ *targeting the natural rate of interest*

$$(1+i)\frac{\bar{P}_1}{P_2} = 1 + r^n,$$

*and, assuming the payroll tax* $\tau$ *is used to eliminate any markups* ($\mu = 1$)*, optimal UI benefits* $b$ *continuing to satisfy the general equilibrium Baily-Chetty formula*

$$\underbrace{\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon_b^{P(unemp)}}_{Incentives} = \underbrace{\frac{\frac{\partial u^u}{\partial c_1^u} - \frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}}}_{Insurance}.$$

Hence, I conclude that optimal UI may fluctuate in response to macroeconomic shocks, as these generically would cause the elasticities in the Baily-Chetty formula to vary — but it remains characterized by the simple trade-off between incentives and insurance. In this sense, UI is a *second-best* instrument in stabilization: it is *not* the first tool policymakers should turn to for macroeconomic stabilization purposes. I find a stabilization role for UI in the next case, when monetary policy is unable to stabilize the economy on its own.

**Binding zero lower bound.** In this case, let us first define the relative price wedge $\tau_{1,2}$.

[24]Notably, this means that the Ramsey optimal allocation can be achieved even if no payroll tax is available ($\tau = 0$). This illustrates a general principle true in other models of nominal rigidities (e.g., Farhi and Werning (2013)): policymakers can face a *relaxed* planning problem as compared to the problem they face with flexible prices and wages, given the additional margin of control they gain from monetary policy.

[25]The redistributive role of mark-ups will be small if the employed are also disproportionate owners of firm equity, as is true in practice.

**Definition 1.3.** *At the Ramsey optimum, the* relative price wedge

$$\tau_{1,2} \equiv 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}} p_2,$$

*where $\lambda_{RC1}$ and $\lambda_{RC2}$ are the multipliers on the resource constraints in planning problem (1.34).*

The relative price wedge captures the wedge between the social marginal rate of transformation across dates, given by the ratio of shadow prices $\frac{\lambda_{RC2}}{\lambda_{RC1}}$, and the marginal rate of substitution across dates perceived by private agents, given by the relative price $p_2$ (i.e., the real interest rate). At a first best allocation, these will be equalized — but with nominal rigidity and a zero lower bound, they may not, as formalized in the following result.

**Proposition 1.7.** *At the Ramsey optimum,*

$$\tau_{1,2} \propto \lambda_{ZLB},$$

*where $\lambda_{ZLB}$ is the multiplier on the zero lower bound in planning problem (1.34).*[26] *Hence,*

$$\tau_{1,2} \begin{cases} = 0 \text{ if } (ZLB) \text{ is slack,} \\ \geq 0 \text{ if } (ZLB) \text{ binds.} \end{cases}$$

As such, the relative price wedge emerges as a sufficient statistic for the distortion created by a binding zero lower bound at the Ramsey optimum. When the zero lower bound binds and the real interest rate is "too high", the relative price wedge makes this statement precise: the real interest rate is too high relative to the social marginal rate of transformation across dates. In this situation, the Ramsey planner would like private agents to reallocate consumption from the future to the present, but cannot induce them to do so in view of a binding zero lower bound. This role of a relative price wedge in summarizing the consequences of constraints on monetary policy is introduced in the theory of macroprudential regulation advanced by Farhi and Werning (2015); as I show here, it similarly motivates a stabilization role for UI.

[26] I characterize the exact relationship between $\tau_{1,2}$ and $\lambda_{ZLB}$ in the proof of this proposition.

In particular, a positive relative price wedge has two effects on the Ramsey optimal risk-sharing condition from the flexible price and wage benchmark:

**Proposition 1.8.** *Ramsey optimal risk-sharing is characterized by*

$$\frac{\frac{\partial v^e}{\partial w_1^e}}{1 \underbrace{-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}_{\text{AD ext.}} - \frac{1}{p(\theta)s}\Big(\underbrace{(1-\tau_{1,2})\frac{\partial x_1}{\partial s}}_{\substack{\Delta\text{ cost of}\\ \text{fiscal ext.}}} + p_2\frac{\partial x_2}{\partial s}\Big)\frac{\partial s}{\partial w_1^e}} = \frac{\frac{\partial v^u}{\partial w_1^u}}{1 \underbrace{-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}}_{\text{AD ext.}} - \frac{1}{1-p(\theta)s}\Big(\underbrace{(1-\tau_{1,2})\frac{\partial x_1}{\partial s}}_{\substack{\Delta\text{ cost of}\\ \text{fiscal ext.}}} + p_2\frac{\partial x_2}{\partial s}\Big)\frac{\partial s}{\partial w_1^u}} \tag{1.35}$$

*where the production-inclusive excess supply functions $x_1$ and $x_2$ are as defined in (1.23) and (1.24).*

These two effects on Ramsey optimal risk-sharing directly imply two corresponding changes in the formula for optimal UI, as formalized in the following implementation result:

**Proposition 1.9.** *The Ramsey optimum is implemented by monetary policy $\{i, P_2\}$ up against its constraints*

$$i = 0, P_2 = \bar{P}_2$$

*and, assuming the payroll tax $\tau$ is used to eliminate any markups ($\mu = 1$), optimal UI benefits $b$ satisfying a* generalized Baily-Chetty formula

$$\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon_b^{P(unemp)} \underbrace{\left(1-\tau_{1,2}\left(1-\frac{\partial c_1^e}{\partial w_1^e}-\frac{z_1^u}{b}\right)\right)}_{\Delta\text{ cost of disincentives}} = \frac{\frac{\partial u^u}{\partial c_1^u}-\frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}} + \underbrace{\tau_{1,2}\left(\frac{\partial c_1^u}{\partial w_1^u}-\frac{\partial c_1^e}{\partial w_1^e}\right)}_{\text{AD externality}}, \tag{1.36}$$

*for $\tau_{1,2}$ small, where $-\frac{z_1^u}{b}$ is the debt / UI ratio for unemployed agents in the short-run.*[27]

First, in (1.35) a novel aggregate demand externality distorts optimal risk-sharing across the employed and unemployed. Given short-run macroeconomic slackness summarized by a positive relative price wedge $\tau_{1,2}$, there is a difference between the social and private

[27] To ease interpretation of the formula, I take a first order Taylor approximation of both sides around $\tau_{1,2} = 0$. The exact formula for general $\tau_{1,2}$ is presented in the proof of this proposition.

valuation of income arising from an agent's propensity to spend, and thus help stabilize the economy, in the short-run.

In (1.36), the aggregate demand externality means that the social value of UI differs from its private insurance value in the standard Baily-Chetty formula. The wedge is governed by the difference in MPCs between the unemployed and employed.[28] When this difference is positive, the social value of transfers to the unemployed through the UI system exceeds the private value of consumption insurance. This distinction between the private and social valuation of transfers in the presence of slackness is an application of Farhi and Werning (2015)'s core insight, developed in the context of macroprudential regulation, to the present framework focused on social insurance.

Second, in (1.35) short-run macroeconomic slackness changes the welfare cost of the general equilibrium fiscal externality. This further distorts optimal risk-sharing across the employed and unemployed. Consider the case when $\frac{\partial x_1}{\partial s} > 0$. This means that reduced search, and thus employment, tends to reduce the supply of resources by more in the short-run than it does the demand on those resources. The more depressed is short-run demand, as summarized by a higher relative price wedge $\tau_{1,2}$, the less socially costly is the reduction in production induced by reduced search.

In (1.36), this changes the social cost of disincentives in the Baily-Chetty formula. Slackness unambiguously reduces the disincentive cost of UI when, as is empirically realistic, the unemployed are net borrowers in the short-run, which ensures that $\frac{\partial x_1}{\partial s} > 0$.[29] This reduction in the social cost of disincentives shares intuition with the "rat race" effect of Landais *et al.* (2015b,a), who argue that inefficiently low tightness reduces the cost of

[28]In the present environment, this difference in marginal propensities to consume arises from differences in the utility functions $u^e(c_1^e, c_2^e)$ and $u^u(c_1^u, c_2^u)$, to illustrate these ideas in the simplest way. In a robustness exercise in section 1.2.6, I demonstrate that this formula continues to hold when agents are hand-to-mouth, generating more realistic sources of MPC variation. And in my infinite horizon analysis studied numerically in section 1.3, precautionary behavior generates differences in MPCs which are sizeable and drive the aggregate demand effects of UI.

[29]When the unemployed are net debtors, it must be that the difference in the level of employed and unemployed consumption is smaller than their difference in labor income, and thus marginal product. In this case, incrementally lower employment will reduce the economy's supply of resources by more than it does the demand on those resources in the short-run — that is, $\frac{\partial x_1}{\partial s} > 0$.

disincentivizing labor supply and thus motivates optimally higher UI in a general matching framework. The present setting formalizes why the economy can find itself in a situation of inefficiently low tightness, owing to nominal rigidity, a binding zero lower bound, and thus low aggregate demand.

Taken together, in the empirically realistic case, a two-way interaction between UI and aggregate demand motivates *optimally higher* UI benefits than the Baily-Chetty benchmark when the economy is slack. On the one hand, UI has a novel macroeconomic stabilization role to play in view of a positive aggregate demand externality from transfers to unemployed. On the other hand, macroeconomic slackness itself reduces the social cost of disincentives due to the provision of UI.

In Appendix A.3, I complete the description of the Ramsey optimum by characterizing the optimality condition with respect to tightness $\theta$. I use this to demonstrate that the relative price wedge $\tau_{1,2}$ is itself revealed in a deviation of the *labor wedge* (as studied by Chari *et al.* (2007), Shimer (2009), and many others) from its natural level.[30] This formalizes the intuition that a binding zero lower bound, reflected in a positive relative price wedge, leads to inefficient employment in the competitive equilibrium.

### 1.2.5 Positive impact of UI: the *UI multiplier*

I now move beyond welfare and evaluate how higher UI generosity affects output and employment. I define the *UI multiplier* to be the marginal impact of an increase in unemployed agents' income on output, as an analog to the standard government spending multiplier. With fully sticky prices, the core channel is an effect of redistribution on aggregate demand, which indeed forms the basis for the aggregate demand externality identified in the normative analysis. A more subtle result is that supply-side parameters also matter, however, contrary to the standard Keynesian intuition.

[30] By natural level, I mean the labor wedge at the Ramsey optimum with flexible prices and wages. As I show in Appendix A.3, the natural level is not zero, owing to the presence of search frictions and heterogeneity (induced by incomplete markets and moral hazard) which distinguish my environment from the neoclassical benchmark.

**Notation and policy of interest.** I first introduce new notation which simplifies the exposition of these positive results. Define the short-run real disposable income of agents

$$y_1^e \equiv \frac{1}{P_1}\left((1-\tau)W - P_1 t + (\Pi + \Pi^r - T^r)\right),$$
$$y_1^u \equiv \frac{1}{P_1}\left(P_1 b + (\Pi + \Pi^r - T^r)\right), \tag{1.37}$$

so that each agent's real wealth levels defined in (1.15) and (1.16) can be summarized as

$$w_1^e = y_1^e + p_2 y_2^e,$$
$$w_1^u = y_1^u + p_2 y_2^u.$$

In the sticky price equilibrium, the income of unemployed agents $y_1^u$ in (1.37) simplifies to

$$y_1^u = b + (1 - \mu^{-1}\alpha) f(p(\theta)s - k\theta s) \tag{1.38}$$

where, recall, $\alpha$ is the return to scale in production and $\mu \equiv \frac{\bar{P}_1}{P^I}$ is the retailer mark-up.

In this section, I will treat the income of unemployed agents $y_1^u$ rather than UI benefits $b$ as the primary policy instrument of interest. As is evident from (1.38), $y_1^u$ differs from $b$ due to unemployed agents' claim on firm equity. In response to variation in UI benefits $b$, there will be changes in firm profits which further change the level and distribution of agents' income, generating "second-round feedback" effects which are sensitive to the fact that all agents hold symmetric equity shares in this short-run/long-run environment. To avoid complicating the formulas based on such second-round effects, and since in practice unemployed workers likely hold smaller equity shares than employed workers, I instead directly treat the disposable income $y_1^u$ as the policy instrument of interest. One interpretation of the resulting multipliers is that they correspond to an increase in $b$, assuming that the payroll tax $\tau$ is used to keep markups unchanged at $\mu = 1$ (as I assume in section 2.3) and that the economy's profit share is zero ($\alpha = 1$). In that case, (1.38) implies $y_1^u = b$.

Now let employment and output in the short-run be $n_1$ and $y_1$, respectively:

$$n_1 \equiv p(\theta)s, \tag{1.39}$$

$$y_1 \equiv f(p(\theta)s - k\theta s). \tag{1.40}$$

The goal of this section will be to characterize the impact of changes in unemployed workers' income $y_1^u$ and monetary policy $\{i, P_2\}$ on these short-run aggregates.

**Three aggregate relations.** I derive the marginal impact on employment and the UI multiplier by characterizing the micro-level responses to changes in policy, and then aggregating up to general equilibrium. After doing so, I find that three aggregate relations jointly summarize the response of output $y_1$, employment $n_1$ and search $s_1$ to policy:

$$\text{Aggregate demand: } \frac{dy_1}{y_1} = \mu^{AD}_{y_1^u}\frac{dy_1^u}{y_1^u} - \mu^{AD}_{p_2}\left(di - \frac{dP_2}{P_2}\right) + \mu^{AD}_{n}\frac{dn_1}{n_1}, \tag{1.41}$$

$$\text{Technology: } \frac{dy_1}{y_1} = \mu^{tech}_{n_1}\frac{dn_1}{n_1} + \mu^{tech}_{s}\frac{ds}{s}, \tag{1.42}$$

$$\text{Labor supply: } \frac{ds}{s} = -\mu^{LS}_{y_1^u}\frac{dy_1^u}{y_1^u} + \mu^{LS}_{p_2}\left(di - \frac{dP_2}{P_2}\right). \tag{1.43}$$

In Appendix A.4, I formally derive these relations. Here, I discuss in detail four of the coefficients which are most important to build intuition: $\{\mu^{AD}_{y_1^u}, \mu^{AD}_{p_2}, \mu^{tech}_{s}, \mu^{LS}_{y_1^u}\}$. I then combine these relations to characterize the effects on employment and output.

*Aggregate demand.* The coefficient $\mu^{AD}_{y_1^u}$ captures the *redistribution effect on aggregate demand*:

$$\mu^{AD}_{y_1^u} \equiv \underbrace{\frac{1}{1-\frac{\partial c_1^e}{\partial w_1^e}}}_{\substack{\text{Keynesian}\\ \text{cross}}} \left[ \underbrace{\frac{(1-n_1)y_1^u}{y_1}}_{\substack{\text{Rel. importance}\\ \text{of unemp. to AD}}} \underbrace{\left(\frac{\partial c_1^u}{\partial w_1^u} - \frac{\partial c_1^e}{\partial w_1^e}\right)}_{\text{Diff in MPCs}}\right].$$

This is the key coefficient of interest. It means that a marginal increase in the income of the unemployed affects output according to the difference in MPCs between the unemployed and employed. This effect underlies the aggregate demand externality in the normative analysis. In terms of affecting output, it is scaled up by the relative fraction of aggregate income accruing to the unemployed, which has the intuitive implication that transfers to the unemployed will have greater stimulus in times of high unemployment. It is further scaled by $(1 - \frac{\partial c_1^e}{\partial w_1^e})^{-1}$, reflecting a standard Keynesian cross, wherein higher output raises disposable income, demand, and thus output, and so on.

A coefficient of secondary interest is $\mu_{p_2}^{AD}$, which captures the aggregate demand stimulus from monetary policy:

$$\mu_{p_2}^{AD} \equiv \underbrace{\frac{1}{1-\frac{\partial c_1^e}{\partial w_1^e}}}_{\substack{\text{Keynesian}\\ \text{cross}}} \left[\frac{1}{y_1} \underbrace{\left(n_1\left(\frac{\partial c_1^{e,h}}{\partial p_2} - \frac{\partial c_1^e}{\partial w_1^e}\frac{z_1^e}{p_2}\right) + (1-n_1)\left(\frac{\partial c_1^{u,h}}{\partial p_2} - \frac{\partial c_1^u}{\partial w_1^u}\frac{z_1^u}{p_2}\right)\right)}_{\text{Weighted avg. interest rate sensitivities for each agent}}\right].$$

Each agent's interest rate sensitivity is governed by a substitution effect, which depends on the compensated demand sensitivity $\frac{\partial c_1^{i,h}}{\partial p_2}$ for Hicksian demand $c_1^{i,h}(v^i(w_1, p_2), p_2)$, as well as an income-cum-wealth effect, which depends on a given agent's net asset position $z^i$. The latter reflects the redistribution channel of monetary policy, as in Auclert (2015)'s analysis. The economy's interest rate sensitivity is intuitively governed by a weighted average of these effects across agents. In times of high unemployment, economy-wide sensitivity will thus tilt towards that of unemployed agents.

These aggregate demand effects do *not* fully describe the equilibrium effects on output, however, owing to the presence of employment $n_1$ in the relation. This term, not present in representative agent models, captures the effect of extensive margin changes in employment on the level of aggregate demand since employed and unemployed agents consume different amounts. We thus exploit the technological relationship between output and employment to make further progress.

*Technology.* Output $y_1$, employment $n_1$, and search effort $s$ are related by (1.39) and (1.40), which implies (1.42). In particular, the effect of search on production (holding fixed employment) is

$$\mu_s^{tech} \equiv \alpha\left(\frac{(\frac{1}{\eta}-1)\frac{k}{q(\theta)}}{1-\frac{k}{q(\theta)}}\right),$$

where $\alpha$ is the return to scale in production, $\eta - 1$ is the elasticity of the vacancy-filling rate with respect to tightness, and $\frac{k}{q(\theta)}$ summarizes recruiting costs per hire. Given positive recruiting costs $\frac{k}{q(\theta)}$, a decrease in search effort $s$ will thus reduce equilibrium output.[31] Intuitively, a reduction in search acts like a decrease in the productivity of labor, as firms need to engage in more, costly recruiting to hire the same number of workers.[32] Contrary to the standard Keynesian intuition, the labor supply response to policy thus matters even in this demand-determined world.

*Labor supply.* The general equilibrium behavior of labor supply could potentially be quite complex, owing to the endogenous response of the wealth distribution to changes in policy. However, in Appendix A.4 I show that the labor supply response is considerably simplified when considering a local change around the flexible price and wage Ramsey optimum. In that case

$$\mu^{LS}_{y_1^u} \equiv \frac{1-n_1}{(n_1)^2}\left(\frac{y_1^u}{b}\right)\varepsilon_b^{P(unemp)},$$

where $\varepsilon_b^{P(unemp)}$, the micro-elasticity of the probability of unemployment with respect to an increase in UI, is defined in (1.27). Hence, the micro-elasticity tightly characterizes the general equilibrium response of search to changes in income $y_1^u$, similar to its sufficiency in the normative Baily-Chetty formula. A rise in unemployed agents' income $y_1^u$ unambiguously reduces equilibrium search.

**UI multiplier and discussion.** We now can characterize the UI multiplier:

**Proposition 1.10.** *Around the flexible price and wage Ramsey optimum, the marginal effects of*

[31] This requires $1-\frac{k}{q(\theta)}>0$, which must be true in a well-defined equilibrium since $1-\frac{k}{q(\theta)}=\frac{1}{p(\theta)s}(p(\theta)s-k\theta s)$.

[32] This effect shares intuition with models in the "labor hoarding" literature, in which firm productivity changes with labor utilization. The difference here is that productivity-augmenting effort of workers occurs on the extensive margin, rather than the intensive margin.

*policy on employment and output are given by*

$$\frac{dn_1}{n_1} = -\mu^{n_1}_{p_2}\left(di - \frac{dP_2}{P_2}\right) + \mu^{n_1}_{y^u_1}\frac{dy^u_1}{y^u_1},$$
$$\frac{dy_1}{y_1} = -\mu^{y_1}_{p_2}\left(di - \frac{dP_2}{P_2}\right) + \mu^{y_1}_{y^u_1}\frac{dy^u_1}{y^u_1},$$

*where*

$$\mu^{n_1}_{y^u_1} = \underbrace{\mu^{n_1}_{p_2}}_{\substack{\text{Effect of}\\ \text{1\% cut in } i\\ \text{on } n_1}} \underbrace{\left(\frac{\mu^{AD}_{y^u_1}+\mu^{tech}_s\mu^{LS}_{y^u_1}}{\mu^{AD}_{p_2}+\mu^{tech}_s\mu^{LS}_{p_2}}\right)}_{\substack{\text{Rel. stimulus of}\\ y^u_1 \text{ vs. } i}}, \tag{1.44}$$

*and the* UI multiplier *is given by*

$$\mu^{y_1}_{y^u_1} = \underbrace{\mu^{tech}_{n_1}\mu^{n_1}_{y^u_1}}_{\substack{\text{Employment}\\ \text{stimulus}}} - \underbrace{\mu^{tech}_s\mu^{LS}_{y^u_1}}_{\substack{\text{Loss from}\\ \text{recruiting}}}. \tag{1.45}$$

*for aggregate demand coefficients in (1.41), technological coefficients in (1.42), and labor supply coefficients in (1.43).*

We can understand these results in two steps. First consider the case when hiring costs per hire $\frac{k}{q(\theta)}$ are small, which implies $\mu^{tech}_s$ is small and also can be used to show $\mu^{tech}_{n_1} \approx \alpha$.[33] If the labor supply elasticity $\varepsilon^{P(unemp)}_b$ is not too big, we have $\mu^{tech}_s\mu^{LS}_{y^u_1} \approx 0$ and can also show $\mu^{tech}_s\mu^{LS}_{p_2} \approx 0$. In that case, the UI multiplier in (1.45) becomes

$$\mu^{y_1}_{y^u_1} \approx \alpha\mu^{n_1}_{y^u_1} \approx \alpha\mu^{n_1}_{p_2}\left(\frac{\mu^{AD}_{y^u_1}}{\mu^{AD}_{p_2}}\right) = \underbrace{\alpha\mu^{n_1}_{p_2}}_{\substack{\text{Effect of}\\ \text{1\% i cut}\\ \text{on } y_1}} \underbrace{\left(\frac{(1-n_1)y^u_1\left(\frac{\partial c^u_1}{\partial w^u_1} - \frac{\partial c^e_1}{\partial w^e_1}\right)}{n_1\left(\frac{\partial c^{e,h}_1}{\partial p_2} - \frac{\partial c^e_1}{\partial w^e_1}\frac{z^e_1}{p_2}\right) + (1-n_1)\left(\frac{\partial c^{u,h}_1}{\partial p_2} - \frac{\partial c^u_1}{\partial w^u_1}\frac{z^u_1}{p_2}\right)}\right)}_{\text{Relative AD stimulus from redistribution}}.$$

Hence, the redistribution effect on aggregate demand $\mu^{AD}_{y^u_1}$ is *approximately sufficient* to characterize the multiplier on output. Normalizing by the effect of monetary stimulus is a convenient way to collect general equilibrium effects (the Keynesian cross and the effect of

[33] If we take this short-run/long-run model literally, $\frac{k}{q(\theta)} = \frac{k\nu}{q(\theta)\nu} = \frac{k\nu}{p(\theta)e}$ is the ratio of recruiters to employment. Citing the 1997 National Employer Survey, Landais *et al.* (2015a) report this at 2.5%, while the calibration of Shimer (2010) implies this is 0.5%. Of course, the absence of any incumbent workers makes this mapping inappropriate — and, indeed, is an important reason why I move to an infinite horizon model when I consider the quantitative application in section 1.3 — but it suggests that small hiring costs are an important benchmark case of interest.

extensive margin changes in employment on the level of demand) so that we can focus on the core mechanism.

Why do supply-side elasticities of UI emphasized in prior work, such as a disincentive effect or wage elasticity, not appear in this approximate formula? First, the disincentive effect of UI on search effort — which indeed survives in general equilibrium, as implied by the labor supply relation (1.43) — is swamped by a general equilibrium response of labor demand to the change in aggregate demand. Since firms post more vacancies to offset any reduction in search among workers, measuring only the disincentive effect of UI will miss its aggregate effects. Second, the effect on wages is irrelevant, conditional on overall disposable income for each agent. While changes in wages may affect the source of income earned by agents (labor vs. profit), the overall level of disposable income is what matters for aggregate demand.[34] Aggregate demand, in turn, drives labor demand for standard Keynesian reasons, with no role played by wages.

At the same time, the UI multiplier is *not* completely determined by the redistribution effect on aggregate demand for positive recruiting costs $\frac{k}{q(\theta)}$. In (1.45), the general equilibrium reduction in search (through $\mu^{LS}_{y^u_1}$) raises recruiting costs and thus reduces output (through $\mu^{tech}_s$). This reveals a subtle difference from the standard Keynesian intuition: while firm labor demand will endogenously offset a reduction in worker search effort due to higher UI, this is not a costless one-for-one switch. In a frictional labor market, the increase in recruiting costs tends to reduce output.

Indeed, recruiting costs rise sharply as tightness rises, and thus the vacancy-filling rate collapses:

$$\frac{k}{q(\theta)} \to \infty \text{ as } \theta \to \infty.$$

This underscores the limits of the earlier approximation result for small hiring costs. It suggests why, at the extreme, complete insurance need not maximize output even in a

[34]The wage response may change the distribution of disposable income across unemployed and employed agents through a redistributive role of markup variation. This depends on the allocation of equity shares across agents. As noted in footnote 25, this redistributive role of markups is likely to be small in practice. As discussed earlier, to avoid focusing on this effect, I treat the disposable income of the unemployed as the policy instrument of interest.

demand-determined world. As such, it provides intuition for why disincentives still matter in my generalized Baily-Chetty formula for optimal UI.

### 1.2.6 Robustness of the formulas

The preceding subsections characterized two key results: optimal UI in the presence of inefficient fluctuations, and the effect of a marginal increase in unemployed workers' income on equilibrium output. In this subsection, I demonstrate that these results are robust to a number of alternative settings: sticky wages rather than prices; hand-to-mouth rather than unconstrained unemployed agents; and a small open economy facing a fixed exchange rate. Here, I provide intuition on what changes from the baseline case. The formal analysis of each case is in Appendix A.5.

#### Sticky wages

With sticky wages, producers' optimal vacancy posting condition in (1.3) and retailers' optimal price-setting in (1.6) leads to labor demand relation

$$f'(p(\theta)s - k\theta s)\left(1 - \frac{k}{q(\theta)}\right) = \frac{\bar{W}}{P_1}. \tag{1.46}$$

The left-hand side is the marginal product of labor, accounting for hiring costs. The right-hand side is the real wage. Re-arranging to solve for $P_1$ and combining with the Fisher equation in (1.17), we find that the real interest rate in this economy is

$$\begin{aligned} 1 + r &= \left( \frac{\bar{W} \Big/ \left( f'(p(\theta)s - k\theta s)\left(1 - \frac{k}{q(\theta)}\right)\right)}{P_2} \right)(1 + i) \\ &\geq \left( \frac{\bar{W} \Big/ \left( f'(p(\theta)s - k\theta s)\left(1 - \frac{k}{q(\theta)}\right)\right)}{\bar{P}_2} \right), \end{aligned} \tag{1.47}$$

where the inequality accounts for the constraints on monetary policy. Unlike the sticky price case, the real interest rate is not fully determined by monetary policy: a reduction in the marginal product of labor (accounting for hiring costs) raises producers' real marginal cost and drives up the price level, raising the real interest rate. For the same reason, the constraint on the real interest rate posed by the zero lower bound is now endogenous.

Normatively, this leads to an additional change in the disincentive cost of UI when monetary policy is constrained. If production is characterized by decreasing returns to scale ($\alpha < 1$), the disincentive effect of UI raises the marginal product of labor by reducing employment, which *relaxes* the binding constraint on the real interest rate in (1.47). This effect further pushes towards higher optimal UI relative to the Baily-Chetty benchmark.

Positively, this generates scope for the supply-side effects of UI to partially choke off aggregate demand. In particular, the economy can be characterized by an upward-sloping aggregate supply curve relating employment and the price level, as is standard in models with sticky wages. *Conditional* on employment, a reduction in search effort amounts to an upward shift in the aggregate supply curve, as it raises recruiting costs and thus firm marginal costs. For small hiring frictions, this effect may again be second-order relative to the aggregate demand impact of transfers; but it again highlights why disincentives continue to matter in the determination of optimal UI.

#### Hand-to-mouth unemployed

Let us return to the sticky price case, but now suppose unemployed agents are characterized by hand-to-mouth behavior owing to a loss of access to credit markets. In this case, their optimization problem (1.14) is replaced by

$$\begin{aligned} v^u &= \max_{c_1^u, c_2^u} u^u(c_1^u, c_2^u) \text{ s.t.} \\ (RC)^u &: c_1^u + p_2 c_2^u = w_1^u, \\ (HTM)^u &: c_2^u = y_2^u. \end{aligned} \tag{1.48}$$

where the $(HTM)^u$ constraint is new. This generates indirect utility $v^u(w_1^u, p_2, y_2^u)$ and consumption demands $c_1^u(w_1^u, p_2, y_2^u), c_2^u(w_1^u, p_2, y_2^u)$. It leads to three economic implications in equilibrium:

$$\frac{\partial c_1^u}{\partial w_1^u} = 1, \tag{1.49}$$

$$\frac{\partial c_1^u}{\partial p_2} = 0, \tag{1.50}$$

$$z_1^e = z_1^u = 0. \tag{1.51}$$

The first two reflect the fact that without access to credit markets, the consumption of the unemployed perfectly tracks short-run income, but is fully insensitive to the real interest rate. The last is an implication of equilibrium in the asset market: since the unemployed cannot access credit markets, the net asset position of the employed must be zero.

The resulting formulas for optimal UI and the UI multiplier are unchanged from the benchmark case, except with (1.49)-(1.51) plugged in everywhere. The hand-to-mouth behavior of the unemployed makes the aggregate demand externality and relative stimulus from redistribution unequivocally positive. Now it is unambiguous that in the presence of a binding zero lower bound, optimal UI exceeds the Baily-Chetty benchmark.

**Small open economy with a fixed exchange rate**

I now consider a very different environment: a small open economy (SOE) of the sort studied by Obstfeld and Rogoff (2000), Schmitt-Grohe and Uribe (2012), and Farhi and Werning (2014), with applications to peripheral European economies in the recent Eurozone downturn. In a static setting, this SOE has two classes of goods: non-traded goods produced by a domestic sector, and traded goods which are treated as endowments. There is nominal wage rigidity in the non-traded sector coupled with a fixed exchange rate and a price of traded goods (in foreign currency) which is fixed on world markets. This combination prevents adjustment in the relative price of non-tradeables, which can lead to real distortions in the presence of macroeconomic shocks.

Unlike the above authors, I enrich the production of non-traded goods with search and matching frictions, giving rise to involuntary unemployment.[35] I further assume the presence of a government-run UI scheme, owing to the absence of private markets to insure against unemployment risk, but limited by moral hazard. The formal characterization of the economic environment and equilibrium is provided in Appendix A.5.3. In this context, does UI have any macroeconomic stabilization role to play when the SOE is buffeted by macroeconomic shocks which cannot be addressed by domestic monetary policy, owing to the fixed exchange rate?

It does — and despite the considerable differences from the benchmark model, in fact *all* of the earlier results apply with a simple relabeling. Short-run and long-run consumption are replaced by non-traded and traded consumption. The relative price wedge which motivates a macroeconomic stabilization role for UI concerns the relative price of non-traded and traded goods, rather than the relative price of consumption across dates (the real interest rate). And the aggregate demand effects of UI are governed by the difference in marginal propensities to spend on domestic non-tradeables, rather than on short-run consumption.[36]

**Summing up.** Across the extensions considered here, the core results of my benchmark model regarding the normative and positive role of UI in stabilization remain robust.

First, in the presence of nominal rigidities (either in prices or wages) and constraints on monetary policy (due to a zero lower bound or fixed exchange rate), optimal UI departs from the classic insurance-incentives trade-off familiar from public finance. An aggregate demand externality gives UI a role to play in macroeconomic stabilization, while macroeconomic slackness itself changes the relative cost of disincentives. These effects become sharply signed when unemployed agents are hand-to-mouth, unambiguously motivating higher optimal UI generosity relative to the Baily-Chetty benchmark in the presence of macroeconomic

[35] Schmitt-Grohe and Uribe (2012) provide an alternative way of capturing unemployment in such a SOE, assuming a rationing rule with downward nominal wage rigidity in an otherwise Walrasian labor market.

[36] The one difference from the closed economy at the zero lower bound is that there may be a *negative* relative price wedge. The same conditions on sufficient statistics which motivate optimally higher generosity when the relative price of non-traded goods is "too high" (and the economy is depressed) also motivate optimally lower generosity when the relative price of non-traded goods is "too low" (and the economy is overheated).

slackness.

Second, under the same nominal rigidities, a marginal increase in the income of the unemployed affects employment and output through the redistribution effect on aggregate demand. This is governed by the difference in MPCs between the unemployed and employed, and underlies the aggregate demand externality in the normative analysis. The disincentive effect of UI matters through its effect on recruiting costs; while this effect may be small, its presence suggests why disincentives still matter in the formula for optimal UI in a demand-determined world.

## 1.3 Quantitative application: U.S. during the Great Recession

I turn now to an analysis of the benefit extensions in the U.S. during the Great Recession — an expansion of generosity unprecedented in the history of the UI program. From a base of 26 weeks of benefit duration in almost all states, benefits were substantially extended from August 2008 through December 2013, reaching 99 weeks in some states at the depth of the recession. What effect did these extensions have on equilibrium employment and welfare?

To study these policies, I generalize the model from the theoretical analysis to an infinite horizon setting, which allows me to study the effects of policies affecting the duration of benefits in a long-lasting recession. An important implication of the calibrated model in steady-state is that the MPC of unemployed agents rises sharply with duration of unemployment. This means that in a Bewley-Huggett-Aiyagari (BHA) framework with Diamond-Mortensen-Pissarides (DMP) labor market dynamics calibrated to match salient features of the U.S. economy, the long-term unemployed are an extremely promising "tag" for high MPCs.

My simulation of transitional dynamics suggests that the benefit extensions had quantitatively large, positive effects on aggregate demand. Relative to a less generous path of benefits capped at 9 months of duration in the calibrated model, the observed extensions prevent a further rise in the unemployment rate of 2–5 percentage points and generate a strict Pareto improvement. Consistent with the theoretical analysis, these effects appear driven by the high MPCs of the long-term unemployed, governing the initial stimulus from transfers. In an infinite horizon environment, the initial stimulus is further amplified by the dynamic response of precautionary saving.

### 1.3.1 Overview of 2008–2013 benefit extensions

I first provide background on the 2008–2013 benefit extensions before moving forward with the quantitative analysis.

Almost all states had maximum benefit durations of 26 weeks prior to the Great Recession.[37] Other aspects of the programs, such as eligibility for UI benefits, the replacement rate (fraction of pre-unemployment wages paid in benefits), and funding structures, differed more substantially across states (United States Department of Labor (2008)).

Over the period 2008–2013, two programs together led to a dramatic expansion in benefit durations across the country: the Extended Benefits program (EB), and the Emergency Unemployment Compensation Act of 2008 (EUC08). EB, which has been in place since 1970, automatically extends the duration of benefits in a particular state when its unemployment rate rises above a particular threshold. Over 2009-2012, EB led to an increase in benefit durations of up to 20 weeks in some states. EUC08, which was signed into federal law on June 30, 2008, substantially extended these benefit durations specifically in response to the Great Recession. A first tier of benefits provided 13 — and then, starting in November 2008, 20 — additional weeks of benefits to workers in all states. A further three tiers provided up to 33 additional weeks of benefits for states with high unemployment rates. In total, EUC08 led to an increase in benefit durations of up to 53 weeks.

Taken together, benefit durations were extended up to 99 weeks in some states at the depth of the Great Recession. Figure 1.1 illustrates the median benefit duration across all U.S. states over time.[38] For visual comparison, the figure extends backwards to incorporate

[37]Two states offered higher potential durations: Montana (28 weeks) and Massachussetts (30 weeks).

[38]This is the median across all U.S. states as reported by Farber and Valetta (2013), weighting each state by its population as measured by BLS' monthly estimates of the civilian noninstitutional population. I thank John Coglianese for providing me with this data.

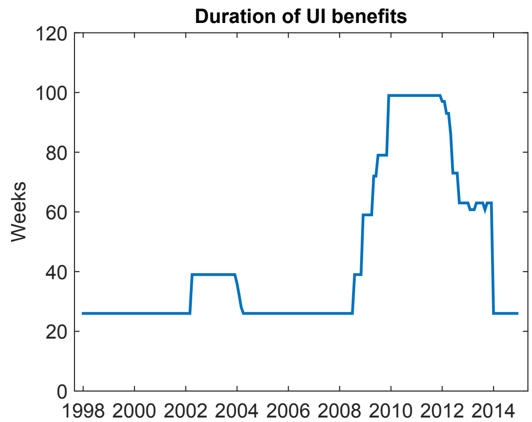



**Figure 1.1:** *Median potential duration of benefits across U.S. states*

the UI response to the 2001-02 recession. As is suggested by the figure, and confirmed by examination of the historical record extending back to the creation of the UI program during the Great Depression, the benefit extensions over 2008–2013 marked an unprecedented expansion of generosity.

### 1.3.2 Economic environment

I now enrich the environment from section 1.2.1 to enable a more realistic evaluation of the 2008-13 benefit extensions. The primary change is the move to an infinite horizon, which allows me to more fully explore the rich interplay between incomplete markets (à la BHA) and search and matching (à la DMP) in determining the effects of changes in finite duration UI policy. In addition, I add three other elements which allow me to obtain a more realistic calibration of the U.S. economy: (i) duration-dependent match efficiency, (ii) an explicit borrowing constraint, and (iii) sources of non-own-labor income (such as spousal income in practice) which cushion the effects of job loss.

In this dynamic framework, a given period can be broken down into four phases: search, matching, consumption and production, and separation. I characterize the search and matching processes in more detail below. Exogenous separations, standard in such models, occur at the end of the period, and ensure that there is a positive unemployment rate in steady-state.

The economic actors remain the same as before: measure one each of workers, intermediate-good producers engaged in recruiting and production, and retailers who can accommodate demand-driven output if prices are sticky. I revisit their optimization problems, and then the policy and general equilibrium structure, before defining a flexible price equilibrium in this setting.

**Agent optimization revisited.** There is no longer a representative ex-ante worker; instead, workers at the beginning of each period are heterogeneous on three dimensions: the real value of assets ($z$), employment status ($e$ or $u$), and the number of prior periods the worker has been unemployed in the current spell ($d$) if she is unemployed. These, correspondingly, become state variables in a recursive representation of workers' problem. At the beginning of period $t$, employed and unemployed workers respectively face

$$\tilde{v}_t^e(z_t) = v_t^e(z_t), \tag{1.52}$$

$$\tilde{v}_t^u(z_t, d_t) = \max_{s_t} (p(\theta_t; d)s_t) v_t^e(z_t) + (1 - p(\theta_t; d)s_t) v_t^u(z_t, d_t) - \psi(s_t), \tag{1.53}$$

where $v_t^e(z)$ and $v_t^u(z, d)$ are value functions for workers in the *middle* of the period, and are conditional on employment status in the *middle* of period $t$. Hence, at the beginning of the period, employed workers are already secured in their employment, while unemployed workers need to search, facing job-finding probability per unit effort $p(\theta; d)$ which I characterize in more detail below.[39] These middle-of-period value functions in turn solve

$$\begin{aligned} v_t^e(z_t) = \max_{\{c_{tj}^e\}, z_{t+1}^e} u(c_t^e) + \beta_t \left[(1 - \delta_t)\tilde{v}_{t+1}^e(z_{t+1}^e) + \delta_t \tilde{v}_{t+1}^u(z_{t+1}^e, 0)\right] \text{ s.t.} \\ (RC)_t^e(z_t) : \int_0^1 P_t(j) c_{tj}^e dj + M_t(P_{t+1} z_{t+1}^e) \leq Y_t^e + P_t z_t, \\ (BC)_t^e(z_t) : z_{t+1}^e \geq \underline{z}_t, \end{aligned} \tag{1.54}$$

[39] Note that this formulation of the problem already embeds an implicit assumption that incumbent workers will prefer employment to unemployment (i.e., will not voluntarily quit). Since I assume there is no fixed disutility from employment, this will of course be true in equilibrium, so for brevity I directly assume it here.

and

$$
\begin{aligned}
v_t^u(z_t, d_t) = \max_{\{c_{tj}^u\}, z_{t+1}^u} u(c_t^u) + \beta_t \tilde{v}_{t+1}^u(z_{t+1}^u, d_t + 1) \text{ s.t.} \\
(RC)_t^u(z_t, d_t) : \int_0^1 P_t(j) c_{tj}^u dj + M_t(P_{t+1} z_{t+1}^u) \leq Y_t^u(d_t) + P_t z_t, \\
(BC)_t^u(z_t, d_t) : z_{t+1}^u \geq \underline{z}_t,
\end{aligned} \tag{1.55}
$$

where $c_t^e$ and $c_t^u$ are CES aggregators over varieties as defined in (1.2), but I have now assumed that flow utility is time-separable and identical across agents.[40] Agents can continue to trade (only) a one-period riskless nominal bond at price $M_t \equiv \frac{1}{1+i_t}$ — but now, their assets are bounded below owing to a real borrowing constraint of $\underline{z}_t$ in period $t$. I defer a specification of agents' per-period incomes $Y_t^e$ and $Y_t^u(d)$ until I discuss government policy below.

Intermediate good producers face a richer dynamic problem in view of the stock of incumbent workers they carry forward from one period to the next, and imperfect substitutability of incumbent and new workers owing to hiring costs. I specialize here to the case with exogenous real wages within the per-period bargaining set between workers and firms. In the present environment with heterogeneity across workers owing to incomplete markets, this greatly simplifies the characterization of producer optimization (and equilibrium) relative to the alternatives.[41] At the same time, building on the point of Hall (2005), it remains consistent with bilateral efficiency in an environment without commitment to long-run wage contracts. Denoting the exogenous real wage $w_t$, the representative producer with $n_t$ incumbent workers then faces

$$
\begin{aligned}
J_t(n_t) = \max_{\nu_t} P_t^I f_t(n_t + q(\theta_t)\nu_t - k\nu_t) - P_t w_t(n_t + q(\theta_t)\nu_t) + M_t J_{t+1}(n_{t+1}) \text{ s.t.} \\
(Evolution)_t(n_t) : n_{t+1} = (1 - \delta_t)(n_t + q(\theta_t)\nu_t),
\end{aligned} \tag{1.56}
$$

[40] I now index consumption of a particular variety $j$ as a subscript, to avoid confusion with the state variables in parenthesis which identify a particular agent.

[41] Under Nash bargaining between workers and firms, heterogeneity across workers gives them heterogeneous outside options, generating an equilibrium wage schedule as characterized in Krusell *et al.* (2010). Under competitive search, workers are further heterogeneous in their attitudes towards to risk, leading to sorting across a continuum of submarkets indexed by differing degrees of labor market tightness, as pointed out by Acemoglu and Shimer (1999).

where $q(\theta_t)$ is the vacancy-filling rate as before, and $v_t$ is the number of vacancies posted. I assume that producers discount future profits at the nominal discount factor $M_t$.[42]

Finally, consider retailers: since they still operate with a pure pass-through technology and can update prices each period, they continue to simply face a sequence of static problems. In particular, retailer $j$ faces

$$\begin{aligned}\Pi^r_{tj} = \max_{P_{tj}, y_{tj}, x_{tj}} P_{tj} y_{tj} - (1+\tau^r) P^I_t x_{tj} \text{ s.t.}& \\ (Tech)_{tj} : x_{tj} = y_{tj},& \\ (Demand)_{tj} : y_{tj} = \left(\frac{P_{tj}}{P_t}\right)^{-\varepsilon} c_t.&\end{aligned} \tag{1.57}$$

Relative to (1.6), the only change beyond notation is that aggregate consumption $c_t$ will account for the rich heterogeneity across agents in employment status, duration of unemployment, and asset holdings in the present setting.

**Policy and general equilibrium revisited.** I begin with labor market policy. I enrich the modeling of UI to account for duration-dependent UI benefits, as observed in practice. In particular, at time $t$, I assume a generic schedule of real benefits $b_t(d)$, financed by real lump-sum taxes on employed agents $t_t$. I no longer model additional intervention via payroll taxes.[43]

Given the rich heterogeneity across workers in employment status, duration of unemployment, and assets in this enriched framework, we need new notation to capture the distribution of workers across the idiosyncratic state space. At the beginning of period $t$, I let $\tilde{\varphi}^e_t(z)$ and $\tilde{\varphi}^u_t(z,d)$ denote the conditional distribution of employed and unemployed agents across assets and duration of unemployment, respectively, and $\tilde{\lambda}^e_t$ denote the fraction

[42] The proper specification of the firm's objective in such incomplete markets environments is not obvious, as agents' imperfectly insured idiosyncratic risk means that they will value future profit streams differently (see, e.g., Geanakoplos *et al.* (1990)). I make the present assumption for simplicity.

[43] As noted by Chetty (2006), the payroll taxes which finance the UI program in the U.S. are assessed on an earnings base which renders them inframarginal for most workers. As a result, I model such taxes as lump-sum via the tax $t_t$, and leave the analysis of other, income-contingent taxes to future work. The lump-sum tax financing UI remains distortionary through its effect on labor supply, owing to incomplete markets.

of employed agents. I let $\{\varphi_t^e(z), \varphi_t^u(z,d), \lambda_t^e\}$ denote the analogs in the middle of period $t$. Given this distribution of workers, government budget balance in its UI program at date $t$ is given by

$$\lambda_t^e t_t = (1-\lambda_t^e)\left[\sum_{d=0}^{\infty}\left(\int_z \varphi_t^u(z,d)dz\right) b_t(d)\right]. \tag{1.58}$$

Beyond government policy in the labor market, I assume instruments for monetary policy and policy targeted at retailers which are analogous to those in the short-run/long-run case. Monetary policy in this dynamic environment can be specified with a nominal interest rate rule in the usual way (see, e.g., Woodford (2003)). An invariant tax on retailers $\tau^r = -\frac{1}{\varepsilon}$ coupled with lump-sum financing $T_t^r$ on all agents continues to be a passive instrument which offsets the distortion from monopolistic competition. Government budget balance in policy targeted at retailers is then

$$T_t^r + \tau^r P_t^I \int_0^1 x_{tj} dj = 0. \tag{1.59}$$

In view of the above government policy, I can now characterize agents' per-period incomes $Y_t^e$ and $Y_t^u(d)$. I add an ingredient not present in the short-run/long-run model: a uniform, non-random endowment $\omega_t$ earned by all agents, capturing spousal and other household income as in the data. Then, agents' incomes are

$$\begin{aligned} Y_t^e &= P_t w_t - P_t t_t + (\Pi_t + \Pi_t^r - T_t^r) + P_t \omega_t, \\ Y_t^u(d) &= P_t b_t(d) + (\Pi_t + \Pi_t^r - T_t^r) + P_t \omega_t. \end{aligned}$$

where $\Pi_t$ is the per-period profit earned by the representative producer in (1.56),

$$\Pi_t = P_t^I f_t(n_t + q(\theta_t)v_t - kv_t) - P_t w_t(n_t + q(\theta_t)v_t),$$

and $\Pi_t^r$ aggregates over each retailer's profit as defined in (1.57).[44]

[44]I assume for simplicity that agents cannot trade equity securities, resulting in an invariant, uniform distribution of equity shares across the population. An alternative approach is to allow agents to trade these securities. While in principle this would require an additional state variable in equity holdings, I could follow Krusell *et al.* (2010) and take advantage of a no-arbitrage equilibrium relationship linking the nominal interest rate to the nominal return on equity shares. This would allow me to model asset holdings $z$ as bonds plus equity. Since I calibrate the model with constant returns to scale in production in section 1.3.3, profit income in

I now complete the general equilibrium structure of this economy by detailing the matching process and market clearing conditions. Matching differs from the short-run/long-run model owing to the assumption that match efficiencies are duration-specific. This feature of the matching process allows me to accommodate the possibility of structural duration dependence in job-finding rates, as would be implied by employer screening and as is suggested by recent resume audit studies (Ghayad (2013), Kroft *et al.* (2013), and Eriksson and Rooth (2014)). When I study the UI extensions shortly, this feature also allows me to accommodate the possibility of potential hysteresis effects of extended joblessness and extended benefits (see, e.g., Ball (2009)). I assume in particular a general job-finding probability per unit effort for unemployed agents at duration $d$ of

$$p(\theta_t; d) = (\bar{m}(0))^{1-\eta} \frac{\bar{m}(d)}{\bar{m}(0)} \theta_t^{\eta}, \tag{1.60}$$

where tightness $\theta_t \equiv \frac{v_t}{\bar{s}_t}$ and aggregate (weighted) search

$$\bar{s}_t \equiv (1 - \tilde{\lambda}_t^e) \sum_{d=0}^{\infty} \frac{\bar{m}(d)}{\bar{m}(0)} \int_z s_t(z,d) \tilde{\varphi}_t^u(z,d) dz. \tag{1.61}$$

In this formulation, the parameters $\{\bar{m}(1), \bar{m}(2), \ldots\}$ control the relative efficiency of matching, as (1.60) implies

$$\frac{p(\theta_t; d'')}{p(\theta_t; d')} = \frac{\bar{m}(d'')}{\bar{m}(d')},$$

while $\bar{m}(0)$ controls the overall level of match efficiency in the economy. Indeed, the aggregate number of matches implied by the above formulation is

$$m(\bar{s}_t, \bar{v}_t) = (\bar{m}(0))^{1-\eta} \bar{s}_t^{1-\eta} \bar{v}_t^{\eta},$$

and the vacancy-filling probability facing firms is given by

$$q(\theta_t) = (\bar{m}(0))^{1-\eta} \theta_t^{\eta - 1}.$$

When $\bar{m}(d) = \bar{m}$ for all $d$, this matching process collapses to that in the short-run/long-run

---

the calibrated economy is roughly proportional to hiring costs and is very small. I thus expect the approaches to yield virtually identical simulation results.

model.

Lastly, final goods and intermediate goods market clearing at date $t$ are given by

$$\lambda_t^e \left( \int_z c_{tj}^e(z) \varphi_t^e(z) dz \right) + (1 - \lambda_t^e) \left( \sum_{d=0}^{\infty} \int_z c_{tj}^u(z,d) \varphi_t^u(z,d) dz \right) = y_{tj} + \omega_t \ \forall j, \tag{1.62}$$

$$\int_0^1 x_{tj} dj = f_t(n_t + q(\theta_t)\nu_t - k\nu_t), \tag{1.63}$$

respectively. And bond market clearing at date $t$ is given by

$$\lambda_t^e \left( \int_z z_{t+1}^e(z) \varphi_t^e(z) dz \right) + (1 - \lambda_t^e) \left( \sum_{d=0}^{\infty} \int_z z_{t+1}^u(z,d) \varphi_t^u(z,d) dz \right) = 0. \tag{1.64}$$

We are now ready to characterize a flexible price equilibrium in the present dynamic environment with search frictions and heterogeneous agents.

**Definition 1.4.** *A* flexible price equilibrium *is a sequence of*

- *worker value functions* $\{\tilde{v}_t^e, \tilde{v}_t^u, v_t^e, v_t^u\}$ *and policies* $\{s_t, \{c_{tj}^e\}, z_{t+1}^e, \{c_{tj}^u\}, z_{t+1}^u\}$;
- *representative producer value functions* $J_t$ *and policies* $\nu_t$;
- *retailer profit functions* $\Pi_{tj}^r$ *and policies* $\{P_{tj}, y_{tj}, x_{tj}\}$;
- *labor market tightness* $\theta_t$, *employment* $n_t$, *and nominal prices and profits* $\{P_t^I, \Pi_t\}$;
- *and probability measures characterized by* $\{\tilde{\lambda}_t^e, \tilde{\varphi}_t^e, \tilde{\varphi}_t^u, \lambda_t^e, \varphi_t^e, \varphi_t^u\}$

*such that, given policy* $\{b_t(d), t_t; i_t; \tau^r, T_t^r\}$ *and exogenous real wages* $\{w_t\}$:

1. *workers solve (1.52)-(1.55);*
2. *producers solve (1.56);*
3. *retailers solve (1.57);*
4. *tightness is consistent with worker and firm behavior* ($\theta_t = \frac{\nu_t}{\bar{s}_t}$, *given* $\bar{s}_t$ *in (1.61));*
5. *goods and bond markets clear at each date according to (1.62)-(1.64);*
6. *the government's budget is balanced according to (1.58) and (1.59);*

7. *and the probability measures characterized by* $\{\tilde{\lambda}^e_t, \tilde{\varphi}^e_t, \tilde{\varphi}^u_t, \lambda^e_t, \varphi^e_t, \varphi^u_t\}$ *are consistent with labor market clearing* $\tilde{\lambda}^e_0 = n_0$ *at date 0, and consistent with the above policies and stochastic elements of the model for all future dates.*

### 1.3.3 Calibration and properties of the stationary RCE

I will assume that the economy starts in a *stationary recursive competitive equilibrium (RCE)*, wherein macroeconomic policy and aggregates are constant. In this subsection, I numerically solve and characterize the stationary RCE as calibrated to match salient features of the U.S. economy prior to the onset of the Great Recession, with technical details provided in the Computational Appendix. The key feature of the stationary RCE is sharply rising MPCs by duration of unemployment — an endogenous outcome, not a calibrated target.

#### Calibrating the stationary RCE

The definition of a stationary RCE is standard and identical to Definition 1.4, without time subscripts.

I first specify functional forms and certain assumed parameter values for use in the calibration, as summarized in Table 1.1. I assume isoelastic forms for search costs and flow utility from consumption for workers. Consistent with the approach and empirical evidence of Kroft *et al.* (forthcoming), I assume that match efficiencies by unemployment duration through the first 8 months of unemployment are related by an exponential function, and are flat thereafter. The elasticity of job-finding with respect to tightness is taken from the central estimate of Petrongolo and Pissarides (2001). For simplicity, I assume constant returns to scale in production for intermediate good firms. Finally, I assume an exogenous separation rate as calculated by Chodorow-Reich and Wieland (2015).

I assume a stepwise UI benefit schedule which is consistent with regular UI benefits in the U.S. Through the first 6 months of an unemployment spell, agents receive a UI benefit which is 50% of the prevailing wage rate in the economy. After that point, agents have exhausted their UI benefits, but receive other social assistance (welfare, food stamps, etc.)

**Table 1.1:** *Functional forms and assumed parameters*

| **Side of economy** | **Functional form** | **Assumed parameters** |
|---|---|---|
| Search | $\psi(s) = s^{\xi+1}$ | |
| Matching | $p(\theta; d) = (\bar{m}(0))^{1-\eta} \frac{\bar{m}(d)}{\bar{m}(0)} \theta^{\eta}$, $\bar{m}(d) = \begin{cases} \bar{m}(0)\exp(-\lambda d) \text{ for } d < 8 \\ \bar{m}(7) \text{ for } d \geq 8 \end{cases}$ | $\eta = 0.7$ (PP [2001]) |
| Consumption | $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ | $\sigma = 4$ |
| Production | $f(n + q(\theta)v - kv) = a(n + q(\theta)v - kv)^{\alpha}$ | $a = 0.5, \alpha = 1$ |
| Separation | exogenous at rate $\delta$ | $\delta = 0.066$ (CRW [2015]) |
| UI & social assistance | $b(d) = \begin{cases} b^{UI} \text{ for } d < 6 \\ b^{SA} \text{ for } d \geq 6 \end{cases}$ | $b^{UI} = 0.5w$ $b^{SA} = 0.1w$ (RV [2014]) |

which I assume to be 10% of the wage rate.[45] In practice of course UI and social assistance are distinct programs; for parsimony, I model them here together, since they represent fungible uses of the fiscal authority's tax revenue.

I then solve for the stationary RCE at a monthly frequency, calibrating the remaining parameters to target six macroeconomic aggregates characterizing the U.S. economy before the Great Recession, and two behavioral elasticities highlighted as important by the earlier theoretical analysis. The targeted and simulated moments are summarized in Table 1.2. I furthermore include the value of the economic parameter which I primarily vary in order to target the given moment.[46]

Among macroeconomic aggregates, I use the discount factor $\beta$ to target a real interest rate of 2%, characterizing the steady-state of the U.S. economy prior to 2008. Consistent

[45]The UI replacement rate is consistent with the replacement rate for the average worker as reported by the Department of Labor's Employment and Training Administration, after accounting for additional savings in taxes. The assumption for social assistance is set to be roughly consistent with the fall in household income upon UI exhaustion reported by Rothstein and Valletta (2014) in Table 3. In practice social assistance is not indexed to wages, but I simply present it this way for ease of interpretation; it has no effects on my simulation results.

[46]In the usual way, changing any one parameter affects all of the moments in question, so the mapping is not one-to-one. Nonetheless, this mapping provides a useful guide to the underlying parameters which vary most when I pursue sensitivity analysis in the next section.

**Table 1.2:** *Targeted moments and calibration results*

| Moment | Target | Achieved | Parameter | Value |
|---|---|---|---|---|
| *Macro aggregates* | | | | |
| Real interest rate (ann.) | 2% | 1.9% | $\beta$ | 0.996 |
| Unemployment rate | 5.5% | 5.3% | $w/a$ | 0.97 |
| Long-term unemp. rate ($d \geq 6$) | 1% | 1.0% | $\lambda$ | 0.23 |
| Conventional market tightness | 0.634 (HM [2008]) | 0.634 | $\bar{m}(0)$ | 0.57 |
| Initial HH income of unemp. / emp. | 0.75 (RV [2014]) | 0.75 | $\omega/a$ | 0.91 |
| Recruiting / employment | 0.025 (LMS [2015]) | 0.025 | $k$ | 0.32 |
| *Behavioral elasticities* | | | | |
| Duration elast. to benefit duration | 0.1 (KM [2002]) | 0.10 | $\xi$ | 3.8 |
| Avg unemp. MPC - avg emp. MPC | 0.06 | 0.059 | $\underline{z}/a$ | 4.6 |

with the long-run patterns presented in Kroft *et al.* (forthcoming), I target an unemployment rate of 5.5% and long-term unemployment rate of 1%, where the latter captures those who are unemployed for at least 6 months (roughly 26 weeks, in practice). I use the exogenous real wage $w$ to target the former through its impact on firm labor demand, and use the decay of match efficiencies by duration $\lambda$ to target the latter. Importantly, I find that match efficiencies must fall through an unemployment spell to rationalize the observed fraction of long-term unemployed in the U.S. economy, consistent with empirical evidence on negative duration dependence in job-finding rates.[47] Defining conventional market tightness to be the ratio of aggregate vacancies and aggregate unemployment at the beginning of each period, I use the level of match efficiency $\bar{m}(0)$ to target conventional tightness of 0.634 as reported by Hagedorn and Manovskii (2008).[48] Recognizing that the income of households — at which level consumption decisions are made — does not fall as sharply as individual

[47]In an alternative calibration where I eliminate duration dependence in match efficiencies ($\lambda = 0$) and use the remaining parameters to target the remaining moments, the calibrated model implies a long-term unemployed rate of 0.3%. Intuitively, agents search considerably harder as they proceed through an unemployment spell, leading to (counterfactual) positive duration dependence in job-finding. I discuss this and other features of labor market dynamics in the calibrated stationary RCE in Appendix A.6.

[48]This is distinct from the concept of tightness $\theta$ which matters for job-finding and vacancy-filling in the model. $\theta$ accounts for the search effort and match-efficiencies of the unemployed, and is thus unobservable in the data.

income upon job loss, I use the exogenous endowment $\omega$ earned by all agents to target an initial drop in household income of 25%, consistent with the evidence of Rothstein and Valletta (2014). Finally, I use the hiring costs on the firm side, $k$, to target a recruiting to employment ratio of 2.5% as reported by Landais *et al.* (2015a).

In terms of behavioral elasticities, I focus on targeting the key supply- and demand-side statistics which the theoretical analysis in section 1.2 suggests are important. On the supply-side, the micro-level elasticity of job-finding with respect to an increase in UI generosity was the key parameter in the general equilibrium Baily-Chetty, and generalized Baily-Chetty, formulas. In the present setting, as in the real world, UI generosity involves both the level (replacement rate) and duration of benefits. Since the duration of benefits is the policy instrument which varied during the Great Recession, I focus on targeting the observed elasticity of unemployment duration to potential duration of benefits, reported by Krueger and Meyer (2002) to be 0.1. I use the elasticity of workers' disutility to marginal increases in search, $\xi$, to target this moment.

On the demand-side, the difference in MPCs between the unemployed and employed was the key statistic driving the aggregate demand externality in the generalized Baily-Chetty formula, and the redistribution effect in the UI multiplier. There is not much evidence on heterogeneous MPCs by employment status. To make progress, I use the borrowing limit $\underline{z}$ to target a difference in average MPCs between the unemployed and employed of 0.06 monthly, or 0.18 quarterly. A difference in quarterly MPCs of 0.18 is roughly halfway between the point estimates of Johnson *et al.* (2006) and Parker *et al.* (2013) concerning the difference in non-durable MPCs between medium- and low-income households, where I proxy for the unemployed with low-income.[49] Given the considerable uncertainty regarding the true value of the difference in MPCs between the unemployed and employed, I consider a range of alternative targets in sensitivity analysis in the next section.

In Appendix A.9, I pursue an alternative approach of calibrating the borrowing limit

[49]Observed income is an imperfect proxy for employment status; differences in the former are likely driven by differences in permanent income, whereas job loss is primarily a loss in temporary income.

$\underline{z}$ to match other moments describing the consumption behavior of the unemployed (and about which we have more evidence), and then leaning on the model to ask what MPCs are *implied* by the calibration. First, I target estimates of the path of consumption levels upon unemployment (Gruber (1997), Chodorow-Reich and Karabarbounis (forthcoming), and Ganong and Noel (2015)). Second, I target an estimate of the micro-elasticity of the consumption drop upon unemployment to the UI replacement rate (Browning and Crossley (2001)). Finally, I target an estimate of the elasticity of unemployment duration to severance payments, scaled by the elasticity of unemployment duration to UI duration (Card *et al.* (2007)). The resulting calibrations imply differences in MPCs, and thus transitional dynamics, consistent with the range I study in the following analysis.

**Characterizing the stationary RCE**

**MPCs through an unemployment spell.** The consumption dynamics of the calibrated stationary RCE *endogenously* generate sharply rising MPCs as agents proceed through an unemployment spell. Coupled with the insights of the theoretical analysis from prior sections, this previews why I find large, positive aggregate demand effects of the UI benefit extensions in the next section.

Rising MPCs through an unemployment spell are an outcome of the consumption policy functions for agents as a function of their assets, coupled with the evolution of agents' asset holdings through an unemployment spell. Figure 1.2 illustrates these features of the stationary RCE by comparing the consumption policy functions and marginal distributions over asset holdings for employed agents and unemployed agents in their first month of unemployment ($d = 0$), in their fourth month ($d = 3$), and just after the expiration of unemployment benefits ($d = 6$).

First, at a given level of wealth, the consumption policy functions imply a generally rising MPC the longer an agent remains unemployed. In the presence of negative structural duration dependence in job-finding rates, an unemployed worker sees her conditional probability of finding a job fall as she proceeds through unemployment. This implies greater

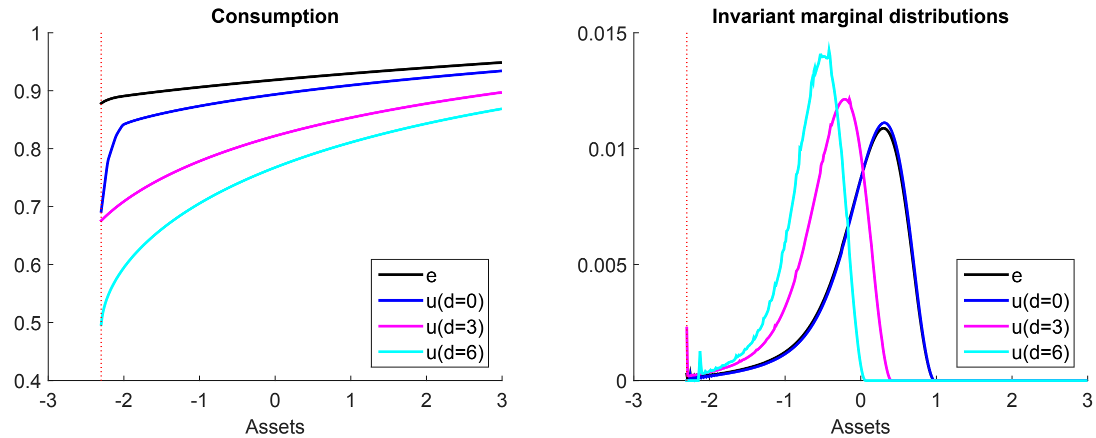



**Figure 1.2:** *Consumption policy functions and invariant marginal distributions*

downside risk, and means that incentives to precautionary save rise. Holding fixed wealth, this tends to depress the level of consumption, but raises the sensitivity of consumption to cash-on-hand.[50]

Second, the marginal distributions of agents reveal that they optimally decumulate assets through an unemployment spell, pushing them deeper into the region of the state space with higher MPCs. Intuitively, an unemployment spell corresponds to a loss in temporary income relative to permanent income. Agents thus tend to borrow against future income while unemployed, despite the precautionary concerns highlighted above.

Together, these forces mean that in a BHA framework with DMP labor market dynamics calibrated to match the incidence and duration of unemployment in the U.S. economy, the long-term unemployed are an extremely promising "tag" for high MPCs. Table 1.3 summarizes the 1-month and implied 3-month MPCs out of unexpected, transitory income for unemployed agents by duration, as compared to those of employed agents. The model implies that relative to the MPC of the employed, the MPC of the long-term unemployed

[50]One region where this does not hold is at the very lowest levels of wealth, when comparing unemployed agents in their first month of unemployment ($d = 0$) with those a few months into it ($d = 3$). This has to do with the level of job-finding rates and income. At low levels of wealth, unemployed agents at $d = 0$ are optimally up against the borrowing constraint since they expect to exit unemployment fairly soon, and thus borrow considerably against future income. As an unemployment spell drags on, however, job-finding rates become low, and benefit exhaustion draws near. Hence, unemployed agents at $d = 3$ no longer borrow so aggressively, so in the lowest region of wealth their MPC will be lower than that of agents at $d = 0$.

**Table 1.3:** *MPCs of unemployed by duration, less MPC of employed*

| **(Avg unemp. MPC by dur.) - (Avg emp. MPC)** | **1-month** | **3-month** |
|---|---|---|
| Short-term unemployed ($d = 0 - 2$) | 0.01 | 0.03 |
| Medium-term unemployed ($d = 3 - 5$) | 0.04 | 0.12 |
| Long-term unemployed ($d \geq 6$) | 0.24 | 0.56 |

is 24 percentage points higher at a 1-month horizon, and 56 percentage points higher at a 3-month horizon. Later, I demonstrate that these differences remain large across calibrations I study in the sensitivity analysis.

I conjecture that several changes to the model might change this result. First, additional heterogeneity among workers could render the long-term unemployed a fundamentally different population than the employed or short-term unemployed. This is intimately related to the reason for duration dependence in job-finding rates: I allow for structural duration dependence, whereas an alternative literature argues that heterogeneity among job-seekers is more important in explaining this phenomenon (e.g., Ahn and Hamilton (2015) and Alvarez *et al.* (2015)). If heterogeneity is important, and the long-term unemployed are characterized by low temporary *and* permanent income, then their MPC may not be so much higher than that of the employed.

Second, additional sources of idiosyncratic risk could change the dynamic evolution of wealth among the unemployed. In the present setting, long-term unemployment is "as bad as it gets". As such, a long-term unemployed agent will borrow heavily against future income, pushing her closer to the borrowing constraint and driving up her MPC. In practice, additional risks, such as health risks or the loss of spousal income, may change this outlook for the long-term unemployed. The desire to maintain a buffer stock of savings could mitigate borrowing, and thus reduce the MPC.

Third, the addition of an illiquid asset in the model as in Kaplan and Violante (2014) may introduce new complexities into agents' consumption decisions through an unemployment spell. At short to medium unemployment durations, an agent's MPC may rise with duration as is the case in the present model. But at longer durations, facing the exhaustion of UI and a low prospect of future employment, an agent may find it worthwhile to pay the transaction cost of tapping into her illiquid asset (e.g., downsizing her home or selling her car). The release of liquidity may then reduce the MPC out of cash on hand for the long-term unemployed. This hypothesis parallels Kaplan and Violante (2014)'s finding that MPCs can be higher in milder recessions relative to severe ones.

Given these alternative hypotheses, empirical work measuring the MPC profile by duration of unemployment would be extremely valuable. Measurement of this profile could help identify whether adding the above model ingredients to the present framework is first-order. Given its important role in driving the aggregate demand effects of UI extensions in the simulations of the next subsection, directly targeting credible estimates of this profile may in fact be sufficient to characterize the dynamics implied by calibrated versions of the richer models sketched above.

**Average MPC in the economy.** In addition to the profile of MPCs by duration of unemployment, the economy's average MPC may be of interest. As is the case with many other models in the BHA tradition, the calibrated economy displays a wealth distribution which is not nearly as dispersed as it is in the data. As a result, the simulated economy-wide average MPC is only 0.02 monthly, or 0.05 quarterly, far below an average quarterly MPC for the U.S. economy around 0.25 (see Carroll *et al.* (2015) for a thorough review of the empirical literature).

I expect that two changes to the model could enable it to better match the economy's steady-state (liquid) wealth distribution and thus average MPC. First, adding government bonds or capital could absorb households' positive net savings and thus be used to target the empirical mean of liquid wealth. Second, modeling preference heterogeneity as in Carroll *et al.* (2015) or an illiquid asset as in Kaplan *et al.* (2016) could match the empirical dispersion of liquid wealth.

These changes should certainly be explored in future work — but may only raise the stimulative effect of benefit extensions relative to the basic framework here. As is clarified by the analytical results in the first half of the paper, the difference in MPCs across the population governs the initial stimulus from more generous UI. The level of MPCs instead governs general equilibrium amplification of the initial stimulus through the Keynesian cross. Hence, I conjecture that an enriched model capable of delivering a higher economy-wide average MPC would raise the stimulative effect of benefit extensions I find in this paper, all else equal.

### 1.3.4 Transitional dynamics and evaluation of UI policy

I now characterize the economy's transitional dynamics in response to a macroeconomic shock causing a large recession in view of sticky prices and a binding zero lower bound. I calibrate the shock to induce a binding zero lower bound for 90 months and rise in the unemployment rate of 4.5 percentage points, as has been roughly true in the U.S. during the Great Recession. I then compare outcomes under the observed path of UI benefit extensions with those under counterfactual paths. Technical details are again provided in the Computational Appendix.

Relative to less generous policies, I find large, positive effects of the benefit extensions on employment and welfare operating through aggregate demand. The result is traced to this paper's unique combination of DMP labor market dynamics, heterogeneity in MPCs by unemployment duration, and collapse in aggregate demand owing to sticky prices and a binding zero lower bound.

#### Equilibrium given a macroeconomic shock

I assume that an unexpected macroeconomic shock occurs at date 1, which I associate with July 2008. Once the shock is realized, I assume there is no remaining uncertainty over the future path of macroeconomic aggregates and economic policy, as in similar experiments performed in Guerrieri and Lorenzoni (2015), Auclert (2015), and Kaplan *et al.* (2016).

Like the first two papers, I assume a straightforward form of nominal rigidity under which I can evaluate stabilization policy in this context: retailer prices are permanently fixed at

$$P_{tj} = \bar{P} \; \forall j, t. \tag{1.65}$$

Each retailer $j$ facing problem (1.57) will then simply produce and accommodate desired demand, provided it can earn non-negative profits:

$$x_{tj} = y_{tj} = \begin{cases} c_t \text{ provided } \bar{P} \geq (1+\tau^r)P_t^I, \\ 0 \text{ otherwise.} \end{cases} \tag{1.66}$$

While this is of course an extreme treatment of nominal rigidity, it can be justified on three grounds. First, the shock modeled below will be temporary; since the economy will converge to the original steady-state in the long-run, retailers' incentive to update prices will eventually vanish. Second, inflation has indeed been quite low during the Great Recession. Finally and most importantly, in standard New Keynesian models, fiscal policy is further expansionary at the zero lower bound owing to the creation of inflation expectations which lower the real interest rate (Woodford (2010)). This relates to the "paradox of toil", as characterized in Eggertsson (2010) and Eggertsson and Krugman (2012), and implies that UI extensions may be even more expansionary in an environment with partial price stickiness. Other researchers have debated the plausibility of this channel and the implicit assumptions on which it is based (Kiley (2014), Wieland (2014), and Cochrane (2015)). Assuming fully sticky prices allows me to sidestep this debate and ensures that all of my results below are *not* generated through this channel.

We thus obtain the following definition of equilibrium.

**Definition 1.5.** *A* fully sticky price equilibrium *is a set of value functions, policies, tightness, employment, nominal prices and profits, and probability measures such that, given policy and exogenous real wages, conditions 1-2 and 4-7 of Definition 1.4 are satisfied, and condition 3 is replaced by (1.65) and (1.66).*

I will study this equilibrium with an initial distribution of agents by employment status,

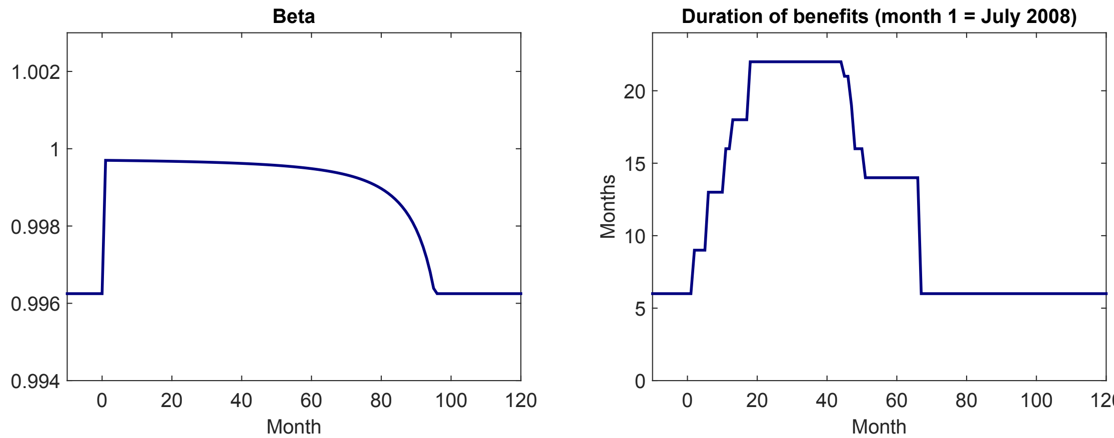



**Figure 1.3:** *A ZLB-inducing shock and UI policy*

unemployment duration, and assets inherited from the stationary RCE characterized earlier. I now discuss the exogenous shock and policy response which drive dynamics in this economy.

In my benchmark analysis, I assume that the relevant macroeconomic shock at date 1 is to preferences: a rise in the discount factor raises desired saving in the economy, driving down the real interest rate. The entire path of the discount factor after the initial shock is depicted in the first panel of Figure 1.3.[51] Such a shock is easy to work with and has also been used by other researchers studying the zero lower bound (e.g., Krugman (1998), Eggertsson and Woodford (2003)).

The path of UI policy from period 1 onwards matches that observed in the U.S. from July 2008 onwards. The second panel of Figure 1.3 summarizes the simulated duration of benefits in months, which is simply identical to Figure 1.1 after assuming 4.5 weeks per month.

Given this shock and policy, the endogenous behavior of macroeconomic aggregates in the *natural allocation* — the equilibrium under flexible prices — is summarized by the

[51] I choose the path of the discount factor to match two features of the observed macroeconomic dynamics during the Great Recession. First, the initial rise in the discount factor is chosen to match the observed 4.5% increase in the unemployment rate in the equilibrium with sticky prices and a binding zero lower bound. Second, the speed with which the discount factor returns to steady-state is chosen to match a binding zero lower bound for roughly 90 months, consistent with exit in December 2015 (given date 1 corresponding to July 2008).

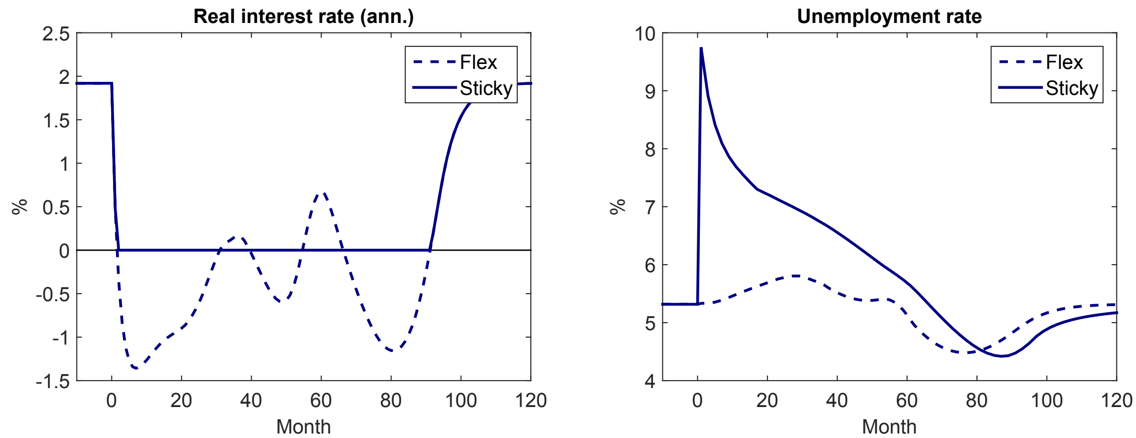



**Figure 1.4:** *Macroeconomic aggregates given actual UI policy*

dotted lines in Figure 1.4. Except for two brief periods when the natural rate exceeds zero, around 3 and then 5 years after the initial shock, the natural rate of interest in the first panel is negative for roughly 90 months.[52] The shock, coupled with the dramatic expansion in generosity of UI, has implications for the aggregate unemployment rate in the second panel, but it remains within roughly one percentage point of its steady-state level.

With fully sticky prices, I assume that the monetary authority follows an interest rate rule which implies that the nominal rate targets the natural rate of interest once the economy is permanently away from a binding zero lower bound, and is at the zero lower bound otherwise.[53] The resulting path of the real interest rate is summarized by the solid line in the first panel of Figure 1.4.

In view of the resulting binding zero lower bound for 90 months, the economy undergoes a severe recession, as indicated by the dramatic rise in the unemployment rate in the second

[52]The fluctuations in the natural rate in these two episodes result from the significant change in the generosity of UI around these times: the triggering off of benefits provided by the EB program in the first half of 2012, and termination of the EUC program in December 2013.

[53]This path of the nominal rate may not be optimal — indeed, in a representative agent framework Krugman (1998), Eggertsson and Woodford (2003), and Werning (2012) demonstrate the gains from policy commitments beyond the liquidity trap, such as maintaining a zero nominal rate. Characterizing optimal monetary policy in the present heterogeneous agent environment remains an open question for future work. I choose the present path for simplicity.

panel of Figure 1.4.[54] This headline rate rises to roughly 10% in view of the collapse in aggregate demand, consistent with the maximum unemployment rate in the U.S. during the Great Recession. Of course, the path of the unemployment rate differs from that observed in practice; whereas perfect foresight of a binding zero lower bound implies peak unemployment at month 1 in the simulated model, in practice the unemployment rate did not peak until late 2009, well after the downturn had begun.

Beyond the headline behavior in the unemployment rate, low demand induced by a binding zero lower bound has consequences for a number of additional labor market aggregates whose behavior was not targeted in calibrating the discount factor shock. I discuss these in Appendix A.7. Notably, vacancies, and thus a conventional measure of tightness, collapse relative to the natural allocation. This occurs for two reasons. First, a collapse in aggregate demand reduces labor demand through standard Keynesian channels. Second, similar to a point made by Hall (2015), a real interest rate which exceeds that in the natural allocation makes hiring, a form of investment in this frictional labor market, more costly.

With the transitional dynamics under the observed 2008-13 UI policy in place, we can now turn to an evaluation of counterfactual paths.

#### 2008-13 extensions vs. counterfactual capped at 9 months

I begin by comparing the observed path of extensions to counterfactual UI policy capped at 9 months of duration. The latter is consistent with the greatest degree of average generosity across U.S. states prior to the Great Recession. An alternative interpretation of this counterfactual is that (roughly speaking) it would have allowed for the duration extensions provided by the EB program, which is legislated into law, but would have eliminated the additional emergency measures taken by the federal government through

[54] Note that labor market variables exhibit damped oscillations in response to shocks in my simulation. For ease in visualizing outcomes, I smooth the labor market series by plotting every other point. This damped oscillatory property of the model appears related to the value of hiring costs $k$. In alternative calibrations where this is reduced, such behavior disappears.

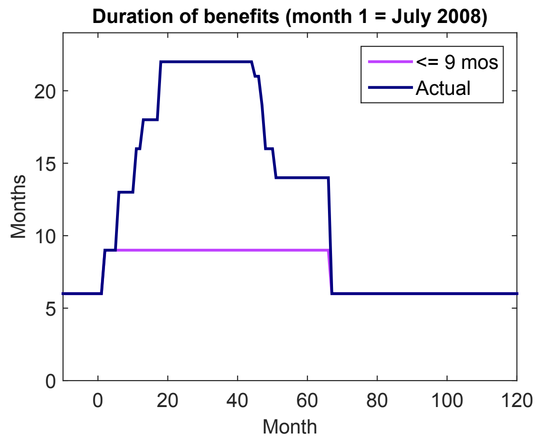



**Figure 1.5:** *Actual vs. counterfactual benefit durations*

the EUC08 program. Figure 1.5 compares the observed and counterfactual path of benefit durations.

To evaluate outcomes under the counterfactual policy, I first re-solve for the natural allocation under this fiscal policy stance, and then apply the nominal interest rate policy rule discussed in the prior subsection to characterize a counterfactual path for the nominal interest rate. Figure 1.6 illustrates the path of the natural rate and nominal interest rate given sticky prices under this counterfactual scenario. For ease in comparison, I also plot the same objects under the actual UI policy (reproduced from Figure 1.4). While the path of natural rates differ, owing to considerable differences in the generosity of UI, the path of the nominal interest rates in each case are virtually identical, owing to the binding zero lower bound.

Figure 1.7 summarizes the key quantitative, positive takeaway of my analysis: under sticky prices and a binding zero lower bound, the greater generosity of UI under the observed path prevents a substantial rise in the unemployment rate. In this benchmark calibration, I find the observed extensions prevent an initial rise in the unemployment rate of 5 percentage points, and that this difference in simulated unemployment rates remains positive for over 3 years after the initial shock.[55] This is consistent with large, positive effects

[55]Note that the initial increase in unemployment under the counterfactual path exceeds the fraction who

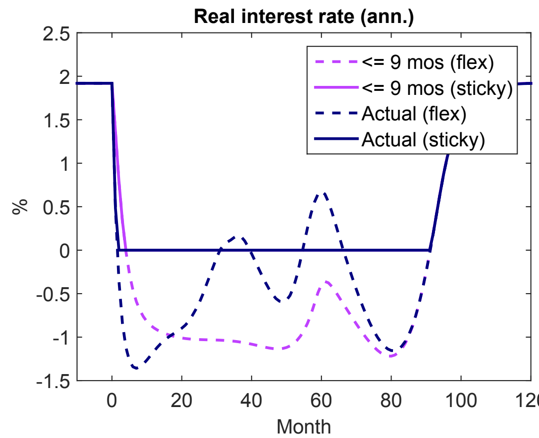



**Figure 1.6:** *Interest rate paths under actual vs. counterfactual durations*

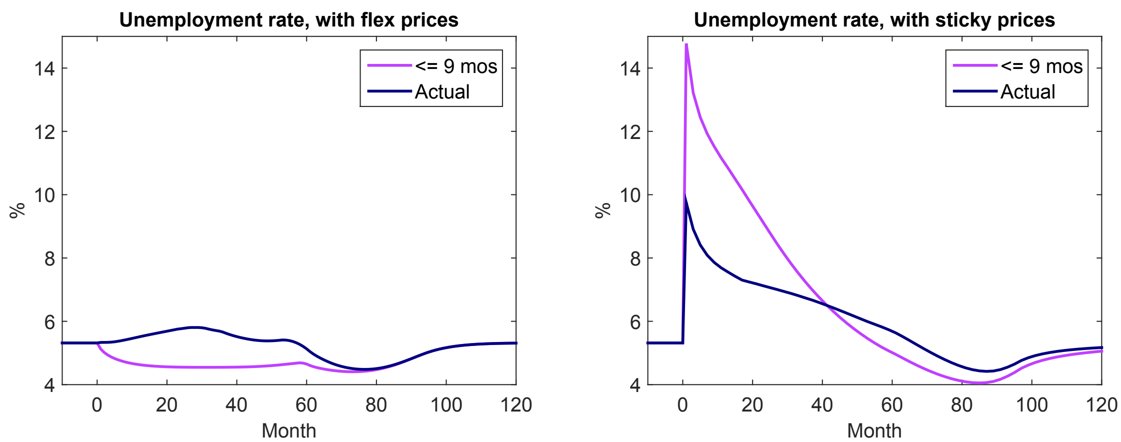



**Figure 1.7:** *Unemployment rate under actual vs. counterfactual durations*

on aggregate demand.

Notably, the unemployment differences between the actual and counterfactual UI scenarios are reversed from those in a flexible price environment, where the effect of UI on disincentives dominates. In particular, in the flexible price case, the unemployment rate reaches a level over 1 percentage point *higher* under the more generous, actual UI policy. Prior analyses of unemployment insurance in calibrated macroeconomic models

would exogenously separate from their firms in month 1. This reflects the collapse in aggregate demand, and thus desired hiring, under the counterfactual policy. Firms in fact would want to shed some workers in this case. To accommodate this and thus obtain a well-defined interior equilibrium, I enrich the model to allow firms to costlessly lay off workers, as described in Appendix A.8. Layoffs only occur in month 1, owing to my assumption of perfect foresight.

have focused on such flexible price frameworks (e.g., Krusell *et al.* (2010), Nakajima (2012), and Mitman and Rabinovich (2015)). In this sense, accounting for the consequences of nominal rigidities and constraints on monetary policy during the Great Recession reverses our understanding of the general equilibrium effects of UI.

The time-path of the unemployment differences between the actual and counterfactual UI scenarios suggests the importance of precautionary saving in amplifying the stimulus from the more generous UI schedule. Future UI extensions, by reallocating income to high-MPC agents and thus stimulating the economy, raise job-finding rates. In prior periods, this reduces incentives to precautionary save for all agents, further raising aggregate demand, stimulating the economy, and thus reducing incentives to precautionary save at prior dates, and so on. As evidence of this amplification, the difference in unemployment rates is biggest in the first month after the shock — even though, as seen in Figure 1.5, the duration of UI is no different under each scenario at that point. This amplification mechanism builds on recent work by Ravn and Sterk (2014), Challe *et al.* (2014), and Werning (2015) linking precautionary saving with aggregate demand.

Beyond the unemployment comparison, we can further compare welfare under the two policy regimes. For each agent, we can compare indirect utility at the beginning of date 1, after realization of the macroeconomic shock, under the actual and counterfactual policy. Formally, we define

$$dW^e(z) = \tilde{v}_1^e(z)|_{\text{actual}} - \tilde{v}_1^e(z)|_{\text{counterfactual}},$$
$$dW^u(z,d) = \tilde{v}_1^u(z,d)|_{\text{actual}} - \tilde{v}_1^u(z,d)|_{\text{counterfactual}},$$

where the first corresponds to employed agents and the second to unemployed agents at the beginning of date 1. We can then aggregate up using utilitarian weights for agents within each employment and duration category to compute

$$dW^e \equiv \int_z dW^e(z)\tilde{\varphi}_1^e(z)dz,$$
$$dW^u(d) \equiv \frac{1}{\int_z \tilde{\varphi}_1^u(z,d)dz}\int_z dW^u(z,d)\tilde{\varphi}_1^u(z,d)dz.$$

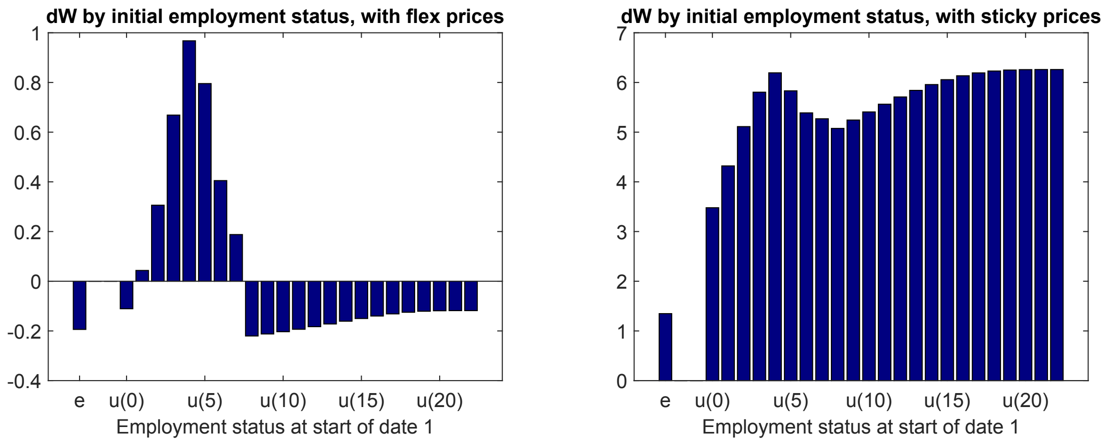



**Figure 1.8:** *Welfare under actual vs. counterfactual durations*

Plotting these aggregates, Figure 1.8 demonstrates that the greater generosity of UI under the actual path leads to welfare gains for the average agent across *all* categories. Remarkably, even initially employed agents who are likely to bear the brunt of paying for the benefit extensions gain from greater generosity. Intuitively, the effects of greater redistribution on aggregate demand raise these agents' job finding rates should they lose their job in the future, raising their continuation utility by enough that they gain from the policy change — consistent with the aggregate demand externality emphasized by the earlier theoretical analysis.

In fact, a closer examination of welfare changes within each employment and duration category reveals that the actual policy leads to a *strict Pareto improvement* as compared to the counterfactual policy under sticky prices. Even the wealthiest, employed agents gain from the observed extensions, owing to strong aggregate demand externalities from transfers to the unemployed.

Aggregating these welfare changes with utilitarian weights across all agents, the greater generosity of UI under the actual path leads to a 3.2% increase in social welfare. In money metric terms, this is equivalent to the welfare gain provided by a 0.4% per-period rise in consumption for all agents in all states, at all dates, as summarized in the second column of

**Table 1.4:** *Utilitarian social welfare under actual policy relative to counterfactual*

| Metric | Flexible prices | Sticky prices + ZLB |
|---|---|---|
| $\%\Delta SW$ | -0.3% | +3.2% |
| Equiv $\%\Delta c$ | -0.03% | +0.4% |

Table 1.4.[56]

Again, these welfare results are reversed from the case with flexible prices, highlighting the importance of nominal rigidities and constraints on monetary policy to these results. With flexible prices, in Figure 1.8, only the short-run unemployed who will directly gain from the benefit extensions beyond 9 months see their welfare rise from the actual policy. The initially employed and long-term unemployed see considerable welfare losses, owing to the direct cost of the policy and the general equilibrium effects on labor demand. On balance, as shown in the first column of Table 1.4, utilitarian social welfare falls under the actual benefit extensions relative to the counterfactual.

**Sensitivity.** I now characterize the sensitivity of these employment and welfare results to alternative calibrations. I focus on two simulated moments highlighted above: the change in peak unemployment and the change in social welfare between the observed extensions and counterfactual policy. I define these moments so that a positive value is consistent with a stabilizing effect of greater generosity provided by the observed extensions. In the baseline case characterized above, these were simulated to be 5 percentage points and 3.2%, respectively.

I focus on varying the calibration to target alternative values for three empirical moments about which there is uncertainty. First, as noted earlier, we know least about the heterogeneity in MPCs by employment status. While in the baseline I targeted a difference

[56] This is a simple money metric to compute numerically, as it keeps the labor supply allocation unchanged. Of course, for the same reason, it is not an implementable (incentive compatible) counterfactual.

in average 1-month MPCs between the unemployed and employed of 0.06, here I consider smaller differences of 0.02 and 0.04 to assess whether the sizable effects of the extensions on aggregate demand survive.[57,58] Second, applied researchers have obtained a range of estimates of the unemployment duration elasticity to potential UI duration. While in the baseline I targeted an elasticity of 0.1, consistent with the central tendency of the estimates surveyed by Krueger and Meyer (2002), here I consider alternatives of 0.05 and 0.15.[59] Third, estimates of recruiting costs in the economy vary depending on the source and definition used. While in the baseline I targeted a fraction of recruiters to employment of 2.5%, here I consider alternatives of 1.5% and 0.5%. The latter, while quite low, is consistent with the calibration of Shimer (2010). Appendix A.10 summarizes the calibrations in each case.

Consistent with the theoretical analysis, Figures 1.9 and 1.10 demonstrate that the targeted difference in average MPCs has a very significant impact on the simulated effect of benefit generosity on employment and welfare, while the targeted duration elasticity to benefit duration, and fraction of employment spent on recruiting, play smaller roles.[60] Moreover, in all cases the extensions to 22 months prevent a further deterioration in employment, and raise social welfare, relative to the case with benefits capped at 9 months.

[57]One motivation for the $[0.02, 0.06]$ range comes from Japelli and Pistaferri (2014), which to my knowledge is the only paper measuring heterogeneity in MPCs by employment status. In a survey of Italian households, these authors find that unemployed households report an MPC which exceeds that of employed households by 0.07. Survey respondents were not asked over what horizon they intended to spend a temporary income windfall. While we must be cautious about extrapolating from Italy to the United Status, the $[0.02, 0.06]$ range is roughly consistent with the evidence from this paper accounting for uncertainty in the horizon of reported spending (3-month vs. 1-month).

[58]The range $[0.02, 0.06]$ is also consistent with the range of differences in average 1-month MPCs between the unemployed and employed implied by the alternative calibrations described in Appendix A.9, which try to match the estimates of Gruber (1997), Chodorow-Reich and Karabarbounis (forthcoming), Ganong and Noel (2015), Browning and Crossley (2001), and Card *et al.* (2007).

[59]This is a smaller range than I would like; Meyer (1990) and Katz and Meyer (1990) report a duration elasticity with respect to potential duration as high as 0.5. I am unable to solve for a stationary RCE with a duration elasticity exceeding 0.18 given the particular functional forms and other parameter values I have assumed.

[60]The one exception is that the fraction of recruiters to employment appears to have a fairly big impact on the welfare results. When I reduce hiring costs $k$ to vary this moment, the equilibrium labor market dynamics exhibit smaller oscillatory behavior in view of the point made in footnote 54. Since welfare is affected by idiosyncratic employment volatility for individual agents, I conjecture that this is responsible for the sensitivity results.

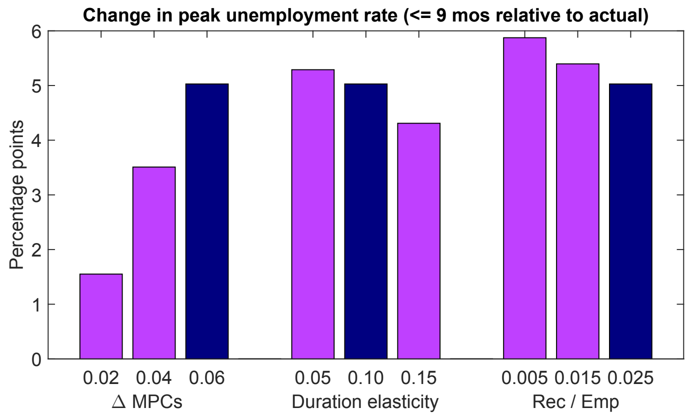



**Figure 1.9:** *Sensitivity of employment effect of benefit extensions to alternative calibrations*

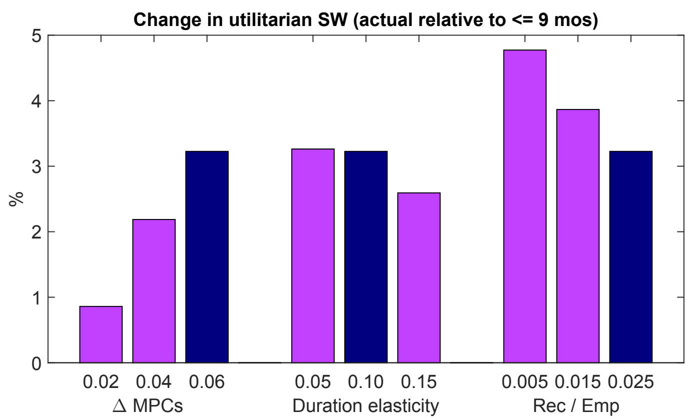



**Figure 1.10:** *Sensitivity of welfare effect of benefit extensions to alternative calibrations*

Given the lack of substantial empirical evidence regarding MPC heterogeneity by employment status, I use the range of estimates in Figures 1.9 and 1.10 to conclude that the observed benefit extensions prevent a further rise in the unemployment rate by 2–5 percentage points, and raise social welfare by 1-4%, relative to the counterfactual. Across this range of calibrations, I still find that the observed policy generates a strict Pareto improvement relative to the counterfactual, owing to the aggregate demand externality from marginal extensions covering the long-term unemployed.

**Robustness to positive wage elasticity.** In my counterfactual analysis, I keep real wages unchanged from their (flat) path assumed in the simulation of the observed benefit extensions. This is consistent with available micro evidence that the re-employment wage elasticity to changes in benefit generosity is indistinguishable from zero (e.g., Card *et al.* (2007), Lalive (2007), and van Ours and Vodopivec (2008)). In recent work, however, Hagedorn *et al.* (2015a) have estimated large and positive effects of the 2008-13 benefit extensions on local labor market wages.

Allowing wages to respond to changes in UI policy would not have any direct effect on labor demand in my simulation. With fully sticky prices, nominal rigidity renders output demand-determined through standard Keynesian channels. The wage response to benefits would affect output only through implied redistribution from markup variation, which depends on the allocation of equity shares across agents, and which in practice is small when firm profits are small. I demonstrate this formally in Appendix A.12, where a real wage elasticity of the order of magnitude in Hagedorn *et al.* (2015a) has a very minor effect on the comparison between the observed UI extensions and counterfactual durations capped at 9 months under sticky prices.

Accommodating partial price stickiness would change this result, but I conjecture would only *increase* the stimulative impact of the benefit extensions under standard assumptions on price-setting. As discussed earlier, the "paradox of toil" implies that the upward pressure on wages would, in an economy at the zero lower bound, create inflation expectations which reduce the real interest rate and thus further stimulate aggregate demand (Eggertsson (2010) and Eggertsson and Krugman (2012)). Indeed, this channel has been found by other researchers to motivate higher stimulus from transfers at the zero lower bound (Christiano *et al.* (2015) and Giambattista and Pennings (2015)). I conclude that an equilibrium wage response, if one exists, may not reverse the stabilizing role played by UI benefit extensions in the present setting.

**Magnitudes, measurement, and future refinements.** The preceding analysis suggests that two forces drive the large, positive effects of the benefit extensions on employment and welfare relative to the less generous counterfactual: (i) a sharply rising profile of MPCs by duration of unemployment, driving the initial stimulus from transfers to the long-term unemployed and (ii) dynamic amplification arising from the effect on precautionary saving.[61] The important role played by each suggests important avenues for future work to refine the magnitude of the results here.

First, future empirical work should measure the consumption response of unemployed agents to transitory income shocks at various durations of unemployment. As discussed earlier, the model-implied profile of MPCs by duration may be sensitive to the introduction of additional heterogeneity, idiosyncratic risks, or assets. Hence, estimating this MPC profile could narrow down, if not change, the range of effects I find in this paper. Measurement of this MPC profile should join the consumption drop upon unemployment, the disincentive elasticity to benefit changes, and the wage elasticity to benefit changes as a key object of interest in the applied literature on UI.

Second, future quantitative work should assess the sensitivity of the results to macroeconomic uncertainty. The dynamic amplification operating through precautionary saving likely leans heavily on the assumption of perfect foresight; in practice, the duration of the recession and binding zero lower bound, and the re-authorizations and expansions of UI generosity, were not fully anticipated. Such uncertainty is likely to reduce the dynamic amplification from precautionary saving and thus reduce the employment and welfare gains from the benefit extensions simulated here.

More generally, it would be useful to decompose the direct contribution of MPCs and the amplification through precautionary saving in driving the overall employment and

[61] In addition, there is likely a third stimulative force operating through the direct effect of more generous UI on savings behavior. An expansion in generosity for the long-term unemployed will reduce the idiosyncratic downside risk faced by the short-term unemployed and employed, reducing their incentives to precautionary save. This is a partial equilibrium effect, in contrast to the dynamic amplification channel operating in general equilibrium through changes in job-finding rates; indeed, in Appendix A.11, I make this third force precise in a partial equilibrium analysis of a short-term unemployed agent in a dynamic setting. I focus on an experiment in which benefit generosity is raised should she remain unemployed in the following period, while taxes financing the policy change are raised should she gain employment in the following period. This renders her expected future income unchanged; yet, when her flow utility is consistent with a positive coefficient of prudence, her present consumption will rise. In fact, relative to other forms of fiscal stimulus such as conventional government spending, this third force may render UI extensions a more powerful tool in stabilization owing to the considerable downside risk posed by job loss.

welfare results I obtain. Such a decomposition first requires an analytical understanding of sufficient statistics characterizing the effects on precautionary saving. In ongoing research, I am extending the short-run/long-run model of section 1.2 to a three-period framework which would permit me to characterize the effects on precautionary saving in closed form and isolate statistics governing their size.

Finally, two more mechanical exercises would be useful to improve the estimates of the effects of the 2008-13 benefit extensions. First, the framework should account for incomplete eligibility and take-up of UI, which would likely reduce the stimulative effects found here, perhaps substantially. Second, the framework should account for the debt finance of extensions; since the model is non-Ricardian, debt finance is likely to moderately increase the stimulative effects found here.

#### Additional counterfactuals on the duration and replacement rate margin

We can use the simulated model to consider two final policy experiments of interest. First, we can compare the observed extensions to a broader set of counterfactuals operating on the duration margin: to what extent were the observed extensions optimal? Second, we can compare the observed extensions to counterfactuals keeping durations at the regular 6 months, but varying the replacement rate of benefits: how would these policies compare?

**Broader set of counterfactual durations.** I first consider a broader set of counterfactual paths operating along the duration margin. At the extremes, benefits could have been limited to the regular duration of 6 months, or could have been extended to cover any unemployed worker at the depth of the Great Recession. A less extreme set of changes would have been to cap benefits at 16 months, or extend them to 28 months. Along with the baseline counterfactual of durations capped at 9 months, I summarize the full set of counterfactual durations in Figure 1.11. Unemployment and welfare under this class of counterfactual policies is summarized in Figure 1.12.

The analysis of these counterfactuals reveals that within this restricted class, indefinite

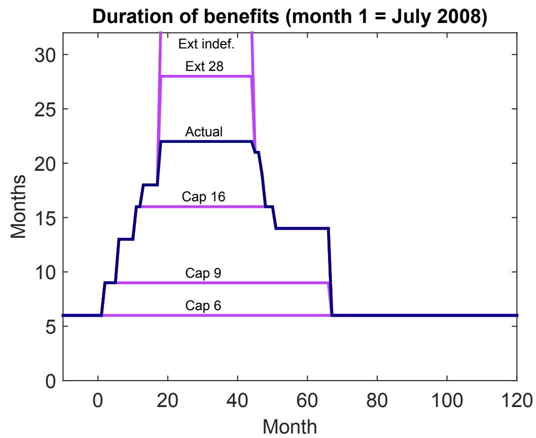



**Figure 1.11:** *Benefit durations under broader set of counterfactuals*

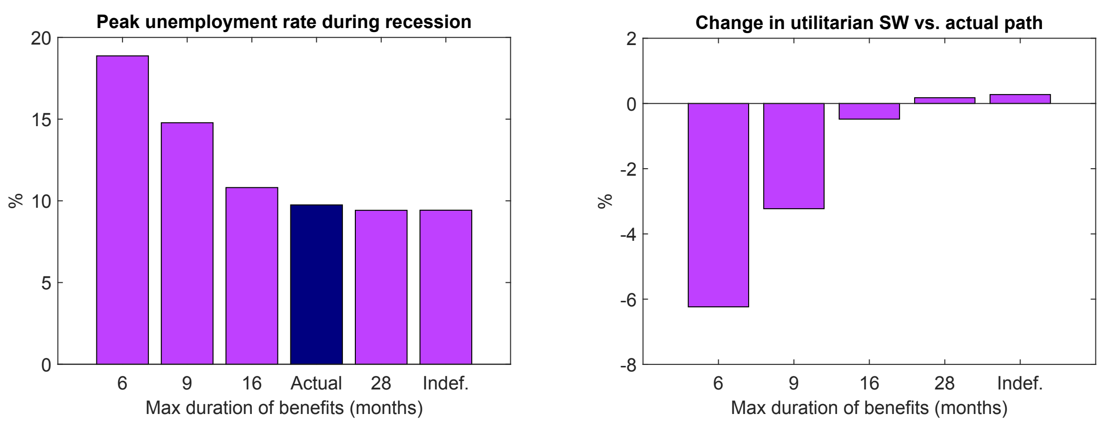



**Figure 1.12:** *Outcomes under broader set of counterfactuals*

benefit extensions are optimal, while the absence of any extensions make an already bad recession considerably worse. Importantly, it is worth remembering that indefinite benefit extensions for a finite period of calendar time does *not* eliminate the incentives to search: since the replacement rate remains unchanged, and the extensions are only temporary, the value from employment still exceeds that from unemployment.

A subtle implication of these counterfactuals is an asymmetry result: relative to doing nothing, the observed policy delivers very significant employment and welfare gains; but relative to extending benefits indefinitely, the observed policy achieves almost all of the gains in employment and welfare. This result follows from a rich interplay between the pattern of unemployment rates and MPCs by duration during the simulated recession. First, while there are many agents who become unemployed for 6 to 22 months during the simulated downturn, there are considerably fewer who remain unemployed for more than 22 months. Second, while the MPC tends to rise through an unemployment spell, it is bounded above by one. Taken together, the aggregate demand effects of extensions from 6 to 22 months are considerably larger than those from 22 to indefinite duration. This interaction between the fraction of agents covered by the marginal policy change, and the MPC of the marginal recipients of transfers, is precisely what is suggested by my theoretical result on the size of the UI multiplier in the simpler short-run/long-run model.

**Duration vs. level of benefits.** I now compare the observed policy to a class of counterfactuals where the level, rather than duration, of benefits is made more generous. In particular, I consider five counterfactual policies in which the duration of benefits is the regular 6 months, while the replacement rate rises from 50% to reach 60%, 70%, 80%, 90%, or 100%. For each counterfactual, I assume that the time-path of replacement rates mirrors that of the benefit durations under the observed policy. I summarize the counterfactual replacement rates in Figure 1.13. Unemployment and welfare under this class of counterfactual policies is summarized in Figure 1.14.

The observed duration extensions deliver greater employment and utilitarian social welfare than each of these policy changes on the level margin. This result is consistent with a dominant role for the differences in MPCs: while the replacement rate increases apply to any agent who becomes unemployed, rather than only the long-term unemployed, the duration extensions target an endogenously high-MPC population. This result is particularly notable because the calibration has "stacked the deck" against the duration extension: in the stationary RCE, the simulated duration elasticity to a change in the replacement rate is 0.16, whereas the central tendency from empirical estimates is 0.5 (Krueger and Meyer (2002)). Thus, even with a disincentive response to replacement rate changes which may be too low relative to that observed in the real world, the simulated model still implies that the

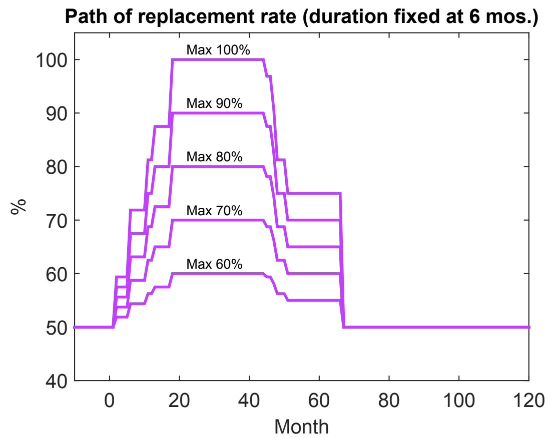



**Figure 1.13:** *Counterfactual replacement rates*

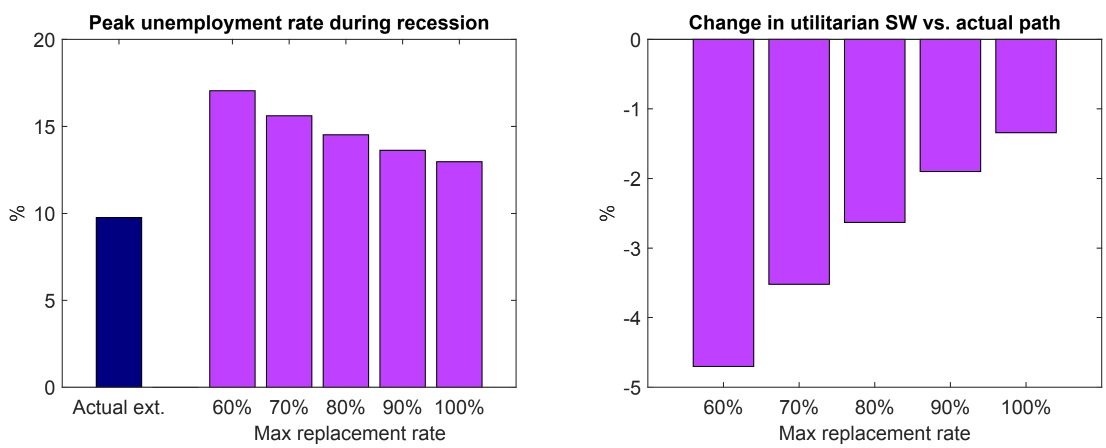



**Figure 1.14:** *Outcomes under counterfactual replacement rates*

observed duration extensions had larger, positive effects on employment and welfare due to their effects on aggregate demand.

While further research on the optimal policy is surely needed, my results suggest that policymakers should continue to focus on duration extensions rather than replacement rate generosity as the primary lever within the UI system to vary in macroeconomic stabilization, given the high MPCs of the long-term unemployed.

## 1.4 Conclusion

This paper contributes to our understanding of the role that the UI system, a key part of the social safety net in advanced economies, can play in macroeconomic stabilization of short-run fluctuations.

Building on the classic analysis of optimal UI in public finance, I theoretically demonstrate that the interaction between UI and aggregate demand naturally motivates higher generosity when the economy is slack. In the presence of nominal rigidities, a redistribution effect on aggregate demand drives the marginal impact of higher UI on output. When the economy is slack, this means that UI takes on a novel macroeconomic stabilization role arising from the aggregate demand externality caused by transfers to the unemployed. Moreover, low aggregate demand itself changes the social cost of disincentivizing labor supply. When the unemployed have a higher MPC than the employed and are net debtors, these channels imply a positive effect of redistribution on output, and optimal generosity which is higher than the classic public finance formula would imply.

In a calibrated infinite-horizon generalization of this model, I apply these insights to speak directly to the policy debate over the unprecedented extension of UI generosity in the U.S. during the Great Recession. My simulations suggest that the observed path of extensions substantially helped to prevent a deeper recession and raise welfare. Relative to a counterfactual path capped at 9 months of duration, I find that the observed extensions to 22 months prevent a further rise in the unemployment rate of 2–5 percentage points and generate a strict Pareto improvement. Consistent with the theoretical analysis, these effects appear driven by the fact that the benefit extensions target transfers to the high-MPC long-term unemployed. In an infinite horizon environment, this initial stimulus is further amplified by the dynamic response of precautionary saving.

My characterization of UI in stabilization can guide research on the role of other social insurance and cash transfer programs over the business cycle. The analysis of these programs will add considerable color to our understanding of fiscal policy in macroeconomic stabilization — a welcome development, in view of the very significant role that these social programs play in actual government budgets (McKay and Reis (2015)). I leave the analysis of these other programs, and the comparison of UI and these programs to more standard government purchases, to future research.

# Chapter 2

# Labor Market Frictions in a Monetary Union

## 2.1 Introduction

The prolonged slump in Southern European economies in recent years has brought a spotlight back on labor market frictions in Europe. Several decades ago, diverging labor market performance on either side of the Atlantic led to a wave of research asking if European labor market institutions might be responsible. In the last few years, and now with many European countries bound by the Euro, unemployment rates exceeding 15% in Greece, Spain, and Portugal have again renewed academic and policy interest in whether differing institutional environments in these countries might be partly to blame. On measures such as mandated severance payments, depicted in Figure 2.1, these countries are indeed considerably more generous than other Eurozone members.

Existing analyses do not provide a sufficiently rich framework to think through the interplay between the functioning of a particular country's labor market and its membership in a monetary union. While many researchers have accounted for stickiness in nominal wages, virtually none have explored whether the search, matching, or bargaining features of

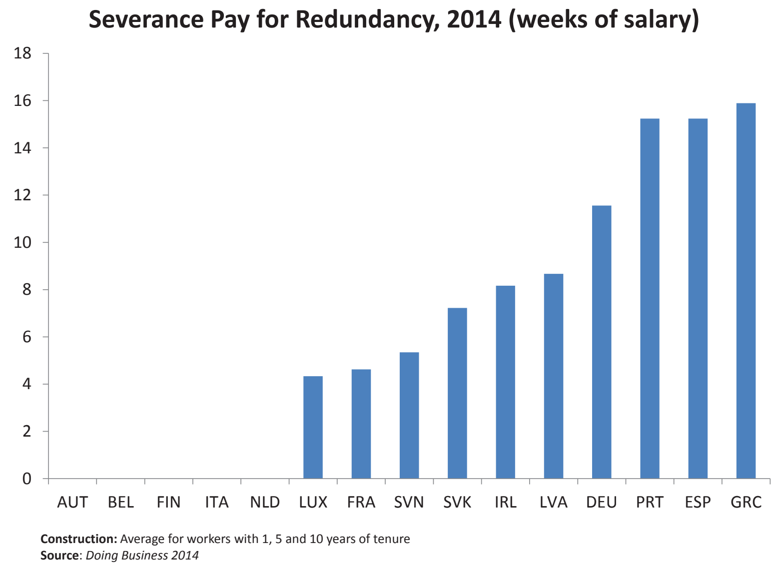



**Figure 2.1:** *Mandated severance pay in the Eurozone*

real-world labor markets matter for macroeconomic outcomes across the union.[1] This paper takes a first step to fill that gap.

Within a standard model of a two-country monetary union subject to country-specific TFP shocks, I embed labor market frictions in the tradition of Diamond (1981), Mortensen (1982), and Pissarides (1984) (hereafter, DMP). In this setting, I revisit two of the central questions in the theory of monetary unions in the presence of non-trivial and heterogeneous labor market frictions across countries. First, when will such a monetary union face distortions? Is it possible that Mundell (1961)'s classic business cycle synchronicity criterion may be modified — that symmetric shocks may interact with asymmetric labor markets to generate inefficiencies in a monetary union? Second, how should second-best optimal monetary policy be conducted? Should it target smaller employment fluctuations in the more sclerotic, or the more fluid, labor market in the union?

[1]Among recent analyses of monetary unions, Gali and Monacelli (2008), Kehoe and Pastorino (2014), and Chari *et al.* (2015) assume a Walrasian labor market, while Benigno (2004), Schmitt-Grohe and Uribe (2012, 2013), Farhi *et al.* (2013), and Farhi and Werning (2014) allow for sticky wages but otherwise assume frictionless labor markets. An emerging literature studies monetary unions in which members' labor markets are characterized by DMP frictions, which my paper joins. I situate my paper within this literature later in this section.

Under benchmark preferences and technologies in the literature, I obtain two sharp results: *institutional irrelevance* for the question of when a monetary union will face distortions, and *relative accommodation* of the more sclerotic member of the union at the second-best optimum. Institutional irrelevance is a consequence of invariant employment to TFP shocks in the flexible price and wage allocation, rendering the terms of trade between countries just a function of relative TFP regardless of the level of or cross-country heterogeneity in DMP frictions. Given asymmetric shocks, relative accommodation of the more sclerotic union member — that is, the member with greater hiring costs and lower labor market flows[2] — is a consequence of both greater welfare losses and greater inflationary/deflationary pressure from output fluctuations in that member. Together, these results suggest that while labor market frictions may not bear on the optimality of the Eurozone as a currency area, they should inform stabilization policy by union-wide institutions such as the ECB.

I begin by building a static, two-country model of a currency union with DMP frictions. The model merges standard elements from the international macro and macro/labor traditions. From the first, production of distinct intermediate goods uses domestic labor and country-specific TFP, international trade is in differentiated final goods, and domestic money prices of imported varieties are mediated by the nominal exchange rate which is assumed fixed.[3] From the second, recruiting workers is costly, workers and vacancies in each country are brought together by a matching function, and wages split the bilateral surplus from a match. Importantly, I specify preferences and technologies consistent with widely-used benchmarks in each literature: utility from consumption which is separable from labor, log over aggregate consumption, and Cobb-Douglas over goods produced in each country (as in Cole and Obstfeld (1991)), and recruiting costs expressed in forgone incumbent worker time in production, thus scaling with domestic TFP (as in Shimer (2010)).

I first study the equilibrium with flexible prices and wages in each member of union —

[2]This definition of sclerotic vs. fluid labor markets follows in the tradition of Blanchard and Gali (2010).

[3]I model a monetary union as characterized by a fixed exchange rate between two countries, nesting the case of a common currency (where the exchange rate is one).

the natural allocation — and find that it is characterized by three properties important to understanding the stabilization problem under sticky prices. First, equilibrium employment in each country is invariant to domestic TFP. This generalizes the benchmark results of Blanchard and Gali (2010) and Shimer (2010) to the open economy, and relies on the assumption of Cole and Obstfeld (1991) preferences and recruiting costs which scale with domestic TFP. Second, equilibrium employment in each country is invariant to all foreign economic conditions. This generalizes the benchmark result in Clarida *et al.* (2002) to a labor market characterized by DMP frictions, and relies again on the assumed preferences and fact that recruiting costs use only domestic resources. Third and finally, the natural allocation is constrained efficient if, in each country, the distortion from monopolistic competition in the retail sector is offset with a production subsidy and surplus sharing in the labor market satisfies the Hosios (1990) condition.[4,5] The latter offsets search externalities in each labor market just as in the closed economy because of my maintained assumption of no labor mobility across union members — a better approximation for Eurozone countries than it is for U.S. states.

Given nominal rigidity in prices and TFP shocks across the union, I can then tractably study the stabilization problem facing a union-wide central bank by studying a linear-quadratic approximation to the Ramsey policy problem. Suppose that only some retailers in each country can set prices after the realization of TFP shocks; as the other retailers accommodate demand at preset prices, union-wide monetary policy is non-neutral. Under the important assumption that the natural allocation is constrained efficient as described above, a linear-quadratic approximation to the Ramsey policy problem suffices to characterizes the optimal allocation up to first-order from steady-state. The linear constraints of this problem consist of relations linking producer-price inflation and output in each country —

[4]The constrained efficient allocation is that chosen by a planner subject only to the global economy's technological constraints in producing and recruiting.

[5]As a result, a competitive search equilibrium will continue to induce efficiency in the labor market (generalizing Moen (1997) to the open economy). In the body of the paper I instead assume random search and Nash bargaining.

the Phillips Curve in this one-period environment — as well as a relation linking inflation and output across countries owing to the fixed exchange rate. Approximated in this way, the coefficients on policymakers' objective, each country's Phillips Curve, and the constraint imposed by the fixed exchange rate encode *all* the necessary information to characterize the effect of labor market frictions on stabilization challenges facing a monetary union.

With this formulation of the stabilization problem in hand, I first obtain a sharp institutional irrelevance result: the constrained efficient allocation is achievable if and only if TFP shocks are symmetric across countries, regardless of the level of or heterogeneity in DMP frictions. Intuitively, a sufficient statistic for inefficiency in a monetary union is movement in the terms of trade in the natural allocation. If the natural terms of trade adjust given a set of TFP shocks, a monetary union with sticky prices will be unable to replicate this movement in relative prices without some distortions in prices, output, or both. Under the benchmark preference and technology assumptions described above, employment in each country is invariant to domestic and foreign TFP, and thus relative production is simply a function of relative TFP, in the natural allocation. It immediately follows that the natural terms of trade will adjust if and only if TFP shocks are asymmetric across countries. As such, Mundell (1961)'s business cycle synchronicity criterion for optimal currency areas is robust to the presence of rich and diverse labor market frictions across countries.

In the presence of asymmetric shocks necessitating some distortions in a monetary union, I then find that optimal policy is characterized by relative accommodation of the more sclerotic labor market. In particular, greater hiring costs or lower labor market flows in a particular union member tend to raise the deadweight costs of inefficient hiring in that country. This increases the welfare cost of output distortions and the slope of the Phillips Curve in that country — which, in fact, are tightly linked when the natural allocation is constrained efficient, reflecting the connection between the curvature of social welfare and the sensitivity of firms' real marginal costs to output in that case. Since the more sclerotic labor market features greater welfare losses and greater price pressure from a given distortion in output, the optimal policy accommodates that market by targeting both a smaller output gap and smaller inflation/deflation at the second-best optimum.

I conclude by tracing out tentative implications of my results for the Eurozone, motivating fruitful directions for future work. My institutional irrelevance result clarifies that the existence and heterogeneity of labor market frictions across member states need not have anything to do with the optimality of the Eurozone as a currency area. But my relative accommodation result suggests that the fluidity of labor markets should indeed inform the conduct of stabilization policy by the ECB when perfect stabilization is unattainable — and may in fact mean that the historical conduct of policy has been suboptimal. To more thoroughly explore these results and their consequences for the Eurozone, future work should extend the model to an infinite horizon and perform a quantitative calibration, taking advantage of the rich dynamics introduced by the DMP modeling of the labor market and a long tradition in connecting such models to the data.

This paper marries two active literatures: from international macro, the literature on monetary unions, and from macro/labor, the literature connecting technological and institutional features of labor markets with the level and cyclical behavior of unemployment. The questions of this paper are inspired by the seminal contributions of Mundell (1961), McKinnon (1963), and Kenen (1969) on Optimal Currency Areas, and Ljungqvist and Sargent (1998), Nickell and Layard (1999), and Blanchard and Wolfers (2000) on European institutions and unemployment. The model I develop combines elements of Obstfeld and Rogoff (1995) and Gali and Monacelli (2008)'s more modern analyses of fixed exchange rates and monetary unions, and Blanchard and Gali (2010) and Shimer (2010)'s analysis of labor market dynamics in the presence of DMP frictions.

In focusing on the consequences of cross-country heterogeneity on stabilization policy in a monetary union, this paper is distinct from, but complementary with, well-known work by Benigno (2004). Benigno (2004) studies stabilization of a monetary union in which the degree of nominal rigidity differs across countries, whereas I focus on heterogeneity in the search and matching features of labor markets. These affect the stabilization problem through different channels — the cost of inflation in the first case, versus the cost of output fluctuations in the second. They also admit distinct empirical counterparts — for instance, the observed duration of nominal contracts in the first case, versus the institutional and technological costs involved in hiring and firing workers in the second. Our analyses are complementary, however, in that both imply the optimality of targeting policy to accommodate the more rigid member of the monetary union. And indeed, the key insight of Benigno (2004) is nested and holds within my model: optimal policy converges to targeting zero inflation in one union member as prices converge to full flexibility in the other.

Motivated by policy interest in Eurozone labor markets in recent years, several other authors have recently studied the consequences of DMP frictions in a monetary union; relative to these, my analysis is distinguished by revisiting the classic normative insights of the Optimal Currency Area literature in such an environment. Campolmi and Faia (2011) and Abbritti and Mueller (2013) instead focus on the positive features of such a union, obtaining empirical and quantitative results consistent with the log-linearized equilibrium relations of this paper. Eggertsson *et al.* (2014) analyze Ramsey optimal policy in a similar environment, but focus on the constraint posed by a union-wide zero lower bound rather than adjustment in the terms of trade. Cacciatore *et al.* (2016) also analyze optimal policy, but focus instead on the policy trade-offs in using union-wide monetary policy to undo steady-state distortions from inefficient labor or product markets among members.

In section 2.2 I outline the economic environment. In section 2.3 I study the natural allocation, and in section 2.4 I develop and characterize the stabilization problem under partially sticky prices, obtaining the main results of the paper. In section 2.5 I discuss the implications of my analysis for the Eurozone, as well as directions for future work. Finally, in section 2.6 I conclude.

## 2.2 Environment: a monetary union with DMP frictions

In this section I characterize a global economy consisting of two countries bound by a fixed exchange rate (as implicitly the case in a monetary union), each subject to country-specific TFP shocks, and each featuring a labor market characterized by DMP frictions. The model ingredients merge standard elements of the international macro and macro/labor literatures.

### 2.2.1 Primitives

I first specify tastes, technologies, endowments, and market structures in a one-period global economy comprised of Home and Foreign. Throughout the paper, I use asterisks to denote variables chosen by or endowed to Foreign agents.

Each country is comprised of measure one agents organized into measure one households. Owing to complete domestic asset markets and ex-ante symmetry across households, I follow Merz (1995) and Andolfatto (1996) in focusing on a representative household in each country with preferences

$$u(c,n) = \log c - \chi \frac{n^{1+\varphi}}{1+\varphi}, \quad u^*(c^*,n^*) = \log c^* - \chi^* \frac{{n^*}^{1+\varphi^*}}{1+\varphi^*},$$

where $c$ and $c^*$ denote consumption for each member of the household, and $n \in [0,1]$ and $n^* \in [0,1]$ denote the fraction of the household which is employed. Consumption, in turn, is a Cobb-Douglas aggregator of consumption of goods produced in each country (with home bias $\gamma > \frac{1}{2}$), each of which is a CES aggregator over that country's measure one of varieties (with common elasticity of substitution $\varepsilon$ only for expository simplicity):

$$c = (c_H)^{\gamma}(c_F)^{1-\gamma}, \quad c^* = (c_H^*)^{1-\gamma}(c_F^*)^{\gamma}, \tag{2.1}$$

$$c_H = \left[\int_0^1 c_H(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_H^* = \left[\int_0^1 c_H^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.2}$$

$$c_F = \left[\int_0^1 c_F(j^*)^{\frac{\varepsilon-1}{\varepsilon}} dj^*\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_F^* = \left[\int_0^1 c_F^*(j^*)^{\frac{\varepsilon-1}{\varepsilon}} dj^*\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.3}$$

I consider two classes of firms in each country: perfectly competitive intermediate good producers and monopolistically competitive final good retailers. Producers recruit workers in their frictional domestic labor market, produce an intermediate good with linear technology and TFP $a$ and $a^*$, and sell only to domestic retailers. Retailers, in turn, transform their country's intermediate good into differentiated varieties with a pure-pass through technology and sell them globally.

The labor market in each country is the novel element of my framework relative to the monetary union literature. Following Shimer (2010), producers must use $k$ and $k^*$ incumbent workers to manage the recruiting process for a single vacancy. Under my maintained assumption that labor is immobile across borders, with $u$ and $u^*$ unemployed workers searching for employment and $v$ and $v^*$ vacancies posted by producers, the Home and Foreign economies see

$$m(u,v) = \bar{m}(u)^{1-\eta}(v)^{\eta}, \quad m^*(u^*,v^*) = \bar{m}^*(u^*)^{1-\eta^*}(v^*)^{\eta^*}$$

aggregate matches, respectively. Workers search randomly across posted vacancies, and wages are Nash bargained ex-post with worker bargaining shares $\beta$ and $\beta^*$ in Home and Foreign, respectively.[6]

Finally, I specify endowments and international asset markets. I assume that fractions $(1-\delta)n_0$ and $(1-\delta^*)n_0^*$ of each country's representative household have already matched with firms at the start of the period, where I interpret $n_0$ and $n_0^*$ as the fictitious prior period's employment rate, and $\delta$ and $\delta^*$ as each economy's separation rate.[7] I assume that households start with equal ownership of their domestic firms but hold zero net foreign assets because international asset markets are non-existent. The latter will be without loss of generality, given that the seminal result of Cole and Obstfeld (1991) holds when vacancy-posting requires domestic resources only.

### 2.2.2 Defining the equilibrium

Having specified the global economy's tastes, technologies, endowments, and market structures, I now define a competitive equilibrium.

I begin by defining prices facing agents in the global economy. Let starred prices denote

[6] All of the stabilization results in section 2.4 would be unchanged if firms instead posted wages and workers direct their search, since these results will assume that the Hosios (1990) condition holds.

[7] The distinction between $\{n_0, n_0^*\}$ and $\{\delta, \delta^*\}$ will become meaningful when, in my analysis of macroeconomic stabilization in section 2.4, I assume that the steady-state realization of TFP shocks $\{a, a^*\}$ is consistent with $\{n = n_0, n^* = n_0^*\}$. Then, $\delta$ and $\delta^*$ will capture the magnitude of flows in each labor market even in this one-period model.

nominal quantities in Foreign's unit of account. Then $P_H(j)$ and $P_H^*(j)$ are the prices faced by residents of Home and Foreign for variety $j$ produced in Home; $P_F(j^*)$ and $P_F^*(j^*)$ are the prices faced by these same residents for variety $j^*$ produced in Foreign; $P^I$ and $P^{I*}$ are the price of intermediate goods in each country; and $W$ and $W^*$ are the prevailing wages in each country. Given the nested CES structure of household preferences, it will prove useful to define the following price indices:

$$P_H = \left[\int_0^1 P_H(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}, \quad P_F = \left[\int_0^1 P_F(j^*)^{1-\varepsilon} dj^*\right]^{\frac{1}{1-\varepsilon}}, \tag{2.4}$$

$$P_H^* = \left[\int_0^1 P_H^*(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}, \quad P_F^* = \left[\int_0^1 P_F^*(j^*)^{1-\varepsilon} dj^*\right]^{\frac{1}{1-\varepsilon}}. \tag{2.5}$$

In addition to market prices, labor market tightness will play an equilibrating role in each country's frictional labor market. Tightness in each country is defined as

$$\theta = \frac{v}{u}, \quad \theta^* = \frac{v^*}{u^*},$$

where $\{u, u^*, v, v^*\}$ are aggregate unemployment and vacancies, giving rise to vacancy-filling probabilities per vacancy posted

$$q(\theta) = \frac{m(u,v)}{v}, \quad q^*(\theta^*) = \frac{m^*(u^*, v^*)}{v^*},$$

and job-finding probabilities for each unemployed agent

$$p(\theta) = \frac{m(u,v)}{u}, \quad p^*(\theta^*) = \frac{m^*(u^*, v^*)}{u^*}.$$

Finally, I define the policy instruments assumed available to a union-wide policymaker throughout the paper. In terms of fiscal instruments, I assume that policymakers can assess ad-valorem taxes $\tau^r$ and $\tau^{r*}$ on retailers' purchases of intermediate goods, and assess lump-sum taxes $T$ and $T^*$ on domestic residents. In terms of monetary instruments, I assume that policymakers can set the monetary bases $\bar{M}$ and $\bar{M}^*$ in each country. Money will serve as a unit of account in each country, and I will complement agents' optimization problems described below with ad-hoc money market clearing conditions in equilibrium.

We are now in a position to specify agents' optimization. Recalling that representative households have $(1-\delta)n_0$ and $(1-\delta^*)n_0^*$ workers already matched at the start of the period, and assuming for simplicity that incumbent and new workers are paid the same wage,[8] the households will choose consumption and labor force participation among their unemployed members according to

$$\begin{aligned} v((1-\delta)n_0) &= \max_{\{c_H(j)\}_j,\{c_F(j^*)\}_{j^*},u\in[0,1-(1-\delta)n_0]} \log c - \chi\frac{n^{1+\varphi}}{1+\varphi} \text{ s.t.} \\ (RC): \int_0^1 P_H(j)c_H(j)dj &+ \int_0^1 P_F(j^*)c_F(j^*)dj^* \leq Wn + \Pi + \int_0^1 \Pi^r(j)dj - T, \\ &(Evol): n = (1-\delta)n_0 + p(\theta)u, \end{aligned} \tag{2.6}$$

$$\begin{aligned} v^*((1-\delta^*)n_0^*) &= \max_{\{c_H^*(j)\}_j,\{c_F^*(j^*)\}_{j^*},u^*\in[0,1-(1-\delta^*)n_0^*]} \log c^* - \chi^*\frac{(n^*)^{1+\varphi^*}}{1+\varphi^*} \text{ s.t.} \\ (RC)^*: \int_0^1 P_H^*(j)c_H^*(j)dj &+ \int_0^1 P_F^*(j^*)c_F^*(j^*)dj^* \leq W^*n^* + \Pi^* + \int_0^1 \Pi^{r*}(j^*)dj^* - T^*, \\ &(Evol)^*: n^* = (1-\delta^*)n_0^* + p^*(\theta^*)u^*, \end{aligned} \tag{2.7}$$

given the consumption aggregators defined in (2.1) - (2.3) and firm profits $\{\Pi, \{\Pi^r(j)\}_j, \Pi^*, \{\Pi^{r*}(j^*)\}_{j^*}\}$ characterized below. Assuming ex-ante symmetry across producers, constant returns to scale in production and recruiting means that we can focus on representative producers facing

$$\Pi((1-\delta)n_0) = \max_v P^I a\left[(1-\delta)n_0 + q(\theta)v - kv\right] - W\left[(1-\delta)n_0 + q(\theta)v\right], \tag{2.8}$$

$$\Pi^*((1-\delta^*)n_0^*) = \max_{v^*} P^{I*}a^*\left[(1-\delta^*)n_0^* + q^*(\theta^*)v^* - k^*v^*\right] - W^*\left[(1-\delta^*)n_0^* + q^*(\theta^*)v^*\right]. \tag{2.9}$$

[8] This is innocuous; given risk-sharing within the household, the wage paid to incumbent workers has no effect on the equilibrium allocation, as is well known in the macro/labor literature.

Finally, retailer $j \in [0,1]$ in Home and $j^* \in [0,1]$ in Foreign face

$$\begin{aligned}\Pi^r(j) = \max_{y(j),x(j),P_H(j)} P_H(j)y(j) - (1+\tau^r)P^I x(j) \text{ s.t.}\\ (Tech)(j) : y(j) = x(j),\\ (Demand)(j) : y(j) = \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} c_H + \left(\frac{P_H^*(j)}{P_H^*}\right)^{-\varepsilon} c_H^*,\\ (PCP)(j) : P_H^*(j) = \frac{P_H(j)}{E},\end{aligned} \tag{2.10}$$

$$\begin{aligned}\Pi^{r*}(j^*) = \max_{y(j^*),x(j^*),P_F^*(j^*)} P_F^*(j^*)y^*(j^*) - (1+\tau^{r*})P^{I*} x^*(j^*) \text{ s.t.}\\ (Tech)^*(j^*) : y^*(j^*) = x^*(j^*),\\ (Demand)^*(j^*) : y^*(j^*) = \left(\frac{P_F^*(j^*)}{P_F^*}\right)^{-\varepsilon} c_F^* + \left(\frac{P_F(j^*)}{P_F}\right)^{-\varepsilon} c_F,\\ (PCP)^*(j^*) : P_F(j^*) = EP_F^*(j^*),\end{aligned} \tag{2.11}$$

respectively, where the $(Demand)$ constraints reflect the standard solution of households' lower-stage optimization problem in (2.6) and (2.7) given CES price indices defined in (2.4) and (2.5), the nominal exchange rate $E$ is units of Home currency per unit of Foreign's currency, and the $(PCP)$ constraints reflect the assumption of producer-currency pricing in the global economy (which is also already baked into the specification of retailers' profit functions).

I now turn to market clearing and budget balance. For intermediate goods, it must be that

$$\int_0^1 x(j)dj = a\left[(1-\delta)n_0 + q(\theta)v - kv\right], \tag{2.12}$$

$$\int_0^1 x^*(j^*)dj^* = a^*\left[(1-\delta^*)n_0^* + q^*(\theta^*)v^* - k^*v^*\right]. \tag{2.13}$$

For final goods, it must be that

$$c_H(j) + c_H^*(j) = y(j), \tag{2.14}$$

$$c_F(j^*) + c_F^*(j^*) = y^*(j^*) \tag{2.15}$$

for each $j \in [0,1]$ and $j^* \in [0,1]$. Each country's fiscal budget balance requires

$$\int_0^1 \tau^r P^I x(j) dj + T = 0, \tag{2.16}$$

$$\int_0^1 \tau^{r*} P^{I*} x^*(j^*) dj^* + T^* = 0. \tag{2.17}$$

Finally, I characterize the money market. I assume ad-hoc money market equilibrium conditions

$$\bar{M} = \frac{\psi}{\gamma} P_H c_H, \tag{2.18}$$

$$\bar{M}^* = \frac{\psi^*}{\gamma} P_F^* c_F^*, \tag{2.19}$$

which could be microfounded by giving agents separable utility from real money balances.[9] I capture the monetary union between Home and Foreign by assuming that policymakers maintain a fixed exchange rate

$$E = \bar{E} \tag{2.20}$$

between the two.

We are now ready to define a flexible price and wage equilibrium.

**Definition 2.1.** *A* flexible price and wage equilibrium *is an allocation* $\{\{\{c_H(j)\}_j, \{c_F(j^*)\}_{j^*}, u, v, \theta, n, \{x(j)\}_j, \{y(j)\}_j\}, \ \{\{c_H^*(j)\}_j, \{c_F^*(j^*)\}_{j^*}, u^*, v^*, \theta^*, n^*, \{x^*(j^*)\}_{j^*}, \{y^*(j^*)\}_{j^*}\}$ *and set of nominal wages, prices, and profits* $\{\{W, P^I, \{P_H(j)\}_j, \{P_H^*(j)\}_j, \Pi, \{\Pi^r(j)\}_j\}, \ \{W^*, P^{I*}, \{P_F(j^*)\}_{j^*}, \{P_F^*(j^*)\}_{j^*}, \Pi^*, \{\Pi^{r*}(j^*)\}_{j^*}\}\}$ *such that, given policy* $\{\{\tau^r, T, \bar{M}\}, \{\tau^{r*}, T^*, \bar{M}^*\}, \bar{E}\}$*:*

1. *households solve (2.6) and (2.7);*

2. *producers solve (2.8) and (2.9);*

3. *retailers solve (2.10) and (2.11);*

4. *wages are Nash bargained (as detailed in Appendix C.2);*

[9] Ad-hoc money market equilibrium is commonly assumed in the background of dynamic monetary models at the "cashless limit", as in Woodford (2003) and Gali (2008). I adopt it in the present static setting so that my results on optimal stabilization policy are unaffected by utility fluctuations from real money balances.

5. *tightness is consistent with aggregate vacancies and job-seekers* ($\theta = \frac{v}{u}, \theta = \frac{v^*}{u^*}$)*;*

6. *intermediate and final goods markets clear according to (2.12)-(2.15);*

7. *fiscal budgets are balanced according to (2.16) and (2.17);*

8. *and the money market is assumed to "clear" and be consistent with fixed exchange rate* $\bar{E}$ *according to (2.18)-(2.20).*

In addition to the flexible price and wage equilibrium, this paper is concerned with the equilibrium when not all retailers are able to update their prices in response to TFP shocks. In particular, suppose that only measure $\iota$ and $\iota^*$ of retailers in Home and Foreign can freely set prices according to (2.10) and (2.11), respectively (without loss of generality, index retailers such that these are the "first" $\iota$ and $\iota^*$ retailers). The remaining retailers must remain committed to preset prices

$$P_H(j) = \bar{P}_H, \quad P_F^*(j^*) = \bar{P}_F^*. \tag{2.21}$$

Continuing to assume producer-currency pricing, these retailers will accommodate consumption demand at posted prices provided they earn non-negative profits:

$$x(j) = y(j) = \begin{cases} \left(\frac{\bar{P}_H}{P_H}\right)^{-\varepsilon} c_H + \left(\frac{\bar{P}_H/E}{P_H^*}\right)^{-\varepsilon} c_H^* \text{ provided } \bar{P}_H \geq (1+\tau^r)P^I, \\ 0 \text{ otherwise,} \end{cases} \tag{2.22}$$

$$x^*(j^*) = y^*(j^*) = \begin{cases} \left(\frac{\bar{P}_F^*}{P_F^*}\right)^{-\varepsilon} c_F^* + \left(\frac{E\bar{P}_F^*}{P_F}\right)^{-\varepsilon} c_F \text{ provided } \bar{P}_F^* \geq (1+\tau^{r*})P^{I*}, \\ 0 \text{ otherwise.} \end{cases} \tag{2.23}$$

This motivates the following definition of a sticky price equilibrium.

**Definition 2.2.** *A* sticky price equilibrium *is an allocation and set of nominal wages, prices, and profits such that, given policy, conditions 1-2 and 4-8 of Definition 2.1 are satisfied; the first* $\iota$ *and* $\iota^*$ *of retailers solve (2.10) and (2.11); and the remaining* $1-\iota$ *and* $1-\iota^*$ *of retailers solve (2.21)-(2.23).*

### 2.2.3 Characterizing equilibrium

I now derive equilibrium conditions for general final goods prices $\{P_H(j)\}_j$, $\{P^*_H(j)\}_j$, $\{P_F(j^*)\}_{j^*}$, and $\{P^*_F(j^*)\}_{j^*}$ provided that these are consistent with producer-currency pricing. The resulting conditions hold both in the flexible price and wage equilibrium as well as the sticky price equilibrium. In the next two sections of the paper, I complete the description of each by characterizing equilibrium final-goods prices in each case. A reader uninterested in the derivation of equilibrium can proceed directly to the next two sections.

First consider trade balance in equilibrium absent international risk-sharing:[10]

$$\int_0^1 P_F(j^*)c_F(j^*)dj^* = \int_0^1 P_H(j)c^*_H(j)dj.$$

Now consider households' consumption allocation problems in (2.6) and (2.7). The standard solution given the nested CES structure of preferences implies

$$c_H(j) = \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} c_H, \quad c_F(j^*) = \left(\frac{P_F(j^*)}{P_F}\right)^{-\varepsilon} c_F, \tag{2.24}$$

$$c^*_H(j) = \left(\frac{P^*_H(j)}{P^*_H}\right)^{-\varepsilon} c^*_H, \quad c^*_F(j^*) = \left(\frac{P^*_F(j^*)}{P^*_F}\right)^{-\varepsilon} c^*_F, \tag{2.25}$$

which implies that trade balance can be written

$$P_F c_F = P_H c^*_H.$$

Defining the *terms of trade*

$$s \equiv \frac{P_H}{P_F} = \frac{P^*_H}{P^*_F},$$

where the equality follows from producer-currency pricing, we have that the equilibrium

[10]Formally, this can be derived as follows. Given Home households' budget constraint in (2.6), substitute on the right-hand side for domestic firm profits, the consistency of tightness with aggregate vacancies and job-seekers, fiscal budget balance (2.16), and intermediate goods market clearing (2.12), obtaining

$$\int_0^1 P_H(j)c_H(j)dj + \int_0^1 P_F(j^*)c_F(j^*)dj^* = \int_0^1 P_H(j)y(j)dj.$$

This simply reflects the identity that aggregate consumption must equal aggregate income, absent international risk-sharing. Substituting on the right-hand side for final goods market clearing for each domestic variety $j$ in (2.14), and cancelling terms involving each $c_H(j)$, yields the desired result.

terms of trade satisfy

$$s = \frac{c_F}{c_H^*}.$$

Since the nested CES structure of preferences further implies that

$$c_F = \left(\frac{1-\gamma}{\gamma}\right)\frac{P_H}{P_F}c_H, \quad c_H^* = \left(\frac{1-\gamma}{\gamma}\right)\frac{P_F^*}{P_H^*}c_F^*, \tag{2.26}$$

we can equivalently say that the equilibrium terms of trade satisfy

$$s = \frac{c_F^*}{c_H}. \tag{2.27}$$

Turn now to equilibrium in the labor market. Producers' optimization problems (2.8) and (2.9) require that

$$P^I a - W = P^I \frac{ka}{q(\theta)},$$
$$P^{I*}a^* - W^* = P^{I*}\frac{k^* a^*}{q^*(\theta^*)}$$

for an interior optimum in vacancy posting to exist. As shown in Appendix C.2, the Nash bargained wage in each country is

$$W = -P\frac{u_n}{u_c} + \frac{\beta}{1-\beta}(P^I a - W),$$
$$W^* = -P^*\frac{u_n^*}{u_c^*} + \frac{\beta^*}{1-\beta^*}(P^{I*}a^* - W^*),$$

where $P$ and $P^*$ are the standard upper-level price indices

$$P = \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}(P_H)^\gamma(P_F)^{1-\gamma}, \quad P^* = \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}(P_H^*)^{1-\gamma}(P_F^*)^\gamma.$$

Combining these with interiority in vacancy posting, we obtain labor market equilibrium conditions

$$P^I a = -P\frac{u_n}{u_c} + \frac{1}{1-\beta}P^I\frac{ka}{q(\theta)},$$
$$P^{I*}a^* = -P^*\frac{u_n^*}{u_c^*} + \frac{1}{1-\beta^*}P^{I*}\frac{k^*a^*}{q^*(\theta^*)}.$$

Given preferences as in Cole and Obstfeld (1991), we can re-express the marginal rate of substitution in each country in terms of that between labor and domestic consumption:

$$-P\frac{u_n}{u_c} = \frac{\chi}{\gamma}P_H c_H(n)^{\varphi},$$
$$-P^*\frac{u_n^*}{u_c^*} = \frac{\chi^*}{\gamma}P_F^* c_F^*(n^*)^{\varphi^*}.$$

Hence, labor market equilibrium can be summarized by

$$P^I a = \frac{\chi}{\gamma}P_H c_H(n)^{\varphi} + \frac{\beta}{1-\beta}P^I\frac{ka}{q(\theta)}, \tag{2.28}$$
$$P^{I*}a^* = \frac{\chi^*}{\gamma}P_F^* c_F^*(n^*)^{\varphi^*} + \frac{\beta^*}{1-\beta^*}P^{I*}\frac{k^*a^*}{q^*(\theta^*)}. \tag{2.29}$$

Moreover, since the equilibrium wage exceeds the marginal rate of substitution between labor and consumption in each country, is is straightforward to show that households will optimally ensure all initially unemployed agents participate in the labor market:

$$u = 1-(1-\delta)n_0,$$
$$u^* = 1-(1-\delta^*)n_0^*.$$

Equilibrium employment in each market is thus

$$n = (1-\delta)n_0 + p(\theta)(1-(1-\delta)n_0), \tag{2.30}$$
$$n^* = (1-\delta^*)n_0^* + p^*(\theta^*)(1-(1-\delta^*)n_0^*). \tag{2.31}$$

Lastly, consider final good market clearing (2.14) and (2.15) for each variety produced by each country. Integrating over varieties, we obtain

$$\int_0^1 c_H(j)dj + \int_0^1 c_H^*(j)dj = \int_0^1 y(j)dj,$$
$$\int_0^1 c_F(j^*)dj^* + \int_0^1 c_F^*(j^*)dj = \int_0^1 y^*(j^*)dj^*.$$

Substituting in households' optimal consumption of each variety in (2.24) and (2.25), and making use of retailers' production technologies $y(j) = x(j)$ and $y^*(j^*) = x^*(j^*)$, intermediate good market clearing in (2.12) and (2.13), and the consistency of tightness with aggregate vacancies and job-seekers, we obtain

$$\left(\int_0^1 \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} dj\right) c_H + \left(\int_0^1 \left(\frac{P_H^*(j)}{P_H^*}\right)^{-\varepsilon} dj\right) c_H^* = an - kav,$$
$$\left(\int_0^1 \left(\frac{P_F(j^*)}{P_F}\right)^{-\varepsilon} dj^*\right) c_F + \left(\int_0^1 \left(\frac{P_F^*(j^*)}{P_F^*}\right)^{-\varepsilon} dj^*\right) c_F^* = a^*n^* - k^*a^*v^*.$$

Defining the price dispersion indices

$$D_H \equiv \int_0^1 \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} dj = \int_0^1 \left(\frac{P_H^*(j)}{P_H^*}\right)^{-\varepsilon} dj, \tag{2.32}$$
$$D_F^* \equiv \int_0^1 \left(\frac{P_F(j^*)}{P_F}\right)^{-\varepsilon} dj^* = \int_0^1 \left(\frac{P_F^*(j^*)}{P_F^*}\right)^{-\varepsilon} dj^*, \tag{2.33}$$

where the equalities make use of producer-currency pricing, we have

$$c_H + c_H^* = \frac{1}{D_H} a\left(n - kv\right),$$
$$c_F + c_F^* = \frac{1}{D_F^*} a^*\left(n^* - k^*v^*\right).$$

Substituting in for import consumption $c_H^*$ and $c_F$ using (2.26) and (2.27), and using the optimal full participation of unemployed agents in the labor market coupled with tightness consistent with aggregate vacancies and job-seekers, we obtain

$$c_H = \gamma \frac{1}{D_H} a\left(n - k(1-(1-\delta)n_0)\theta\right), \tag{2.34}$$
$$c_F^* = \gamma \frac{1}{D_F^*} a^*\left(n^* - k^*(1-(1-\delta^*)n_0^*)\theta^*\right). \tag{2.35}$$

In the next two sections, I will complement equilibrium conditions (2.24)-(2.35) with conditions on final good prices to characterize the flexible price/wage and sticky price equilibria, respectively.

## 2.3 Natural allocation: equilibrium with flexible prices and wages

In this section I characterize the *natural allocation*, the quantities and relative prices in the equilibrium with flexible prices and wages. I first summarize the welfare-relevant components of the natural allocation by a system of nine, transparent equations in nine unknowns. I then study this system to outline three key properties of the natural allocation, and define a notion of sclerotic vs. fluid labor markets in steady-state, setting the stage for the stabilization results in the next section.

### 2.3.1 Summarizing the natural allocation

When retailers are free to set prices after the realization of TFP shocks in each country, the solutions to (2.10) and (2.11) imply standard optimal price-setting conditions

$$P_H(j) = \frac{\varepsilon}{\varepsilon-1}(1+\tau^r)P^I, \quad P_F^*(j^*) = \frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})P^{I*}. \tag{2.36}$$

Given identical final goods prices, workers' optimally consume identical amounts of varieties produced by a given country (as seen in (2.24) and (2.25)), so we can focus on characterizing the aggregators $\{c_H, c_F, c_H^*, c_F^*\}$. Plugging in (2.36) into the goods market equilibrium conditions in (2.34) and (2.35) and the labor market equilibrium conditions in (2.28) and (2.29), and re-considering equilibrium conditions (2.26), (2.27), (2.30), and (2.31), we obtain the following characterization of the natural allocation (denoted with $n$ superscripts).

**Lemma 2.1.** *In the natural allocation,* $\{\{c_H^n, c_F^n, \theta^n, n^n\}, \{c_H^{*n}, c_F^{*n}, \theta^{*n}, n^{*n}\}, s^n\}$ *satisfy*

$$\begin{cases} \frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^r)}a = \frac{\chi}{\gamma}c_H^n(n^n)^{\varphi} + \frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^r)}\left(\frac{1}{1-\beta}\right)\frac{ka}{q(\theta^n)}, \\ n^n = (1-\delta)n_0 + p(\theta^n)(1-(1-\delta)n_0), \\ c_H^n = \gamma a\left[n^n - k(1-(1-\delta)n_0)\theta^n\right], \end{cases} \tag{2.37}$$

$$\begin{cases} \frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})}a^* = \frac{\chi^*}{\gamma}c_F^{*n}(n^{*n})^{\varphi^*} + \frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})}\left(\frac{1}{1-\beta^*}\right)\frac{k^*a^*}{q^*(\theta^{*n})}, \\ n^{*n} = (1-\delta^*)n_0^* + p^*(\theta^{*n})(1-(1-\delta^*)n_0^*), \\ c_F^{*n} = \gamma a^*\left[n^{*n} - k^*(1-(1-\delta^*)n_0^*)\theta^{*n}\right], \end{cases} \tag{2.38}$$

$$\begin{cases} s^n = \frac{c_F^{*n}}{c_H^n}, \\ c_F^n = \frac{1-\gamma}{\gamma}s^n c_H^n, \\ c_H^{*n} = \frac{1-\gamma}{\gamma}\frac{1}{s^n}c_F^{*n}. \end{cases} \tag{2.39}$$

Lemma 2.1 makes evident two immediate features of the natural allocation. First, it is fully determined without reference to nominal prices and wages in the global economy, or the (fixed) exchange rate between countries, reflecting a standard real/nominal dichotomy. Second, it is bloc recursive: domestic consumption and labor market conditions in each country are fully determined on their own, after which the terms of trade and consumption of imported goods are determined. I discuss the implications of this result, and other key properties of the system (2.37)-(2.39), next.

### 2.3.2 Three properties of the natural allocation

An analysis of (2.37)-(2.39) reveals three key properties of the natural allocation which are important in understanding the stabilization results of the next section. I present them here as the first proposition of the paper, and then discuss each in further detail.

**Proposition 2.1.** *Three properties characterize the natural allocation:*

1. *Employment in each country is invariant to domestic TFP.*
2. *Employment in each country is invariant to foreign structural parameters and foreign TFP.*
3. *Define the constrained efficient allocation as that chosen by a planner maximizing union-wide utilitarian social welfare subject to the global economy's technological constraints in producing and recruiting. Then the natural allocation is constrained efficient if and only if in each country retailers are subsidized to offset the distortion from monopolistic competition ($\tau^r = \tau^{r*} = -\frac{1}{\varepsilon}$) and the Hosios (1990) condition holds ($\beta = 1 - \eta$, $\beta^* = 1 - \eta^*$).*

Property 1 generalizes the benchmark results of Blanchard and Gali (2010) and Shimer (2010) in the macro/labor literature to the open economy. These authors demonstrate that in a real business cycle framework with DMP labor market frictions, log preferences over consumption and recruiting costs which scale with TFP mean that equilibrium employment is invariant to TFP. This itself generalizes the standard result from a frictionless RBC model in which, under log preferences, income and substitution effects from changes in TFP exactly offset to leave equilibrium hours unchanged. Along with the earlier work of Shimer (2005), these' authors' neutrality result has given rise to a large literature on the "unemployment volatility puzzle", seeking other ways to explain the observed cyclical fluctuations in unemployment. My result implies that their neutrality result *further* generalizes to the open economy in the presence of Cole and Obstfeld (1991) preferences.

Property 2 generalizes a benchmark result of Clarida *et al.* (2002) in the international macro literature to a frictional labor market. These authors demonstrate that in a two-country global economy with trade in intermediate goods, domestic real marginal cost is invariant to foreign output in the presence of Cole and Obstfeld (1991) preferences. In this case, greater foreign output has offsetting impacts on the terms of trade and domestic consumption, rendering domestic real marginal cost unchanged. They demonstrate that this invariance result has important implications — for instance, only away from this case are there gains from international monetary policy coordination. My result implies that their separability result generalizes to a frictional labor market provided that recruiting costs only use domestic resources and scale with domestic TFP.

Finally, property 3 implies that standard efficiency results from the monetary, international macro, and macro/labor literatures continue to induce efficiency in this richer environment which combines all three. Relative to the constrained efficient allocation, there are three sets of possible distortions in the competitive equilibrium: monopoly power among retailers in each country, missing international risk-sharing markets, and search externalities imposed on others in a frictional labor market. A subsidy provided to retailers can undo the first, a standard result in the monetary literature. As shown by Cole and Obstfeld (1991), the assumed preferences undo the second since relative price movements in goods markets provide perfect cross-country insurance; this result generalizes to the frictional labor market when recruiting requires only domestic resources. Finally, participants in the labor market only impose search externalities on others in the domestic market in view of the assumed lack of cross-country labor mobility, implying that the Hosios (1990) condition ensures that these externalities are internalized just as in the closed economy.

### 2.3.3 Steady-state and sclerotic vs. fluid labor markets

Before turning to sticky prices and the union's stabilization problem, I define the union's *steady-state*. Suppose that the random process governing TFP $\{a, a^*\}$ means that it fluctuates around some $\{\bar{a}, \bar{a}^*\}$. I denote the equilibrium allocation in this steady-state with bars, and assume its distinctive characteristic is that employment is unchanged from the fictional prior period:

**Assumption 2.1.** *$\{n_0, n_0^*\}$ are such that $\bar{n} = n_0$, $\bar{n}^* = n_0^*$.*

Given the equilibrium evolution of employment in (2.30) and (2.31), this assumption means that in steady-state

$$\delta\bar{n} = p(\bar{\theta})(1-(1-\delta)n), \quad \delta^*\bar{n}^* = p^*(\bar{\theta}^*)(1-(1-\delta^*)n^*),$$

allowing me to interpret the separation rates $\{\delta, \delta^*\}$ as the magnitude of steady-state flows in each labor market even in this one-period environment.

Following Blanchard and Gali (2010), in what follows I will focus on the distinction between a *sclerotic* and a *fluid* labor market, where the former is characterized as having greater hiring costs (higher $k$) and lower labor market flows (lower $\delta$). Using comparative statics on the natural allocation characterized in Lemma 2.1, it can be shown that a more sclerotic labor market will tend to exhibit lower steady-state employment than a more fluid labor market:

**Lemma 2.2.** *Steady-state employment in a given country falls as:*

- *hiring costs in that country rise ($\frac{d\bar{n}}{dk} < 0$, $\frac{d\bar{n}^*}{dk^*} < 0$);*
- *labor market flows in that country fall ($\frac{d\bar{n}}{d\delta} > 0$, $\frac{d\bar{n}^*}{d\delta^*} > 0$), at least in the neighborhood of small steady-state unemployment ($1-\bar{n}, 1-\bar{n}^*$ close to zero).*

In a dynamic environment, $\{k, k^*, \delta, \delta^*\}$ will also mediate the continuation value of a match in each country, a channel which is missing here and which underscores the importance of advancing the present framework to an infinite horizon in future work.

Nonetheless, even in this one-period environment, $\{k, k^*, \delta, \delta^*\}$ will affect the equilibrium cost of output fluctuations and thus my results on macroeconomic stabilization in a direction which, I conjecture, is robust to an infinite horizon.

## 2.4 Stabilization: sticky prices, trade-offs, and optimal policy

In this section I study the equilibrium with partially sticky prices, obtaining the main results of the paper on macroeconomic trade-offs and optimal second-best monetary policy. I first characterize a linear-quadratic approximation to the Ramsey policy problem. I then solve this problem to obtain two main results: (i) *institutional irrelevance* for the question of when the constrained efficient allocation is achievable, and (ii) *relative accommodation* of the more sclerotic labor market — the one with greater hiring costs or smaller labor market flows — at the second-best optimum.

### 2.4.1 Sticky price equilibrium and the Ramsey policy problem

Let $\mathcal{P}_H$ and $\mathcal{P}_F^*$ be the final goods prices chosen by the $\iota$ Home and $\iota^*$ Foreign retailers who can set prices after the realization of TFP shocks, respectively. In all of the analysis which follows, I will assume that retailer subsidies are fixed at $\tau^r = \tau^{r*} = -\frac{1}{\varepsilon}$. Following (2.36), these retailers will uniformly choose

$$\mathcal{P}_H = \frac{\varepsilon}{\varepsilon - 1}(1 + \tau^r)P^I = P^I, \quad \mathcal{P}_F^* = \frac{\varepsilon}{\varepsilon - 1}(1 + \tau^{r*})P^{I*} = P^{I*}.$$

The remaining $1 - \iota$ and $1 - \iota^*$ retailers in each country will accommodate demand at posted prices $\bar{P}_H$ and $\bar{P}_F^*$, respectively. Per Definition 2.2 and the other conditions derived in section 2.2.3, it is straightforward to summarize the sticky price equilibrium with a system of conditions in which the real and nominal sides of the global economy are now jointly determined, given a particular choice of union-wide monetary policy $\{\bar{M}, \bar{M}^*\}$ consistent with $E = \bar{E}$ in a monetary union.

Here, I proceed directly to characterize the set of implementable real allocations given the possible choices of $\{\bar{M}, \bar{M}^*\}$ consistent with $E = \bar{E}$. In addition to summarizing the essence of the equilibrium, these conditions serve as constraints in the optimal policy problem described below.

**Lemma 2.3.** *The real allocation* $\{\{c_H, c_F, \theta, n\}, \{c_H^*, c_F^*, \theta^*, n^*\}, s\}$ *forms part of a sticky price equilibrium if and only if there exist nominal prices/indices* $\{\{\mathcal{P}_H, P_H, D_H\}, \{\mathcal{P}_F^*, P_F^*, D_F^*\}\}$ *such that:*

$$\begin{cases} \frac{\mathcal{P}_H}{P_H} a = \frac{\chi}{\gamma} c_H (n)^{\varphi} + \frac{\mathcal{P}_H}{P_H} \left(\frac{1}{1-\beta}\right) \frac{ka}{q(\theta)}, \\ n = (1-\delta) n_0 + p(\theta)(1-(1-\delta)n_0), \\ c_H = \gamma \frac{1}{D_H} a \left[n - k(1-(1-\delta)n_0)\theta\right], \end{cases} \tag{2.40}$$

$$\begin{cases} \frac{\mathcal{P}_F^*}{P_F^*} a^* = \frac{\chi^*}{\gamma} c_F^* (n^*)^{\varphi^*} + \frac{\mathcal{P}_F^*}{P_F^*} \left(\frac{1}{1-\beta^*}\right) \frac{k^* a^*}{q^*(\theta^*)}, \\ n^* = (1-\delta^*) n_0^* + p^*(\theta^*)(1-(1-\delta^*)n_0^*), \\ c_F^* = \gamma \frac{1}{D_F^*} a^* \left[n^* - k^*(1-(1-\delta^*)n_0^*)\theta^*\right], \end{cases} \tag{2.41}$$

$$\begin{cases} s = \frac{c_F^*}{c_H}, \\ c_F = \frac{1-\gamma}{\gamma} s c_H, \\ c_H^* = \frac{1-\gamma}{\gamma} \frac{1}{s} c_F^*, \end{cases} \tag{2.42}$$

$$\begin{cases} P_H = \left[\iota (\mathcal{P}_H)^{1-\varepsilon} + (1-\iota)(\bar{P}_H)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \\ D_H = \iota (\frac{\mathcal{P}_H}{P_H})^{-\varepsilon} + (1-\iota)(\frac{\bar{P}_H}{P_H})^{-\varepsilon}, \end{cases} \tag{2.43}$$

$$\begin{cases} P_F^* = \left[\iota^* (\mathcal{P}_F^*)^{1-\varepsilon} + (1-\iota^*)(\bar{P}_F^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \\ D_F^* = \iota^* (\frac{\mathcal{P}_F^*}{P_F^*})^{-\varepsilon} + (1-\iota^*)(\frac{\bar{P}_F^*}{P_F^*})^{-\varepsilon}, \end{cases} \tag{2.44}$$

$$\frac{P_H}{\bar{E} P_F^*} = s. \tag{2.45}$$

I organize these constraints into six blocks to facilitate comparison with the natural allocation described in Lemma 2.1. (2.40) and (2.41) clearly generalize (2.37) and (2.38) to the case with potential dispersion in retailer prices (given $\tau^r = \tau^{r*} = -\frac{1}{\varepsilon}$). (2.42) is identical to (2.39). (2.43) and (2.44) summarize prices in each country in the sticky price equilibrium, and (2.45) is the fundamental constraint on implementable allocations imposed by a fixed exchange rate.

Now consider the problem faced by the union's central bank seeking to choose $\{\bar{M}, \bar{M}^*\}$

to maximize union-wide utilitarian social welfare in response to TFP shocks. Given Lemma 2.3, Ramsey optimal monetary policy will implement the real allocation maximizing

$$U \equiv u(c,n) + u^*(c^*, n^*) \tag{2.46}$$

subject to implementability constraints (2.40)-(2.45).

### 2.4.2 Linear-quadratic approximation to the Ramsey problem

I follow the monetary economics literature in adopting the following notation for any endogenous variable $z$: let $\bar{z}$ denote its value under the steady-state realization of TFP shocks, $\hat{z} \equiv \log z - \log \bar{z}$ denote the log deviation from steady-state, $\hat{z}^n \equiv \log z^n - \log \bar{z}$ denote the log deviation of $z$ under the natural allocation from steady-state, and $\tilde{z} \equiv \hat{z} - \hat{z}^n$ denote the log deviation of $z$ from its value under the natural allocation. I will refer to $\tilde{z}$ as the "gap" in $z$.

I now specify definitions of inflation, production, and output which will substantially ease the analysis of the stabilization problem in the present environment. Define (producer-price) inflation in each country

$$\pi_H \equiv \log P_H - \log \bar{P}_H,$$

$$\pi_F^* \equiv \log P_F^* - \log \bar{P}_F^*.$$

This definition captures the idea that the sticky prices $\bar{P}_H$ and $\bar{P}_F^*$ reflect retailer prices in the fictional "prior" period to the one-period model under consideration. Define the effective units of labor engaged in production (*inclusive* of equilibrium recruiting activity)

$$f(n) \equiv n - k(1-(1-\delta)n_0)p^{-1}\left(\frac{n-(1-\delta)n_0}{1-(1-\delta)n_0}\right),$$

$$f^*(n^*) \equiv n^* - k^*(1-(1-\delta^*)n_0^*)p^{*-1}\left(\frac{n^*-(1-\delta^*)n_0^*}{1-(1-\delta^*)n_0^*}\right),$$

such that, given the evolution of employment in (2.30) and (2.31) and consistency of tightness with aggregate vacancies and job search, in equilibrium the aggregate output of intermediate goods in each country is characterized by the technological relation

$$x \equiv a[(1-\delta)n_0 + q(\theta)v - kv] = af(n),$$
$$x^* \equiv a^*[(1-\delta^*)n_0^* + q^*(\theta^*)v^* - k^*v^*] = a^*f^*(n^*).$$

I now make the following assumptions:

**Assumption 2.2.** *The natural allocation is constrained efficient (implemented as described in property 3 of Proposition 2.1).*

**Assumption 2.3.** *The TFP shocks $\{a, a^*\}$ are small around the steady-state $\{\bar{a}, \bar{a}^*\}$.*

**Assumption 2.4.** *The preset prices $\bar{P}_H$ and $\bar{P}_F^*$ are consistent with a zero-inflation steady-state under the Ramsey optimal policy characterized below.*

Following Woodford (2003), assumptions 2.2-2.4 ensure that the Ramsey optimal allocation can be approximated up to first order in deviations from steady-state by studying a linear approximation to the implementability constraints (2.40)-(2.45) and quadratic approximation to social welfare (2.46).

I first discuss the quadratic approximation to utilitarian social welfare.

**Lemma 2.4.** *Up to second order around steady-state, (2.46), making use of (2.40)-(2.44), implies*

$$U - \bar{U} = -\frac{1}{2}\left[(\lambda_\pi \pi_H^2 + \lambda_x \tilde{x}^2) + (\lambda_\pi^* (\pi_F^*)^2 + \lambda_x^* (\tilde{x}^*)^2)\right] + tips \tag{2.47}$$

*where $tips$ denotes terms independent of policy,*

$$\lambda_\pi \equiv \left(\frac{1-\iota}{\iota}\right)\varepsilon,$$
$$\lambda_\pi^* \equiv \left(\frac{1-\iota^*}{\iota^*}\right)\varepsilon^*,$$
$$\lambda_x \equiv 1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f},$$
$$\lambda_x^* \equiv 1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}},$$

*and the $\epsilon$ terms denote elasticities of the technologies $f$ and $f^*$:*

$$\epsilon_n^f \equiv \frac{f'(n)n}{f(n)}, \quad \epsilon_n^{f'} \equiv \frac{f''(n)n}{f'(n)},$$
$$\epsilon_{n^*}^{f^*} \equiv \frac{f^{*\prime}(n^*)n^*}{f^*(n^*)}, \quad \epsilon_{n^*}^{f^{*\prime}} \equiv \frac{f^{*\prime\prime}(n^*)n^*}{f^{*\prime}(n^*)}.$$

As is standard in the monetary economics literature, the welfare loss from relative price dispersion in each country is rising in the stickiness of prices and the elasticity of substitution across varieties $\varepsilon$. The welfare loss from output distortions in each country is rising in the inverse Frisch elasticity of labor supply $\{\varphi, \varphi^*\}$, as this controls the disutility of incremental work; is falling in the returns to scale in production $\{\epsilon_n^f, \epsilon_{n'}^{f'}\}$, as this informs how many more workers are needed to produce the incremental unit; and is rising in the (absolute value of the) elasticity of marginal production with respect to employment $\{-\epsilon_n^{f'}, -\epsilon_{n'}^{f'}\}$, as this governs how much the productivity of the last worker falls with higher employment.

I now turn to the linear approximation to the implementability constraints.

**Lemma 2.5.** *Up to first order around steady-state, (2.40)-(2.45) imply*

$$\pi_H = \mu \tilde{x}, \tag{2.48}$$

$$\pi_F^* = \mu^* \tilde{x}^*, \tag{2.49}$$

$$\hat{s}^n = (\pi_H + \tilde{x}) - (\pi_F^* + \tilde{x}^*), \tag{2.50}$$

*where*

$$\mu \equiv \frac{\iota}{1-\iota}\left[1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f}\right],$$
$$\mu^* \equiv \frac{\iota^*}{1-\iota^*}\left[1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}}\right].$$

(2.48) and (2.49) relate producer-price inflation with the output gap in each economy, and thus constitute the Phillips Curves in this one-period environment. The slopes $\{\mu, \mu^*\}$ are rising in the degree of price flexibility $\{\iota, \iota^*\}$, as well as a term summarizing the increase in firms' real marginal costs in response to an increase in output. We see that the latter term in fact is identical to the welfare cost of output fluctuations $\{\lambda_x, \lambda_{x^*}\}$, just as it is in the benchmark without DMP frictions (e.g., Woodford (2003) and Gali (2008)). This follows from the constrained efficiency of the natural allocation in assumption 2.2 (in particular, the assumption that the Hosios (1990) condition holds), which creates a tight link between the first derivative of the social welfare function with respect to output and the real marginal cost faced by firms in the competitive equilibrium.

(2.50) summarizes the key constraint imposed on countries by being part of a monetary union. The combination of sticky prices and a fixed exchange rate hinders adjustment in the terms of trade in the competitive equilibrium. If the *natural terms of trade* $\hat{s}^n$ would adjust in response to macroeconomic shocks, this leads to costly inflation/deflation, output distortions, or both.

Taken together, (2.47)-(2.50) imply the linear-quadratic approximation to the Ramsey problem:

$$\begin{aligned} \min_{\pi_H, \tilde{n}, \pi_F^*, \tilde{n}^*} (\lambda_\pi \pi_H^2 + \lambda_x \tilde{x}^2) + (\lambda_\pi^* (\pi_F^*)^2 + \lambda_x^* (\tilde{x}^*)^2) \text{ s.t.} \\ \pi_H = \mu \tilde{x}, \\ \pi_F^* = \mu^* \tilde{x}^*, \\ \hat{s}^n = (\pi_H + \tilde{x}) - (\pi_F^* + \tilde{x}^*). \end{aligned} \tag{2.51}$$

Relative to the exact Ramsey problem, there are two advantages of this approximation. First, it lays bare that a sufficient statistic for inefficiency in a monetary union is movement in the natural terms of trade $\hat{s}^n$. Second, the coefficients on policymakers' objective and each country's Phillips Curve encode all the necessary information to characterize the effect of frictional and heterogeneous labor markets on the second-best optimal policy. In fact, in view of the tight link between the welfare cost of output fluctuations $\{\lambda_x, \lambda_x^*\}$ and the slopes of the Phillips Curves $\{\mu, \mu^*\}$, the effect of frictional labor markets on the second-best policy will operate through the *single* statistic in each country

$$\phi \equiv 1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f}, \quad \phi^* \equiv 1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}},$$

summarizing both the curvature in social welfare and the curvature of the cost function faced by firms as a function of domestic output. I will explore each of these points in more depth in the next two sub-sections, obtaining the paper's main results on institutional irrelevance and relative accommodation in a monetary union.

### 2.4.3 Inefficiency in a monetary union: *institutional irrelevance*

I first ask whether frictional and heterogeneous labor markets affect *when* a monetary union will face distortions.[11]

(2.51) makes evident that a nonzero deviation in the natural terms of trade from steady-state is a sufficient statistic for the existence of distortions in a monetary union. The following *institutional irrelevance* result finds that DMP frictions have no effect on movements in the natural terms of trade, and thus the circumstances under which such a union will face distortions.

**Proposition 2.2.** *In the natural allocation,*

$$\hat{s}^n = \hat{a}^* - \hat{a}.$$

*Thus, the constrained efficient allocation can be achieved iff* $\hat{a} = \hat{a}^*$ *regardless of the values of* $\{k, \delta, \beta = \eta\}$ *and* $\{k^*, \delta^*, \beta^* = \eta^*\}$.

Thus, under the benchmark set of preferences and technologies used in this paper, Mundell (1961)'s business cycle synchronicity criterion for optimal currency areas is unchanged despite arbitrary levels of and heterogeneity in DMP frictions across countries. Intuitively, recall that properties 1 and 2 in Proposition 2.1 mean that in the natural allocation, employment in each country is invariant to TFP shocks originating at home and from abroad. Hence, relative production across countries is simply a function of relative TFP, so the relative price of goods produced by each country — the terms of trade — is also

[11] By studying the linear-quadratic approximation, I can only assess the existence of first-order distortions in prices and output. But an examination of the Ramsey problem demonstrates that Proposition 2.2 in fact holds exactly.

just a function of relative TFP. There is thus no interaction between shocks and institutions determining relative prices in the natural allocation.

The arguments underlying Proposition 2.2 are useful in clarifying when institutions *will* interact with shocks to generate distortions in a monetary union. In particular, departures from the underlying assumptions giving rise to properties 1 and 2 of the natural allocation — Cole and Obstfeld (1991) preferences and the specification of recruiting costs as scaling with domestic TFP and invariant to foreign TFP — would break this benchmark result.

### 2.4.4 Second-best monetary policy: *relative accommodation*

I now ask whether frictional and heterogeneous labor markets affect *how* second-best optimal monetary policy should be conducted in the presence of asymmetric shocks.

To answer this question, it is useful to first solve the stabilization problem (2.51):

**Lemma 2.6.** *Up to first order deviations from steady-state, the Ramsey optimal allocation is characterized by*

$$\tilde{x} = \omega \hat{s}^n, \qquad \pi_H = \mu \omega \hat{s}^n,$$
$$\tilde{x}^* = -\omega^* \hat{s}^n, \qquad \pi_F^* = -\mu^* \omega^* \hat{s}^n,$$

*where*

$$\omega \equiv \frac{\left(\lambda_\pi^* (\mu^*)^2 + \lambda_x^*\right)(\mu + 1)}{\left(\lambda_\pi (\mu)^2 + \lambda_x\right)(\mu^* + 1)^2 + \left(\lambda_\pi^* (\mu^*)^2 + \lambda_x^*\right)(\mu + 1)^2} > 0,$$
$$\omega^* \equiv \frac{\left(\lambda_\pi (\mu)^2 + \lambda_x\right)(\mu^* + 1)}{\left(\lambda_\pi (\mu)^2 + \lambda_x\right)(\mu^* + 1)^2 + \left(\lambda_\pi^* (\mu^*)^2 + \lambda_x^*\right)(\mu + 1)^2} > 0.$$

Intuitively, given an appreciation in the natural terms of trade ($\hat{s}^n > 0$), sluggish price adjustment and the (implicitly) fixed exchange rate in the monetary union means that Home-produced goods will be "too cheap" relative to Foreign-produced goods. At the second-best optimum, the union must accept an inefficient boom at Home ($\tilde{x}, \pi_H > 0$) and recession in Foreign ($\tilde{x}^*, \pi_F^* < 0$).

In the present static environment, the Ramsey optimal allocation can be implemented equivalently by a rule targeting a weighted average of inflation rates or output gaps across the union. I focus here on the former representation, given that the ECB in fact only has a mandate for price stability, and (largely for that same reason) inflation targeting rules have been a focus of prior work on monetary unions such as Benigno (2004).

**Lemma 2.7.** *The Ramsey optimal allocation can be implemented by an inflation targeting rule*

$$\xi \pi_H + (1-\xi)\pi_F^* = 0$$

*with weight on Home producer-price inflation*

$$\xi \equiv \frac{\mu^* \omega^*}{\mu \omega + \mu^* \omega^*} = \frac{\frac{\lambda_\pi (\mu)^2 + \lambda_x}{\mu(\mu+1)}}{\frac{\lambda_\pi (\mu)^2 + \lambda_x}{\mu(\mu+1)} + \frac{\lambda_\pi^* (\mu^*)^2 + \lambda_x^*}{\mu^*(\mu^*+1)}}.$$

I now ask how heterogeneity in labor market frictions affects this optimal policy rule. As derived and discussed in section 2.4.2, the effect of frictional labor markets will operate through the single curvature parameter for each country

$$\phi \equiv 1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f}, \quad \phi^* \equiv 1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}}, \tag{2.52}$$

which governs both the welfare cost of output fluctuations (the curvature of social welfare in output) and the slope of the Phillips Curve (the curvature of firms' cost function in output) in each country:

$$\lambda_x = \phi, \quad \lambda_x^* = \phi^*,$$
$$\mu = \left(\frac{\iota}{1-\iota}\right)\phi, \quad \mu^* = \left(\frac{\iota^*}{1-\iota^*}\right)\phi^*.$$

This considerably simplifies our ability to characterize, and gain intuition behind, the effect of labor market frictions on the optimal policy rule. First, it is straightforward to show that the weight on a given country in the optimal inflation targeting rule unambiguously falls in its curvature parameter:

**Lemma 2.8.** *The optimal inflation targeting rule is characterized by*

$$\frac{d\xi}{d\phi} > 0, \quad \frac{d\xi}{d\phi^*} < 0.$$

The weight on Home in the equivalent optimal output-gap targeting rule is characterized by the same comparative statics. In this sense, optimal policy relatively accommodates the economy with a greater welfare cost of employment fluctuations and steeper Phillips Curve.

Now we need only characterize how the labor market frictions of interest — hiring costs $\{k, k^*\}$ and magnitude of flows $\{\delta, \delta^*\}$ — affect these curvature parameters. The next lemma expresses the curvature parameters in closed form.

**Lemma 2.9.** *The curvature parameters defined in (2.52), governing both the welfare cost of output fluctuations and the slope of the Phillips Curve in each country, are given by*

$$\phi \equiv 1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f} = 1 + \frac{\varphi + \frac{\frac{1}{\eta}\frac{k}{q(\theta)}}{1-\frac{1}{\eta}\frac{k}{q(\theta)}}\left(\frac{1-\eta}{\delta\eta}\right)}{\frac{1-\frac{1}{\eta}\frac{k}{q(\theta)}}{1-\delta\frac{k}{q(\theta)}}}, \quad \phi^* \equiv 1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}} = 1 + \frac{\varphi + \frac{\frac{1}{\eta^*}\frac{k^*}{q^*(\theta^*)}}{1-\frac{1}{\eta^*}\frac{k^*}{q^*(\theta^*)}}\left(\frac{1-\eta^*}{\delta^*\eta^*}\right)}{\frac{1-\frac{1}{\eta^*}\frac{k^*}{q^*(\theta^*)}}{1-\delta^*\frac{k^*}{q^*(\theta^*)}}}.$$

To gain intuition behind these expressions, note that as hiring costs disappear ($k, k^* \to 0$), it is straightforward to show that

$$1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f} \to 1 + \varphi, \quad 1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}} \to 1 + \varphi^*.$$

That is, the welfare cost of output fluctuations and the slope of the Phillips Curves converge to those in the frictionless benchmarks of Woodford (2003) and Gali (2008) with log utility over consumption, constant returns to scale in production, and a zero discount factor (in view of the fact that the present model is a static one). In these benchmarks, the curvature in welfare and firms' cost function arises purely from the increase in the marginal rate of substitution between labor and consumption given an increase in output.

Comparing these limiting cases to the expressions in Lemma 2.9, we see that recruiting costs in a frictional labor market have two consequences. First, they reduce the returns to scale in the economy's production function (lower $\epsilon_n^f$ and $\epsilon_{n^*}^{f^*}$ below one). Second, they raise the extent to which an increase in production reduces the marginal productivity of labor (raise $\epsilon_n^{f'}$ and $\epsilon_{n^*}^{f^{*\prime}}$ above zero). Both of these effects follow from the fact that recruiting costs siphon resources away from production, and thus generate deadweight costs of adjusting output.

Given property 2 of Proposition 2.1, the curvature parameter for one country is invariant to primitives in the other. Hence, we only need to characterize the comparative statics of each country's curvature parameter with respect to its own magnitude of hiring costs and labor market flows. Doing so, we obtain the paper's second main result on *relative accommodation* in a monetary union.

**Proposition 2.3.** *The welfare cost of output fluctuations and the slope of the Phillips Curve in a given country rises as*

- *that country's hiring costs rise* $\left(\frac{d\phi}{dk} > 0, \frac{d\phi^*}{dk^*} > 0\right)$,
- *that country's labor market flows get smaller* $\left(\frac{d\phi}{d\delta} < 0, \frac{d\phi^*}{d\delta^*} < 0\right)$, *at least for the case of small hiring costs* $\{k, k^*\}$ *and steady-state unemployment* $\{1-\bar{n}, 1-\bar{n}^*\}$.

*Hence, the weight on Home in the optimal inflation targeting rule* $\xi$ *rises as its labor market grows more sclerotic (*$k$ *rises or* $\delta$ *falls) or Foreign's labor market grows more fluid (*$k^*$ *falls or* $\delta^*$ *rises).*

Intuitively, higher hiring costs or smaller flows in a given country amplify the deadweight cost of inefficient hiring decisions, raising the welfare cost of output distortions and steepening the slope of the Phillips Curve. Facing greater welfare losses and greater inflationary/deflationary pressure from a given distortion in output, policy then targets smaller distortions in the more sclerotic union member at the second-best optimum.

This relative accommodation result is distinct from, but complementary with, the results in Benigno (2004). Benigno (2004) focuses on cross-country heterogeneity in the degree of nominal rigidity, whereas Proposition 2.3 focuses on heterogeneity in the costs with which firms and workers search and match. These affect the stabilization problem through distinct theoretical channels: the cost of relative price dispersion in the first case, versus the cost of output fluctuations in the second. They also suggest distinct empirical counterparts, as I discuss in the next section. At the same time, the results are complementary in that both imply the optimality of targeting policy to accommodate the more rigid member of the union. And indeed, the key insight of Benigno (2004) is nested and holds within my model despite the presence of rich and diverse labor markets across countries: holding fixed the degree of Foreign price stickiness ($\iota^*$), as Home prices approach perfect flexibility ($\iota \to 1$), optimal monetary policy fully seeks to stabilize inflation in Foreign ($\xi \to 0$).

## 2.5 Implications for the Eurozone and future directions for research

In this section I trace out the implications of my results for macroeconomic stabilization in the Eurozone, motivating fruitful directions for future work building on the framework here.

Broadly, the current analysis suggests that while labor market frictions may not directly bear on the optimality of the Eurozone as a currency area, they should be incorporated in the design of stabilization policy by union-wide institutions such as the ECB. In particular, the institutional irrelevance result clarifies that the existence and heterogeneity of labor market frictions across member states need not have anything to do with the ECB's ability to stabilize the union. However, the relative accommodation result implies that the institutional and technological features of labor markets across member states should inform the conduct of stabilization policy by the ECB when perfect stabilization is unattainable.

The theoretical mechanisms underlying my result on relative accommodation of the more sclerotic union member find support in European data and in other models. In particular, Campolmi and Faia (2011) report a positive correlation between employment protection laws and inflation volatility across Eurozone members, consistent with the model's key prediction that the sensitivity of marginal cost rises with hiring costs and falls with the magnitude of labor market flows. In a richer model studied numerically, Abbritti and Mueller (2013) also find that greater restrictions on flows into and out of unemployment steepen the Phillips curve for members of a monetary union.

This begs the provocative question: has the ECB, and more broadly Brussels, been getting stabilization policy wrong? In recent years, many commentators have argued that the policy of union-wide institutions has tended to favor the interests of core, Northern members at the expense of peripheral, Southern members. Yet on some labor market measures, such as the magnitude of legislated severance payments described in the Introduction, peripheral countries appear to have greater frictions in the matching process between workers and firms. A cursory application of the ideas in this paper would suggest that in this respect, Eurozone policy in recent years has been insufficiently accommodative of these countries.

To more fully address this question, future work should focus on extending the framework developed here to an infinite horizon and calibrating it to match salient labor market features of Eurozone economies. A dynamic extension will make full use of the conceptual advantages of a search and matching modeling of the labor market, where worker flows into and out of unemployment can be tractably studied. A quantitative calibration will allow a more formal evaluation of whether heterogeneity in the fluidity of Eurozone labor markets has a first-order effect on stabilization trade-offs and optimal policy — particularly as compared to other dimensions of heterogeneity studied elsewhere in the literature, such as the duration of prices or wages (Benigno (2004)) or debt levels across countries (Aguiar *et al.* (2015)).

Two other lines of inquiry would also be particularly interesting to expand beyond the specific mechanisms and trade-offs explored in this paper. First, it would be useful to study an enriched economic environment with endogenous layoffs and incomplete markets. This would permit a deeper analysis of heterogeneity in specific institutions such as severance payments and unemployment insurance across a monetary union, building on the seminal contributions of Acemoglu and Shimer (1999) and Blanchard and Tirole (2008), as well as some of my own work (Kekre (2016)). Second, it would be interesting to reverse the question asked by the present paper, and assess how the stabilization choices of union-wide bodies might affect the design of national labor market policies themselves. Might it be possible that by accommodating more sclerotic members of a monetary union, a union-wide central bank will blunt incentives by these national legislatures to reform?

## 2.6 Conclusion

In this paper I have analyzed the consequences of cross-country heterogeneity in labor market frictions for macroeconomic trade-offs and stabilization policy in a monetary union. I embedded DMP frictions into an otherwise standard model of a two-country union with sticky prices subject to TFP shocks, and used a linear-quadratic approximation to the Ramsey optimal monetary policy problem to establish two key results: institutional irrelevance for the question of when a monetary union will face distortions, and relative accommodation of the more sclerotic member of the union at the second-best optimum. Applied to the Eurozone, my results suggest that labor market frictions may not bear on the optimality of the single currency, but may be quite important in guiding optimal stabilization policy by union-wide institutions such as the ECB.

# Chapter 3

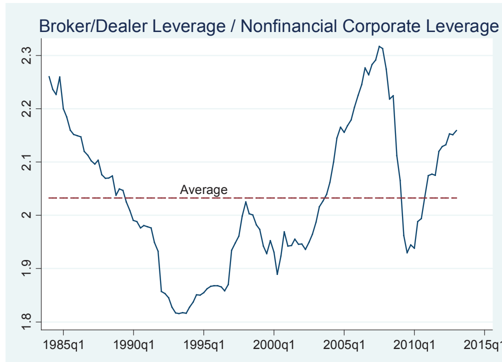
# Firm vs. Bank Leverage over the Business Cycle

## 3.1 Introduction

Since the 2008-09 financial crisis, the capital structure of financial intermediaries has come under heavy scrutiny in the public and academic debate. Relative to non-financial corporates, intermediary leverage has been shown to be unique both in its level and its cyclicality over time. As Figure 3.1 shows using Fed Flow of Funds data, the leverage of broker/dealers has been slightly more than twice that of corporates over the past thirty years. As Figure 3.2 shows using cyclical variation in the same data, the leverage of broker/dealers covaries positively with log real GDP — it is *procylical* — while that of corporates is *countercyclical*.[1] As Adrian and Shin (2010b,a, 2011) have emphasized in an early series of papers documenting these facts, such procyclicality may be an important mechanism by which the financial sector exacerbates economic fluctuations.

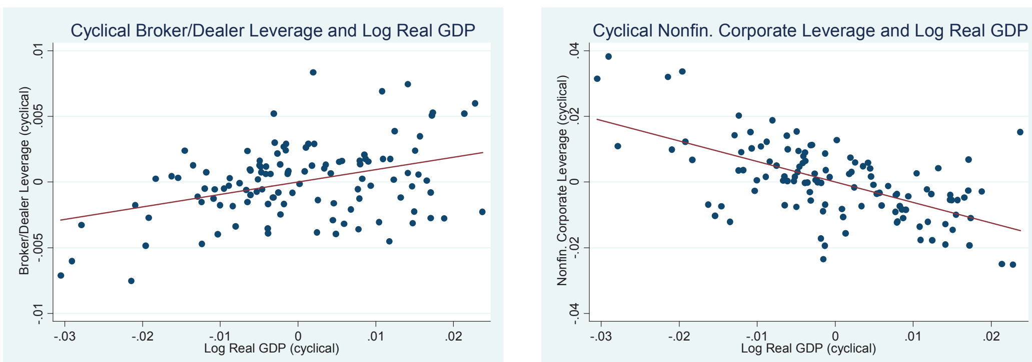
Why do intermediaries ("banks") and non-financial corporates ("firms") manage their leverage so differently over the business cycle? While there is an established literature

[1] Leverage is computed as *Debt*/*Assets*, consistent with its definition in the model which follows. Cyclical variation is computed by running each series through a Hodrick-Prescott filter with smoothing parameter $\lambda = 1600$.




**Construction:** leverage = debt / assets
**Source**: *Fed Flow of Funds*, 1984Q1-2013Q1

**Figure 3.1:** *Broker/dealer vs. non-financial corporate leverage: level*




**Construction:** leverage = debt / assets, and cyclical components obtained using HP filter (λ= 1600)
**Source**: *Fed Flow of Funds* and Bureau of Economic Analysis, 1984Q1-2013Q1

**Construction:** leverage = debt / assets, and cyclical components obtained using HP filter (λ= 1600)
**Source**: *Fed Flow of Funds* and Bureau of Economic Analysis, 1984Q1-2013Q1

**Figure 3.2:** *Broker/dealer vs. non-financial corporate leverage: cyclicality*

on the level of bank leverage and importance of debt in the liability structure of banks more generally[2], the difference in leverage cyclicality remains a puzzle. In this paper, I provide one theory which can rationalize the difference in leverage of firms and banks over the business cycle. I use this framework to explain additional cross-sectional facts in the data. In an evaluation of social welfare, I then speak directly to current policy debates over macroprudential regulation in banking.

I argue that the endogenous diversification of banks on the asset side of their balance sheet can rationalize the procyclicality of bank leverage and countercyclicality of firm leverage

[2]Among other papers, see Diamond and Dybvig (1983), Diamond (1984), Gorton and Pennacchi (1990), Calomiris and Kahn (1991), Flannery (1994), and Diamond and Rajan (2000).

in general equilibrium. Following Diamond (1984), banks are characterized as delegated monitors who diversify across their investments so that they may issue safe debt, the cheapest form of external finance in an environment with asymmetric information and costs of financial distress. In an economic upturn characterized by an improvement in the underlying distribution of project quality, firms see the lemons discount in equity issuance fall relative to the cost of financial distress, driving a reduction in firm leverage. In contrast, as diversified intermediaries, banks facing the same macroeconomic shock see their capacity to issue safe debt expand, driving an increase in bank leverage.

This perspective on banks and firms can rationalize additional cross-sectional patterns in the data and, normatively, implies that the procyclicality of leverage is an efficient property of banking. Within the banking sector, banks with access to deposit insurance exhibit less procyclical leverage than other types of banks, and are re-intermediated in downturns, consistent with empirical evidence comparing commercial banks with broker/dealers. Among firms, bank-dependent firms exhibit more procyclical leverage than non-bank-dependent firms because the improved funding conditions for banks are inherited by the former set of firms in general equilibrium, consistent with empirical evidence on LBO targets and smaller firms. Finally, the issuance of safe debt is an efficient response to the informational and technological frictions in the economy. As such, recent proposals for countercyclical capital requirements in banking may be misplaced, since the procyclicality of bank leverage is constrained efficient even if the level of bank leverage is not.

I begin by specifying a general equilibrium environment with three sets of agents — households, entrepreneurs, and bankers — in which two chains of financing and investment co-exist. In the first chain, entrepreneurs directly issue bond or equity securities to household investors so that they may invest in a proprietary, fixed scale investment technology. I interpret these entrepreneurs as corresponding to the non-financial corporate sector in the U.S., where little financing is obtained through the banking sector. In the second chain, a distinct set of entrepreneurs can directly issue bonds to household investors or take out a bank loan from bankers so that they may invest in their own proprietary, fixed scale investment technology. Bankers, in turn, issue bond or equity securities to household investors. I interpret these entrepreneurs as corresponding to mortgage borrowers in the U.S., who ultimately obtain financing through securitization markets or loans held on balance sheet by the banking sector. In general equilibrium, the expected rates of return earned by (risk-neutral) household investors across securities must be equalized.

To obtain concrete financing predictions and clarify the role of bankers in the economy, I then assume three sets of frictions relative to the Modigliani and Miller (1958) benchmark. First, I assume that entrepreneurs have private information regarding the quality of their investment technology. Second, I assume that through loans to entrepreneurs, bankers can eliminate this informational asymmetry at an upfront monitoring cost — but, paralleling the assumption for entrepreneurs, bankers' monitoring cost is private information. Third, I assume that through bond finance of either entrepreneurs or bankers, households can eliminate the informational asymmetry but will incur an ex-post cost of financial distress should the entrepreneur or banker default.

In this environment, firm leverage is countercyclical because the lemons discount in equity issuance falls relative to the costs of financial distress from bond issuance in a boom. With a continuum of firms whose investment quality is drawn from a continuous distribution, in equilibrium firms optimally finance themselves in a manner consistent with intuition of Myers and Majluf (1984)'s pecking-order: low-quality firms forego investment, medium-quality firms issue equity and invest, and high-quality firms issue debt and invest. In an economic upturn characterized by an improvement in the distribution of firm quality, the lemons discount associated with equity issuance falls for the marginal bond issuer, driving a relative increase in equity issuance vs. bond issuance in the corporate sector. This mechanism is similar to one proposed by Choe *et al.* (1993) in their explanation of procylical seasoned equity offerings over the business cycle.

In contrast to firms, bank leverage is procyclical because banks are endogenously diversified across idiosyncratic risk and their capacity to issue safe debt, the cheapest form of external finance, increases in a boom. Consistent with the results in Diamond (1984), bankers lend to a *pool* of entrepreneurs and, by tranching debt on the liability side of their balance sheet, issue default-free debt that is free of asymmetric information and costs of financial distress. The remainder of their capital structure is pinned down by focusing on an equilibrium in which bankers separate from less skilled bankers by issuing enough costly, risky bonds, while also being consistent with free entry into banking. In an economic upturn, the improvement in the quality of entrepreneur projects leads to an increase in the safe debt capacity of bankers, driving the procyclicality of bank leverage.

From a positive perspective, extensions of this framework can explain additional cross-sectional patterns in the data. First, the model clarifies why commercial banks may exhibit less procyclical leverage than broker/dealers, a point made by He *et al.* (2010) and Hall and Krueger (2012).[3] Commercial banks are distinguished from these other intermediaries by their participation in government-provided deposit insurance. In the model, deposit insurance means that banks' safe debt capacity need not fall in a downturn, preventing them from needing to delever. Because their weighted average cost of capital is unchanged, it also means that the interest rate they must charge on loans need not rise, so their assets do not fall. This is consistent with the reintermediation of commercial banks discussed in Gatev and Strahan (2006) and Ivashina and Scharfstein (2010).

Second, the model clarifies why bank-dependent firms may exhibit more procyclical leverage than other firms, consistent with the findings of Axelson *et al.* (2013) for LBO targets and Leary (2009) for small firms. Bank-dependent firms can be modeled as a set of entrepreneurs who can finance themselves using bonds, bank loans, or equity. For such entrepreneurs, a general equilibrium effect implied by the model is that the higher safe debt capacity and thus improved funding conditions for banks translate to a lower interest rate on bank loans in a boom. This force offsets the lower lemons discount from equity issuance in a boom, and means that their leverage will be more procylical than that of other firms,

[3]In particular, these authors argue that commercial bank leverage is countercyclical when measured using market leverage. As I show in section 3.5, commercial bank leverage remains procyclical, but is indeed less procyclical than broker/dealer leverage, using the marked-to-market book leverage measure obtained using the Fed Flow of Funds.

holding the initial level of leverage the same.

From a normative perspective, the present framework motivates a role for a binding leverage cap in banking, but does not justify any use of this cap to offset the procyclicality of bank leverage. The equilibrium under consideration features excessive risky debt issuance as skilled bankers attempt to signal their quality to the market. The source of the inefficiency is agents' coordination at a Pareto inferior equilibrium among the many possible equilibria arising between entrepreneurs, bankers, and households, common to signaling models in the tradition of Spence (1973). In particular, at a high interest rate on bank loans, skilled bankers have an incentive to separate from unskilled bankers by issuing costly risky debt, raising their cost of capital and supporting the high interest rate in equilibrium. A cap on bank leverage prevents skilled bankers from signaling their quality to the market, and if sufficiently low can break coordination at the 'high interest rate - high leverage' equilibrium, generating a utilitarian welfare increase. However, even in the resulting constrained efficient allocation, bank safe debt capacity is procyclical. In this way, procyclical leverage is an efficient property of banking, even though its high level of leverage may not be.

In future work, the present framework should be extended to include a role for pecuniary externalities, which may enrich the normative conclusions in realistic and important ways. If banks delever by selling existing assets rather than foregoing investment, and the associated asset prices affect collateral constraints facing others in the economy, the procyclicality of bank leverage may introduce welfare-reducing pecuniary externalities. This would likely introduce a rich trade-off involved in the determination of time-varying capital requirements: on the one hand, some degree of countercyclicality in capital requirements might be socially beneficial to mitigate the size of the pecuniary externalities; on the other, capital requirements which are too countercyclical might choke off the safe debt issuance of banks, inefficiently constraining their response to the informational and technological frictions affecting the economy.

Relative to existing work at the intersection of macroeconomics and finance rationalizing procyclical bank leverage, this paper is unique in being able to jointly explain firm and bank financing decisions in general equilibrium, and in an environment where both debt and equity can be issued. Theories of endogenous collateral constraints in Geanakoplos (2010) and Simsek (2013) can naturally explain procyclical leverage in banking, but do not readily clarify why banks differ from firms. The frameworks in Stein (2012) and Gennaioli *et al.* (2013) suggest that the diversified nature of banks distinguishes their balance sheet behavior from that of non-financial firms, but do not allow banks to issue equity alongside debt. Adrian and Shin (2014) microfound a Value-at-Risk constraint giving rise to procyclical bank leverage; however, it remains unclear why firms do not also manage their balance sheets according to such a constraint, or whether the procyclicality of bank leverage would survive if banks could expand their balance sheets with equity instead of only debt.

In section 3.2, I outline the general equilibrium environment. In section 3.3 I focus on the financing chain between entrepreneurs and households to illustrate the determination of firm leverage and its countercyclicality, and in section 3.4 I focus on the financing chain involving bankers, illustrating the economic role played by banks as intermediaries and characterizing bank leverage and its procyclicality. In section 3.5, I extend the benchmark model to shed light on additional cross-sectional predictions of the framework. Finally, in section 3.6, I examine welfare and the role for macroprudential regulation in banking, and in section 3.7, I conclude.

## 3.2 Economic environment

In this section I present the benchmark two-period environment in which I will study the leverage of firms and banks in general equilibrium. I first introduce three sets of agents — households, entrepreneurs, and bankers — who interact along two distinct financing chains. I then specify three frictions relative to Modigliani and Miller (1958): asymmetric information, costs of financial distress, and a monitoring function for bankers. Together, these motivate concrete financing predictions in the subsequent sections, both in steady-state and in response to macroeconomic shocks.

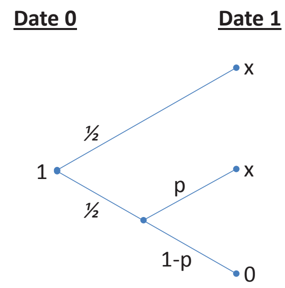



**Figure 3.3:** *Payoff profile of entrepreneur p*

### 3.2.1 Primitives

There is a fixed measure $H$ households, measure one entrepreneurs, and an elastic supply of bankers which can enter the economy. Households and entrepreneurs are endowed with $c^h < 1$ and $c^e < 1$ units of the consumption good ("dollars"), respectively, while bankers have no endowment for simplicity. The representative agent of each type has preferences

$$u(c_0^i, c_1^i) = c_0^i + \beta c_1^i$$

over consumption at dates 0 and 1. Given that agents have identical discount factors, the motivation for borrowing and lending at date 0 of this economy arises from technologies, not preferences.

In particular, entrepreneurs are distinguished by their access to a fixed-scale investment technology requiring one dollar at date 0. I assume that entrepreneurs are subject to aggregate and idiosyncratic risk. With probability $\frac{1}{2}$, all entrepreneur projects pay off $x$ at date 1. With probability $\frac{1}{2}$, this is not the case, and instead an entrepreneur's project pays off with idiosyncratic probability $p \sim F(p)$. The payoff profile of entrepreneur $p$ is summarized in Figure 3.3.

At this point, bankers simply serve as passive intermediaries who can receive capital from households and invest in entrepreneurs. In two subsections, I will introduce frictions relative to the Modigliani and Miller (1958) benchmark motivating a more active role for bankers in equilibrium.

Finally, in terms of assumptions on primitives, I will assume throughout this paper that household wealth ($Hc^h$) is large. Provided it is sufficiently high, households' risk neutrality will pin down their required rate of return in equilibrium in all of the results which follow:

**Proposition 3.1.** *Let $1 + r$ denote the required rate of return on any contract traded with households in equilibrium. Assuming that aggregate household wealth ($Hc^h$) is sufficiently large, $1 + r = \frac{1}{\beta}$.*

### 3.2.2 First best allocation

First consider the contracting problem when entrepreneur quality is common knowledge, entrepreneurs can sign any financial contract with households or bankers, and bankers can sign any financial contract with households. We then obtain the following intuitive result.

**Proposition 3.2.** *When entrepreneur quality $p$ is common knowledge and any financial contract can be written between entrepreneurs, bankers, and households,*

- *all entrepreneurs with $p \geq p_{npv}$ receive financing to invest in their projects, where*

$$p_{npv} := (\frac{1}{2} + \frac{1}{2}p_{npv})x = 1 + r;$$

- *the form of financial contracts between entrepreneurs and households, entrepreneurs and bankers, and bankers and households is indeterminate;*

- *the size of funds intermediated through the banking sector is indeterminate.*

The first part of this result simply reflects the fact that with perfect information on entrepreneur project quality, only positive-NPV projects receive financing. The second and third parts of this result reflect the Modigliani and Miller (1958) Theorem.

### 3.2.3 Frictions

Throughout the rest of the paper, I restrict entrepreneurs and bankers to issue only a mix of

- a *simple debt contract* $(z^D, D)$, where $z^D$ is the amount of the loan and $D$ is the uncontingent face value of the loan; or

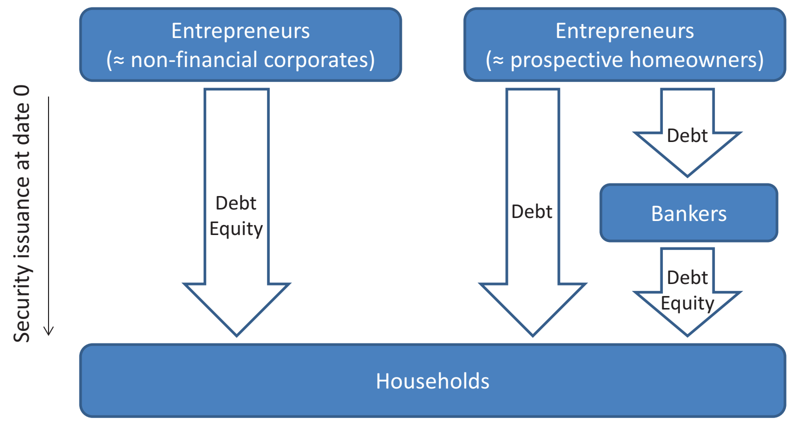



**Figure 3.4:** *Benchmark general equilibrium structure of economy*

- a *simple equity contract* $(z^E, s)$, where $z^E$ is the amount raised and $s$ is the uncontingent ownership share provided,

where multiple debt securities at various seniorities can be issued.

I furthermore assume that there exist two distinct types of entrepreneurs whose relationship with other agents in the economy is summarized in Figure 3.4. The first, which I associate with nonfinancial corporates, can issue debt or equity directly to the household sector. The second, which I associate with prospective homeowners, can issue debt directly to households or debt through the banking sector. Bankers, in turn, can then issue debt or equity directly to the household sector. These assumptions are made for realism in the U.S., where to first order bank balance sheets are dominated by intermediation in real estate rather than corporate credit markets.

Taken together, the above assumptions do not change the implementability of the first best allocation, nor do they pin down financing decisions in equilibrium; I now describe three sets of frictions relative to the Modigliani and Miller (1958) benchmark which do.

**Assumption 3.1.** *Entrepreneurs observe their idiosyncratic quality $p$, while households do not.*

**Assumption 3.2.** *Each banker draws a monitoring cost $m \sim G(m)$ upon entering the industry. At cost $m$ per dollar lent, the banker can observe an entrepreneur's $p$. Bankers observe their monitoring cost $m$, while households do not.*

**Assumption 3.3.** *By lending through debt contracts to entrepreneurs or bankers, households can observe an entrepreneur's $p$ or banker's $m$. The monitoring is costless ex-ante, but ex-post leads to a cost of financial distress of $d$ per dollar lent in the event of default.*

The private information of entrepreneurs in Assumption 3.1 is standard in the vast literature on the pecking-order hypothesis since Myers and Majluf (1984). Given the resulting asymmetric information problem, Assumption 3.2 introduces a role for bankers as delegated monitors as in Diamond (1984). Unlike that paper, however, I further assume that bankers have private information about their own monitoring technology, paralleling the assumption for entrepreneurs.

Assumption 3.3 will lead to determinate, interior financing decisions between debt and equity for both entrepreneurs and bankers in equilibrium. Letting debt issued to households be free of asymmetric information simplifies the characterization of equilibrium while retaining the intuition of Myers and Majluf (1984)'s pecking-order, and acts in favor of debt finance. In practice, the elimination of asymmetric information may occur through bond credit quality ratings, or the writing and enforcement of covenants on bonds (Shavell and Weiss (1979)). The costs of financial distress in debt issuance, on the other hand, act in favor of equity finance. These costs can be justified theoretically (Townsend (1979)) and also exist in practice, as documented by a long line of work in the corporate finance literature on the static "trade-off hypothesis" (Myers (1984)) and on fire sales (Shleifer and Vishny (1997)).

Assumptions 3.2 and 3.3 together will also lead to determinate, interior financing decisions between debt issued to households, and debt issued to banks, for entrepreneurs along the second intermediation chain illustrated in Figure 3.4. Bankers' monitoring is assumed to incur greater up-front costs, capturing the compensation required for delegated monitors. But bankers' monitoring eliminates the ex-post cost of financial distress, motivated by the idea in Bolton and Scharfstein (1996) that a single banker is able to renegotiate loans under default in a way that diffuse debtholders cannot.

In terms of functional forms for the distributions $F(p)$ and $G(m)$ characterizing entrepreneurs and bankers, I assume that over the continuum of entrepreneurs,

$$p \sim U[p_{min}, p_{max}]$$

where $0 \le p_{min} < p_{max} \le 1$, while for bankers

$$m = \begin{cases} m^s \text{ with probability } \lambda \text{ (banker is ``skilled''),} \\ m^u = \infty \text{ with probability } 1-\lambda \text{ (banker is ``unskilled'').} \end{cases}$$

Given that monitoring is infinitely costly for unskilled bankers, they will not face a meaningful portfolio choice problem on the asset side of their balance sheet, and can only invest in marketable securities which households could have invested in themselves.

The timing of events along each financing and intermediation chain described in Figure 3.4 is summarized in Figures 3.5 and 3.6. Since I will continue to assume that, as in Proposition 3.1, household wealth is sufficiently large to pin down households' required rate of return $1 + r = \frac{1}{\beta}$, there are no interactions between these two chains in equilibrium. As such, we can study each separately.

Finally, note that rather than studying a fully dynamic model, for tractability I characterize the "cyclicality" of firm and bank leverage by characterizing their comparative statics in response to a fundamental macroeconomic shock. In particular, the macroeconomic shock of interest is to the quality of underlying projects in the economy — a more granular version of what, in the aggregate data, would show up as a TFP shock. Formally, we have:

**Definition 3.1.** *Let* $\tilde{p} = \frac{p - p_{min}}{p_{max} - p_{min}}$ *denote a transformation of p distributed over* $[0, 1]$, *in which case* $\tilde{p} \sim U[0, 1] = Beta(\alpha = 1, \beta = 1)$. *An* economic boom *is characterized by an increase in* $\alpha$.

Graphically, an economic boom shifts the density of $p$ away from $p_{min}$ and towards $p_{max}$, as shown in Figure 3.7. In the next two sections I focus on characterizing each of the two financing and intermediation chains described above, and how these react to such a macroeconomic shock.

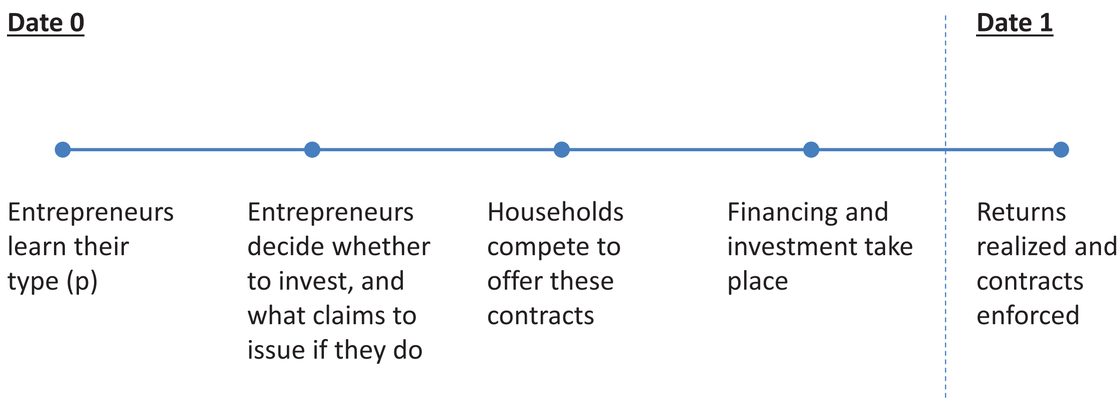



**Figure 3.5:** *Timing along chain between entrepreneurs and households*

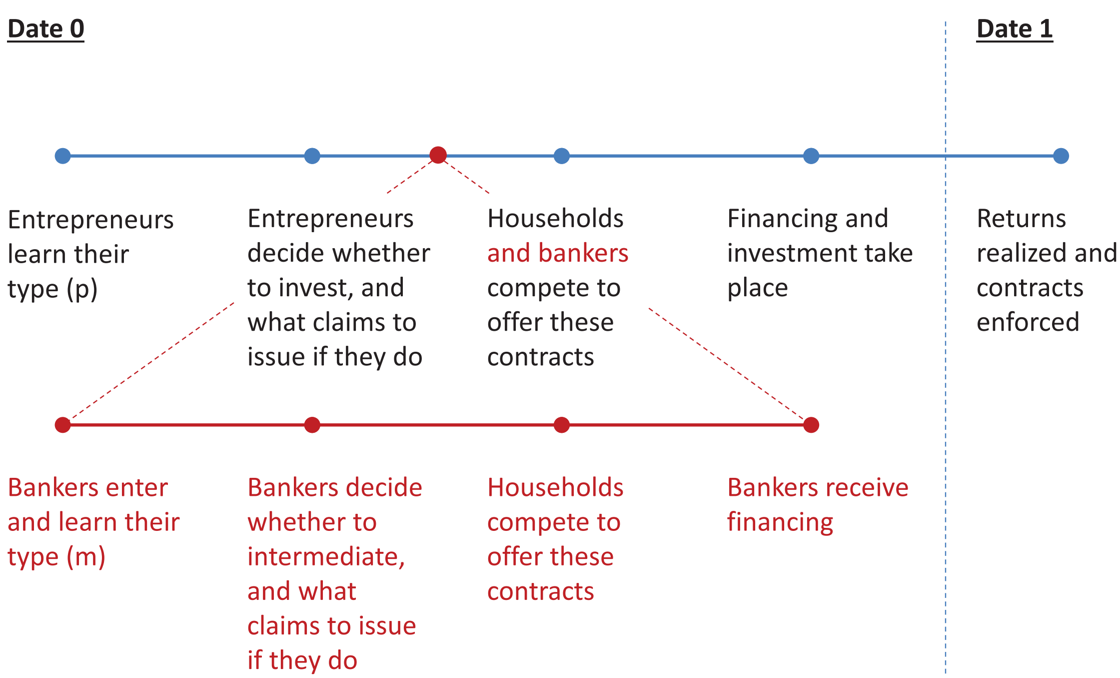



**Figure 3.6:** *Timing along chain between entrepreneurs, bankers, and households*

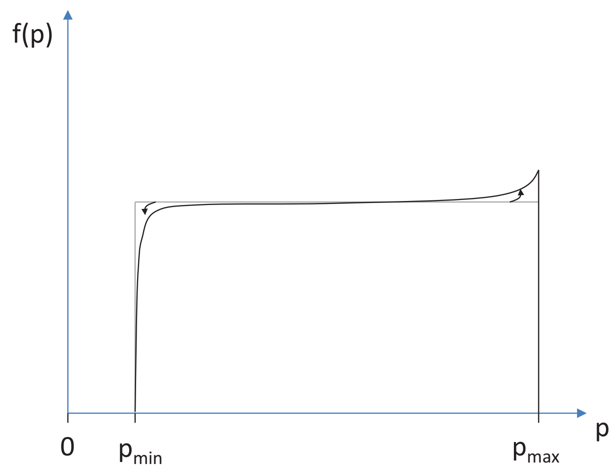



**Figure 3.7:** *Distribution of entrepreneur types at baseline and in an economic boom*

## 3.3 Explaining countercyclical firm leverage

In this section I demonstrate that the entrepreneurs who do not rely on bank intermediation for financing, which I map to non-financial corporates in practice, exhibit countercyclical leverage. In steady-state, entrepreneurs sort into bond finance, equity finance, or non-investment depending on their idiosyncratic quality. In a macroeconomic boom where the idiosyncratic quality of entrepreneur projects improves, leverage in this sector falls because the lemons discount in equity issuance falls relative to the costs of financial distress.

### 3.3.1 Equilibrium

Entrepreneurs and households play a signaling game. I first conjecture an equilibrium.

Consider the actions available to any entrepreneur $p$ at date 0:

- $\mathbf{1}\{\text{invest}\}(p)$: an indicator of whether or not to invest
- $z^D(p)$: the amount raised using bonds
- $z^E(p)$: the amount raised using equity

Note that because the entrepreneur's project only pays off one of two values (0 or $x$), we can restrict ourselves without loss of generality to considering a single debt tranche in equilibrium. Households will form beliefs of entrepreneurs' quality conditional on the choices of financing they observe. These beliefs must be such that the equilibrium face values of debt $D$ and equity shares $s$ ensure that households break even. This motivates the definition of a pure strategy Perfect Bayesian Equilibrium for the signaling game between entrepreneurs and households:

**Definition 3.2.** *Strategies* $\{\mathbf{1}\{invest\}(p), z^E(p), z^D(p)\}_p$ *constitute an* equilibrium *of the signaling game if:*

- *each entrepreneur* $p$ *chooses* $\{\mathbf{1}\{invest\}(p), z^E(p), z^D(p)\}$ *optimally given* $D(z^D; p)$ *and* $s(z^E, z^D)$*;*
- *schedules* $D(z^D; p)$ *and* $s(z^E, z^D)$ *satisfy households' participation constraints given their beliefs* $\mu(p|z^E, z^D)$*;*
- *households 'on-path' beliefs are consistent with entrepreneurs' equilibrium strategies and satisfy Bayes' Rule.*

Note that because bonds are free of asymmetric information, the face value of bonds $D(z^D; p)$ is conditional on the amount $z^D$ raised and an entrepreneur's true type. In contrast, equity is subject to asymmetric information, so equity shares $s(z^E, z^D)$ are conditional only on the signals received by investors. An implicit assumption I make is that equity investors cannot observe the promised payment on debt raised by an entrepreneur in the market, but only the amount raised.

There are multiple equilibria in this game. I focus attention on an equilibrium defined by the following conjectured strategies for entrepreneurs:

- $p \in [p_2, p_{max}]$ invest, fully commit their own capital $c^e$, and issue $z^D = 1 - c^e$ in debt
- $p \in [p_0, p_2)$ invest, fully commit their own capital $c^e$, and issue $z^E = 1 - c^e$ in equity
- $p \in [p_{min}, p_0)$ do not invest

Within the set of equilibria admitted by this model, this appears to be a reasonable one to study for two reasons. First, it is consistent with Myers and Majluf (1984)'s pecking

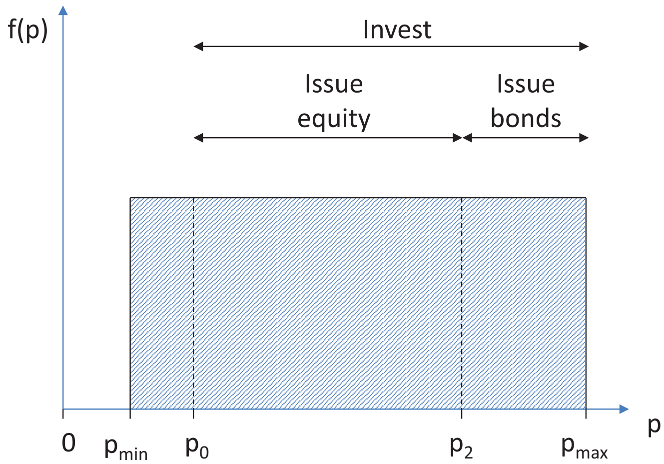



**Figure 3.8:** *Entrepreneur financing in conjectured equilibrium*

order. Second, frictions render external finance expensive for high quality entrepreneurs, so they will want to commit all of their capital to their project. The conjectured equilibrium is depicted in Figure 3.8.

To characterize this equilibrium, we must simply characterize the indifference points $p_2$ and $p_0$.

If entrepreneur $p_2$ issues bonds, his payoff is given by

$$\pi^e(z^D = 1 - c^e, z^E = 0; p_2) = [v(p_2) - (1+r)(1-c^e)] - \frac{1}{2}(1-p_2)d(1-c^e),$$

where $v(p) \equiv (\frac{1}{2} + \frac{1}{2}p)x$ is the expected return on any $p$'s project. Hence, $p_2$'s payoff to bond issuance is that under the first best (in brackets), less the distortion from costs of financial distress.

If entrepreneur $p_2$ issues equity, his payoff is given by

$$\pi^e(z^D = 0, z^E = 1 - c^e; p_2) = [v(p_2) - (1+r)(1-c^e)] - (s^E - s(p_2))v(p_2),$$

where

$$\begin{aligned} s^E &:= s^E v(p^E) = (1+r)(1-c^e), \\ p^E &\equiv E[p|p \in [p_0, p_2]], \\ s(p) &:= s(p)v(p) = (1+r)(1-c^e). \end{aligned}$$

That is, $s^E$ gives the equilibrium equity share demanded by investors given that the average equity issuer has quality $p^E$, while $s(p_2)$ is the equity share that *would* be demanded by an investor in entrepreneur $p_2$ under perfect information. Again, $p_2$'s payoff to equity issuance is that under the first best, less the distortion caused by asymmetric information in equity issuance.

Indifference requires that $(s^E - s(p_2))v(p_2) = \frac{1}{2}(1-p_2)d(1-c^e)$, or

$$(1+r)\left(\frac{v(p_2)}{v(p^E)} - 1\right) = \frac{1}{2}(1-p_2)d. \tag{3.1}$$

This implies an upward-sloping $p_2(p_0)$ schedule as depicted in Figure 3.9. Intuitively, at a higher marginal equity issuer $p_0$, the average quality of equity issuers $p^E$ rises, reducing the lemons discount associated with equity issuance and thus raising the marginal bond issuer $p_2$.

Let us now turn to the indifference condition of entrepreneur $p_0$. If he issues equity, his payoff is given by

$$\begin{aligned} \pi^e(z^D = 0, z^E = 1-c^e; p_0) &= [v(p_0) - (1+r)(1-c^e)] + (s(p_0) - s^E)v(p_0) \\ &= (1-s^E)v(p_0), \end{aligned}$$

where the last term on the first line is now positive, as this entrepreneur receives a subsidy relative to the first best. If he does not invest and instead invests at the market interest rate $1+r$, his payoff is

$$\pi^{noinvest} \equiv (1+r)c^e.$$

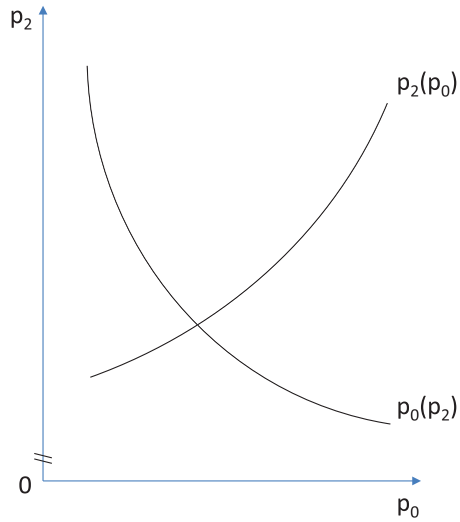



**Figure 3.9:** *Characterization of equilibrium*

Indifference requires that $(1-s^E)v(p_0) = (1+r)c^e$, or

$$\left(1 - \frac{(1+r)(1-c^e)}{v(p^E)}\right) v(p_0) = (1+r)c^e. \tag{3.2}$$

This implies a downward-sloping $p_0(p_2)$ schedule as depicted in Figure 3.9. Intuitively, at a higher marginal bond issuer $p_2$, the average quality of equity issuers $p^E$ rises, increasing the subsidy associated with equity issuance for a given $p < p_2$ and thus reducing the marginal equity issuer $p_0$. Note that for internal funds $c^e$ sufficiently high and the worst possible entrepreneur quality $p_{min}$ sufficiently low, entrepreneur $p_{min}$ will strictly prefer to invest at the market interest rate $1+r$ rather than invest in his project, despite the subsidy he receives from equity issuance.

The characterization of equilibrium cutoffs in Figure 3.9 makes evident that if an equilibrium exists, it will be unique. The following parametric assumptions ensure that an equilibrium indeed exists, and the subsequent proposition formalizes this result.

**Assumption 3.4.** *Assume that:*

- $\max\{\frac{1+r+\frac{1}{2}(1-p_{min})d}{2(1+r)+\frac{1}{2}(1-p_{min})d}, \frac{v(p_{min})}{1+r}\} < c^e < 1 < \frac{v(p_{max})}{1+r}$;

- *given $p_{npv}$ defined in Proposition 3.2,* $0 < d < \frac{(1+r)\left(\frac{v(p_{max})}{v(E[p|p \in [p_{npv}, p^{max}]])} - 1\right)}{\frac{1}{2}(1-p_{max})}$.

**Proposition 3.3.** *Under the conditions of Assumption 3.4, there exists an equilibrium in which*

- $p \in [p_2, p_{max}]$ *invest, fully commit their own capital* $c^e$*, and issue* $z^D = 1 - c^e$ *in debt,*

- $p \in [p_0, p_2)$ *invest, fully commit their own capital* $c^e$*, and issue* $z^E = 1 - c^e$ *in equity,*

- $p \in [p_{min}, p_0)$ *do not invest.*

*The cutoffs* $\{p_2, p_0\}$ *jointly solve (3.1) and (3.2), where* $p_E = E[p|p \in [p_0, p_2]]$.

As noted earlier, firms optimally finance themselves in a manner consistent with the intuition of Myers and Majluf (1984)'s pecking-order: low-quality firms forego investment, medium-quality firms issue equity and invest, and high-quality firms issue debt and invest.

### 3.3.2 Macro shock, firm assets, and firm leverage

I now characterize the behavior of firm assets and leverage in response to an economic boom. Assets in this sector are given by the total investment in projects, while leverage is given by the total amount of debt issued relative to assets.

**Lemma 3.1.** *Total assets among firms is given by*

$$a^e \equiv 1 - F(p_0)$$

*where* $F(p)$ *is the CDF of* $p$ *in the population of entrepreneurs. Leverage (debt/assets) in this sector is given by*

$$l^e \equiv \frac{1 - F(p_2)}{1 - F(p_0)}(1 - c^e).$$

Recalling the distributional assumption that $\tilde{p} \equiv \frac{p - p_{min}}{p_{max} - p_{min}} \sim Beta(\alpha, \beta = 1)$ with $\alpha = 1$ in steady-state (i.e., $\tilde{p} \sim U[0, 1]$), we then obtain the following main result of this section.

**Proposition 3.4.** *In the entrepreneurial sector, assets expand in a boom (*$\frac{da^e}{d\alpha} > 0$*). If initial leverage is low enough, leverage falls in a boom (*$\frac{dl^e}{d\alpha} < 0$*).*

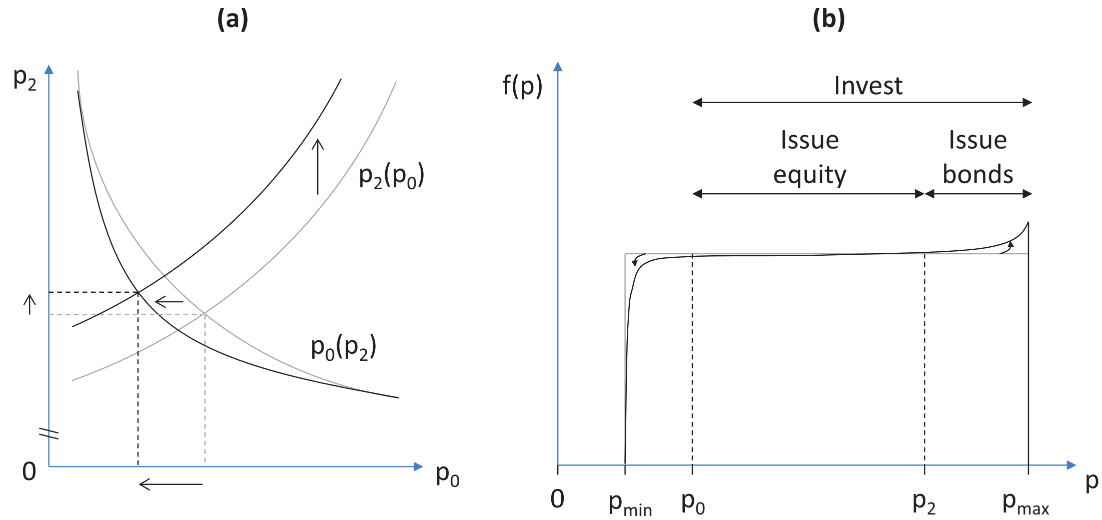



**Figure 3.10:** *Impact of an economic boom*

We can understand the intuition behind this result by examining Figure 3.10. There are three effects of a rightward shift in the density of project quality on the equilibrium:

1. At a given $p_0$, the increase in average equity issuer quality $p^E$ reduces the lemons discount in equity issuance for any $p > p_0$, raising the marginal bond issuer $p_2$: $p_2(p_0)$ shifts up in panel (a).

2. At a given $p_2$, the increase in average equity issuer quality $p^E$ increases the subsidy in equity issuance for any $p < p_2$, reducing the marginal equity issuer $p_0$: $p_0(p_2)$ shifts to the left in panel (a).

3. At given $p_0$ and $p_2$, there are relatively more high-quality entrepreneurs than low-quality entrepreneurs: the density function $f(p)$ shifts to the right in panel (b).

As can be seen graphically, these effects imply that total assets in the entrepreneurial sector rise. The first and second effects related to asymmetric information in equity issuance imply that leverage falls; the third effect implies that leverage rises. When initial leverage $l^e$ is sufficiently low, the first effect dominates the overall movement in leverage.

Thus, I conclude that if leverage among firms is initially not too high — as is the case in practice, at least relative to banks — firm leverage is countercyclical because the lemons discount associated with equity issuance falls relative to the costs of financial distress.

## 3.4 Explaining procyclical bank leverage

In this section I study the investment and financing chain where banks serve as intermediaries, finding that banks exhibit procyclical leverage. In steady-state, banks diversify across idiosyncratic risk by lending to a pool of entrepreneurs, allowing them to tranche out and issue safe debt, the cheapest form of external finance. In response to the same macroeconomic boom as was studied in the prior section, the safe debt capacity of banks rises, driving an increase in equilibrium bank leverage at the same time that non-financial corporate leverage is falling.

### 3.4.1 Equilibrium

With entrepreneurs restricted to issuing debt contracts (either bonds or loans) free of information frictions, the only signaling along this chain occurs between bankers and their household investors. Bankers signal the quality of their monitoring technology through their choice of financing. Implicitly, this presumes that households cannot observe the precise nature of assets in each banker's portfolio, since Assumption 2 implies skilled and unskilled bankers will finance bank loans and bonds, respectively.[4]

More formally, the actions available to any entrepreneur $p$ at date 0 are:

- $\mathbf{1}\{\text{invest}\}(p)$: an indicator of whether or not to invest
- $z^B(p)$: the amount raised using bank loans
- $z^D(p)$: the amount raised using bonds

[4] While this assumption is obviously extreme and is made for internal consistency, authors such as Flannery (1994) and Brunnermeier and Oehmke (2013) stress balance sheet opacity as a distinguishing feature of banks.

As before, because the entrepreneur's project only pays off one of two values (0 or $x$), we can restrict ourselves without loss of generality to considering a single debt tranche in equilibrium. Unlike before, that debt can be in the form of bank loans or bonds.[5]

Given bankers' ability to freely scale up/down their investment by intermediating funds from households, they may diversify and tranche. Anticipating that I will focus on equilibria in which bankers fully diversify across idiosyncratic risk (discussed further in Lemma 3.3 below), the payoff profile of entrepreneurs in Figure 3.3 implies that we need only consider two possible debt tranches issued by bankers in equilibrium. Hence, the actions available to banker $m$ at date 0 are:

- $\mathbf{1}\{\text{invest}\}$(m): an indicator of whether or not to intermediate funds
- $z^S(m)$: the amount raised using senior debt
- $z^D(m)$: the amount raised using junior debt
- $z^E(m)$: the amount raised using equity

where $z^S + z^D + z^E$ gives the total investment by the banker.

Define $1 + r^b$ to be the equilibrium rate of return on bank loans, an endogenous variable which must clear the market for such loans. Assuming that households still provide at least some bond financing directly, and continuing to assume that household wealth is large, the required return on bonds will remain $1 + r$. The definition of equilibrium is then analogous to that in the previous section, except now bankers are doing the signaling and there are potentially two tranches of debt they issue with equilibrium face value schedules $D^S(z^S, z^D, z^E; m)$ and $D^D(z^S, z^D, z^E; m)$.

**Definition 3.3.** *The strategies* $\{\mathbf{1}\{invest\}(p), z^B(p), z^D(p)\}_p$ *and* $\{\mathbf{1}\{invest\}(m), z^S(m), z^D(m), z^E(m)\}_m$ *constitute an* equilibrium *if:*

- *each entrepreneur* $p$ *chooses* $\{\mathbf{1}\{invest\}(p), z^B(p), z^D(p)\}$ *optimally given* $1 + r^b$*;*

[5] It is straightforward to show that it is never optimal for the entrepreneur to issue some of both.

- *each banker $m$ chooses $\{\mathbf{1}\{invest\}(m), z^S(m), z^D(m), z^E(m)\}$ optimally given $1+r^b$, $D^S(z^S, z^D, z^E; m)$, $D^D(z^S, z^D, z^E; m)$, and $s(z^S, z^D, z^E)$;*

- *the schedules $D^S(z^S, z^D, z^E; m)$, $D^D(z^S, z^D, z^E; m)$ and $s(z^S, z^D, z^E)$ are such that households' participation constraints are satisfied given their beliefs over bankers $\mu(m|z^S, z^E, z^D)$;*

- *households 'on-path' beliefs are consistent with bankers' equilibrium strategies and satisfy Bayes' Rule; and*

- *the market for bank loans clears.*

Before conjecturing a particular equilibrium, let us first tease out the implications of this definition of equilibrium. It is straightforward to show that because bank loans require costly monitoring, we have the following result.

**Lemma 3.2.** *In any equilibrium where some entrepreneurs borrow using bank loans,*
$1 + r^b \geq (1+r)(1+m^s)$.

Intuitively, if $1 + r^b < (1+r)(1+m^s)$, the ROA to a skilled banker providing loan financing to an entrepreneur would be $\frac{1+r^b}{1+m^s} < (1+r)$. Since $1+r$ is the required return to household investors in banks, it is easy to verify that the skilled banker would be unable to raise equity in any candidate equilibrium with equity issuance, and would make negative expected profits in any candidate equilibrium with all debt financing (and would thus not provide loan financing to that entrepreneur in the first place).

Given this result, if bank loans are used in equilibrium, it must be that only low quality entrepreneurs find bank loans worthwhile relative to bonds. Intuitively, the cost of financial distress priced into bonds is highest for entrepreneurs with low quality, as they are more likely to default. Hence, if both instruments are used in equilibrium by entrepreneurs:

- $p \in [p_2, p_{max}]$ invest, fully commit their own capital $c^e$, and issue $z^D = 1 - c^e$ in bonds

- $p \in [p_1, p_2)$ invest, fully commit their own capital $c^e$, and issue $z^B = 1 - c^e$ in bank loans

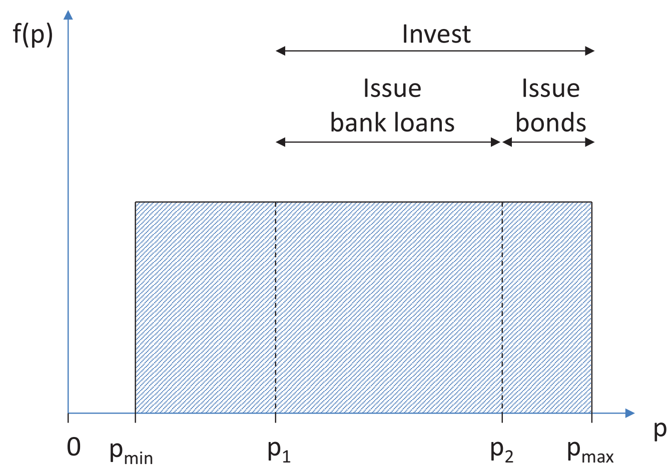



**Figure 3.11:** *Entrepreneur financing in conjectured equilibrium*

- $p \in [p_{min}, p_1)$ do not invest

As before, since both forms of external finance are costly relative to internal funds, entrepreneurs will fully commit their own capital to their project. The conjectured equilibrium between entrepreneurs and bankers is depicted in Figure 3.11.

Given this demand for bank loans and bonds from the entrepreneurial sector, what sort of portfolios will bankers hold in equilibrium? The following result implies that bankers will pool across entrepreneurs, diversifying away idiosyncratic risk from their portfolios.

**Lemma 3.3.** *If beliefs are such that:*

- $\mu(m|z^S, z^D, z^E) \leq \mu(m|z^S I, z^D I, z^E I)$ *for any* $I > 1$*, and*

- $\mu(m|z^S, z^D, z^E) \leq \mu(m|z^{S\prime}, z^{D\prime}, z^E)$ *for any* $z^{S\prime} > z^S$ *such that* $z^S + z^D = z^{S\prime} + z^{D\prime}$*,*

*then in any equilibrium it is weakly optimal for skilled bankers to diversify and issue a senior tranche with as much safe debt as possible (their "safe debt capacity").*

Intuitively, suppose that beliefs are "reasonable" in the sense that scaling up the balance sheet or replacing junior debt with senior debt does not reduce households' beliefs that a banker is skilled. Then relative to the case where the banker intermediates between households and just one entrepreneur chosen at random, pooling allows the banker to tranche out safe debt free of the ex-post cost of financial distress. Given the beliefs just described, the banker can do so without worsening the pricing of equity he may be issuing. In the remainder of the paper I focus on equilibria in which bankers are fully diversified and tranche out as much safe debt as they can, implicitly assuming beliefs are reasonable in the sense described in Lemma 3.3. For that same reason, I also characterize banks' capital structure $\{z^S, z^D, z^E\}$ per dollar of assets in all that follows.[6]

With this motivation in mind, let us derive the payoffs on a skilled banker's portfolio of bank loans. If $1 + r^b$ is the required return on bank loans, the face value of a single loan to entrepreneur $p$ in the amount $1 - c^e$ must be

$$\frac{(1 + r^b)(1 - c^e)}{\frac{1}{2} + \frac{1}{2}p}.$$

The payoff on a portfolio of a continuum of such loans thus pays off:

- In the aggregate 'up' state, $E\left[\frac{(1+r^b)(1-c^e)}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]\right]$
- In the aggregate 'down' state, $E\left[p\left(\frac{(1+r^b)(1-c^e)}{\frac{1}{2}+\frac{1}{2}p}\right)|p \in [p_1, p_2]\right]$

The total resources committed by the banker to such a portfolio would be $(1 - c^e)(1 + m^s)$, so that the resulting return on a single dollar of assets is:

- In the aggregate 'up' state, $E\left[\frac{1}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]\right]\frac{1+r^b}{1+m^s}$
- In the aggregate 'down' state, $E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]\right]\frac{1+r^b}{1+m^s}$

This is depicted in panel (a) of Figure 3.12, where

$$\gamma_u^s \equiv E\left[\frac{1}{\frac{1}{2} + \frac{1}{2}p}|p \in [p_1, p_2]\right],$$
$$\gamma_d^s \equiv E\left[\frac{p}{\frac{1}{2} + \frac{1}{2}p}|p \in [p_1, p_2]\right].$$

We can do the same thing for an unskilled banker's portfolio of bonds. The resulting

[6] That is, I describe banks' financing strategies subject to the constraint $z^S + z^D + z^E = 1$.

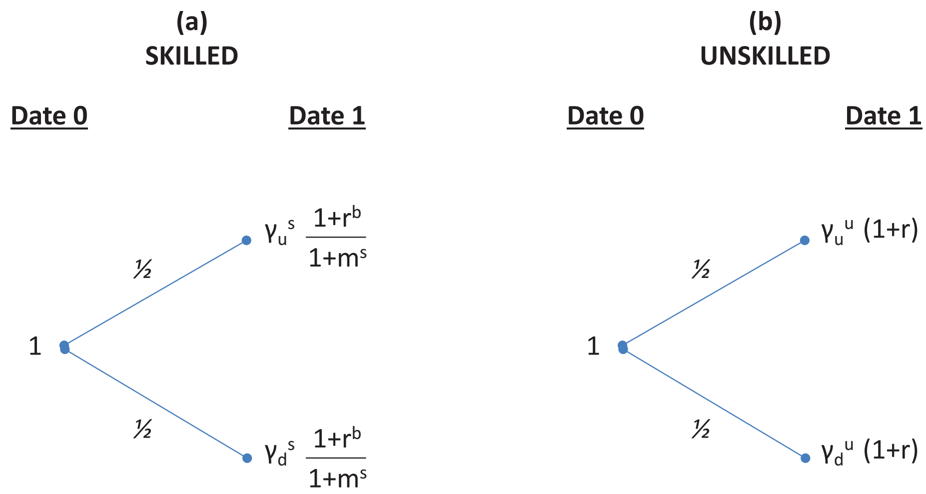



**Figure 3.12:** *Bank portfolios in conjectured equilibrium*

payoff per dollar of assets is given in panel (b) of Figure 3.12, where

$$\gamma_u^u \equiv E\left[\frac{1}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_2, p_{max}]\right] + \frac{\frac{1}{2}d}{1+r}E\left[\frac{1-p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_2, p_{max}]\right],$$

$$\gamma_d^u \equiv E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_2, p_{max}]\right] - \frac{\frac{1}{2}d}{1+r}E\left[\frac{1-p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_2, p_{max}]\right].$$

Note that

$$\gamma_d^s < \gamma_d^u < \gamma_u^u < \gamma_u^s$$

provided $d$ is not too big, a condition which will be assumed below. Hence, the low idiosyncratic quality of entrepreneurs issuing bank loans means that the volatility of skilled bank asset returns exceeds that of unskilled banks, who simply hold bonds.

Finally, note that free entry into banking implies that bankers who actually intermediate must earn zero profits in equilibrium, as the following result formalizes.

**Lemma 3.4.** *In any separating equilibrium,* $\pi^b(z^S, z^D, z^E; m^s) = 0$, $\pi^b(z^S, z^D, z^E; m^u) \leq 0$, *and without loss of generality I assume that unskilled bankers do not intermediate. In any pooling equilibrium, it must be that* $\pi^b(z^S, z^D, z^E; m^s) = \pi^b(z^S, z^D, z^E, ; m^u) = 0$.

Despite the restrictions implied by the previous three results, the signaling between bankers and households admit multiple equilibria in this stage of the game. In the remainder of this subsection, I characterize the set of symmetric *separating* equilibria in which all skilled bankers intermediate, and all unskilled bankers do not. In the next subsection I then characterize the comparative statics around the particular equilibrium in this set with maximal skilled banker leverage, illustrating that it provides price and quantity predictions consistent with the data. In section 3.6, I will characterize the allocation among all separating and pooling equilibria which maximizes utilitarian social welfare, and suggest a role for macroprudential policy as a tool to implement that allocation in equilibrium.

Let us begin characterizing the set of separating equilibria by pinning down the indifference points for entrepreneurs displayed in Figure 3.11. If entrepreneur $p_2$ issues bonds, his payoff is given by

$$\pi^{bond}(p_2) \equiv \pi^e(z^B = 0, z^D = 1 - c^e; p_2) = [v(p_2) - (1+r)(1-c^e)] - \frac{1}{2}(1-p_2)d(1-c^e),$$

as discussed in section 3.3.

If entrepreneur $p_2$ borrows using bank loans, his payoff is given by

$$\pi^{bank}(p_2) \equiv \pi^e(z^B = 1 - c^e, z^D = 0; p_2) = [v(p_2) - (1+r)(1-c^e)] - (r^b - r)(1-c^e).$$

That is, $p_2$'s payoff under bank loan issuance is that under the first best, less the incremental cost of bank loans relative to the market interest rate.

Indifference requires that $\frac{1}{2}(1-p_2)d(1-c^e) = (r^b - r)(1-c^e)$, or

$$\frac{1}{2}(1-p_2)d = r^b - r. \tag{3.3}$$

Let us now turn to the indifference condition of entrepreneur $p_1$. If he borrows with bank loans, his payoff is given by

$$\pi^e(z^B = 1 - c^e, z^D = 0; p_1) \quad = \quad [v(p_1) - (1+r)(1-c^e)] - (r^b - r)(1-c^e).$$

If he does not invest and instead invests at the market interest rate $1 + r$, his payoff is

$$\pi^{noinvest} \equiv (1+r)c^e.$$

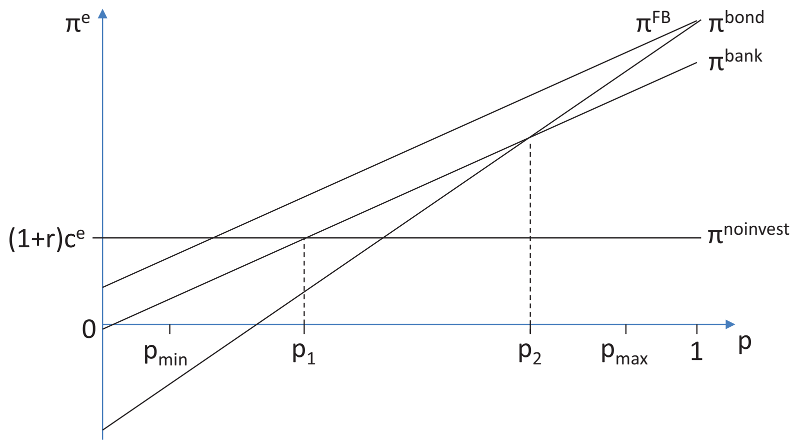



**Figure 3.13:** *Characterization of entrepreneur indifference points $p_1$ and $p_2$*

Indifference requires that $[v(p_1) - (1+r)(1-c^e)] - (r^b - r)(1-c^e) = (1+r)c^e$, or

$$v(p_1) = 1 + r + (r^b - r)(1 - c^e). \tag{3.4}$$

Graphically, the determination of $p_1$ and $p_2$ is shown in Figure 3.13. Relative to the first best level of profit $\pi^{FB}$, the profit from bank loan issuance $\pi^{bank}$ is shifted laterally down by $(r^b - r)(1 - c^e)$, while the profit from bond issuance $\pi^{bond}$ has a higher slope to account for the falling cost of financial distress with entrepreneur quality. As is evident from this graph, for $r^b - r$ not too large, bank loans will profitably be issued by relatively low-quality entrepreneurs.

It remains to characterize equilibrium among banks, which amounts to characterizing $1 + r^b$ and the level of bank leverage $l^b$. The latter is a sufficient statistic for skilled banker financing because I am implicitly assuming beliefs are reasonable in the sense of Lemma 3.3, so skilled bankers will diversify and tranche out as much safe debt as they can. Thus,

$$z^S = \bar{z}^S(r^b; m^s), z^D = l^b - \bar{z}^S(r^b; m^s), z^E = 1 - l^b$$

describes the banker's liabilities per dollar of external finance raised, where

$$\bar{z}^S(r^b; m^s) \equiv \frac{1}{1+r}\gamma_d^s\left(\frac{1+r^b}{1+m^s}\right)$$

is the *safe debt capacity* of a skilled banker.[7]

In a separating equilibrium, it must be that unskilled bankers would earn non-positive profits by imitating. The locus of $\{l^b, 1+r^b\}$ at which unskilled bankers earn zero profits is implicitly defined by

$$\begin{aligned} 0 &= \pi^b(z^S = \bar{z}^S(r^b; m^s), z^D = l^b - \bar{z}^S(r^b; m^s), z^E = 1 - l^b; m^u), \\ &= \begin{cases} (1-s)(1+r-(1+r)l^b) \text{ if } l^b \leq \bar{z}^S(r^b; m^u), \\ (1-s)(1+r-(1+r)\bar{z}^S(r^b; m^s) - (1+r+\frac{1}{2}d)(l^b - \bar{z}^S(r^b; m^s))) \text{ otherwise.} \end{cases} \end{aligned}$$

Here, $\bar{z}^S(r^b; m^u)$ is the safe debt capacity of an unskilled banker

$$\bar{z}^S(r^b; m^u) \equiv \frac{1}{1+r}\gamma_d^u(1+r)$$

and it can be shown that $\bar{z}^S(r^b; m^s) < \bar{z}^S(r^b; m^u)$ for $\frac{1+r^b}{1+m^s}$ not too much larger than $1+r$, a condition which will be satisfied in all that follows.

It must also be that skilled bankers earn zero profits by free entry. The locus of $\{l^b, 1+r^b\}$ at which this occurs is implicitly defined by

$$\begin{aligned} 0 &= \pi^b(z^S = \bar{z}^S(r^b; m^s), z^D = l^b - \bar{z}^S(r^b; m^s), z^E = 1 - l^b; m^s), \\ &= \begin{cases} (1-s)(\frac{1+r^b}{1+m^s} - (1+r)l^b) \text{ if } l^b \leq \bar{z}^S(r^b; m^s), \\ (1-s)(\frac{1+r^b}{1+m^s} - (1+r)\bar{z}^S(r^b; m^s) - (1+r+\frac{1}{2}d)(l^b - \bar{z}^S(r^b; m^s))) \text{ otherwise.} \end{cases} \end{aligned}$$

Both loci are plotted in Figure 3.14.

In segment $A - B$, the ROA of skilled bankers is identical to that of unskilled bankers, $1+r$. In this range, the ROA is identical to the required rate of return on safe debt and equity (which is fairly priced), so as long as no risky debt is issued, $s = 1$ and $\pi^b(\cdot; m^s) = \pi^b(\cdot; m^u) = 0$. If leverage $l^b$ is too high and thus some risky debt must be issued,

[7] Recall that given $r^b$, $p_1$ and $p_2$ are pinned down by (3) and (4), so $\gamma_d^s$ is as well. Hence, $\bar{z}^S$ is a function of $r^b$.

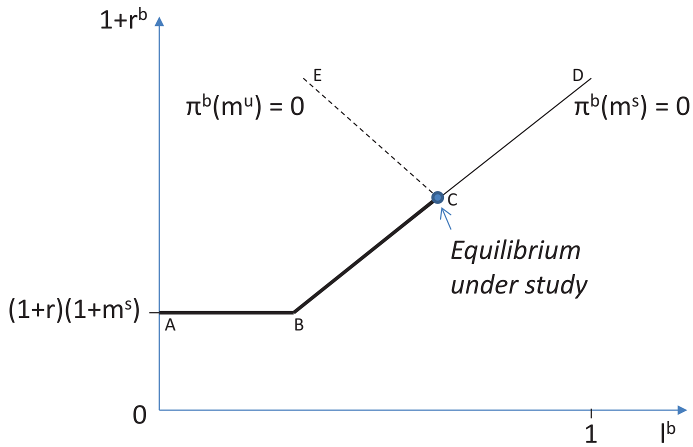



**Figure 3.14:** *Separating equilibria in banking*

it is impossible to deliver equity investors' required return of $1 + r$ given risky debt's ex-post cost of financial distress.

In segment $B - C$, the ROA of skilled bankers $\frac{1+r^b}{1+m^s}$ is higher than that of unskilled bankers $1 + r$. Since the required return on equity and safe debt issued by a skilled banker is $1 + r$, he will earn positive profits unless he issues some risky debt subject to costs of financial distress. The required leverage $l^b$ at which the skilled banker earns zero profits is naturally increasing in his ROA $\frac{1+r^b}{1+m^s}$, and ensures that $s = 1$ so that he earns zero profits. Yet because $s = 1$, an unskilled banker would also earn zero profits by imitating this choice of leverage.

Above the ROA of skilled bankers $\frac{1+r^b}{1+m^s}$ at point $C$, the zero profit loci for skilled and unskilled bankers diverge. For skilled bankers, the zero profit locus continues on from $B - C$ to $C - D$. At an ROA $\frac{1+r^b}{1+m^s}$ sufficiently high, skilled bankers must be fully debt-financed ($l^b = 1$) to imply zero profits; beyond this point, skilled bankers will earn positive profits and this cannot be consistent with an equilibrium with free entry into banking.

Unskilled bankers, on the other hand, find that at point $C$, the level of leverage $l^b$ it would take to imitate skilled bankers would leave no residual value left for equity, even in the aggregate 'up' state of the world. That is, at point $C$,

$$1+r-(1+r)\bar{z}^S(r^b;m^s)-(1+r+\frac{1}{2}d)(l^b-\bar{z}^S(r^b;m^s))=0.$$

Beyond this point, as skilled bankers' ROA $\frac{1+r^b}{1+m^s}$ rises, the set of entrepreneurs issuing bank loans shrinks — both $p_1$ rises and $p_2$ falls in Figure 3.13. For $m^s$ and $d$ sufficiently small, the fall in $p_2$ dominates, implying that the average quality of bank loans falls. This reduces the safe debt capacity of skilled bankers $\bar{z}^S(r^b;m^s)$, and means that *less* total leverage $l^b$ is needed to prevent unskilled bankers from imitating. This generates a *negatively* sloping zero profit locus along $C-E$.

As is evident from this discussion, then, the locus $A-D$ gives a set of potential separating equilibria consistent with free entry into banking. However, over the range $C-D$, the Cho and Kreps (1987) Intuitive Criterion can be used to rule out such equilibria as unreasonable. In particular, to prevent a skilled banker from deviating to lower leverage at a candidate equilibrium in this region, households' beliefs must be sufficiently high that the banker is unskilled. Yet, choices of leverage between the $C-D$ and $C-E$ loci are equilibrium dominated for unskilled bankers. Thus, any candidate separating equilibrium will fail the Intuitive Criterion unless it lies on the $C-E$ locus: but there, skilled bankers will earn strictly positive profits, inconsistent with free entry.

I conclude then that the set of (reasonable) potential separating equilibria lie *just* along the thickened locus $A-B-C$. Until the last section of the paper, I will focus on comparative statics around one such equilibrium, that at point $C$, demonstrating that it generates comparative statics in the banking sector which are consistent with the motivating facts on leverage and assets.

At point $C$,

$$\pi^b(\cdot;m^u)=0 \quad\Rightarrow\quad l^b=\bar{z}^S(r^b;m^s)+(1-\bar{z}^S(r^b;m^s))\frac{1}{1+\frac{\frac{1}{2}d}{1+r}}, \tag{3.5}$$

$$\pi^b(\cdot;m^s)=0 \quad\Rightarrow\quad \frac{1+r^b}{1+m^s}=1+r+\frac{1}{2}d(l^b-\bar{z}^S(r^b;m^s)). \tag{3.6}$$

Equations (3.3), (3.4), (3.5), and (3.6) summarize the equilibrium conditions characterizing this economy. The following parametric assumptions ensure that such an equilibrium actually exists, and the subsequent proposition formalizes this result.

**Assumption 3.5.** *Assume that:*

- $\frac{\partial}{\partial p_2} E[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]] < 1$ *for all* $p_1 \in [p_{npv}, p_{max})$ *and* $p_2 \in (p_{npv}, p_{max}]$ *where* $p_2 > p_1$*;*
- $\frac{1}{2}\frac{1}{1+r}\left(E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_{npv}, p_{max}]\right] - p_{max}\right) < \frac{m^s}{d} < \frac{1}{2}\frac{1}{1+r}\left(\left(\frac{p_{npv}}{\frac{1}{2}+\frac{1}{2}p_{npv}}\right) - p_{npv}\right)$*;*
- $m^s$ *and* $d$ *are sufficiently small.*

**Proposition 3.5.** *Under the conditions of Assumption 3.5, there exists an equilibrium in which:*

- *entrepreneurs sort as follows:*
  - $p \in [p_2, p_{max}]$ *invest, fully commit their own capital* $c^e$*, and issue* $z^D = 1 - c^e$ *in bonds;*
  - $p \in [p_1, p_2)$ *invest, fully commit their own capital* $c^e$*, and issue* $z^B = 1 - c^e$ *in bank loans;*
  - $p \in [p_{min}, p_1)$ *do not invest;*
- *bankers act as follows:*
  - *skilled bankers intermediate, financing every dollar of assets with* $z^S = \bar{z}^S(r^b; m^s)$*,* $z^D = l^b - \bar{z}^S(r^b; m^s)$*, and* $z^E = 1 - l^b$*;*
  - *unskilled bankers do not intermediate;*

*and* $\{p_1, p_2, 1 + r^b, l^b\}$ *are characterized by (3.3), (3.4), (3.5), and (3.6), where* $\bar{z}^S(r^b; m^s) \equiv \frac{1}{1+r}\gamma_d^s\left(\frac{1+r^b}{1+m^s}\right)$ *and* $\gamma_d^s \equiv E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]\right]$.

The following result describes a notable property of this equilibrium.

**Corollary 3.1.** *When the fraction of skilled bankers* $\lambda$ *is sufficiently low, the separating equilibrium defined in Proposition 3.5 is the only equilibrium at that level of* $1 + r^b$*.*

Intuitively, at $\frac{1+r^b}{1+m^s} > 1 + r$, any candidate pooling equilibrium would imply a subsidy in equity issuance for an unskilled banker. If leverage $l^b \leq \bar{z}^S(r^b; m^u)$, an unskilled banker would earn strictly positive profits: he would incur no cost of financial distress, and would receive a subsidy in equity issuance. If leverage $l^b > \bar{z}^S(r^b; m^u)$, an unskilled banker would earn a subsidy from equity issuance but face a cost of financial distress. When $\lambda$ is sufficiently small, the former would be sufficiently small relative to the latter such that the banker would earn negative profits even for $l^b$ just above $\bar{z}^S(r^b; m^u)$.[8] Hence, at this level of $1 + r^b$, there can be no pooling equilibrium consistent with free entry into banking. Thus, the separating equilibrium under study is *unique*.

### 3.4.2 Macro shock, bank assets, and bank leverage

With these ideas in mind, let us consider the behavior of banking sector assets and leverage in response to an economic boom. Bank leverage is an equilibrium object characterized in the previous subsection. Bank assets are the mirror image of total entrepreneurial borrowing using bank loans, with an adjustment for the deadweight cost of monitoring.

**Lemma 3.5.** *Total assets of banks are given by*

$$a^b = (F(p_2) - F(p_1))(1 - c^e)(1 + m^s).$$

*Bank leverage (debt/assets) is given by $l^b$ and is characterized in Proposition 3.5.*

I will consider the same definition of an economic boom as that used in the previous section. We then obtain the following main result of this section.

**Proposition 3.6.** *In the banking sector, leverage rises in a boom ($\frac{dl^b}{d\alpha} > 0$). If the share of bank loans in credit provision is sufficiently high, assets also rise ($\frac{da^b}{d\alpha} > 0$).*

As diversified intermediaries, the rise in banks' safe debt capacity $\bar{z}^S(r^b; m^s)$ drives an increase in bank leverage in a boom. Given an increase in their capacity to issue safe debt, if skilled bankers simply replaced risky debt one-for-one with safe debt, it would be strictly

---

[8] This follows from the fact that the junior debt tranche is of size $z^D = l^b - \bar{z}^S(r^b; m^s) \geq \bar{z}^S(r^b; m^u) - \bar{z}^S(r^b; m^s) > 0$ and the assumption that if there is even a little default on this tranche, the cost of financial distress is still incurred *per dollar lent*. It would not be the case if the cost of financial distress was incurred per dollar defaulted.

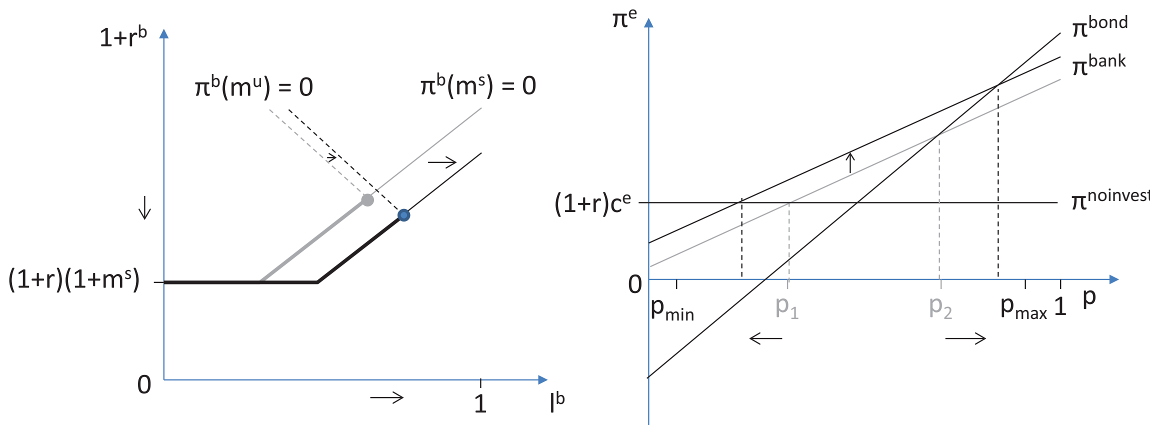



**Figure 3.15:** *Impact of an economic boom*

profitable for unskilled banks to imitate them. Hence, skilled banks ensure overall leverage rises with safe debt capacity.

Moreover, the relative fraction of banks' debt comprised of safe debt increases in a boom, driving an increase in bank assets. Given an increase in their capacity to issue safe debt, skilled bankers need not keep the amount of risky debt unchanged, allowing leverage to rise one-for-one with safe debt. If they did so, unskilled bankers would earn strictly negative profits from imitating them. Since the share of safe debt rises relative to risky debt to ensure that unskilled bankers remain *just* indifferent about imitating them, their weighted average cost of capital falls. Given zero profits in equilibrium, the interest rate on bank loans $1 + r^b$ falls. Indeed, combining (3.5) and (3.6) to obtain

$$1 + r^b = (1+r)(1+m^s)\left(\frac{1 + \frac{\frac{1}{2}d}{1+r+\frac{1}{2}d}}{1 + \frac{\frac{1}{2}d}{1+r+\frac{1}{2}d}\gamma_d^s}\right), \tag{3.7}$$

and recognizing that $\gamma_d^s$ increases with $\alpha$, $1 + r^b$ will fall with $\alpha$. As a result, the marginal bond issuer ($p_2$) rises and the marginal bank loan issuer ($p_1$) falls, driving an increase in bank assets.

Graphically, the rise in banking leverage, fall in the equilibrium cost of bank loans, and resulting rise in banking assets is depicted in Figure 3.15.

Note that the improvement in entrepreneurial project quality means that at given $p_1$ and $p_2$, there are relatively more bond issuers than bank loan issuers. In practice, this is consistent with larger securitization markets in a boom. Nonetheless, to the extent there is enough bank intermediation to begin with, this last effect is dominated by the first two effects, so bank assets also rise in a boom. Intuitively, 'less' of the rightward shift in density occurs in the bond-issuance region.

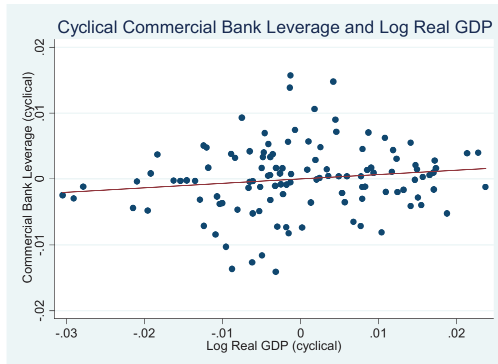
## 3.5 Extensions and applications

In this section I demonstrate that extensions of the present framework can explain additional leverage patterns *within* the banking and non-financial sectors. First, an extension with deposit insurance clarifies why commercial banks may exhibit less procyclical leverage than broker/dealers, a point made by He *et al.* (2010) and Hall and Krueger (2012). Second, an extension where banks can intermediate credit to the non-financial firms modeled in section 3.3 clarifies why bank-dependent firms exhibit more procyclical leverage than other firms, consistent with the findings of Axelson *et al.* (2013) for LBO targets and Leary (2009) for small firms. These extensions lend credibility to the framework of the present paper, as it sheds light on a richer set of cross-sectional patterns than the stylized fact on firm vs. bank leverage it was designed to explain.

### 3.5.1 Commercial banks vs. broker/dealers

There exists a debate in the literature as to whether the procyclicality of leverage is a phenomenon shared by *all* intermediaries. He *et al.* (2010) point out that when measured using market leverage rather than book leverage, the leverage of commercial banks has risen during the 2008-09 crisis. As Hall and Krueger (2012) point out, this would be consistent with — and is indeed necessary to explain — a rise in risk premia observed on assets for which commercial banks were the marginal buyer during the recent crisis.

As shown in Figure 3.16, even when using book leverage, commercial bank leverage is mildly procyclical at best. Quantitatively, the relationship between the cyclical components




**Construction:** leverage = debt / assets, and cyclical components obtained using HP filter (λ= 1600)
**Source**: *Fed Flow of Funds* and Bureau of Economic Analysis, 1984Q1-2013Q1

**Figure 3.16:** *Commercial bank leverage*

of commercial bank leverage and log real GDP is not significantly different from zero, in contrast to a significantly negative relationship for non-financial corporates and significantly positive relationship for broker/dealers, corresponding to the scatterplots in Figure 3.2.[9]

While the simple model in this paper has little to say about the differences between book and market leverage, it can help rationalize a weaker cyclicality of leverage for commercial banks relative to broker/dealers even when measured using book leverage. A key difference between commercial banks and broker/dealers is the fact that the former's deposits are by and large insured. In the context of the model, such deposit insurance means that the safe debt capacity of commercial banks is less sensitive to economic conditions than that of broker/dealers, for whom safe debt capacity may be reflected in their capacity to raise uninsured, collateralized short-term wholesale funds such as repo and ABCP.

Formally, suppose the fiscal authority in the economy under study imposes the following deposit insurance scheme:

- In the aggregate 'up' state, the fiscal authority requires that banks pay a 'deposit insurance fee' of $\tau(\alpha; r^b)$.
- In the aggregate 'down' state, the fiscal authority taxes households $\tau(\alpha; r^b)$ and provides the proceeds to banks to pay their depositors.

[9]This statement holds at any reasonable level of significance ($p = 0.10, 0.05, 0.01$).

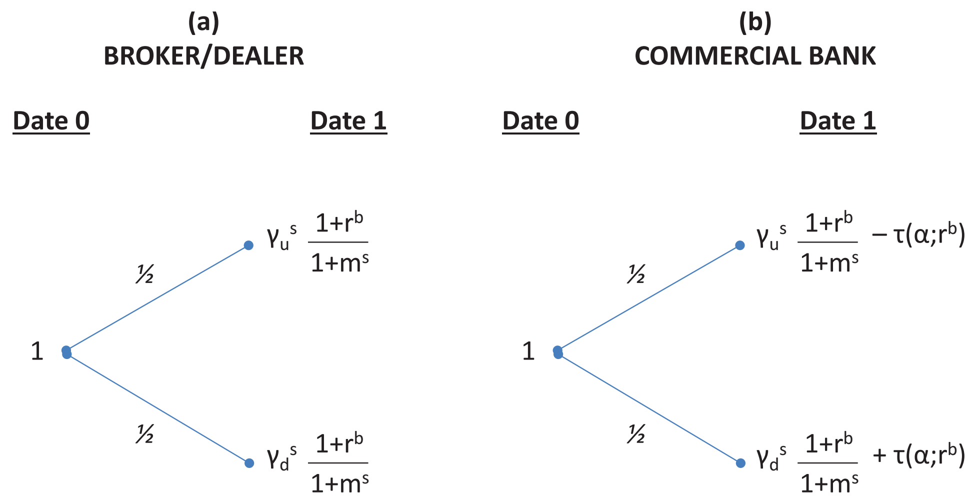



**Figure 3.17:** *Payoffs for broker/dealers vs. commercial banks*

- The lump-sum fee/tax ensures that banks' payoff in the 'down' state is guaranteed at the level that would prevail if $\alpha = 1$ (the benchmark $p \sim U[p_{min}, p_{max}]$ case under consideration):

$$\tau(\alpha; r^b) = \max\{(1+r)\left(\bar{z}^S(r^b, \alpha = 1; m^s) - \bar{z}^S(r^b, \alpha; m^s)\right), 0\}.$$

Under this deposit insurance scheme, the payoff of (skilled) commercial bankers is given in panel (b) of Figure 3.17. In panel (a), I reproduce the payoffs of (skilled) broker/dealers from the benchmark model in section 3.4. In an economy just composed of skilled and unskilled commercial banks subject to the above deposit insurance scheme, we obtain the following result.

**Proposition 3.7.** *Consider a commercial bank with guaranteed deposits. In an economic downturn (a fall in $\alpha$), $a^b$ rises and $l^b$ is unchanged.*

Intuitively, for commercial banks the guarantee on safe debt means that bank leverage need not change. Moreover, because these banks' weighted average cost of capital is unchanged, the interest rate on bank loans need not rise. Given that the interest rate on commercial bank loans does not rise, the marginal bond issuer $p_2$ and bank loan issuer

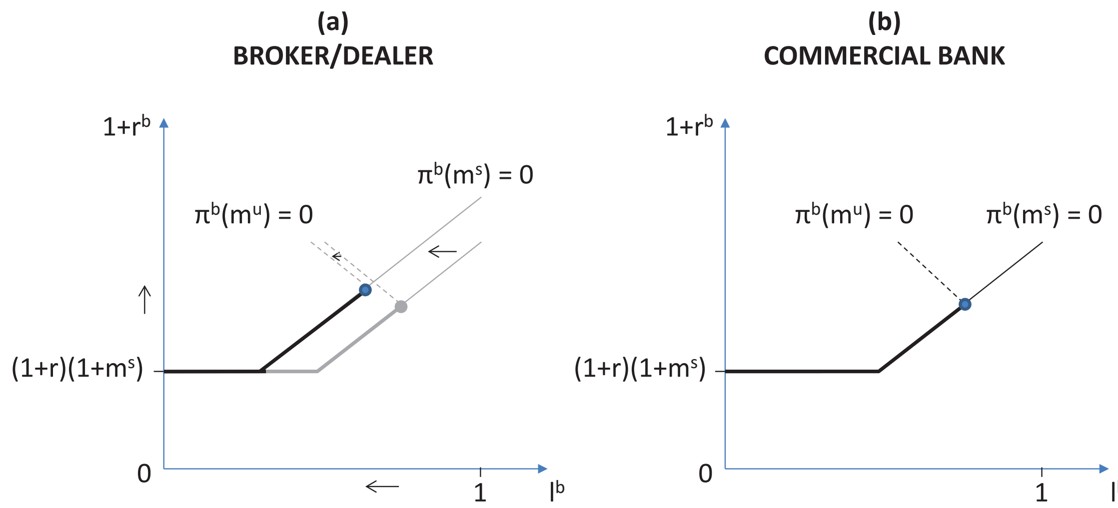



**Figure 3.18:** *Impact of an economic downturn*

$p_0$ do not change. However, because there are relatively worse quality entrepreneurs in the economy, the demand for bank loans rises and thus commercial bank assets rise. This difference between commercial banks and broker/dealers is illustrated in Figure 3.18. Notably, the behavior of commercial bank assets is consistent with the empirical literature on commercial bank reintermediation during downturns (Gatev and Strahan (2006), Ivashina and Scharfstein (2010)), and is obtained without any 'flight to quality' effect.

### 3.5.2 Bank-dependent firms vs. non-bank-dependent firms

I now consider an extension of the financing chain involving bank intermediation in which entrepreneurs can issue equity along with bonds and bank loans. There are now two stages of signaling: between entrepreneurs and investors, and between bankers and investors.

I conjecture an equilibrium which is a natural extension of the ones discussed in the previous sections, depicted in panel (a) of Figure 3.19. In particular, now the worst quality entrepreneurs issue equity, moderate quality entrepreneurs issue bank loans, and high quality entrepreneurs issue bonds. We obtain the following statement of equilibrium.

**Proposition 3.8.** *Under Assumptions 3.4 and 3.5, there exists an equilibrium in which:*

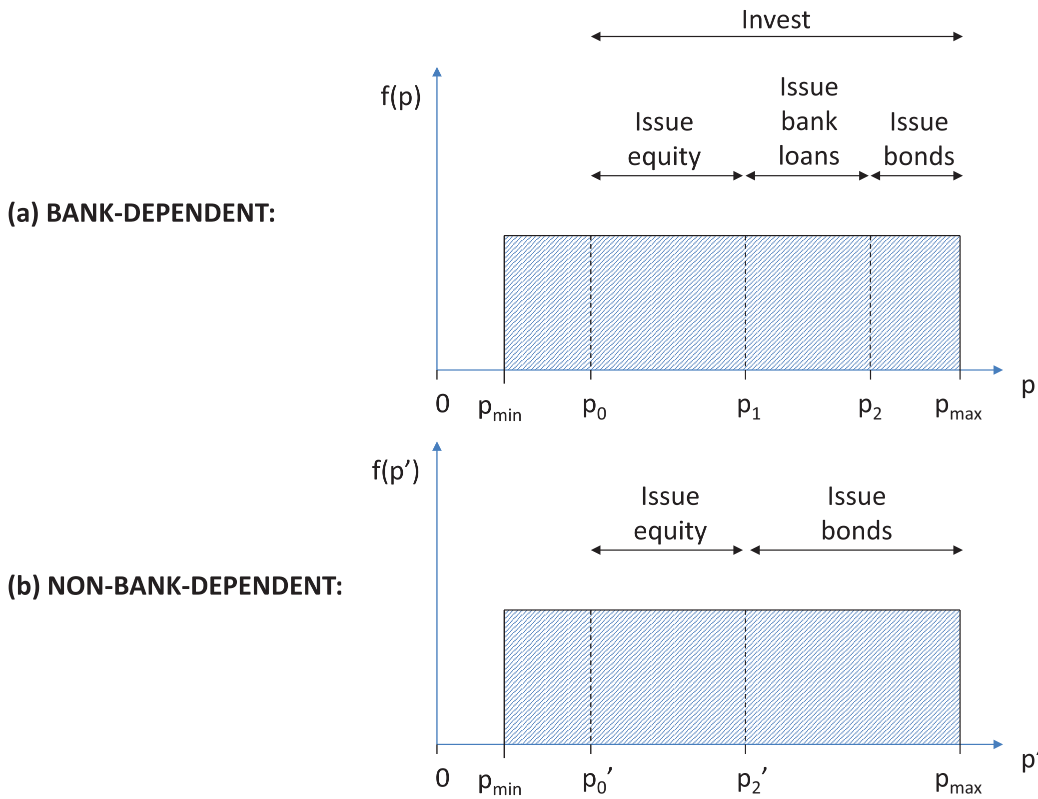



**Figure 3.19:** *Bank-dependent vs. non-bank-dependent firms*

- *entrepreneurs sort as follows:*
  - $p \in [p_2, p_{max}]$ *invest, fully commit their own capital* $c^e$*, and issue* $z^D = 1 - c^e$ *in bonds;*
  - $p \in [p_1, p_2)$ *invest, fully commit their own capital* $c^e$*, and issue* $z^B = 1 - c^e$ *in bank loans;*
  - $p \in [p_0, p_1)$ *invest, fully commit their own capital* $c^e$*, and issue* $z^E = 1 - c^e$ *in equity;*
  - $p \in [p_{min}, p_1]$ *do not invest;*
- *bankers act as follows:*
  - *skilled bankers intermediate, financing every dollar of assets with* $z^S = \bar{z}^S(r^b; m^s)$*,* $z^D = l^b - \bar{z}^S(r^b; m^s)$*, and* $z^E = 1 - l^b$*;*
  - *unskilled bankers do not intermediate;*

*and* $\{p_0, p_1, p_2, 1+r^b, l^b\}$ *are characterized by the system:*

$$\begin{cases} \frac{1}{2}(1-p_2)d = r^b - r \\ (1+r)\left(\frac{v(p_1)}{v(p_0)}\right) = 1 + r^b \\ \left(1 - \frac{(1+r)(1-c^e)}{v(p^E)}\right) v(p_0) = (1+r)c^e \\ l^b = \bar{z}^S(r^b; m^s) + (1-z^S)\frac{1}{1+\frac{\frac{1}{2}d}{1+r}} \\ \frac{1+r^b}{1+m^s} = 1 + r + \frac{1}{2}d(l^b - \bar{z}^S(r^b; m^s)) \end{cases},$$

*where* $p^E \equiv E[p|p \in [p_0, p_1]]$*,* $\bar{z}^S(r^b; m^s) = \frac{1}{1+r}\gamma_d^s\left(\frac{1+r^b}{1+m^s}\right)$*, and* $\gamma_d^s = E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]\right]$.

We can now compare these firms' behavior to those of firms with the same *level* of leverage, but which do not rely on any bank financing. That is, consider the entrepreneurs in panel (b) of Figure 3.19. For the cutoff between equity and bank loans for bank-dependent firms to be the same as the cutoff between equity and bonds for non-bank-dependent firms, the latter must have a lower cost of financial distress but prohibitively high monitoring costs. This is formalized below.

**Corollary 3.2.** *Consider two groups of firms across which banks cannot diversify. Bank-dependent firms are defined as in the previous proposition, while non-bank-dependent firms face a cost of financial distress* $d'$ *but prohibitively expensive monitoring costs* $m^{s\prime}, m^{u\prime} \to \infty$*, such that these firms are only equity and bond-financed. Then there exists a unique* $d' < d$ *such that* $p_0 = p_0'$*,* $p_1 = p_2'$*, and thus* $l^e = l^{e\prime}$*.*

Controlling for the initial leverage in this way, we can ask how an economic boom differentially affects the financing cutoffs for bank-dependent and non-bank-dependent firms, and thus the cyclicality of their leverage over the business cycle. We obtain the following key result.

**Proposition 3.9.** *Leverage is less countercyclical among bank-dependent firms (*$\frac{dl^e}{d\alpha} > \frac{dl^{e\prime}}{d\alpha}$*), and is in fact procyclical for bank-dependent firms (*$\frac{dl^e}{d\alpha} > 0$*).*

Intuitively, in an economic upturn, bank-dependent firms find that borrowing from banks is relatively more attractive since the increase in banks' safe debt capacity flows

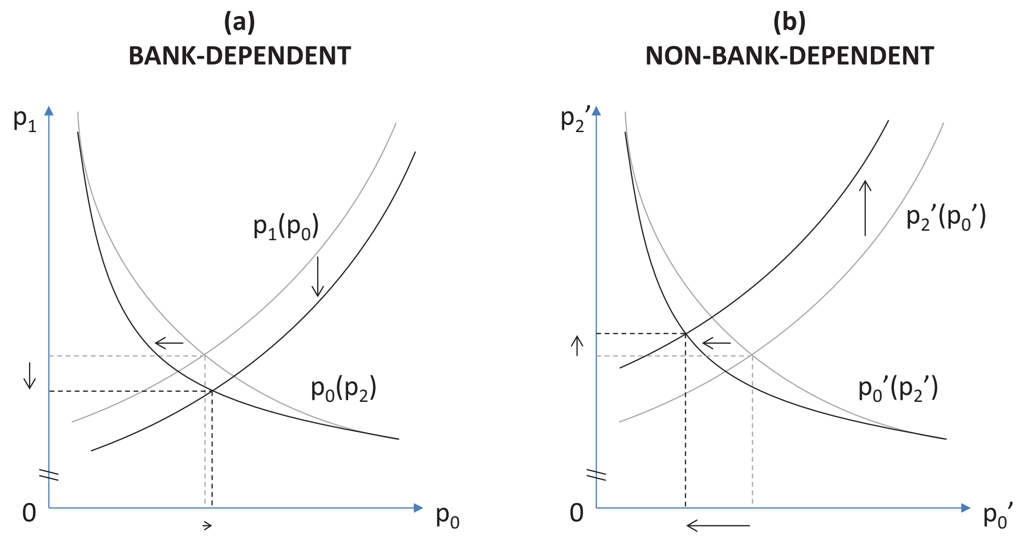



**Figure 3.20:** *Impact of an economic boom*

through in general equilibrium into a lower cost of bank loans. As a result, leverage is less countercyclical for bank-dependent firms than it is for other firms. In fact, given the assumed functional forms and parametric assumptions, the fall in the cost of bank loans dominates the fall in the lemons discount from equity issuance, such that the leverage of bank-dependent firms is procyclical. These differences are depicted graphically in Figure 3.20, where we see in panel (a) that the region of bank-dependent firms which issue equity shrinks in a boom, while in panel (b) the region of non-bank-dependent firms which issue equity widens in a boom.

These cross-sectional predictions on firm leverage are consistent with the findings of Axelson *et al.* (2013) that LBO targets exhibit more procyclical leverage than a matched sample of other non-financial firms, as LBO financing is heavily dependent on the syndicated loan market. They are also consistent with the findings in Leary (2009) that smaller, bank-dependent firms exhibit more procyclical leverage than larger, less bank-dependent firms.

Proposition 3.9 also makes an interesting and testable cross-country prediction. In particular, it would predict that holding the initial level of leverage the same, firms in bank-dependent economies (e.g., Europe) should exhibit more procyclical leverage than those in non-bank-dependent economies (e.g., the U.S.). I leave an empirical test of this prediction to future work.

## 3.6 Welfare and policy

Having demonstrated the positive explanatory power of the framework developed in this paper, I now study its normative implications. I first characterize the constrained efficient allocation chosen by a planner constrained by the same technological and informational frictions as agents in the economy. I then show that there is a role for macroprudential policy in limiting the level, but not cyclicality, of bank leverage to implement the constrained efficient allocation.

### 3.6.1 Constrained efficiency

Consider a planner focused on the intermediation chain involving bankers studied in section 3.4, and who is restricted to selecting one among the feasible allocations supported as competitive equilibria following Definition 3.3. This planner then is constrained by the same technological and informational frictions as agents in the economy, but has the power to choose any one of the multiple equilibria.

I define a *constrained efficient* allocation to be one chosen by this planner to maximize ex-ante utilitarian social welfare.[10] Given linear and identical preferences across agents, this welfare metric is simply given by the NPV of investment, less monitoring costs and expected costs of financial distress, plus initial wealth in the economy. The following result characterizes the equilibria which achieve the constrained efficient allocation according to this definition.

**Proposition 3.10.** *In the constrained efficient equilibria, entrepreneurs sort as in Proposition 3.5, bankers act as follows:*

[10]It is well known that in signaling models in the tradition of Spence (1973), the set of multiple equilibria often can be Pareto ranked. My definition of the constrained efficient allocation in this setting identifies a particular equilibrium or set of equilibria on the Pareto frontier — namely, that/those which maximize ex-ante utilitarian social welfare.

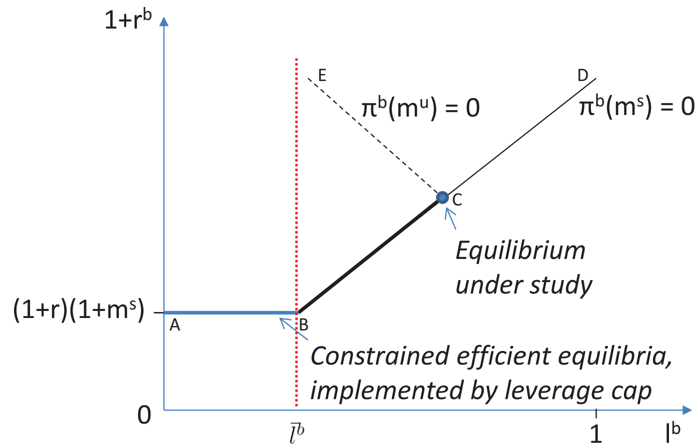



**Figure 3.21:** *Constrained efficiency and implementation in banking*

- *skilled bankers intermediate, financing every dollar of assets with* $z^{S,ce} \leq \bar{z}^S(r^{b,ce}; m^s) \equiv \frac{1}{1+r}\gamma_d^s\left(\frac{1+r^{b,ce}}{1+m^s}\right)$, $z^{D,ce} = 0$, *and* $z^{E,ce} = 1 - z^{S,ce}$;
- *unskilled bankers may or may not intermediate;*

*and* $\{p_1^{ce}, p_2^{ce}, 1 + r^{b,ce}\}$ *are characterized as follows:*

$$\begin{cases} \frac{1}{2}(1 - p_2^{ce})d = r^{b,ce} - r \\ v(p_1^{ce}) = 1 + r + (r^{b,ce} - r)(1 - c^e) \\ 1 + r^{b,ce} = (1 + r)(1 + m^s) \end{cases} .$$

These equilibria are depicted in Figure 3.21, and are compared with the benchmark equilibrium under study in section 3.4 (at point C).

In the constrained efficient equilibria, the equilibrium interest rate on bank loans simply compensates bankers for the cost of monitoring. The ROA of skilled bankers is thus $1 + r$, and their equilibrium financing is simply composed of safe debt (deposits) and equity. Because skilled bankers' ROA is identical to that of unskilled bankers, there is no incentive for the latter to imitate the former, and the equity of skilled bankers will be fairly priced. Hence, bankers' weighted average cost of capital is $1 + r$, ensuring that the zero profit condition is satisfied. Indeed, within the class of financing structures using only safe debt and equity, bank financing is indeterminate following the Modigliani and Miller (1958)

Theorem.

Relative to the constrained efficient allocation, the equilibrium studied in section 3.4 features fewer positive-NPV entrepreneurs receiving financing and greater deadweight costs of bond financing. As such, not only does the constrained efficient allocation yield higher ex-ante utilitarian social welfare — it Pareto dominates the equilibrium of section 3.4. Moving from the latter to the former, previously bank-financed and newly bank-financed entrepreneurs see their expected profit strictly increase, while all other entrepreneurs find that their profit has not changed. Bankers and households continue to earn zero profits and $(1+r)c^h$, respectively.

Coordination at the inefficient equilibrium of section 3.4 is sustained by asymmetric information. In particular, at a high interest rate on bank loans, skilled bankers attempt to signal their quality to the market by issuing costly, risky debt which unskilled bankers cannot profitably replicate. The act of issuing costly, risky debt raises skilled bankers' weighted average cost of capital, supporting the high interest rate on bank loans in an equilibrium with free entry into banking. No skilled banker seeks to issue less risky debt, and thus charge a lower interest rate on bank loans and capture the market, for fear that doing so would signal he is unskilled.

### 3.6.2 Implementation through macroprudential regulation

In practice, the observation that banks issue massive quantities of short-term financing suggests that we may indeed live in a world that has coordinated on the inefficient equilibrium discussed above. While the distinction between safe and risky short-term debt is admittedly imprecise, if we interpret at least some of these short-term debt securities as costly attempts to signal quality to the market, this model suggests that there can exist a socially excessive incentive to do so.

Starting from the inefficient equilibrium, Figure 3.21 suggests that a simple macroprudential tool can implement the constrained efficient allocation: a cap on bank leverage. Similar to results in other signaling models where banning signaling can generate a Pareto improvement, a leverage cap can break coordination at a 'high interest rate - high leverage' equilibrium. Formally, we have:

**Proposition 3.11.** *A banking leverage cap of* $\bar{l}^b = \bar{z}^S(r^{b,ce}; m^s)$ *can eliminate all equilibria in the banking sector except those leading to the constrained efficient allocation.*

At the same time, because banks' safe debt capacity remains procyclical in the constrained efficient allocation, the model does not justify the use of macroprudential tools to limit this cyclicality. The intuition is unchanged from the equilibrium of section 3.4: banks are diversified intermediaries whose worst-case payoff rises when idiosyncratic project quality in the economy improves.

Indeed, within the set of constrained efficient equilibria, there are reasons outside the model to think that letting banks issue as much safe debt as possible may be uniquely constrained efficient. In particular, there appears to exist a special demand for banks' safe liabilities — whether because they provide the transaction services of money (a la Stein (2012)), or because they cater to a specific investor clientele (pension funds, etc.) in a world of segmented markets.

To that end, the following result is notable.

**Proposition 3.12.** *The safe debt capacity of banks in the constrained efficient equilibria* $\bar{z}^S(r^{b,ce}; m^s)$ *is procyclical:* $\frac{d}{d\alpha}\bar{z}^S(r^{b,ce}; m^s) > 0$. *Thus, implementing the constrained efficient equilibrium with maximal safe debt issuance requires a time-varying, procyclical leverage cap.*

Other authors have also advocated for time-variation in bank regulation; what is notable about this result is that bank leverage regulation should be *looser* in good times, to accommodate time variation in the safe debt capacity in the banking sector. To be sure, proponents of countercyclical leverage requirements (e.g., Hanson *et al.* (2011)) have in mind dynamic trade-offs and pecuniary externalities which the present model does not capture. Nonetheless, the analysis here demonstrates that time variation in banks' capacity to issue safe debt is an important force which should also be taken into account in the analysis of macroprudential regulation.

## 3.7 Conclusion

This paper jointly characterizes the investment and financing of non-financial firms and banks in general equilibrium. It is designed to explain the differential cyclicality of firm and bank leverage over the business cycle. The core idea is that while firm leverage falls in an economic boom because of a lower lemons discount in equity issuance relative to the costs of financial distress, bank leverage rises because banks, as diversified intermediaries, see their capacity to issue safe debt expand.

I use this framework to then illuminate several other stylized facts in the data and questions of welfare and policy. First, the model rationalizes the weaker procyclicality of commercial bank leverage as arising from deposit insurance: such insurance reduces the volatility in safe debt capacity for commercial banks relative to that for uninsured intermediaries such as broker/dealers. Second, the model rationalizes evidence on the procyclicality of leverage for LBO targets and small firms: such bank-dependent firms inherit the improved funding conditions of banks in a boom due to a general equilibrium effect on the cost of bank loans. Finally, the model motivates a role for macroprudential regulation of the level of bank leverage, but not its cyclicality. The latter follows from the fact that bank issuance of safe debt is a constrained efficient response to informational and technological frictions in the economy, and their capacity to issue safe debt is procyclical.

A fruitful direction for future work is to enrich the present framework with durable goods, permitting the modeling of pecuniary externalities. This might introduce a rich trade-off in the determination of time-varying capital requirements for banks: on the one hand, preventing banks from shedding assets and delevering in a downturn might avoid socially costly pecuniary externalities, but on the other, forcing banks to maintain higher capital ratios in a boom might complicate their issuance of socially beneficial safe debt. Analyzing this trade-off through an enriched version of the present framework may shed light on comparative statics and empirical moments which can guide policymakers in balancing these objectives to optimally regulate bank capital over the cycle.

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# Appendix A

# Appendix to Chapter 1

## A.1 Proofs of results in the main text

### Proposition 1.1: equivalence of implementable allocations

*Proof.* The text almost completely proves the $\Rightarrow$ direction. Suppose there exists an allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$ as part of a flexible price and wage equilibrium. Then in the same competitive equilibrium, define $w_1^e$ and $w_1^u$ as in (1.15) and (1.16). By the definition of worker optimization in the competitive equilibrium, $c_t^i = c_t^i(w_1^i, p_2)$ and $s = s(w_1^e, w_1^u, p_2, \theta)$ for Marshallian demand and labor supply functions defined in (1.14) and (1.18), respectively.

It only remains to prove that the resource constraints (1.19) and (1.20) are indeed satisfied in the above equilibrium. Optimal price-setting by retailers will clearly lead to identical posted prices $P_1(j) = P_1$, which implies that lower-stage worker optimization will imply $c_1^i(j) = c_1^i$. Hence, date 1 final goods market clearing in (1.7), the pass-through technology of retailers in (1.6), and intermediate goods market clearing in (1.9) imply that (1.19) must be satisfied. And date 2 final goods market clearing in (1.8) immediately implies that (1.20) is satisfied.

I turn to the $\Leftarrow$ direction. Suppose there exist $\{w_1^e, w_1^u\}$ such that, for a particular allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$, resource constraints (1.19) and (1.20) are satisfied and the stated implementability constraints are also satisfied. The goal is to show that this allocation and relative price form part of a flexible price and wage equilibrium for some prices, wages, and profits $\{\{P_1(j)\}, P^I, P_2, W, \Pi, \{\Pi^r(j)\}\}$, tightness schedule $\theta(W)$, and policy $\{b, t, \tau; i, P_2; T^r, \tau^r\}$ per Definition 1.1.

The proof is constructive, demonstrating that appropriate values of prices and policies ensure that agent first-order conditions, agent resource constraints, and market clearing conditions are satisfied at the given allocation and relative price. The first two imply that agents are solving their appropriate optimization problem, given my maintained assumption that their first-order conditions are sufficient to characterize optimality. I will refer to first-order conditions obtained in my analysis of equilibrium in Appendix A.2.

I start with firms. Given $s$ and $\theta$, define $\nu \equiv \theta s$. Then, producers' first-order condition with respect to $\nu$ pins down $\frac{W}{P^I}$. Producers' first-order condition with respect to $W$ pins down $\frac{\theta(W)}{\theta'(W)}$ (locally). Without loss of generality, I then set $\tau^r = -\frac{1}{\varepsilon}$. Retailers' first-order condition with respect to $P_1(j)$ pins down $\frac{P_1(j)}{P^I} = 1 \ \forall j$. Defining the CES aggregator in (1.13), this implies that $P_1(j) = P_1 \ \forall j$ and that $\frac{P^I}{P_1} = 1$. The constraints of problem (1.6) then pin down $x(j) = y_1(j) = (p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u$. Given $\tau^r$, government budget balance in policy targeted at retailers (1.12) pins down $\frac{T^r}{P^I}$.

There is now a standard indeterminacy with flexible prices and wages: we can arbitrarily pick $P_1$. Given relative price (inverse real interest rate) $p_2$, this requires monetary policy $\{i, P_2\}$ satisfying the Fisher equation in (1.17). Having picked $P_1$, the above results pin down $P^I$, $W$, and $T^r$. Moreover, the objective functions of producers and retailers pin down profits $\Pi$ and $\Pi^r(j)$.

I turn now to worker optimization. Given $\frac{\theta(W)}{\theta'(W)}$ (locally), local worker indifference across submarkets pins down $\tau$. Then, given wealth levels $w_1^e$ and $w_1^u$, $t$ and $b$ are pinned down by (1.15) and (1.16), respectively. Worker optimization of the second-stage problem is satisfied with $c_1^i(j) = c_1^i$. And since the implementability constraints $c_t^i = c_t^i(w_1^i, p_2)$ and $s = s(w_1^e, w_1^u, p_2, \theta)$ are assumed satisfied at the given allocation, it follows that $\{c_1^i, c_2^i\}$ and $s$ solve (1.14) and (1.18). Given that $t$ and $b$ were constructed to satisfy (1.15) and (1.16), it follows that $\{c_1^i, c_2^i\}$ and $s$ solve the original problems (1.4) and (1.5). Agents' net asset positions in the short-run $z_1^e$ and $z_1^u$ are then pinned down by the date 2 budget constraints in (1.4). As all macroeconomic aggregates are pinned down at this point, the full schedule $\theta(W)$ can be defined by worker indifference across submarkets indexed by alternative $W$.

Since the given allocation satisfies (1.19) and (1.20), goods market clearing at each date is ensured. All that remains is to check that agents' date 1 budget constraints in (1.4) are satisfied, that bond market clearing (1.10) is satisfied, and that government budget-balance in the labor-market (1.11) is satisfied. The first two are implied by Walras' law for agents' utility maximization problem. The second two are implied by Walras' law holding for the economy's excess demand function. □

### Proposition 1.2: Ramsey optimal risk-sharing

*Proof.* The first-order conditions to the Ramsey planning problem with respect to $w_1^e$ and $w_1^u$ are

$$\begin{aligned} p(\theta)s\frac{\partial v^e}{\partial w_1^e} + \lambda_{RC1}\left(\frac{\partial x_1}{\partial s}\frac{\partial s}{\partial w_1^e} - p(\theta)s\frac{\partial c_1^e}{\partial w_1^e}\right) \\ + \lambda_{RC2}\left(\frac{\partial x_2}{\partial s}\frac{\partial s}{\partial w_1^e} - p(\theta)s\frac{\partial c_2^e}{\partial w_1^e}\right) &= 0, \\ (1-p(\theta)s)\frac{\partial v^u}{\partial w_1^u} + \lambda_{RC1}\left(\frac{\partial x_1}{\partial s}\frac{\partial s}{\partial w_1^u} - (1-p(\theta)s)\frac{\partial c_1^u}{\partial w_1^u}\right) \\ + \lambda_{RC2}\left(\frac{\partial x_2}{\partial s}\frac{\partial s}{\partial w_1^u} - (1-p(\theta)s)\frac{\partial c_2^u}{\partial w_1^u}\right) &= 0, \end{aligned}$$

given multipliers on the resource constraints $\lambda_{RC1}$ and $\lambda_{RC2}$, and the definition of the production-inclusive excess supply functions $x_1(s,\theta)$ and $x_2(s,\theta)$ in (1.23) and (1.24). As in the standard partial equilibrium analysis, these first-order conditions make use of the Envelope theorem to ignore the change in the planner's objective from the labor supply response to marginal changes in wealth.

Straightforward algebraic manipulation then yields:

$$\frac{\partial v^e}{\partial w_1^e} = \frac{\lambda_{RC2}}{p_2}\left[\left(\frac{\lambda_{RC1}}{\lambda_{RC2}}\right)p_2\frac{\partial c_1^e}{\partial w_1^e} + p_2\frac{\partial c_2^e}{\partial w_1^e}\right.$$
$$\left. - \frac{1}{p(\theta)s}\left(\left(\frac{\lambda_{RC1}}{\lambda_{RC2}}\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s}\right)\right)\frac{\partial s}{\partial w_1^e}\right], \tag{A.1}$$

$$\frac{\partial v^u}{\partial w_1^u} = \frac{\lambda_{RC2}}{p_2}\left[\left(\frac{\lambda_{RC1}}{\lambda_{RC2}}\right)p_2\frac{\partial c_1^u}{\partial w_1^u} + p_2\frac{\partial c_2^u}{\partial w_1^u}\right.$$
$$\left. - \frac{1}{1-p(\theta)s}\left(\left(\frac{\lambda_{RC1}}{\lambda_{RC2}}\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s}\right)\right)\frac{\partial s}{\partial w_1^u}\right]. \tag{A.2}$$

Now, define the relative price wedge $\tau_{1,2} \equiv 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}}p_2$ as in Definition 1.3. Then

$$\left(\frac{\lambda_{RC1}}{\lambda_{RC2}}\right)p_2\frac{\partial c_1^i}{\partial w_1^i} + p_2\frac{\partial c_2^i}{\partial w_1^i} = 1 - \left(1 - \left(\frac{\lambda_{RC1}}{\lambda_{RC2}}\right)p_2\right)\frac{\partial c_1^i}{\partial w_1^i},$$
$$= 1 - \tau_{1,2}\frac{\partial c_1^i}{\partial w_1^i},$$

where the first line uses the identity $\frac{\partial c_1^i}{\partial w_1^i} + p_2\frac{\partial c_2^i}{\partial w_1^i} = 1$ implied by partial differentiation of the budget constraint in agent $i$'s ex-post problem (1.14). We can plug this result and the definition of $\tau_{1,2}$ into (A.1) and (A.2), and then solve out for $\frac{\lambda_{RC2}}{p_2}$ to obtain

$$\frac{\frac{\partial v^e}{\partial w_1^e}}{1 - \tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e} - \frac{1}{p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}} = \frac{\lambda_{RC2}}{p_2}$$
$$= \frac{\frac{\partial v^u}{\partial w_1^u}}{1 - \tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u} - \frac{1}{1-p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}}. \tag{A.3}$$

Finally, using the result of Proposition 1.7, $\tau_{1,2} = 0$ in the present planning problem without any ZLB constraint, for clearly this is equivalent to the solution of the problem with a slack constraint. Plugging this in above, the Ramsey optimal risk-sharing result with flexible prices and wages immediately follows. □

**Lemma 1.1: implementing the size of transfers**

*Proof.* By producer optimization in vacancy posting and a zero equilibrium markup in the flexible price and wage equilibrium, as detailed in Appendix A.2, we have

$$w = f'(p(\theta)s - k\theta s))\left(1 - \frac{k}{q(\theta)}\right),$$

where $w$ is the real wage. It follows that

$$\begin{aligned}
\omega &= (w + p_2(y_2^e - y_2^u)) - ((c_1^e + p_2 c_2^e) - (c_1^u + p_2 c_2^u)), \\
&= ((w + \pi + p_2 y_2^e) - (c_1^e + p_2 c_2^e)) - ((\pi + p_2 y_2^u) - (c_1^u + p_2 c_2^u)), \\
&= ((w + \pi + p_2 y_2^e) - ((1-\tau)w - t + \pi + p_2 y_2^e)) - ((\pi + p_2 y_2^u) - (b + \pi + p_2 y_2^u)), \\
&= \tau w + t + b, \\
&= \frac{1}{p(\theta)s} b,
\end{aligned}$$

where $\pi \equiv \frac{1}{P_1}(\Pi + \Pi^r - T^r)$, the third line uses agents' budget constraints in (1.15) and (1.16) accompanying optimization problem (1.14), and the final line uses government budget balance in (1.11). □

**Proposition 1.3: a general equilibrium Baily-Chetty formula**

*Proof.* We make use of two facts at the micro level. First, workers' ex-post problem (1.14) implies

$$\frac{\partial v^i}{\partial w_1^i} = \frac{\partial u^i}{\partial c_1^i}, \tag{A.4}$$

a standard application of the Envelope theorem. Second, the first-order condition of workers' ex-ante problem (1.18) implies that

$$\frac{\partial s}{\partial w_1^e} = -\frac{\frac{\partial v^e}{\partial w_1^e}}{\frac{\partial v^u}{\partial w_1^u}} \frac{\partial s}{\partial w_1^u}, \tag{A.5}$$

which captures the tight link between the labor supply response to changes in wealth when employed versus unemployed.

Then, manipulating the Ramsey optimal risk-sharing condition from Proposition 1.2, we obtain:

$$\frac{\frac{\partial v^e}{\partial w_1^e}}{1-\frac{1}{p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}} = \frac{\frac{\partial v^u}{\partial w_1^u}}{1-\frac{1}{1-p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}}$$

$$\Rightarrow \left[1-\frac{1}{1-p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}\right] = \frac{\frac{\partial v^u}{\partial w_1^u}}{\frac{\partial v^e}{\partial w_1^e}}\left[1-\frac{1}{p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}\right],$$

$$\Rightarrow 1-\frac{1}{1-p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u} = \frac{\frac{\partial v^u}{\partial w_1^u}}{\frac{\partial v^e}{\partial w_1^e}}+\frac{1}{p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u},$$

$$\Rightarrow -\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u} = \frac{\frac{\partial v^u}{\partial w_1^u}-\frac{\partial v^e}{\partial w_1^e}}{\frac{\partial v^e}{\partial w_1^e}},$$

$$\Rightarrow -\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u} = \frac{\frac{\partial u^u}{\partial c_1^u}-\frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}},$$

where the third line uses (A.5) and the last line uses (A.4).

Now, on the left-hand side,

$$\begin{aligned}
-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u} &= -\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)(p(\theta)\omega)\frac{\partial s}{\partial w_1^u},\\
&= -\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\frac{1}{s}b\right)\frac{\partial s}{\partial w_1^u},\\
&= -\left(\frac{1}{p(\theta)s}\right)^2\left(\frac{p(\theta)s}{1-p(\theta)s}\right)\left(\frac{1}{s}b\right)\frac{\partial s}{\partial w_1^u},\\
&= \left(\frac{1}{p(\theta)s}\right)^2\varepsilon_b^{P(unemp)},
\end{aligned}$$

where the first line uses the definition of the size of transfers $\omega$ in (1.25), the second line uses the implementation of transfers in Lemma 1.1, and the last line uses the definition of the micro-elasticity $\varepsilon_b^{P(unemp)}$ in (1.27). The general equilibrium Baily-Chetty formula follows. □

### Proposition 1.4: equivalence of implementable allocations

*Proof.* As with the proof of Proposition 1.1, the text almost completely proves the ⇒ direction. Given an allocation and relative price as part of a fully sticky price equilibrium, then in the same equilibrium define $w_1^e$ and $w_1^u$ as in (1.15) and (1.16). By the definition of worker optimization in the competitive equilibrium, $c_t^i = c_t^i(w_1^i, p_2)$ and $s = s(w_1^e, w_1^u, p_2, \theta)$ for Marshallian demand and labor supply functions defined in (1.14) and (1.18), respectively.

Unlike the flexible price and wage case, retailers no longer update prices; nonetheless, the assumption of identical pre-set prices $P_1(j) = \bar{P}_1$ ensures that lower-stage worker optimization still implies $c_1^i(j) = c_1^i$. Hence, date 1 final goods market clearing in (1.7), the pass-through technology of retailers in (1.6), and intermediate goods market clearing in (1.9) again imply that the resource constraint (1.19) must be satisfied. And date 2 final goods market clearing in (1.8) again implies that the resource constraint (1.20) is satisfied.

Finally, as described in the text, the real interest rate must be bound below by (1.32) in the given competitive equilibrium. It follows that the ZLB implementability constraint (1.33) is satisfied.

I turn to the $\Leftarrow$ direction.

As in the constructive proof developed for Proposition 1.1, $s$ and $\theta$ imply $\nu \equiv \theta s$. Producers' first-order condition with respect to $\nu$ pins down $\frac{W}{P^I}$ and their first-order condition with respect to $W$ pins down $\frac{\theta(W)}{\theta'(W)}$ (locally). Without loss of generality, I again set $\tau^r = -\frac{1}{\varepsilon}$. Now there is no retailer optimality condition to consider; instead, they simply satisfy demand at posted, identical prices $\bar{P}_1$. Since prices are identical, we still have $x(j) = y_1(j) = (p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u$. Given $\tau^r$, government budget balance in policy targeted at retailers (1.12) still pins down $\frac{T^r}{P^I}$.

There is no longer any nominal indeterminacy. Given $\bar{P}_1$, the relative price (inverse real interest rate) $p_2$ must be implemented by a particular monetary policy $\{i, P_2\}$ satisfying the Fisher equation in (1.17). Since the ZLB implementability constraint (1.33) holds, there must exist at least one such combination of $i$ and $P_2$ which also satisfy the constraints on monetary policy in (1.31).

I turn now to worker optimization. Given $\frac{\theta(W)}{\theta'(W)}$ (locally), local worker indifference across submarkets pins down $\frac{W}{\bar{P}_1}(1-\tau)$. But importantly, while the post-tax real wage is pinned down, the decomposition between the gross wage and payroll taxes is not (as in the standard Keynesian analysis with sticky prices). Equivalently, $\tau$ can be used to target any gross real wage $\frac{W}{\bar{P}_1}$ and thus the inverse gross markup $\frac{P^I}{\bar{P}_1}$. Conditional on picking $\tau$, we have $P^I$, $W$, and $T^r$, and then the objective functions of producers and retailers again pin down profits $\Pi$ and $\Pi^r(j)$.

The remainder of the proof proceeds exactly as in that for Proposition 1.1. Importantly, once the redistribution from markups is pinned down by the choice of $\tau$ above, the given wealth levels $w_1^e$ and $w_1^u$ again pin down $t$ and $b$ using (1.15) and (1.16), respectively. □

### Proposition 1.5: optimality of the flexible price and wage allocation

*Proof.* The result follows immediately from the equivalence of Ramsey planning problems under flexible prices (1.21) and sticky prices (1.34), when the $(ZLB)$ constraint is slack. □

### Lemma 1.2: implementing the size of transfers

*Proof.* By producer optimization in vacancy posting as detailed in Appendix A.2, we have

$$w = \mu^{-1} f'(p(\theta)s - k\theta s)) \left(1 - \frac{k}{q(\theta)}\right),$$

where $w$ is the real wage and $\mu^{-1} \equiv \frac{P^I}{\bar{P}_1}$ is the inverse gross markup of retailers. It follows that

$$\begin{aligned}
\omega &= (\mu w + p_2(y_2^e - y_2^u)) - ((c_1^e + p_2 c_2^e) - (c_1^u + p_2 c_2^u)), \\
&= (w + p_2(y_2^e - y_2^u)) - ((c_1^e + p_2 c_2^e) - (c_1^u + p_2 c_2^u)) + (\mu - 1)w, \\
&= \frac{1}{p(\theta)s} b + (\mu - 1)w,
\end{aligned}$$

where the last line uses the same steps as in the proof of Lemma 1.1. □

**Proposition 1.7: a sufficient statistic for the ZLB constraint**

*Proof.* The first-order condition of the Ramsey planning problem with respect to $p_2$ is

$$\begin{aligned} & p(\theta)s\frac{\partial v^e}{\partial p_2} + (1-p(\theta)s)\frac{\partial v^u}{\partial p_2} \\ & \quad + \lambda_{RC1}\left(\frac{\partial x_1}{\partial s}\frac{\partial s}{\partial p_2} - p(\theta)s\frac{\partial c_1^e}{\partial p_2} - (1-p(\theta)s)\frac{\partial c_1^u}{\partial p_2}\right) \\ & \quad + \lambda_{RC2}\left(\frac{\partial x_2}{\partial s}\frac{\partial s}{\partial w_1^e} - p(\theta)s\frac{\partial c_2^e}{\partial p_2} - (1-p(\theta)s)\frac{\partial c_2^u}{\partial p_2}\right) = \lambda_{ZLB}. \end{aligned} \tag{A.6}$$

I focus on simplifying the left-hand side. To do so, it is helpful to use Roy's identity

$$\frac{\partial v^i}{\partial p_2} = -\frac{\partial v^i}{\partial w_1^i}c_2^i,$$

as well as an implication of Roy's identity on the first-order condition of workers' ex-ante problem (1.18)

$$\frac{\partial s}{\partial p_2} = -\frac{\partial s}{\partial w_1^e}c_2^e - \frac{\partial s}{\partial w_1^u}c_2^u,$$

which captures the tight link between the labor supply response to changes in prices and wealth. Using the above two identities, (A.6) can be re-expressed as

$$\begin{aligned} & -c_2^e\left(p(\theta)s\frac{\partial v^e}{\partial w_1^e} + \left(\lambda_{RC1}\frac{\partial x_1}{\partial s} + \lambda_{RC2}\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}\right) \\ & \quad - c_2^u\left((1-p(\theta)s)\frac{\partial v^u}{\partial w_1^u} + \left(\lambda_{RC1}\frac{\partial x_1}{\partial s} + \lambda_{RC2}\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}\right) \\ & \quad - \lambda_{RC1}\left(p(\theta)s\frac{\partial c_1^e}{\partial p_2} + (1-p(\theta)s)\frac{\partial c_1^u}{\partial p_2}\right) - \lambda_{RC2}\left(p(\theta)s\frac{\partial c_2^e}{\partial p_2} + (1-p(\theta)s)\frac{\partial c_2^u}{\partial p_2}\right) = \lambda_{ZLB}. \end{aligned}$$

Then, the first-order conditions for the planning problem with respect to $w_1^e$ and $w_1^u$ (detailed in the proof of Proposition 1.2) can be used to simplify the left-hand side, implying

$$\begin{aligned} & -c_2^e\left(\lambda_{RC1}p(\theta)s\frac{\partial c_1^e}{\partial w_1^e} + \lambda_{RC2}p(\theta)s\frac{\partial c_2^e}{\partial w_1^e}\right) \\ & \quad - c_2^u\left(\lambda_{RC1}(1-p(\theta)s)\frac{\partial c_1^u}{\partial w_1^u} + \lambda_{RC2}(1-p(\theta)s)\frac{\partial c_2^u}{\partial w_1^u}\right) \\ & \quad - \lambda_{RC1}\left(p(\theta)s\frac{\partial c_1^e}{\partial p_2} + (1-p(\theta)s)\frac{\partial c_1^u}{\partial p_2}\right) - \lambda_{RC2}\left(p(\theta)s\frac{\partial c_2^e}{\partial p_2} + (1-p(\theta)s)\frac{\partial c_2^u}{\partial p_2}\right) = \lambda_{ZLB}. \end{aligned}$$

The Slutsky equation can be used to collect terms on the left-hand side, substantially simplifying the identity to

$$-\lambda_{RC1}\left(p(\theta)s\frac{\partial c_1^{e,h}}{\partial p_2}+(1-p(\theta)s)\frac{\partial c_1^{u,h}}{\partial p_2}\right)-\lambda_{RC2}\left(p(\theta)s\frac{\partial c_2^{e,h}}{\partial p_2}+(1-p(\theta)s)\frac{\partial c_2^{u,h}}{\partial p_2}\right)=\lambda_{ZLB}$$

given Hicksian demand functions $c_t^{i,h}(v^i(w_1^i, p_2), p_2)$. Finally, we can use the identity for compensated price derivatives

$$\frac{\partial c_1^{i,h}}{\partial p_2}+p_2\frac{\partial c_2^{i,h}}{\partial p_2}=0$$

to further simplify the above as

$$\begin{aligned}
\frac{\lambda_{RC2}}{p_2}\left(1-\frac{\lambda_{RC1}}{\lambda_{RC2}}p_2\right)\left(p(\theta)s\frac{\partial c_1^{e,h}}{\partial p_2}+(1-p(\theta)s)\frac{\partial c_1^{u,h}}{\partial p_2}\right)&=\lambda_{ZLB},\\
\Rightarrow \frac{\lambda_{RC2}}{p_2}\tau_{1,2}\left(p(\theta)s\frac{\partial c_1^{e,h}}{\partial p_2}+(1-p(\theta)s)\frac{\partial c_1^{u,h}}{\partial p_2}\right)&=\lambda_{ZLB},\\
\Rightarrow \tau_{1,2}=\frac{\lambda_{ZLB}}{\frac{\lambda_{RC2}}{p_2}\left(p(\theta)s\frac{\partial c_1^{e,h}}{\partial p_2}+(1-p(\theta)s)\frac{\partial c_1^{u,h}}{\partial p_2}\right)}&,
\end{aligned}$$

where the second line uses the definition of the relative price wedge in Definition 1.3. Since these compensated cross-price derivatives must be non-negative, we obtain $\tau_{1,2}\propto\lambda_{ZLB}$. □

### Proposition 1.8: Ramsey optimal risk-sharing

*Proof.* The proof proceeds exactly like that for Proposition 1.2. Ramsey optimal risk-sharing with a general relative price wedge $\tau_{1,2}$ is characterized in (A.3). □

### Proposition 1.9: a generalized Baily-Chetty formula

*Proof.* We again make use of two facts at the micro level, (A.4) and (A.5), described in the proof of Proposition 1.3.

Manipulating the Ramsey optimal risk-sharing condition from Proposition 1.8, we obtain:

$$
\begin{aligned}
&\frac{\frac{\partial v^e}{\partial w_1^e}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}-\frac{1}{p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}} = \\
&\frac{\frac{\partial v^u}{\partial w_1^u}}{1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}-\frac{1}{1-p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}} \\
\Rightarrow &\left[1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}-\frac{1}{1-p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}\right] = \\
&\frac{\frac{\partial v^u}{\partial w_1^u}}{\frac{\partial v^e}{\partial w_1^e}}\left[1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}-\frac{1}{p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}\right], \\
\Rightarrow &\, 1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}-\frac{1}{1-p(\theta)s}\left(\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u} = \\
&\frac{\frac{\partial v^u}{\partial w_1^u}}{\frac{\partial v^e}{\partial w_1^e}}\left(1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}\right)+\frac{1}{p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}, \\
\Rightarrow &-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u} = \\
&\frac{\frac{\partial v^u}{\partial w_1^u}}{\frac{\partial v^e}{\partial w_1^e}}\left(1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}\right)-\left(1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}\right), \\
\Rightarrow &-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\frac{(1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\frac{\partial s}{\partial w_1^u} = \frac{\frac{\partial v^u}{\partial w_1^u}}{\frac{\partial v^e}{\partial w_1^e}}-\left(\frac{1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right), \\
\Rightarrow &-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\frac{(1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\frac{\partial s}{\partial w_1^u} = \frac{\frac{\partial u^u}{\partial c_1^u}}{\frac{\partial u^e}{\partial c_1^e}}-\left(\frac{1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right),
\end{aligned}
$$

where the third line uses (A.5) and the last line uses (A.4).

On the right-hand side,

$$
\frac{\frac{\partial u^u}{\partial c_1^u}}{\frac{\partial u^e}{\partial c_1^e}}-\left(\frac{1-\tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)=\frac{\frac{\partial u^u}{\partial c_1^u}-\frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}}+\frac{\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\left(\frac{\partial c_1^u}{\partial w_1^u}-\frac{\partial c_1^e}{\partial w_1^e}\right).
$$

And on the left-hand side,

$$
\begin{aligned}
-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)&\left(\frac{(1-\tau_{1,2})\frac{\partial x_1}{\partial s}+p_2\frac{\partial x_2}{\partial s}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\frac{\partial s}{\partial w_1^u}\\
&=-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\left(\frac{\partial x_1}{\partial s}+\frac{1}{1-\tau_{1,2}}p_2\frac{\partial x_2}{\partial s}\right)\left(\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\right)\frac{\partial s}{\partial w_1^u},\\
&=-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\left(p(\theta)\omega+\frac{\tau_{1,2}}{1-\tau_{1,2}}p_2\frac{\partial x_2}{\partial s}\right)\left(\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\right)\frac{\partial s}{\partial w_1^u},\\
&=-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\left(\left(\frac{1}{s}b+\frac{\tau_{1,2}}{1-\tau_{1,2}}p_2\frac{\partial x_2}{\partial s}\right)\left(\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\right)\frac{\partial s}{\partial w_1^u},\\
&=-\left(\frac{1}{1-p(\theta)s}+\frac{1}{p(\theta)s}\right)\frac{1}{s}b\frac{\partial s}{\partial w_1^u}\left(\left(1+\frac{\tau_{1,2}}{1-\tau_{1,2}}\frac{p_2\frac{\partial x_2}{\partial s}s}{b}\right)\left(\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\right)\right),\\
&=\left(\frac{1}{p(\theta)s}\right)^2\varepsilon_b^{P(unemp)}\left(\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}+\frac{\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\frac{p_2\frac{\partial x_2}{\partial s}s}{b}\right),
\end{aligned}
$$

where the second equality uses the definition of the size of transfers $\omega$ in (1.25), the third equality uses the implementation of transfers in Lemma 1.2 assuming that the payroll tax is used to keep markups at $\mu=1$, and the final equality uses the same steps as in the proof of Proposition 1.3 to collect terms into the micro-elasticity $\varepsilon_b^{P(unemp)}$. Moreover, we have

$$
\begin{aligned}
\frac{p_2\frac{\partial x_2}{\partial s}s}{b}&=\frac{p_2p(\theta)s}{b}\left((y_2^e-y_2^u)-(c_2^e-c_2^u)\right),\\
&=\frac{p_2p(\theta)s}{b}\left((y_2^e-c_2^e)-(y_2^u-c_2^u)\right),\\
&=\frac{p_2p(\theta)s}{b}\left(-\frac{1}{p_2}z_1^e+\frac{1}{p_2}z_1^u\right),\\
&=\frac{p_2p(\theta)s}{b}\frac{1}{p_2}\frac{1}{p(\theta)s}z_1^u,\\
&=\frac{z_1^u}{b},
\end{aligned}
$$

where the first line uses the definition of the long-run production-inclusive excess supply function $x_2(s,\theta)$ in (1.24), the third line uses agents' second period resource constraints in the competitive equilibrium, and the fourth line uses bond market clearing (1.10).

We thus obtain the exact *generalized Baily-Chetty formula*

$$\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon_b^{P(unemp)} \underbrace{\left(\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}} + \frac{\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\frac{z_1^u}{b}\right)}_{\Delta \text{ cost of disincentives}} = \frac{\frac{\partial u^u}{\partial c_1^u} - \frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}} + \underbrace{\frac{\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\left(\frac{\partial c_1^u}{\partial w_1^u} - \frac{\partial c_1^e}{\partial w_1^e}\right)}_{\text{AD externality}}. \quad \text{(A.7)}$$

Using the Taylor approximations for a small relative price wedge

$$\frac{1-\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}} + \frac{\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}}\frac{z_1^u}{b} = 1-\tau_{1,2}\left(1-\frac{\partial c_1^e}{\partial w_1^e}-\frac{z_1^u}{b}\right)+o(||\tau_{1,2}||^2),$$

$$\frac{\tau_{1,2}}{1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}} = \tau_{1,2}+o(||\tau_{1,2}||^2),$$

we obtain the approximate formula in the text. □

### Proposition 1.10: the UI multiplier

*Proof.* I first summarize the arguments used in Appendix A.4 which result in aggregate relations (1.41)-(1.43).

*Micro-level responses.* Standard results in price theory can be used to obtain the micro-level responses

$$dc_1^i = \underbrace{\frac{\partial c_1^i}{\partial w_1^i}dy_1^i}_{\substack{\text{MPC out}\\\text{of income}}} + \left(\underbrace{\frac{\partial c_1^{i,h}}{\partial p_2}}_{\substack{\text{subst.}\\\text{effect}}} - \underbrace{\frac{\partial c_1^i}{\partial w_1^i}\frac{z_1^i}{p_2}}_{\substack{\text{inc.+}\\\text{wealth}\\\text{effect}}}\right)dp_2, \quad \text{(A.21)}$$

$$ds = \underbrace{\frac{ds}{dw_1^e}\left(dy_1^e - \frac{z_1^e}{p_2}dp_2\right)}_{\substack{\text{resp. to wealth}\\\text{when emp.}}} + \underbrace{\frac{ds}{dw_1^u}\left(dy_1^u - \frac{z_1^u}{p_2}dp_2\right)}_{\substack{\text{resp. to wealth}\\\text{when unemp.}}} + \underbrace{\frac{ds}{d\theta}d\theta}_{\substack{\text{resp. to}\\\text{job-finding}}}, \quad \text{(A.22)}$$

given the Marshallian demand function $c_1^i(w_1^i, p_2)$ in (1.14), the Hicksian demand function $c_1^{i,h}(v^i(w_1^i, p_2), p_2)$ implied by (1.14), and the labor supply function $s(w_1^e, w_1^u, p_2, \theta)$ in (1.18).

*Aggregation.* Two identities facilitate aggregation:

$$(p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s), \tag{A.23}$$

$$(p(\theta)s)y_1^e + (1 - p(\theta)s)y_1^u = f(p(\theta)s - k\theta s). \tag{A.24}$$

The first simply reflects goods market clearing. The second says that aggregate income must equal aggregate resources.

Total differentiation of these conditions, combined with the micro-level consumption response in (A.21), gives the aggregate demand relation in (1.41). The micro-level labor supply response in (A.22) alongside (A.24) and the Ramsey optimality conditions from section 1.2.2 give the labor supply relation in (1.43). Finally, the specification of technology gives the technological relation in (1.42).

With (1.41)-(1.43) in hand, the remaining results are straightforward. Plugging the technological relation in (1.42) into the aggregate demand relation in (1.41), we obtain an equation in $\{n_1, s, y_1^u, p_2\}$. Substituting in for $s$ using the labor supply relation in (1.43) and collecting terms, we obtain the marginal effect on employment in (1.44). Substituting this back into (1.42) and using (1.43) to substitute in for $s$, we obtain the UI multiplier in (1.45). □

## A.2 Equilibrium conditions in the two-period model

### A.2.1 Flexible prices and wages

Here I solve for the first-order conditions of agents' optimization problems, which I assume are necessary to characterize their optimal choice of controls in equilibrium.

For producers' problem (1.3), the first-order conditions with respect to $\nu$ and $W$ are, respectively,

$$P^I f'(q(\theta)\nu - k\nu)\left(1 - \frac{k}{q(\theta)}\right) = W, \tag{A.8}$$

$$P^I f'(q(\theta)\nu - k\nu) - W = \frac{1}{\eta - 1}\frac{\theta(W)}{\theta'(W)}. \tag{A.9}$$

For workers' ex-post problem (1.4), the lower-stage optimization problem generates standard demand functions

$$c_1^i(j) = \left(\frac{P_1(j)}{P_1}\right)^{-\varepsilon} c_1^i \tag{A.10}$$

for the standard CES price index $P_1$ defined in (1.13). The first-order conditions of their upper-stage problem with respect to $c_1^i$, $c_2^i$, and $z_1^i$ then imply

$$\frac{\frac{\partial u^i}{\partial c_1^i}}{\frac{\partial u^i}{\partial c_2^i}} = (1+i)\frac{P_1}{P_2}, \tag{A.11}$$

a generalized Euler equation in the case of potentially non-separable utility.

For workers' ex-ante problem (1.5), the first-order condition with respect to $s$ is:

$$p(\theta)\left(v^e - v^u\right) = \psi'(s). \tag{A.12}$$

Finally, retailers' problem (1.6) implies the standard optimal price-setting equation

$$P_1(j) = \frac{\varepsilon}{\varepsilon - 1}(1 + \tau^r)P^I,$$

which, given the maintained assumption $\tau^r = -\frac{1}{\varepsilon}$, implies

$$P_1(j) = P^I. \tag{A.13}$$

I now characterize the implications of assumed worker indifference across submarkets indexed by different nominal wages $\tilde{W}$. To do so, it is helpful to generalize the description of employed workers' ex-post problem in (1.4) to be indexed by the prevailing wage rate in a particular submarket:

$$v^e(\tilde{W}) = \max_{\{c_1^e(j)\}, c_2^e, z_1^e} u^e(c_1^e, c_2^e) \text{ s.t.}$$

$$(RC)_1^e : \int_0^1 P_1(j) c_1^e(j) dj + P_1 z_1^e \leq Y_1^e(\tilde{W}),$$

$$(RC)_2^e : P_2 c_2^e \leq P_2 y_2^e + (1+i) P_1 z_1^e,$$

where

$$Y_1^e(\tilde{W}) = (1-\tau)\tilde{W} - P_1 t + (\Pi + \Pi^r - T^r).$$

Then in this case, worker indifference across submarkets formally requires that

$$(v(\tilde{W}) - v^*)(\theta(\tilde{W}))^{-1} \leq 0, \text{ with equality if } (\theta(\tilde{W}))^{-1} > 0, \tag{A.14}$$

where

$$v(\tilde{W}) = \max_s (p(\theta(\tilde{W}))s) v^e(\tilde{W}) + (1 - p(\theta(\tilde{W}))s) v^u - \psi(s)$$

and

$$v^* \equiv v(W),$$

the indirect utility achieved at the equilibrium wage $W$. In words, (A.14) says that in any off-path submarket which would attract positive measure of workers, workers' expected utility by directing search to that submarket must equal that obtained in the actual submarket operating in equilibrium. If expected utility is less than that obtained in equilibrium, no workers can be expected to apply to such an off-path submarket.

Assuming that the equilibrium wage rate is at an interior point in the implied $\theta(\tilde{W})$ schedule, and this schedule and the optimal policies and multipliers are continuously differentiable at the equilibrium wage, local worker indifference and the Envelope Theorem imply that

$$\eta \frac{P_1}{1-\tau} \frac{1}{\frac{\partial u^e}{\partial c_1^e}} (v^e - v^u) = -\frac{\theta(W)}{\theta'(W)}. \tag{A.15}$$

### A.2.2 Sticky prices

In this case, $P_1(j) = \bar{P}_1$ replaces (A.13) in the above characterization of equilibrium. The remaining optimality conditions are unchanged.

## A.3 Additional normative results[1]

I now characterize additional features of the Ramsey optimal allocation described in sections 1.2.2-1.2.4 of the main text.

### A.3.1 Flexible prices and wages

**Optimality with respect to $\theta$ and a generalized Hosios condition**

Reconsider planning problem (1.21), reproduced here:

$$\max_{w_1^e, w_1^u, \theta, p_2} (p(\theta)s(\cdot))v^e(w_1^e, p_2) + (1 - p(\theta)s(\cdot))v^u(w_1^u, p_2) - \psi(s(\cdot)) \text{ s.t.}$$

$$(RC)_1 : (p(\theta)s(\cdot))c_1^e(\cdot) + (1 - p(\theta)s(\cdot))c_1^u(\cdot) = f(p(\theta)s(\cdot) - k\theta s(\cdot)),$$

$$(RC)_2 : (p(\theta)s(\cdot))c_2^e(\cdot) + (1 - p(\theta)s(\cdot))c_2^u(\cdot) = (p(\theta)s(\cdot))y_2^e + (1 - p(\theta)s(\cdot))y_2^u.$$

The first-order conditions with respect to wealth levels $\{w_1^e, w_1^u\}$ and relative price $p_2$ were shown in Proposition 1.2 to imply a Ramsey optimal risk-sharing condition. The main text did not provide a characterization of the first-order condition with respect to tightness $\theta$, and its implication for optimality in the labor market. I do so here.

In particular, the first-order condition with respect to tightness $\theta$ can be combined with the first-order conditions with respect to $w_1^e$ and $p_2$ to imply

$$\frac{v^e - v^u}{\frac{\partial v^e}{\partial w_1^e}} + \frac{1}{p'(\theta)s}\left(\frac{\partial x_1}{\partial \theta} + p_2 \frac{\partial x_2}{\partial \theta}\right) = 0. \tag{A.16}$$

[1] Proofs of results stated in this appendix are available on request.

Expanding the derivatives of the production-inclusive excess supply functions, this implies

$$\frac{v^e - v^u}{\frac{\partial v^e}{\partial w_1^e}} + f'(p(\theta)s - k\theta s)\left(1 - \frac{1}{\eta}\frac{k}{q(\theta)}\right) - (c_1^e - c_1^u) + p_2\left((y_2^e - y_2^u) - (c_2^e - c_2^u)\right) = 0. \quad \text{(A.17)}$$

(A.17), together with Ramsey optimal risk-sharing in (1.22) and the resource constraints of the planning problem, constitute 4 equations in 4 unknowns $\{w_1^e, w_1^u, p_2, \theta\}$ characterizing the optimum.

I now manipulate the first-order condition (A.17) to more transparently illustrate the economic forces at play. To do so, it is helpful to define the *surplus wedge*.

**Definition A.1.** *At a particular allocation, the* surplus wedge $\tau^{surplus}$ *summarizes the deviation in surplus-sharing between firms and workers from the Hosios condition (Hosios (1990)):*

$$\tau^{surplus} \equiv 1 - \frac{\overbrace{\left(\frac{1}{\frac{\partial v^e}{\partial w_1^e}}(v^e - v^u)\right)}^{\text{Worker surplus}} \Big/ \overbrace{\left(f'(p(\theta)s - k\theta s)\frac{k}{q(\theta)}\right)}^{\text{Firm surplus}}}{\underbrace{(1-\eta)/\eta}_{\text{Relative worker / firm surplus under Hosios}}}. \quad \text{(A.18)}$$

Hosios (1990) showed that in the baseline DMP model, search externalities imposed by agents on others in the matching process are neutralized, and lead to constrained efficiency, if workers receive a share $1 - \eta \equiv \varepsilon_v^{m(s,v)}$ of the match surplus. This result also holds in a setting with risk-averse workers (but complete markets) and decreasing returns to scale in production (Merz (1995)).

This motivates my use of the "wedge" terminology: at a nonzero surplus wedge, the Ramsey optimal allocation is characterized by a deviation from this standard efficiency condition. In particular, a positive surplus wedge is associated with a lower share of surplus accruing to workers relative to the Hosios condition.

Having defined this wedge, we can combine (A.17) with the Ramsey optimal risk-sharing condition (1.22) to characterize a *generalized Hosios condition* at the Ramsey optimum:

**Proposition A.1.** *The Ramsey optimum is characterized by a* generalized Hosios condition

$$\tau^{surplus} = \frac{\frac{\frac{\partial v^u}{\partial w_1^u} - \frac{\partial v^e}{\partial w_1^e}}{\frac{\partial v^e}{\partial w_1^e}}}{\left(f'(p(\theta)s - k\theta s)\frac{k}{q(\theta)}\left(\frac{1-\eta}{\eta}\right)\right)\left(-\frac{1}{1-p(\theta)s}\frac{1}{s}\frac{\partial s}{\partial w_1^u}\right)}.$$

With only partial insurance at the Ramsey optimum, it follows that $\tau^{surplus} > 0$. Hence, the optimum is characterized by optimally *lower* surplus accruing to workers than is implied by the Hosios condition. Since partial insurance was a result of the general equilibrium fiscal externality induced by search behavior in response to wealth transfers, we can conclude that this same externality motivates surplus-sharing in the labor market which deviates from the Hosios condition.

**Implementation**

In the flexible price and wage equilibrium as described in section 1.2.1, labor market equilibrium is characterized by

$$\begin{aligned} f'(p(\theta)s - k\theta s)\frac{k}{q(\theta)} &= f'(p(\theta)s - k\theta s) - \frac{W}{P_1} \\ &= \frac{1}{P_1}\frac{1}{\eta - 1}\frac{\theta(W)}{\theta'(W)} \\ &= \frac{1}{1-\tau}\frac{\eta}{1-\eta}\frac{1}{\frac{\partial v^e}{\partial w_1^e}}(v^e - v^u) \end{aligned}$$

where the first line uses (A.8) and (A.13), the second line uses (A.9), and the last line uses (A.15) and $\frac{\partial v^e}{\partial w_1^e} = \frac{\partial u^e}{\partial c_1^e}$ (by the Envelope Theorem).

Given the definition of the surplus wedge, we can immediately see the following:

**Lemma A.1.** *In a flexible price and wage equilibrium, a particular surplus wedge $\tau^{surplus}$ (as defined in (A.18)) is implemented by a payroll tax $\tau$ satisfying*

$$\tau^{surplus} = \tau.$$

This relationship between the surplus wedge and payroll tax illustrates a subtle point: a payroll tax serves a different function in a search and matching framework than it does in a

Walrasian labor market. In a Walrasian setting, a payroll tax accompanied by a lump-sum rebate to the representative agent unambiguously reduces equilibrium production. In the present setting with search and heterogeneity, a payroll tax financed by a lump-sum rebate to employed workers instead amounts to a change in effective bargaining weights. It is better characterized as an increase in the *progressivity* of the tax schedule, and by shifting surplus towards firms, can lead to an increase in production through labor demand. This point is well known in the literature on taxation in search economies (Pissarides (1998), Boone and Bovenberg (2002), Lehmann and van der Linden (2007)), and is nicely summarized by Lemma A.1.

Coupled with the generalized Hosios condition in Proposition A.1, we immediately pin down the optimal payroll tax (or more precisely, the degree of tax progressivity) at the Ramsey optimum. In particular, a *positive* payroll tax is needed as part of the optimal policy mix in general equilibrium. Lehmann and van der Linden (2007) noted the importance of a payroll tax accompanying a UI benefit to implement constrained efficiency in general equilibrium, and Landais *et al.* (2015b) have noted that their "efficiency term" relates to the Hosios condition. To my knowledge, however, this precise characterization of a positive payroll tax implementing a positive wedge relative to the Hosios condition is novel.

This concludes the full implementation of optimal policy at the Ramsey optimum: labor market policy is fully specified, while monetary policy is indeterminate. In particular, with UI generosity $b$ implementing Ramsey optimal risk-sharing as in Proposition 1.3, and the payroll tax $\tau$ implementing a generalized Hosios condition in the labor market as in Proposition A.1, the lump-sum tax $t$ is pinned down by government budget balance (1.11). Conversely, none of these implementation results pinned down monetary policy $\{i, P_2\}$, which are thus indeterminate. This reflects the *real/nominal dichotomy* in the present setting with flexible prices and wages: while monetary policy determines the short-run price level $P_1$, it has no impact on the real allocation.

In 1.2.1, I pointed out that the conclusions in the main text regarding the role of UI in stabilization were robust to a variety of alternative assumptions on wage-setting in the competitive equilibrium. In 1.2.2, I noted that extending the set of instruments available to policymakers would not change the characterization of the Ramsey optimal allocation. Both points boil down to the following: with changes to wage-setting or changes in the available set of instruments, the set of implementable allocations characterized in Proposition 1.1 is unchanged. It follows that the Ramsey optimality conditions are unchanged, and all that changes is their implementation. I precisely characterize implementation in the flexible price and wage case under a variety of alternative settings below:

- *Nash bargained wages.* Suppose that production is constant returns to scale ($\alpha = 1$) and that wages in the competitive equilibrium are determined by Nash bargaining with worker share $\phi$, rather than competitive search. Continue to assume the same policy instruments as in the benchmark model. Then

$$\tau^{surplus} = 1 - \frac{\phi/(1-\eta)}{(1-\phi)/\eta}(1-\tau).$$

  In particular, only a composite of the tax rate and worker bargaining share is determined, but not the mix between them. Notably, if $\tau = 0$, implementing $\tau^{surplus} > 0$ requires $\phi < 1 - \eta$, a departure from the Hosios (1990) bargaining weight.

- *Hiring/payroll taxes faced by firms.* Suppose that a hiring/payroll tax $\tau^f$ assessed on producers is added to the set of policy instruments. Then

$$\tau^{surplus} = \frac{\tau + \tau^f}{1 + \tau^f}.$$

  In particular, only a composite of the tax rate on firms and tax rate on workers is determined, but not the mix between them.

- *Rigid real wages.* Suppose that real wages are instead rigid at $\bar{w}$. Assume as above that policymakers have access to a payroll tax on firms $\tau^f$ rather than (only) the payroll tax on workers assumed in the main text. Then

$$\tau^{surplus} = 1 - \frac{\left(\frac{1}{\frac{\partial v^e}{\partial w_1^e}}(v^e - v^u)\right) / \left(f'(p(\theta)s - k\theta s) - (1+\tau^f)\bar{w}\right)}{(1-\eta)/\eta}.$$

Rigidity in real wages means that a payroll tax on workers will have no direct impact on labor market equilibrium. But provided policymakers can tax firms instead, they can implement the generalized Hosios condition.

In each case, the size of transfers $\omega$ is implemented by UI benefits $b$ as in Lemma 1.1, and thus optimal UI continues to satisfy the general equilibrium Baily-Chetty formula in Proposition 1.3.

**Connection to the labor wedge**

The surplus wedge $\tau^{surplus}$ is a very useful in helping us immediately understand how the labor market at the Ramsey optimum differs from standard efficiency in a DMP setting without incomplete markets and moral hazard. In this section, I provide an equivalent characterization of Ramsey optimality in the labor market using the conventional *labor wedge*. This is a more well known wedge in macroeconomics, having been studied by Chari *et al.* (2007), Shimer (2009), and many others.

Following Shimer (2010), consider a hypothetical researcher who observes aggregate production $y_1$, employment $n_1$, and consumption $c_1$ generated by the model economy at the Ramsey optimum, but erroneously assumes that the economy is characterized by a neoclassical labor market with a representative agent having a period utility function $u(c)$, disutility of labor $\chi$, an infinite Frisch elasticity of labor supply. Then the researcher will infer a labor wedge — an ad-valorem tax-rate on the representative agent which can rationalize the observed aggregate data — of

$$\tilde{\tau}^l = 1 - \frac{\frac{1}{u'(c_1)}\chi}{\alpha\frac{y_1}{n_1}}.$$

Scaling this simply for convenience, we define the *measured labor wedge* $\tau^l$ as

$$\tau^l \equiv \left(\alpha\frac{y_1}{n_1}\right)\tilde{\tau}^l_1 = \alpha\frac{y_1}{n_1} - \frac{1}{u'(c_1)}\chi.$$

Given the actual structure of the flexible price and wage equilibrium in the present setting, the researcher will measure

$$y_1 \equiv f(p(\theta)s - k\theta s),$$
$$n_1 \equiv p(\theta)s,$$
$$c_1 \equiv (p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u.$$

Then, we can manipulate the planner's first-order condition with respect to $\theta$ (A.17) to obtain:

**Proposition A.2.** *At the Ramsey optimum, the measured labor wedge satisfies*

$$\tau^l = \overbrace{\left(\frac{1-\eta}{\eta}\right)\left(f'(p(\theta)s - k\theta s)\frac{k}{q(\theta)}\right)}^{\text{Effect of search}} + \underbrace{\left[(c_1^e + p_2 c_2^e) - (c_1^u + p_2 c_2^u) - \left(\frac{1}{\frac{\partial v^e}{\partial w_1^e}}(v^e - v^u) + \frac{1}{u'(c_1)}\chi\right)\right]}_{\substack{\text{Effect of heterogeneity (incomplete markets + moral hazard)} \\ \text{and model misspecification}}} \equiv \tau^{l*}$$

This defines the *natural labor wedge* $\tau^{l*}$, the measured labor wedge at the Ramsey optimum with flexible prices and wages. Obviously, the natural labor wedge is not zero, since the labor market is far from neoclassical. Its components summarize how the present setting departs from the neoclassical benchmark due to search frictions, incomplete markets, and moral hazard.

Indeed, the following result demonstrates how, in the limit where search frictions vanish and the moral hazard problem disappears (because agents are risk-neutral, so there is no insurance-incentive trade-off), the natural labor wedge converges to zero as in the neoclassical benchmark:

**Lemma A.2.** *Suppose $u^e(c_1^e, c_2^e) = u(c_1^e) - \chi + \beta u(c_2^e)$ and $u^u(c_1^u, c_2^u) = u(c_1^u) + \beta u(c_2^u)$ so that there is no model misspecification. Then in the limiting case $k \to 0$ (no search frictions) and $u(c) = c$ (no moral hazard),*

$$\tau^{l*} \to 0.$$

The natural labor wedge is a useful benchmark against which we can compare the measured labor wedge in the case with sticky prices. I turn to this case now.

### A.3.2 Sticky prices

#### Relative price wedge and the labor wedge

Consider the Ramsey planning problem in the equilibrium with fully sticky prices, (1.21). In this case, the first-order condition with respect to tightness $\theta$ can be combined with the first-order conditions with respect to $w_1^e$ and $p_2$ to imply

$$\left(1-\tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e}\right)\frac{v^e-v^u}{\frac{\partial v^e}{\partial w_1^e}}+\frac{1}{p'(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial \theta}+p_2\frac{\partial x_2}{\partial \theta}\right)=0. \tag{A.19}$$

Together with Ramsey optimal risk-sharing in (1.35), the resource constraints of the planning problem, and a complementary slackness condition on $\tau_{1,2}$ (which uses the relationship between $\tau_{1,2}$ and $\lambda_{ZLB}$ characterized in the proof of Proposition 1.7)

$$\tau_{1,2}\left(\bar{p}_2-p_2\right)=0,$$

these 5 equations in 5 unknowns $\{w_1^e, w_1^u, p_2, \theta, \tau_{1,2}\}$ fully characterize the optimum.

When the zero lower bound is slack and $\tau_{1,2}=0$, clearly (A.19) and (A.16) are identical, which simply reflects the fact that the planning problems under sticky and flexible prices are identical. I characterize implementation below.

When the zero lower bound binds, however, there are wedges in the first-order condition for optimal $\theta$, just as there were wedges in the Ramsey optimal risk-sharing condition (1.35). We can manipulate this condition to obtain the following relationship between the relative price wedge and the labor wedge:

**Proposition A.3.** *At the Ramsey optimum, the relative price wedge is reflected in a deviation of the labor wedge from its natural level:*

$$\tau_{1,2}=\frac{\tau^l-\tau^{l*}}{\frac{1}{p'(\theta)s}\frac{\partial x_1}{\partial \theta}+\frac{\partial c_1^e}{\partial w_1^e}\left(\frac{v^e-v^u}{\frac{\partial v^e}{\partial w_1^e}}\right)}. \tag{A.20}$$

This formalizes the sense in which a binding zero lower bound, manifest in a positive relative price wedge, is reflected in inefficiency in the labor market in equilibrium.

**Irrelevance of the payroll tax**

In the text, I argued that the payroll tax could be arbitrarily set under sticky prices. The following result explains why.

**Lemma A.3.** *In a fully sticky price equilibrium, a particular surplus wedge $\tau^{surplus}$ (as defined in (A.18)) is implemented by a payroll tax $\tau$ satisfying*

$$\tau^{surplus} = 1 - (1-\tau)\frac{P^I}{\bar{P}_1}.$$

In other words, at arbitrary $\tau$, the inverse gross markup $\frac{P^I}{\bar{P}_1}$ will adjust to implement a particular surplus wedge at the Ramsey optimum, provided that monetary policy and UI are used to achieve the natural rate of interest and implement the Ramsey optimal size of transfers $\omega$, respectively. This simply reflects the standard Keynesian intuition that, given consumption demand, after-tax real wages will adjust to so that firms can hire the necessary labor to produce given demand. It follows that any change in the payroll tax will be offset by a change in real wages to keep after-tax real wages unchanged.

## A.4 Intermediate steps in UI multiplier[2]

**Micro-level responses.** I begin by studying workers' ex-post and ex-ante problems specified in (1.14) and (1.18), respectively. We can use standard tools from price theory to characterize the response of workers' short-run consumption and labor supply to variation in short-run disposable income $\{y_1^e, y_1^u\}$, the inverse real interest rate $p_2$, and market tightness $\theta$.

[2]Proofs of results stated in this appendix are available on request.

**Proposition A.4.** *In the competitive equilibrium, ex-post*

$$dc_1^i = \underbrace{\frac{\partial c_1^i}{\partial w_1^i} dy_1^i}_{\substack{\text{MPC out} \\ \text{of income}}} + \Bigg( \underbrace{\frac{\partial c_1^{i,h}}{\partial p_2}}_{\substack{\text{subst.} \\ \text{effect}}} - \underbrace{\frac{\partial c_1^i}{\partial w_1^i} \frac{z_1^i}{p_2}}_{\substack{\text{inc.+} \\ \text{wealth} \\ \text{effect}}} \Bigg) dp_2 \tag{A.21}$$

*for $i \in \{e, u\}$, and ex-ante*

$$ds = \underbrace{\frac{ds}{dw_1^e} \left( dy_1^e - \frac{z_1^e}{p_2} dp_2 \right)}_{\substack{\text{resp. to wealth} \\ \text{when emp.}}} + \underbrace{\frac{ds}{dw_1^u} \left( dy_1^u - \frac{z_1^u}{p_2} dp_2 \right)}_{\substack{\text{resp. to wealth} \\ \text{when unemp.}}} + \underbrace{\frac{ds}{d\theta} d\theta}_{\substack{\text{resp. to} \\ \text{job-finding}}} \tag{A.22}$$

*given Marshallian demand function $c_1^i(w_1^i, p_2)$ in (1.14), Hicksian demand function $c_1^{i,h}(v^i(w_1^i, p_2), p_2)$ implied by (1.14), and labor supply function $s(w_1^e, w_1^u, p_2, \theta)$ in (1.18).*

The response of consumption in (A.21) is straightforward. Its response to short-run income is governed by the MPC (definitionally). Its response to the real interest rate is governed by the combination of a standard substitution effect and income-cum-wealth effect, where the latter depends on the net asset position of the agent in the short-run, $z_1^i$.

The response of search effort in (A.22) reflects the differences between the present setting and a Walrasian labor market. Labor supply responds to changes in wealth when employed and unemployed, which depend on disposable income and the income-cum-wealth effects from real interest rate changes. These combine the typical wealth and substitution effects in labor supply, since labor supply is search effort on the extensive margin. In addition, labor supply responds positively to an increase in the job-finding rate, which scales up the return to search in a frictional labor market.

**Aggregation.** To aggregate up these behavioral responses, we can exploit the following two identities which must hold in the competitive equilibrium:

$$(p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s), \tag{A.23}$$

$$(p(\theta)s)y_1^e + (1 - p(\theta)s)y_1^u = f(p(\theta)s - k\theta s). \tag{A.24}$$

The first simply reflects goods market clearing. The second says that aggregate income must equal aggregate resources. Total differentiation of these conditions, combined with the micro-level consumption response in (A.21), gives the following general equilibrium result.

**Lemma A.4.** *Define short-run output $y_1 \equiv f(p(\theta)s - k\theta s)$ and short-run employment $n_1 \equiv p(\theta)s$. Then $y_1$, $n_1$, $y_1^u$, and $p_2$ are related as follows:*

$$\frac{dy_1}{y_1} = \mu_{y_1^u}^{AD}\frac{dy_1^u}{y_1^u} + \mu_{p_2}^{AD}dp_2 + \mu_n^{AD}\frac{dn_1}{n_1}, \tag{A.25}$$

*where*

$$\mu_{y_1^u}^{AD} \equiv \frac{1}{1-\frac{\partial c_1^e}{\partial w_1^e}}\left[\frac{(1-n_1)y_1^u}{y_1}\left(\frac{\partial c_1^u}{\partial w_1^u} - \frac{\partial c_1^e}{\partial w_1^e}\right)\right],$$
$$\mu_{p_2}^{AD} \equiv \frac{1}{1-\frac{\partial c_1^e}{\partial w_1^e}}\left[\frac{1}{y_1}\left(n_1\left(\frac{\partial c_1^{e,h}}{\partial p_2} - \frac{\partial c_1^e}{\partial w_1^e}\frac{z_1^e}{p_2}\right) + (1-n_1)\left(\frac{\partial c_1^{u,h}}{\partial p_2} - \frac{\partial c_1^u}{\partial w_1^u}\frac{z_1^u}{p_2}\right)\right)\right],$$
$$\mu_{n_1}^{AD} \equiv \frac{1}{1-\frac{\partial c_1^e}{\partial w_1^e}}\left[\frac{n_1}{y_1}\left((c_1^e - c_1^u) - \frac{\partial c_1^e}{\partial w_1^e}(y_1^e - y_1^u)\right)\right].$$

This defines the aggregate demand relation in the present environment with heterogeneity and search. As described in the main text, the key coefficient of interest is $\mu_{y_1^u}^{AD}$, which summarizes the *redistribution effect on aggregate demand*. A coefficient of secondary interest is $\mu_{p_2}^{AD}$, governing the response of aggregate demand to interest rates.

These coefficients cannot yet be interpreted as multipliers, however, owing to the presence of $n_1$, an endogenous object, in the above formula. This term, not present in representative agent models, captures the effect of extensive margin changes in the measure of employed agents on the level of aggregate demand. But of course, output $y_1$ and employment $n_1$ are related by aggregate technology in the economy $y_1 = f(p(\theta)s - k\theta s)$, which implies

$$\begin{aligned}\frac{dy_1}{y_1} &= \alpha\left(\frac{1-\frac{1}{\eta}\frac{k}{q(\theta)}}{1-\frac{k}{q(\theta)}}\right)\frac{dn_1}{n_1} + \alpha\left(\frac{(\frac{1}{\eta}-1)\frac{k}{q(\theta)}}{1-\frac{k}{q(\theta)}}\right)\frac{ds}{s} \\ &\equiv \mu_{n_1}^{tech}\frac{dn_1}{n_1} + \mu_s^{tech}\frac{ds}{s}\end{aligned} \tag{A.26}$$

As noted in the main text, holding fixed employment $n_1$, a decrease in search effort $s$ acts like a decrease in the productivity of labor. Firms need to engage in more, costly recruiting to hire the same number of workers, leading to a reduction in output $y_1$.

It follows that the labor supply response to a marginal increase in UI matters even in this demand-determined world. The general equilibrium behavior of labor supply in (A.22) could potentially be quite complex. However, the following result implies that the labor supply response is considerably simplified when considering a local change around the Ramsey optimum.

**Lemma A.5.** *Around the flexible price and wage Ramsey optimum, the general equilibrium response of labor supply to marginal changes in transfers and the real interest rate is given by*

$$\frac{ds}{s} = -\mu^{LS}_{y_1^u} \frac{dy_1^u}{y_1^u} - \mu^{LS}_{p_2} dp_2, \tag{A.27}$$

*where*

$$\mu^{LS}_{y_1^u} \equiv \frac{1-n_1}{(n_1)^2}\left(\frac{y_1^u}{b}\right)\varepsilon_b^{P(unemp)},$$
$$\mu^{LS}_{p_2} \equiv -\frac{1-n_1}{(n_1)^2}\left(\frac{\frac{1}{p_2}z_1^u}{b}\right)\varepsilon_b^{P(unemp)},$$

*and* $\varepsilon_b^{P(unemp)}$*, the micro-elasticity of the probability of unemployment with respect to an increase in UI, is defined in (1.27).*

As noted in the main text, the micro-disincentive elasticity of UI tightly characterizes the response of equilibrium search intensity to changes in the income of the unemployed ($y_1^u$) and interest rates ($p_2$). Interestingly, this parallels the sufficiency of the micro-disincentive elasticity in the normative Baily-Chetty formula. Transfers to the unemployed unambiguously reduce equilibrium search intensity. Furthermore, a cut in interest rates (rise in $p_2$) affects labor supply through the implied reallocation of wealth; it will reduce search intensity if and only if the unemployed are net debtors ($z_1^u < 0$).

Finally, note that the Fisher equation with sticky prices (1.30) implies

$$dp_2 = -dr = -\left(di - \frac{dP_2}{P_2}\right). \tag{A.28}$$

where I use the standard approximation that $1 + r \approx 1$ and there is zero inflation in steady-state.

Then, (A.25)-(A.28) imply the three aggregate relations (1.41)-(1.43) described in the main text.

## A.5 Robustness of the theoretical formulas[3]

I characterize optimal UI and the UI multiplier in each case discussed in the main text.

### A.5.1 Sticky wages

Suppose producers have committed to identical pre-set nominal wages $W = \bar{W}$. Then the representative producer's problem (1.3) is replaced by

$$\Pi = \max_{v} P^I f(q(\theta)v - kv) - \bar{W}q(\theta)v \tag{A.29}$$

A sticky wage equilibrium is defined as follows.

**Definition A.2.** *A* sticky wage equilibrium *is an allocation and set of nominal prices and profits such that, given policy, conditions 1, 3, and 4-7 of Definition 1.1 are satisfied, and condition 2 is replaced by producers solving (A.29).*

As noted in the text, the key change from the sticky price case is producers' first-order condition for labor demand, giving (1.46). In view of the assumed constraints on monetary policy in (1.31), this implies a constraint on the real interest rate (1.47). This motivates the following result on the equivalence of implementable allocations.

**Proposition A.5.** *An allocation $\{c_1^e, c_1^u, c_2^e, c_2^u, s, \theta\}$ and relative price $p_2$ ($\Leftrightarrow$ real interest rate $1 + r$) form part of a sticky wage equilibrium if and only if there exist wealth levels $\{w_1^e, w_1^u\}$ satisfying the economy-wide resource constraints*

$$(p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s),$$

$$(p(\theta)s)c_2^e + (1 - p(\theta)s)c_2^u = (p(\theta)s)y_2^e + (1 - p(\theta)s)y_2^u,$$

[3]Proofs of results stated in this appendix are available on request.

*and the ZLB implementability constraint*

$$p_2 \leq \bar{p}_2 \equiv \frac{\bar{P}_2}{\bar{W} \Big/ \left(f'(p(\theta)s - k\theta s)\left(1 - \frac{k}{q(\theta)}\right)\right)}, \tag{A.30}$$

*given implementability constraints* $c_1^e = c_1^e(w_1^e, p_2)$, $c_1^u = c_1^u(w_1^u, p_2)$, $c_2^e = c_2^e(w_1^e, p_2)$, $c_2^u = c_2^u(w_1^u, p_2)$, *and* $s = s(w_1^e, w_1^u, p_2, \theta)$ *as defined in (1.14) and (1.18).*

It proves useful to denote the right-hand side of (A.30) as $\bar{p}_2(s, \theta)$. Then the resulting planning problem implies the following Ramsey optimal risk-sharing condition.

**Proposition A.6.** *Ramsey optimal risk-sharing is characterized by*

$$\frac{\frac{\partial v^e}{\partial w_1^e}}{1 - \tau_{1,2}\frac{\partial c_1^e}{\partial w_1^e} - \frac{1}{p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s} + \tau_{1,2}\sigma^h\frac{\partial \bar{p}_2}{\partial s}\right)\frac{\partial s}{\partial w_1^e}}$$
$$= \frac{\frac{\partial v^u}{\partial w_1^u}}{1 - \tau_{1,2}\frac{\partial c_1^u}{\partial w_1^u} - \frac{1}{1-p(\theta)s}\left((1-\tau_{1,2})\frac{\partial x_1}{\partial s} + p_2\frac{\partial x_2}{\partial s} + \tau_{1,2}\sigma^h\frac{\partial \bar{p}_2}{\partial s}\right)\frac{\partial s}{\partial w_1^u}}$$

*where the production-inclusive excess supply functions* $x_1$ *and* $x_2$ *are as defined in (1.23) and (1.24), the relative price wedge* $\tau_{1,2}$ *is as in Definition 1.3, and*

$$\sigma^h \equiv p(\theta)s\frac{\partial c_1^{e,h}}{\partial p_2} + (1 - p(\theta)s)\frac{\partial c_1^{u,h}}{\partial p_2}$$

*is the weighted average compensated sensitivity of short-run consumption to the real interest rate.*

Since the equilibrium mark-up is identical to its flexible price and wage level (of 1), the implementation of a particular size of transfers $\omega$ is only a function of UI. It follows that Ramsey optimal risk-sharing is implemented by the following generalized Baily-Chetty formula:

**Proposition A.7.** *The Ramsey optimum is implemented by optimal UI benefits b satisfying a* *generalized Baily-Chetty formula*

$$\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon_b^{P(unemp)}\left(1-\tau_{1,2}\left(1-\frac{\partial c_1^e}{\partial w_1^e}-\frac{z_1^u}{b}+\underbrace{\tfrac{\sigma^h \bar{p}_2(1-\alpha)}{b}}_{\substack{\text{Add'l } \Delta \text{ cost} \\ \text{of disincentives}}}\right)\right)$$
$$=\frac{\frac{\partial u^u}{\partial c_1^u}-\frac{\partial u^e}{\partial c_1^e}}{\frac{\partial u^e}{\partial c_1^e}}+\tau_{1,2}\left(\frac{\partial c_1^u}{\partial w_1^u}-\frac{\partial c_1^e}{\partial w_1^e}\right)+o(||\tau_{1,2}||^2).$$

Consistent with the intuition provided in the main text, if production is characterized by decreasing returns to scale ($\alpha < 1$), then the marginal reduction in search caused by higher UI tends to reduce employment, raise the marginal product of labor, and thus relax the zero lower bound constraint. Beyond the effects present with sticky prices — which remain in this formula — this further reduces the welfare cost of disincentives, and pushes toward higher optimal UI.

I turn now to the positive results in this environment. The aggregate demand and labor supply relations in Lemmas A.4 and A.5 are unchanged. But now, as is standard in sticky wage models, there is an upward-sloping *aggregate supply* relation which affects the equilibrium response to policy.

In particular, the Fisher equation with flexible prices means that (A.28) is replaced by

$$dp_2 = -dr = -\left(di - \frac{dP_2}{P_2} + \frac{dP_1}{P_1}\right) \tag{A.31}$$

The response of short-run prices to policy relies on a *labor demand relation*, defined as follows.

**Lemma A.6.** *Given firms' labor demand (1.46) and the technological relation in (A.26), $\{P_1, n_1, s\}$ are related by*

$$\frac{dP_1}{P_1} = \mu_{n_1}^{LD}\frac{dn_1}{n_1} - \mu_s^{LD}\frac{ds}{s}, \tag{A.32}$$

*where*

$$\mu_{n_1}^{LD} \equiv (1-\alpha)\left(\frac{1-\frac{1}{\eta}\frac{k}{q(\theta)}}{1-\frac{k}{q(\theta)}}\right)+\frac{\left(\frac{1}{\eta}-1\right)\frac{k}{q(\theta)}}{1-\frac{k}{q(\theta)}},$$

$$\mu_{s}^{LD} \equiv \alpha\left(\frac{\left(\frac{1}{\eta}-1\right)\frac{k}{q(\theta)}}{1-\frac{k}{q(\theta)}}\right).$$

The labor demand relation implies that *conditional* on employment, a reduction in search will tend to *raise* real marginal costs by raising firms' necessary recruiting costs. This translates into a rise in the price level.

We can combine this with the general equilibrium labor supply response to policy to obtain an *aggregate supply relation* in this economy:

**Lemma A.7.** *Around the flexible price and wage Ramsey optimum, the labor supply relation in Lemma A.5 and labor demand relation in Lemma A.6 imply*

$$\frac{dP_1}{P_1} = \mu_{n_1}^{AS}\frac{dn_1}{n_1} + \mu_{y_1^u}^{AS}\frac{dy_1^u}{y_1^u} - \mu_{p_2}^{AS}\left(di - \frac{dP_2}{P_2}\right), \tag{A.33}$$

*where*

$$\mu_{n_1}^{AS} \equiv \frac{1}{1+\mu_s^{LD}\mu_{p_2}^{LS}}\mu_{n_1}^{LD},$$

$$\mu_{y_1^u}^{AS} \equiv \frac{1}{1+\mu_s^{LD}\mu_{p_2}^{LS}}\mu_s^{LD}\mu_{y_1^u}^{LS},$$

$$\mu_{p_2}^{AS} \equiv \frac{1}{1+\mu_s^{LD}\mu_{p_2}^{LS}}\mu_s^{LD}\mu_{p_2}^{LS}.$$

The positive coefficient on employment $n_1$ defines an upward-sloping aggregate supply curve in the usual way. More importantly, the positive coefficient on income of the unemployed $y_1^u$ implies that transfers will shift the aggregate supply curve upwards and thus potentially choke off any aggregate demand stimulus. This is true even though nominal wages are sticky, through the effect of reduced search on higher recruiting costs. For small hiring frictions, this effect may again be small relative relative to any aggregate demand effects; but it again highlights why disincentives matter in the determination of optimal UI.

Proceeding as in the main text, we obtain the employment and output response to transfers:

**Proposition A.8.** *Around the flexible price and wage Ramsey optimum, the marginal effects of policy on employment and output are given by*

$$\frac{dn_1}{n_1} = -\mu_{p_2}^{n_1}\left(di - \frac{dP_2}{P_2}\right) + \mu_{y_1^u}^{n_1}\frac{dy_1^u}{y_1^u},$$
$$\frac{dy_1}{y_1} = -\mu_{p_2}^{y_1}\left(di - \frac{dP_2}{P_2}\right) + \mu_{y_1^u}^{y_1}\frac{dy_1^u}{y_1^u},$$

*where*

$$\mu_{y_1^u}^{n_1} = \mu_{p_2}^{n_1}\left(\frac{1}{1-\mu_{p_2}^{AS}}\left(\frac{\mu_{y_1^u}^{AD} + \mu_s^{tech}\mu_{y_1^u}^{LS}}{\mu_{p_2}^{AD} + \mu_s^{tech}\mu_{p_2}^{LS}} - \mu_{y_1^u}^{AS}\right)\right), \tag{A.34}$$

*and the* UI multiplier *is given by*

$$\mu_{y_1^u}^{y_1} = \underbrace{\mu_{n_1}^{tech}\mu_{y_1^u}^{n_1}}_{\substack{\textit{Employment}\\ \textit{stimulus}}} - \underbrace{\mu_s^{tech}\mu_{y_1^u}^{LS}}_{\substack{\textit{Loss from}\\ \textit{recruiting}}}. \tag{A.35}$$

*for aggregate demand coefficients* $\{\mu_{y_1^u}^{AD}, \mu_{p_2}^{AD}\}$ *from Lemma A.4, technological coefficients in (A.26), labor supply coefficients* $\{\mu_{y_1^u}^{LS}, \mu_{p_2}^{LS}\}$ *from Lemma A.5, and aggregate supply coefficients* $\{\mu_{y_1^u}^{AS}, \mu_{p_2}^{AS}\}$ *from Lemma A.7.*

Comparing (A.34) to (1.44), the effect on employment not only includes the redistribution effect on aggregate demand but also the disincentive effect on aggregate supply.

### A.5.2 Hand-to-mouth unemployed

As noted in the main text, if the unemployed are unable to access credit markets, their optimization problem (1.14) is replaced by (1.48).

From a normative perspective, the result on equivalence of implementable allocations (Proposition 1.4) and the planning problem (1.34) are unchanged, except that $\{v^u(w_1^u, p_2), c_t^u(w_1^u, p_2)\}$ are replaced by $\{v^u(w_1^u, p_2, y_2^u), c_t^u(w_1^u, p_2, y_2^u)\}$. The Ramsey optimal risk-sharing and generalized Baily-Chetty formulas are thus unchanged from the baseline model in terms of sufficient statistics. The only difference is that in equilibrium, $\frac{\partial c_1^u}{\partial w_1^u} = 1$ and $z_1^u = z_1^e = 0$, which implies that optimal UI unambiguously exceeds that implied by the benchmark

Baily-Chetty formula when the ZLB binds.

From a positive perspective, the coefficient $\mu_{p_2}^{AD}$ in the aggregate demand relation (1.41) becomes

$$\mu_{p_2}^{AD} \equiv \frac{1}{1 - \frac{\partial c_1^e}{\partial w_1^e}} \left[ \frac{n_1}{y_1} \frac{\partial c_1^{e,h}}{\partial p_2} \right].$$

Intuitively, aggregate demand responds to interest rate movements only through a substitution effect for employed agents. There is no income-cum-wealth effect for these agents since there is no equilibrium borrowing and lending in the economy. The unemployed are not interest rate sensitive because they are hand-to-mouth.

The remaining coefficients on the three aggregate relations (1.41)-(1.43) are unchanged, except that again, in equilibrium, $\frac{\partial c_1^u}{\partial w_1^u} = 1$ and $z_1^u = z_1^e = 0$. In terms of sufficient statistics, the formula for the marginal impact of higher $y_1^u$ on employment, and the UI multiplier, are thus unchanged.

### A.5.3 SOE with a fixed exchange rate

Consider a static SOE with two types of goods: nontraded and traded. Nontraded goods are produced by a domestic sector, which is split into intermediate good producers and final good retailers; traded goods are treated as endowments for simplicity. The world price of traded goods is $P_T^*$ in units of the foreign currency, and is taken as given by this SOE.

**Technologies, tastes, and endowments.** The labor market in the non-traded sector is characterized by search and matching frictions, which give rise to domestic unemployment. In particular, intermediate good firms face a vacancy-filling probability $q(\theta)$ when they post a vacancy at cost $k$, where the cost is manifest in foregone worker time producing with production function $f(q(\theta)\nu - k\nu) = a(q(\theta)\nu - k\nu)^{\alpha}$.

Workers put in search effort $s$ to gain employment, and face a job-finding probability per unit effort $p(\theta)$. They have general preferences over nontraded and traded goods, and separable disutility from search effort for simplicity:

$$U = (p(\theta)s)u^e(c^e_{NT}, c^e_T) + (1 - p(\theta)s)u^u(c^u_{NT}, c^u_T) - \psi(s).$$

Consumption of nontraded goods is a CES aggregator over differentiated varieties produced by retailers in that sector.

Finally, all workers share equal ownership of the economy's domestic firms, and receive an endowment of traded goods $y^e_T$ or $y^u_T$ which can depend on idiosyncratic unemployment status.

**Policy, markets, and equilibrium.** In the labor market, the domestic government intervenes with $\{b, t, \tau\}$, where the first two are expressed in units of non-traded goods. In terms of monetary policy, the domestic government would ordinarily have control over the domestic money supply; but under the assumption that the exchange rate is fixed, monetary policy is constrained to implement $E = \bar{E}$ (expressed in units of domestic currency per unit of foreign currency). Finally, domestic retailers face an ad valorem tax $\tau^r$, financed lump-sum with $T^r$, which I assume offsets any distortions from monopolistic competition.

Finally, I close the description of labor and asset markets. Wages in the domestic non-traded sector are nominally sticky at $\bar{W}$. For simplicity, I assume there is no ex-ante international trade. Domestically, there do not exist markets to insure against idiosyncratic unemployment risk.

The resulting optimization problems are as follows. Ex-post worker $i \in \{e, u\}$ faces

$$\begin{aligned} v^i = \max_{\{c^i_{NT}(j)\}, c^i_T} \; & u^i(c^i_{NT}, c^i_T) \text{ s.t.} \\ (RC)^i : \int_0^1 P_{NT}(j)c^i_{NT}(j)dj + \bar{E}P^*_T c^i_T & \leq Y^i_{NT} + \bar{E}P^*_T y^i_t, \end{aligned} \tag{A.36}$$

where short-run disposable incomes excluding the traded good endowments are

$$\begin{aligned} Y^e_{NT} &= (1-\tau)\bar{W} - P_{NT}t + (\Pi + \Pi^r - T^r), \\ Y^u_{NT} &= P_{NT}b + (\Pi + \Pi^r - T^r), \end{aligned}$$

given aggregate retailer profits $\Pi^r \equiv \int_0^1 \Pi^r(j)dj$ defined below. Ex-ante, the representative worker faces

$$v = \max_s (p(\theta)s)v^e + (1 - p(\theta)s)v^u - \psi(s). \tag{A.37}$$

Producers in the non-traded sector face

$$\Pi = \max_\nu P^I f(q(\theta)\nu - k\nu) - \bar{W}q(\theta)\nu \tag{A.38}$$

The problem of retailers in the non-traded sector is again standard. Exploiting the solution to workers' lower-stage optimization problem in (A.36), retailer $j$ faces

$$\begin{aligned} \Pi^r(j) = \max_{P_{NT}(j), y_{NT}(j), x(j)} & P_{NT}(j)y_{NT}(j) - (1 + \tau^r)P^I x(j) \text{ s.t.} \\ & (Tech)(j) : x(j) = y_{NT}(j), \\ (Demand)(j) : y_{NT}(j) = & \left(\frac{P_{NT}(j)}{P_{NT}}\right)^{-\varepsilon} ((p(\theta)s)c^e_{NT} + (1 - p(\theta)s)c^u_{NT}). \end{aligned} \tag{A.39}$$

where $y_{NT}(j)$ and $x(j)$ are final non-traded goods produced and intermediate non-traded goods purchased by $j$, respectively.

I finally summarize market clearing and the government's budget constraint in this economy. Final goods market clearing of each good is given by

$$(p(\theta)s)c^e_{NT}(j) + (1 - p(\theta)s)c^u_{NT}(j) = y_{NT}(j) \ \forall j, \tag{A.40}$$

$$(p(\theta)s)c^e_T + (1 - p(\theta)s)c^u_T = (p(\theta)s)y^e_T + (1 - p(\theta)s)y^u_T, \tag{A.41}$$

while intermediate goods market clearing in the non-traded sector is given by

$$\int_0^1 x(j)dj = f(q(\theta)\nu - k\nu). \tag{A.42}$$

Assuming that the government separately balances its budgets for policy in the labor market and policy targeted at retailers, we lastly have

$$p(\theta)s\left[P_{NT}t + \tau\bar{W}\right] = (1 - p(\theta)s)P_{NT}b, \tag{A.43}$$

$$T^r + \tau^r P^I \int_0^1 x(j)dj = 0. \tag{A.44}$$

The definition of equilibrium is analogous to that in the benchmark model.

**Core normative and positive results.** The key insight is that the present framework is isomorphic to the benchmark model with a simple re-labeling. The relative price of interest is

$$p_T = \frac{EP_T^*}{P_{NT}},$$

the relative price of traded goods to non-traded goods. After proving equivalence of implementable allocations, the Ramsey planning problem with sticky wages and a fixed exchange rate is

$$\begin{aligned} \max_{w^e, w^u, \theta, p_T} (p(\theta)s(\cdot))v^e(w^e, p_T) + (1 - p(\theta)s(\cdot))v^u(w^u, p_T) - \psi(s(\cdot)) \text{ s.t.} \\ (RC)_{NT} : (p(\theta)s(\cdot))c^e_{NT}(\cdot) + (1 - p(\theta)s(\cdot))c^u_{NT}(\cdot) = f(p(\theta)s(\cdot) - k\theta s(\cdot)), \\ (RC)_T : (p(\theta)s(\cdot))c^e_T(\cdot) + (1 - p(\theta)s(\cdot))c^u_T(\cdot) = (p(\theta)s(\cdot))y^e_T + (1 - p(\theta)s(\cdot))y^u_{NT}, \\ (E = \bar{E}) : p_T = \bar{p}_T \equiv \frac{\bar{E}P_T^*}{W_{NT}/(f'(p(\theta)s(\cdot) - k\theta s(\cdot))(1 - \frac{k}{q(\theta)}))} \end{aligned} \tag{A.45}$$

where wealth levels $w^e, w^u$ are expressed in units of non-traded goods. Ramsey optimal risk-sharing and the generalized Baily-Chetty formula (in the sticky wage case) then hold exactly in this environment, with non-traded goods replacing date 1 consumption, traded goods replacing date 2 consumption, and the relative price wedge now defined as

$$\tau_{NT,T} \equiv 1 - \frac{\lambda_{RC,NT}}{\lambda_{RC,T}} p_T. \tag{A.46}$$

From a positive perspective, the formula for the UI multiplier is identical. In terms of non-traded goods, the disposable income of agents excluding their traded endowment is

$$\begin{aligned} y^e_{NT} &\equiv \frac{1}{P_{NT}} \left( (1 - \tau)W - P_{NT}t + (\Pi + \Pi^r - T^r) \right), \\ y^u_{NT} &\equiv \frac{1}{P_{NT}} \left( P_{NT}b + (\Pi + \Pi^r - T^r) \right). \end{aligned}$$

The general equilibrium effect of an increase in $y^u_{NT}$ on employment is identical to that in the sticky wage case of the closed economy model: the redistribution effect is governed

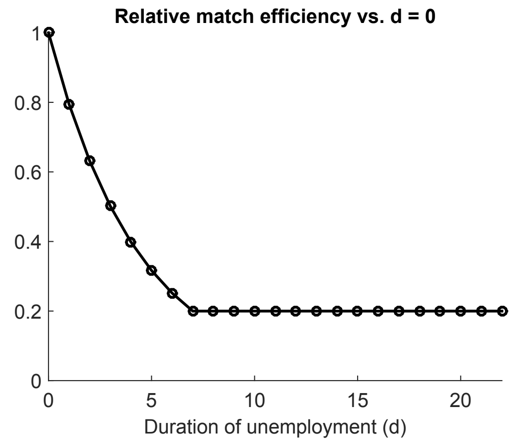


**Figure A.1:** *Relative job-finding rates per unit effort as a function of unemployment duration*

by the difference in marginal propensities to consume on non-traded goods between the unemployed and employed, the normalization by the effect of monetary stimulus is in terms of the expansion provided by an exchange rate depreciation, and reduced search can have a contractionary effect due to the increase in unit labor costs. The resulting output multiplier trades off the net domestic employment impact with the increase in recruiting costs due to reduced search.

## A.6 Labor market dynamics in stationary RCE

In the calibrated stationary RCE, relative job-finding rates per unit effort as a function of unemployment duration are in Figure A.1. As discussed in the main text, the calibration requires such *structural duration-dependence* in the job-finding rate to rationalize the observed long-term unemployment rate in the U.S. economy, given a cost of search effort function which separately is consistent with the measured elasticity of unemployment duration with respect to potential duration of benefits. An alternative approach to rationalize the observed long-term unemployment rate could be to allow for further heterogeneity among job-seekers (e.g., Ahn and Hamilton (2015), Alvarez *et al.* (2015)).

Given these relative match efficiencies and the calibrated cost of effort, the equilibrium search effort policy functions and job-finding probabilities by assets and duration of unem-

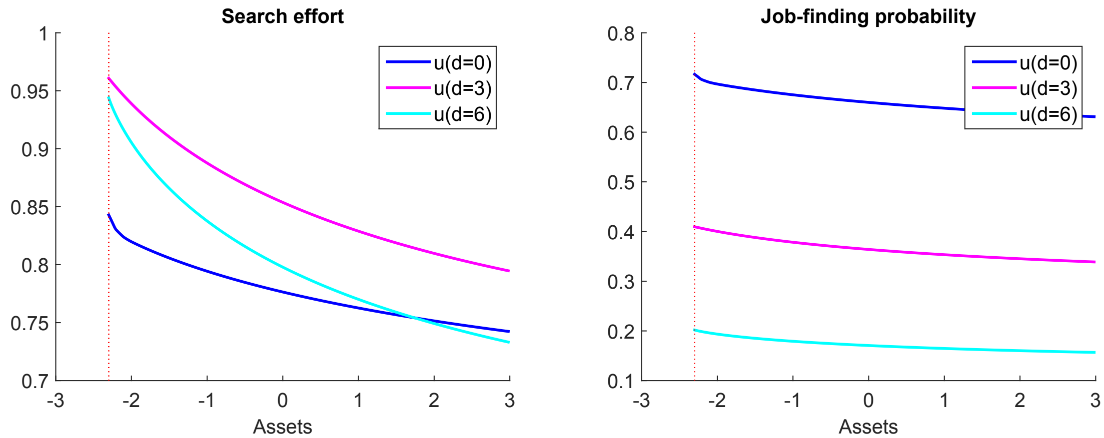


**Figure A.2:** *Search effort policy functions and resulting job-finding probabilities*

ployment are in Figure A.2. There are two important takeaways from these graphs. First, search effort falls monotonically in assets holding fixed duration, but is non-monotonic in duration holding fixed assets. The latter reflects the interplay of two opposing forces on the returns to search as an agent proceeds through an unemployment spell: on the one hand, the difference in welfare when employed and remaining unemployed rises; on the other hand, match efficiencies fall. Second, the equilibrium job-finding probability, combining optimal search with the calibrated match efficiencies, tends to fall with unemployment duration regardless of wealth. This negative duration-dependence in empirically observable job-finding rates is consistent with the evidence in Kroft *et al.* (forthcoming).

## A.7 Labor market dynamics along the transition

Here I discuss two additional features of the economy's transitional dynamics in response to the simulated shock discussed in the main text.

First, the collapse in aggregate demand is reflected in a collapse in labor demand, captured by the behavior of vacancies and thus conventional tightness in Figure A.3. As noted in the text, this occurs not only because of a Keynesian labor demand response to the fall in consumption demand, but also because hiring, a form of investment in this frictional labor market, is made more costly relative to the natural allocation due to a binding zero

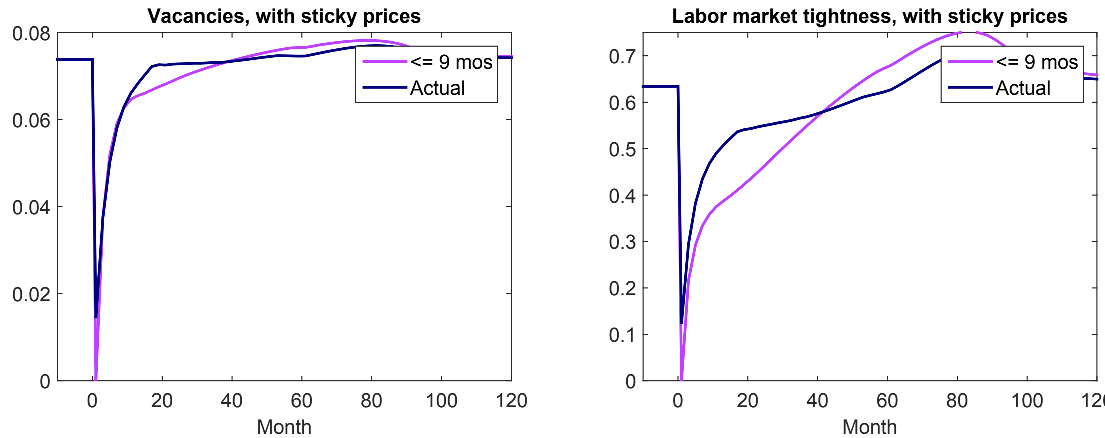


**Figure A.3:** *Vacancies and labor market tightness under actual vs. counterfactual durations*

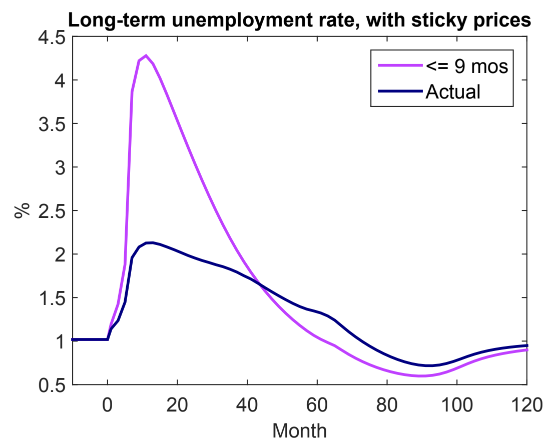


**Figure A.4:** *Long-term unemployment under actual vs. counterfactual durations*

lower bound (Hall (2015)). An implication of the counterfactual analysis is that it does not occur, however, because of any negative supply-side effects of the simulated benefit extensions. In the above graphs, this is reflected in higher equilibrium tightness relative to the case with counterfactually less generous UI durations.

Second, the collapse in labor demand endogenously interacts with the DMP labor market dynamics to generate a rise in the long-term unemployed, as seen in Figure A.4. As is evident, the calibrated model implies a doubling in the long-term unemployment rate. And contrary to the view that extended durations should raise the fraction of long-term unemployment, I find that the increase in long-term unemployment is *smaller* under the observed UI policy relative to a less generous counterfactual, owing to the demand stimulus from transfers to the long-term unemployed.

## A.8 Infinite horizon model accommodating layoffs

To accommodate the possibility of deep recessions under sticky prices and with a binding zero lower bound, I enrich the basic model presented in the text with a layoff technology available to intermediate good producers. In my counterfactual experiments with less generous UI durations, I routinely find that — absent a layoff technology — the collapse in aggregate demand would lead to firms seeking the corner solution of zero vacancies at the beginning of the simulated recession. Introducing a layoff technology thus ensures that in these counterfactual policy experiments, firms remain at an interior optimum and the equilibrium is well defined.

The model with layoffs nests the basic one presented in the text. I assume that at the beginning of period $t$, intermediate good firms can lay off some of their incumbent workers costlessly. I assume that this technology is randomly applied to the stock of incumbent workers, though from the firm's perspective, the assumption of identical real wages across all workers implies that there would anyway be no reason to lay off one over another.

Generalizing problem (1.56), the representative producer then faces

$$\begin{aligned} J_t(n_t) = \max_{\nu_t, l_t} P_t^I f_t(n_t - l_t + q(\theta_t)\nu_t - k\nu_t) - P_t w_t(n_t - l_t + q(\theta_t)\nu_t) \\ + M_t J_{t+1}(n_{t+1}) \text{ s.t.} \\ (Evol)_t(n_t) : n_{t+1} = (1-\delta_t)(n_t - l_t + q(\theta_t)\nu_t), \end{aligned} \tag{A.47}$$

where $l_t$ is the choice of layoffs at the beginning of period $t$.

Generalizing problem (1.52), employed workers at the beginning of period $t$ now face

$$\tilde{v}_t^e(z_t) = (1-\zeta_t)v_t^e(z_t) + \zeta_t \tilde{v}_t^u(z_t, 0), \tag{A.48}$$

where $\zeta_t$ is the layoff rate facing incumbent workers at date $t$, and which is identical for all workers since I am studying an environment with a representative producer. The above problem reflects the fact that, upon a layoff, a given worker is identical to a worker with the same assets who exogenously separated the previous period and is thus unemployed with duration 0. The problem facing unemployed workers at the beginning of period $t$ is unchanged from (1.53).

There are two changes to the way in which this model is closed in general equilibrium. First, there is a consistency condition relating the layoff rate facing workers to the layoff choices of the representative firm:

$$\zeta_t = \frac{l_t}{n_t}. \tag{A.49}$$

Second, aggregate (weighted) search accounts for the search behavior of the newly laid-off workers:

$$\begin{aligned} \bar{s}_t \equiv \int_z s_t(z,0)\left((1-\tilde{\lambda}_t^e)\tilde{\varphi}_t^u(z,0) + \zeta_t \tilde{\lambda}_t^e \tilde{\varphi}_t^e(z)\right) dz \\ +(1-\tilde{\lambda}_t^e)\sum_{d=1}^{\infty}\frac{\bar{m}(d)}{\bar{m}(0)}\int_z s_t(z,d)\tilde{\varphi}_t^u(z,d)dz. \end{aligned} \tag{A.50}$$

I now define a fully sticky price equilibrium with layoffs.

**Definition A.3.** *A* fully sticky price equilibrium with layoffs *is a set of value functions, policies (including layoffs $l_t$), tightness, layoff rate $\zeta_t$, employment, nominal prices and profits, and probability measures such that, given policy and exogenous real wages,*

1. *workers solve (A.48) and (1.53)-(1.55);*
2. *producers solve (A.47);*
3. *retailer prices are fixed as in (1.65), so they accommodate demand as in (1.66);*
4. *tightness is consistent with worker and firm behavior ($\theta_t = \frac{v_t}{\bar{s}_t}$, given $\bar{s}_t$ in (A.50));*
5. *the layoff rate is consistent with worker behavior, as in (A.49);*
6. *goods and bond markets clear at each date according to (1.62)-(1.64);*
7. *the government's budget is balanced according to (1.58) and (1.59);*

8. *and the probability measures characterized by* $\{\tilde{\lambda}_t^e, \tilde{\varphi}_t^e, \tilde{\varphi}_t^u, \lambda_t^e, \varphi_t^e, \varphi_t^u\}$ *are consistent with labor market clearing* $\tilde{\lambda}_0^e = n_0$ *at date 0, and consistent with the above policies and stochastic elements of the model for all* $t \geq 0$.

Relative to the equilibrium definition without layoffs in Definition 1.5, conditions 1, 2, and 4 are generalized, and 5 is new.

## A.9 Calibrating to alternative consumption moments

In the main text, I disciplined the consumption side of the model by targeting a particular difference in average MPCs between the unemployed and employed and, given the relative paucity of empirical evidence on this moment, assessing the sensitivity of the simulation results to a range of targeted values. Here I instead discipline the consumption side of the model with available micro-level evidence on the consumption behavior of the unemployed. I ask what MPC profiles are *implied* by models calibrated to match this evidence, and then characterize the transitional dynamics in response to UI policy. I find that the simulated employment and welfare impacts of the 2008-13 UI extensions are broadly consistent with the range of effects discussed in the main text.

### A.9.1 Existing evidence on consumption of the unemployed

There are three broad sets of moments regarding the consumption behavior of the unemployed which have been the focus of prior empirical work. First, in large part because of its usefulness in calibrating the standard partial equilibrium Baily (1978)-Chetty (2006) formula, the change in consumption *levels* upon unemployment has been estimated by a number of researchers. Second, researchers seeking to understand the consequences of steady-state changes in UI policy have estimated how the consumption drop upon unemployment changes with replacement rate generosity. Third, some researchers have used moments characterizing the job search intensity of unemployed workers to induce the magnitude of liquidity constraints which they face.

Here, I focus on five estimates from this prior work. In terms of the consumption drop upon unemployment, I use Gruber (1997)'s estimate using the PSID that unemployed workers exhibit 6.8% lower consumption than the same workers sampled one year earlier when they were employed; Chodorow-Reich and Karabarbounis (forthcoming)'s estimate using the CE that the relative consumption of a household with all workers unemployed vs. employed for one year is 77%; and Ganong and Noel (2015)'s estimate using account-level data from JP Morgan Chase that the consumption of workers in the month after exhausting benefits is 23% lower than their consumption just prior to job loss. In terms of the sensitivity of the consumption drop to replacement rate generosity, I use Browning and Crossley (2001)'s estimate that the average consumption drop after 6 months of unemployment falls by 0.8 percentage points for a 10 percentage point increase in the replacement rate. Finally, in terms of the link between job search and liquidity, I use Card *et al.* (2007)'s estimate that the average duration of unemployment rises 1.3 times more due to a severance payment of 2 months of wages relative to a 50% increase in the potential duration of benefits.

### A.9.2 Calculation of each moment in the stationary RCE

I begin by replicating, as closely as I can, the empirical strategy used in each of the above papers in the simulated stationary RCE. In particular, it is straightforward to compute the analogs of Gruber (1997) and Ganong and Noel (2015) after simulating agents for 13 and 7 months in the stationary RCE, respectively. Computing the analog of Card *et al.* (2007) is also straightforward, and is just a variation on the other micro-level duration elasticities computed in the main text.

Computing the analogs of the other studies is more nuanced, so I discuss the approaches here. First, to compute the analog of Chodorow-Reich and Karabarbounis (forthcoming), I simulate agents for 12 months in the stationary RCE. Over these 12 months, I then compute the average consumption for each agent $\bar{c}^i$ and fraction of time that agent's household is unemployed

$$d^i \equiv \frac{\sum_{t=1}^{12} 1\{\text{i is unemployed at } t\}}{12 + 12\left(\frac{\omega}{w}\right)},$$

**Table A.1:** *Simulated moments in stationary RCE in section 1.3.3 of main text*

| Study and moment | Estimate | Simulated |
|---|---|---|
| Gruber [2001] cons. drop of unemployed vs. 12 mos prior | -6.8% | -8.5% |
| CRK [2015] cross-sectional unemployed/employed ratio | 0.77 | 0.67 |
| GN [2015] cons. decline through exhaustion | -23% | -21% |
| BC [2001] cons. resp. to 10 pp replacement rate incr. | -0.8 pp | -2.1 pp |
| CCW [2007] duration resp. to severance vs. potential duration | 1.3 | 0.8 |

where the denominator accounts for the fact that non-own-labor income $\omega$ in the simulated model in large part captures spousal income, and thus the fraction of time the entire household spends unemployed should be adjusted accordingly. I then run a cross-sectional OLS regression

$$\log(\bar{c}^i) = \gamma_0 + \gamma_1 d^i + \varepsilon^i,$$

analogous to specification (27) in Chodorow-Reich and Karabarbounis (forthcoming). The estimated ratio of relative consumption between a household unemployed vs. employed for one year is $\exp(\hat{\gamma}_1)$.

Second, to compute the analog of Browning and Crossley (2001), I simulate agents in the stationary RCE who, in the month prior to job loss, learn that their UI replacement rate $b^{UI}/w$ has increased from 50% to 60%. I keep macroeconomic aggregates unchanged, and assess how the consumption drop of agents remaining unemployed after 6 months compares to that of other agents who do not see their replacement rate increase. This recovers a micro-level semi-elasticity of the consumption drop with respect to replacement rate generosity comparable to the quasi-experimental estimate of Browning and Crossley (2001), who identify off of legislative changes which raise some agents' replacement rate while reducing others'.

In Table A.1, I compute these simulated moments for the baseline stationary RCE characterized in subsection 1.3.3 of the main text. The baseline model delivers, roughly speaking, estimates of the consumption drop upon unemployment which are consistent with those in Gruber (1997) and Ganong and Noel (2015). The baseline model implies tighter

**Table A.2:** *Re-calibrated stationary RCE and resulting transitional dynamics*

| Study | Target | Achieved | Stationary RCE $MPC^u - MPC^e$ | Stationary RCE LR $MPC^u - MPC^e$ | Shock and ZLB $\Delta$ peak unemp. | Shock and ZLB %$\Delta$ SW |
|---|---|---|---|---|---|---|
| Gruber [2001] | -6.8% | -7.0% | 0.05 | 0.18 | 3.5pp | 2.1% |
| CRK [2015] | 0.77 | 0.75 | 0.04 | 0.15 | 2.8pp | 1.5% |
| GN [2015] | -23% | -23% | 0.07 | 0.27 | 5.6pp | 3.5% |
| BC [2001] | -0.8 pp | -0.9 pp | 0.02 | 0.07 | 1.6pp | 0.9% |
| CCW [2007] | 1.3 | 1.1 | 0.14 | 0.53 | 6.0pp | 5.1% |

liquidity constraints facing the unemployed, however, than is implied by the estimates of Chodorow-Reich and Karabarbounis (forthcoming) and Browning and Crossley (2001). Conversely, the baseline model implies looser liquidity constraints facing the unemployed than is implied by the estimate of Card *et al.* (2007).

### A.9.3 Implied MPCs and effect of 2008-13 UI extensions

I now recalibrate the stationary RCE to match each of the five empirical estimates discussed in the prior subsections. Columns 4-5 of Table A.2 provide the implied difference in average 1-month MPCs between unemployed and employed workers, as well as long-term unemployed and employed workers, in each calibrated steady-state.

In columns 6-7 of the same table, I then summarize the simulated difference in peak unemployment and utilitarian social welfare under the observed UI extensions relative to a counterfactual capped at 9 months of duration, given a discount factor shock which induces an increase in unemployment of roughly 5% in the sticky price economy at the ZLB. I again define these moments so that a positive value is consistent with a stabilizing effect of greater generosity provided by the observed extensions. These results are to be compared with the 2–5 percentage point reduction in unemployment, and 1–4% social welfare increase, under the observed UI policy given the approach used in the main text.

There are two important takeaways from these results. First, calibrating the model to match this micro-evidence on the consumption behavior of the unemployed implies

**Table A.3:** *Calibration results for sensitivity analyses*

| | | **Δ avg MPCs** | | **Duration elast.** | | **Rec/Emp** | |
|---|---|---|---|---|---|---|---|
| **Parameter** | **Base** | **0.02** | **0.04** | **0.05** | **0.15** | **0.005** | **0.015** |
| $\beta$ | 0.996 | 0.998 | 0.997 | 0.996 | 0.996 | 0.996 | 0.996 |
| $w/a$ | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.99 | 0.98 |
| $\lambda$ | 0.23 | 0.21 | 0.22 | 0.21 | 0.23 | 0.23 | 0.23 |
| $\bar{m}(0)$ | 0.57 | 0.60 | 0.55 | 0.54 | 0.48 | 0.57 | 0.57 |
| $\omega/a$ | 0.91 | 0.90 | 0.90 | 0.90 | 0.90 | 0.92 | 0.91 |
| $k$ | 0.32 | 0.32 | 0.32 | 0.32 | 0.32 | **0.06** | **0.19** |
| $\xi$ | 3.8 | 3.3 | 3.8 | **8.2** | **2.1** | 3.8 | 3.8 |
| $\underline{z}/a$ | 4.6 | **10.0** | **6.4** | 2.3 | 2.3 | 2.3 | 2.3 |

steady-state differences in MPCs between the unemployed and employed consistent with those in the main text. In particular, these calibrations imply that the long-term unemployed have an MPC at least 7% greater than that of the employed at a 1-month horizon. Second, in terms of the transitional dynamics, the quantitative results obtained in the main text remain robust: the extensions to 22 months generate sizable employment and welfare gains relative to the less generous counterfactual.

## A.10 Calibrations for sensitivity analysis

Table A.3 characterizes the analog to Table 1.2 for each of the alternative calibrations. Consistent with the construction of Table 1.2 in the main text, we see that the borrowing constraint ($\underline{z}$) is the major parameter which varies to target the average difference in MPCs; the curvature of the search cost function ($\xi$) is the major parameter which varies to target the duration elasticity; and the magnitude of hiring costs ($k$) is the major parameter which varies to target the recruiting to employment ratio.

In each case, the simulated discount factor shock is laterally shifted up or down from that depicted in Figure 1.3 given the change in the steady-state $\beta$ relative to the baseline simulation. The simulated shock is further scaled up or down by a constant factor to target a peak unemployment rate of 10% under the observed UI extensions, as was observed during the Great Recession.

## A.11 UI and precautionary savings[4]

Consider the following partial equilibrium problem without search effort, for simplicity:

$$v = \max_{c_0, z_0} u(c_0) + \beta \left[ p_1^e v^e(z_0) + (1 - p_1^e) v^u(z_0) \right] \text{ s.t.} \qquad c_0 + m_0 z_0 \leq y_0, \tag{A.51}$$

where at date 1

$$v^i = \max_{c_1^i, c_2^i} u(c_1^i) + \beta u(c_2^i) \text{ s.t.} \qquad c_1^i + m_1 c_2^i \leq y_1^i + m_1 y_2^i + z_0, \tag{A.52}$$

for $i \in \{e, u\}$. This is a natural 3-period generalization of the partial equilibrium problem facing agents in the benchmark model, except for the assumption that utility is time-separable, and labor supply and thus job-finding rates are exogenous.

In this environment, consider an increase in *future* UI which results in $dy_1^u > 0$. Suppose that this policy is accompanied by an increase in taxes on the employed such that

$$dy_1^e = -\frac{(1 - p_1^e)}{p_1^e} dy_1^u, \tag{A.53}$$

which implies that the expected income of an agent at date 1, from the perspective of date 0, is unchanged. Then we obtain:

**Proposition A.9.** *At date 0,*

$$dc_0 = \frac{\partial c_0}{\partial y_0} m_0 \left( \frac{-(1 - p_1^e) u''(c_1^u) \frac{\partial c_1^u}{\partial y_1^u}}{-p_1^e u''(c_1^e) \frac{\partial c_1^e}{\partial y_1^e} - (1 - p_1^e) u''(c_1^u) \frac{\partial c_1^u}{\partial y_1^u}} \right) \left[ \left( \frac{-u''(c_1^e)}{-u''(c_1^u)} \right) \left( \frac{\frac{\partial c_1^u}{\partial y_1^u} - \frac{\partial c_1^e}{\partial y_1^e}}{\frac{\partial c_1^u}{\partial y_1^u}} \right) + \underbrace{\left( \frac{u''(c_1^e) - u''(c_1^u)}{-u''(c_1^u)} \right)}_{\text{Precautionary channel}} \right] dy_1^u.$$

[4]Proofs of results stated in this appendix are available on request.

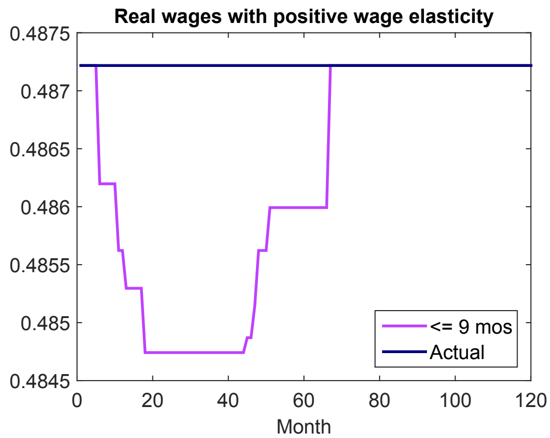


**Figure A.5:** *Real wage paths given positive wage elasticity*

Provided $u'''(c) > 0$ and $c_1^e > c_1^u$, the precautionary channel is positive and drives an increase in date 0 consumption in response to the future policy change.

## A.12 Robustness to positive wage elasticity

As noted in the main text, in the presence of fully sticky prices and unchanged monetary policy, a real wage response to benefit changes only affects macroeconomic aggregates through an effect on aggregate demand induced by re-allocating wealth across agents. In this appendix I demonstrate that the implied reallocation is small, and thus the sticky price results provided in the main text are robust to changes in the real wage when UI policy changes. This is in contrast to flexible price case where an increase in real wages has a large negative effect on labor demand and thus employment.

I consider an elasticity of real wages with respect to UI duration of 0.0057. This is of the same order of magnitude estimated by Hagedorn *et al.* (2015a), and is on the high end of available estimates (e.g., using micro-level variation, Card *et al.* (2007), Lalive (2007), and van Ours and Vodopivec (2008) find that re-employment wages are essentially unchanged in response to benefit duration changes).[5] Continuing to assume that real wages given

[5]In particular, my assumed elasticity is one-half of the estimate in Hagedorn *et al.* (2015a) (Table 5). I use this elasticity to ensure that in the counterfactual dynamics with less generous UI under flexible prices, the

**Table A.4:** *Effect of positive real wage elasticity*

| Metric | No real wage response | | Wage elast = 0.0057 | |
|---|---|---|---|---|
| | Flexible | Sticky + ZLB | Flexible | Sticky + ZLB |
| Peak Δ unemp. counter - observed | -1.3pp | +5.1pp | -4.7pp | +4.9pp |
| % Δ SW observed - counter | -0.3% | +3.2% | -4.7% | +3.0% |

observed UI policy are flat at their steady-state value (a convenient normalization), the path of real wages under counterfactual durations capped at 9 months are now as shown in Figure A.5.

The difference in peak unemployment and social welfare between the observed UI policy and the less generous counterfactual are now summarized in columns 4 and 5 of Table A.4. These can be compared to the baseline model of the main text summarized in columns 2 and 3. With flexible prices, the positive wage elasticity substantially worsens the employment and welfare effects of more generous UI. In contrast, with fully sticky prices, and keeping monetary policy unchanged from that in the main text, the gains from more generous UI are minimally affected.

economy remains at an interior allocation with some unemployment along the transition. The results in the sticky price case are minimally affected if I use the exact Hagedorn *et al.* (2015a) elasticity.

# Appendix B

# Computational Appendix to Chapter 1

In this computational appendix I describe the technical steps involved in the quantitative analysis of the 2008-13 benefit extensions in the second half of the main text. In section B.1 I first characterize key equilibrium conditions in the infinite-horizon environment. In section B.2 I then outline my approach to solving for the stationary RCE, and in section B.3 I outline my approach to solving for the transitional dynamics in response to a macroeconomic shock. In both cases, I use sub-algorithms to iterate workers' value functions backward and to iterate the distribution of workers forward which I outline in sections B.4 and B.5, respectively. Throughout, my approach builds most directly on that in Guerrieri and Lorenzoni (2015), and the code is implemented in MATLAB.

## B.1 Equilibrium conditions in the infinite horizon environment

Here I characterize key equilibrium conditions which guide the computational approach described in the following sections. These conditions follow from the definitions of the flexible price equilibrium in section 1.3.2 and fully sticky price equilibrium in section 1.3.4 of the main text.

First consider agents' optimal choice of controls. For unemployed workers at the beginning of period $t$ facing (1.53), their first-order condition with respect to search effort means that $s_t(z_t, d_t)$ solves

$$p(\theta_t; d)\,(v_t^e(z_t) - v_t^u(z_t, d_t)) \geq \psi'(s_t(z_t, d_t)), \tag{B.1}$$

where the inequality is strict only if $s_t(z_t, d_t) = \frac{1}{p(\theta_t; d)}$ (that is, if the worker puts in enough effort to obtain a job with probability one).

For workers in the middle of period $t$ facing (1.54) and (1.55), their first-order conditions with respect to consumption and assets imply that $\{c_t^e(z_t), c_t^u(z_t, d_t)\}$ and $\{z_{t+1}^e(z_t), z_{t+1}^u(z_t, d_t)\}$ solve the standard Euler equations

$$u'(c_t^e(z_t)) \geq \beta_t(1 + r_t)\mathfrak{v}_{t,z}^e(z_{t+1}^e(z_t)), \tag{B.2}$$

$$u'(c_t^u(z_t, d_t)) \geq \beta_t(1 + r_t)\mathfrak{v}_{t,z}^u(z_{t+1}^u(z_t, d_t), d_t), \tag{B.3}$$

where $\mathfrak{v}_t^e(z_{t+1}^e)$ and $\mathfrak{v}_t^u(z_{t+1}^u, d_t)$ denote the continuation values

$$\mathfrak{v}_t^e(z_{t+1}^e) \equiv (1 - \delta_t)v_{t+1}^e(z_{t+1}^e) + \delta_t \mathfrak{v}_t^u(z_{t+1}^e, 0), \tag{B.4}$$

$$\begin{aligned} \mathfrak{v}_t^u(z_{t+1}^u, d_t) \equiv\, & (p(\theta_{t+1}; d_t + 1)s_{t+1}(z_{t+1}^u, d_t + 1))v_{t+1}^e(z_{t+1}^u) \\ & + (1 - p(\theta_{t+1}; d_t + 1)s_{t+1}(z_{t+1}^u, d_t + 1))v_{t+1}^u(z_{t+1}^u, d_t + 1) \\ & - \psi(s_{t+1}(z_{t+1}^u, d_t + 1)), \end{aligned} \tag{B.5}$$

the inequalities in (B.2) and (B.3) are strict only if $z_{t+1}^e(z_t) = \underline{z}_t$ or $z_{t+1}^u(z_t, d_t) = \underline{z}_t$, respectively, and the real interest rate is defined to be

$$1 + r_t \equiv (1 + i_t)\frac{P_t}{P_{t+1}}$$

given standard CES price indices $P_t$ and $P_{t+1}$. If regularity conditions on the value functions are satisfied, the Envelope Theorem further implies

$$\mathfrak{v}_{t,z}^e(z_{t+1}^e) = (1 - \delta_t)u'(c_{t+1}^e(z_{t+1}^e)) + \delta_t \mathfrak{v}_{t,z}^u(z_{t+1}^e, 0), \tag{B.6}$$

$$\begin{aligned} \mathfrak{v}_{t,z}^u(z_{t+1}^u, d_t) =\, & (p(\theta_{t+1}; d_t + 1)s_{t+1}(z_{t+1}^u, d_t + 1))u'(c_{t+1}^e(z_{t+1}^u)) \\ & + (1 - p(\theta_{t+1}; d_t + 1)s_{t+1}(z_{t+1}^u, d_t + 1))u'(c_{t+1}^u(z_{t+1}^u, d_t + 1)), \end{aligned} \tag{B.7}$$

in the usual way.

Turning to the representative producer facing (1.56), and anticipating the functional form assumption that production is constant-returns-to-scale with productivity process $\{a_t\}$ in the numerical analysis which follows, it is straightforward to show that an interior equilibrium in vacancy posting requires

$$\mu_t^{-1} a_t \left(1 - \frac{k}{q(\theta_t)}\right) - w_t + \frac{1}{1+r_t}(1-\delta_t)\mu_{t+1}^{-1} a_{t+1} \frac{k}{q(\theta_{t+1})} = 0 \tag{B.8}$$

where

$$(\mu_t)^{-1} \equiv \frac{P_t^I}{P_t}$$

is the real price of intermediate goods.

Finally, turning to retailers who can update prices each period according to (1.57), we obtain the standard optimal price-setting condition

$$P_{tj} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)(1+\tau^r)P_t^I.$$

This has two implications. First, given the maintained assumption that $\tau^r = -\frac{1}{\varepsilon}$, it means that in the flexible price equilibrium

$$\mu_t = 1. \tag{B.9}$$

Second, it means that there is no final good price dispersion in the flexible price equilibrium, just as there is no price dispersion in the fully sticky price equilibrium where retailer prices are uniformly fixed at $\bar{P}$ according to (1.65). The absence of price dispersion in both kinds of equilibria will now allow us to simplify workers' budget constraints as well as goods market clearing in equilibrium.

Consider first workers' real budget constraints

$$c_t^e(z_t) + \frac{1}{1+r_t} z_{t+1}^e(z_t) = y_t^e + z_t, \tag{B.10}$$

$$c_t^u(z_t, d_t) + \frac{1}{1+r_t} z_{t+1}^u(z_t, d_t) = y_t^u(d_t) + z_t, \tag{B.11}$$

where agents' real incomes are given by

$$y_t^e = w_t - t_t + \pi_t + \omega_t,$$
$$y_t^u(d) = b_t(d) + \pi_t + \omega_t,$$

and

$$\pi_t \equiv \frac{\Pi_t + \Pi_t^r - T_t^r}{P_t}.$$

Using the definition of producer and retailer profits, as well as government budget balance in policy targeted at retailers (1.59), it is straightforward to show that in equilibrium

$$\pi_t = \int_0^1 \left(\frac{P_{tj}}{P_t}\right) y_{tj} dj - w_t(n_t + q(\theta_t)\nu_t).$$

Moreover, since equilibrium tightness must be consistent with worker and firm behavior,

$$q(\theta_t)\nu_t = p(\theta_t; 0)\bar{s}_t \tag{B.12}$$

given the matching technology assumed in (1.60). It follows that

$$\pi_t = \int_0^1 \left(\frac{P_{tj}}{P_t}\right) y_{tj} dj - w_t(n_t + p(\theta_t; 0)\bar{s}_t).$$

Then, given the absence of equilibrium price dispersion, it is straightforward to use retailers' pure-pass through technology and intermediate goods market clearing in (1.63) to show

$$\pi_t = a_t(n_t + p(\theta_t; 0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t; 0)\bar{s}_t).$$

It follows that in equilibrium, agents' real incomes are given by

$$y_t^e = w_t - t_t + [a_t(n_t + p(\theta_t; 0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t; 0)\bar{s}_t)] + \omega_t. \tag{B.13}$$

$$y_t^u(d) = b_t(d) + [a_t(n_t + p(\theta_t; 0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t; 0)\bar{s}_t)] + \omega_t. \tag{B.14}$$

Now consider final and intermediate goods market clearing in (1.62) and (1.63), respectively. Given the absence of equilibrium price dispersion, it is straightforward to use equilibrium tightness consistent with worker and firm behavior and workers' standard lower-stage demand functions given CES preferences to show that

$$\lambda_t^e \left( \int_z c_t^e(z) \varphi_t^e(z) dz \right) + (1 - \lambda_t^e) \left( \sum_{d=0}^{\infty} \int_z c_t^u(z,d) \varphi_t^u(z,d) dz \right) = a_t(n_t + p(\theta_t; 0)\bar{s}_t - k\theta_t \bar{s}_t) + \omega_t \tag{B.15}$$

in the goods market.

Conditions (B.1)-(B.8) and (B.10)-(B.15) which hold in both the flexible and sticky price equilibria, and condition (B.9) which holds only in the flexible price equilibrium, will play a key role in the algorithms used to numerically characterize the stationary RCE and transitional dynamics. I describe these algorithms in the following two sections.

## B.2 Solving for the stationary RCE

Here I outline my approach to solving for the stationary RCE in section 1.3.3 of the main text. I take as given all of the economic parameters outlined in Tables 1.1 and 1.2. Throughout, I drop time-subscripts because macroeconomic aggregates and policies are time-invariant, by definition.

The goal is to find a fixed point in the triple $\{r, \bar{s}, t\}$. This generalizes the simplest heterogeneous agent model in the BHA tradition, in which the standard algorithm is to search for a fixed point in the real interest rate $r$ only. The complexity here arises because of the DMP structure of the labor market and government intervention in the labor market via UI, which mean that conjectures of $\bar{s}$ and $t$ are needed in order to calculate agents' real income in each employment state.

Two insights simplify the algorithm considerably. First, for the usual reasons, Carroll (2006)'s endogenous gridpoint method substantially speeds up the iterative techniques used to characterize workers' optimal policy functions. Second, given that I focus throughout my analysis on the case where UI schedules and match efficiencies take the functional forms

$$b(d) = \begin{cases} b^{UI} \text{ for } d < \bar{d}^{UI}, \\ b^{SA} \text{ for } d \geq \bar{d}^{UI} \end{cases}, \quad \bar{m}(d) = \begin{cases} \bar{m}(0)\exp(-\lambda d) \text{ for } d < \bar{d}^{\bar{m}}, \\ \bar{m}(\bar{d}^{\bar{m}} - 1) \text{ for } d \geq \bar{d}^{\bar{m}} \end{cases},$$

for some $\{\bar{d}^{UI}, \bar{d}^{\bar{m}}\}$, it is evident from (1.53) and (1.55) that all unemployed workers with duration

$$d \geq \bar{d} \equiv \max\{\bar{d}^{UI}, \bar{d}^{\bar{m}}\}$$

will face identical optimization problems:

$$\tilde{v}^u(z,d) = \tilde{v}^u(z,\bar{d}), \quad v^u(z,d) = v^u(z,\bar{d}) \quad \forall d \geq \bar{d}.$$

As such,

$$s^u(z,d) = s^u(z,\bar{d}), \quad c^u(z,d) = c^u(z,\bar{d}), \quad z_{+1}(z,d) = z_{+1}(z,\bar{d}) \quad \forall d \geq \bar{d},$$

so there are only $\bar{d}+1$ distinct search effort, consumption, and asset policy functions for unemployed agents to characterize, each defined as a function of the asset position $z$.

With this in mind, the algorithm to solve for the stationary RCE is as follows:

1. Initialize small, positive tolerance levels $\{\epsilon_{\bar{z}_{+1}}, \epsilon_{\bar{s}}, \epsilon_t, \epsilon_v, \epsilon_\varphi\}$ and step lengths $\{\Delta_r, \Delta_{\bar{s}}, \Delta_t\}$.

2. Conjecture $\{r, \bar{s}, t\}$.

3. The "supply-side a la DMP" block:

   (a) Use interiority in vacancy posting (B.8), optimal retailer price-setting (B.9), and stationarity to compute $\theta$:

   $$a\left(1 - \frac{k}{q(\theta)}\right) - w + \frac{1}{1+r}(1-\delta)a\frac{k}{q(\theta)} = 0 \Rightarrow \theta = q^{-1}\left[\frac{\left(1 - \frac{1}{1+r}(1-\delta)\right)k}{1 - \frac{w}{a}}\right].$$

   (b) Use $\theta$ along with the consistency of tightness with vacancies and aggregate search (B.12) and stationarity to compute $n$:

   $$n = (1-\delta)(n + p(\theta;0)\bar{s}) \Rightarrow n = \frac{1-\delta}{\delta}p(\theta;0)\bar{s}.$$

   (c) Use $\theta$, $n$, and stationarity to characterize workers' real incomes according to (B.13) and (B.14):

$$y^e = w - t + [a(n + p(\theta;0)\bar{s} - k\theta\bar{s}) - w(n + p(\theta;0)\bar{s})] + \omega,$$

$$y^u(d) = b(d) + [a(n + p(\theta;0)\bar{s} - k\theta\bar{s}) - w(n + p(\theta;0)\bar{s})] + \omega,$$

where, following the earlier discussion, there are only $\bar{d} + 1$ values of $y^u(d)$ we need to store in memory: $\{y^u(0), \ldots, y^u(\bar{d})\}$.

4. The "search-effort-augmented BHA" block:

   (a) Iterate value functions backward using worker optimality conditions (B.1)-(B.7) and resource constraints (B.10)-(B.11) to approximate workers' policy functions $s(z,d)$, $c^e(z)$, $z^e_{+1}(z)$, $c^u(z,d)$, and $z^u_{+1}(z,d)$.

   The value and policy functions are approximated over a $N \times 1$ vector of gridpoints $\boldsymbol{z} = \left(z^1, \ldots, z^N\right)'$, where $z^1 = \underline{z}$. Given values (guessed for the first iteration)

$$\{\boldsymbol{v^e_{t+1}}, \{\boldsymbol{v^{u,d}_{t+1}}\}_{d=0}^{\bar{d}}\}$$

   and policies (for all but the first iteration)

$$\{\{\boldsymbol{s^{u,d}_{t+1}}\}_{d=0}^{\bar{d}}, \boldsymbol{c^e_{t+1}}, \boldsymbol{z^e_{t+2}}, \{\boldsymbol{c^{u,d}_{t+1}}\}_{d=0}^{\bar{d}}, \{\boldsymbol{z^{u,d}_{t+2}}\}_{d=0}^{\bar{d}}\},$$

   the heart of the algorithm uses Carroll (2006)'s endogenous gridpoints method to iterate backward and obtain

$$\{\boldsymbol{v^e_t}, \{\boldsymbol{v^{u,d}_t}\}_{d=0}^{\bar{d}}\},$$
$$\{\{\boldsymbol{s^{u,d}_t}\}_{d=0}^{\bar{d}}, \boldsymbol{c^e_t}, \boldsymbol{z^e_{t+1}}, \{\boldsymbol{c^{u,d}_t}\}_{d=0}^{\bar{d}}, \{\boldsymbol{z^{u,d}_{t+1}}\}_{d=0}^{\bar{d}}\}.$$

   I outline this step in detail in section B.4. In the usual way, this proceeds until the value functions converge given a particular distance function $\phi_v(\cdot)$, such as the Euclidean distance, and tolerance level $\epsilon_v$.

   (b) Iterate the policy functions forward to approximate the invariant distribution characterized by $\{\tilde{\lambda}^e, \tilde{\varphi}^e, \tilde{\varphi}^u, \lambda^e, \varphi^e, \varphi^u\}$.

The marginal distributions will be discretized and approximated over a $M \times 1$ vector of gridpoints which, for the usual reasons, should be finer than the $N \times 1$ vector used in step (a). Moreover, since we do not need to keep track of the specific durations of agents remaining unemployed beyond $\bar{d}$ periods, $\tilde{\boldsymbol{\varphi}}^{\boldsymbol{u},\bar{\boldsymbol{d}}}$ and $\boldsymbol{\varphi}^{\boldsymbol{u},\bar{\boldsymbol{d}}}$ will approximate

$$\tilde{\varphi}^u(z, d \geq \bar{d}) \equiv \sum_{d=\bar{d}}^{\infty} \tilde{\varphi}^u(z, d), \quad \varphi^u(z, d \geq \bar{d}) \equiv \sum_{d=\bar{d}}^{\infty} \varphi^u(z, d),$$

respectively. After interpolating the (time-invariant) policies from (a) over the $M \times 1$ vector of gridpoints, the heart of the algorithm uses transition probabilities implied by these policies to advance the distribution of agents at the beginning of period $t$

$$\{\tilde{\lambda}_t^e, \tilde{\boldsymbol{\varphi}}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\tilde{\boldsymbol{\varphi}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$$

to obtain the distribution of agents in the middle of period $t$

$$\{\lambda_t^e, \boldsymbol{\varphi}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\boldsymbol{\varphi}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$$

and then at the beginning of period $t+1$

$$\{\tilde{\lambda}_{t+1}^e, \tilde{\boldsymbol{\varphi}}_{\boldsymbol{t+1}}^{\boldsymbol{e}}, \{\tilde{\boldsymbol{\varphi}}_{\boldsymbol{t+1}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}.$$

I outline this step in detail in section B.5. In the usual way, this will proceed until the distribution of agents within successive periods converges given a particular distance function $\phi_\varphi(\cdot)$, such as the Euclidean distance, and tolerance level $\epsilon_\varphi$.

5. Assess market clearing and consistency conditions using the above policy functions and distributions, and compute $\{r', \bar{s}', t'\}$ accordingly:

(a) Following its definition, approximate steady-state net asset demand with

$$\begin{aligned}\hat{\bar{z}}_{+1}(r,\bar{s},t) \equiv&\lambda^e\left(\int_z z^e_{+1}(z)\varphi^e(z)dz\right)+(1-\lambda^e)\left(\sum_{d=0}^{\infty}\int_z z^u_{+1}(z,d)\varphi^u(d,z)dz\right),\\ \approx&\lambda^e \boldsymbol{z^{e\prime}_{+1}\varphi^e}+(1-\lambda^e)\sum_{d=0}^{\bar{d}}\boldsymbol{z^{u,d\prime}_{+1}\varphi^{u,d}},\end{aligned}$$

and then set

$$r' = \begin{cases} r-\Delta_r\bar{z}_{+1}(r,\bar{s},t) \text{ if } |\hat{\bar{z}}_{+1}(r,\bar{s},t)|>\epsilon_{\bar{z}_{+1}}, \\ r \end{cases}.$$

(b) Following its definition in (1.61), approximate aggregate search intensity with

$$\begin{aligned}\hat{\bar{s}}(r,\bar{s},t) \equiv&(1-\tilde{\lambda}^e)\sum_{d=0}^{\infty}\frac{\bar{m}(d)}{\bar{m}(0)}\int_z s(z,d)\tilde{\varphi}^u(z,d)dz,\\ \approx&(1-\tilde{\lambda}^e)\sum_{d=0}^{\bar{d}}\frac{\bar{m}(d)}{\bar{m}(0)}\boldsymbol{s^{d\prime}\tilde{\varphi}^{u,d}},\end{aligned}$$

and then set

$$\bar{s}' = \begin{cases} \bar{s}+\Delta_{\bar{s}}(\hat{\bar{s}}(m,\bar{s},t)-\bar{s}) \text{ if } |\hat{\bar{s}}(r,\bar{s},t)-\bar{s}|>\epsilon_{\bar{s}}, \\ \bar{s} \end{cases}.$$

(c) Following (1.58), approximate the lump-sum tax which will balance the government's budget with

$$\begin{aligned}\hat{t}(r,\bar{s},t) =&\frac{1-\lambda^e}{\lambda^e}\sum_{d=0}^{\infty}\left[\left(\int_z \varphi^u(z,d)dz\right)b(d)\right],\\ \approx&\frac{1-\lambda^e}{\lambda^e}\sum_{d=0}^{\bar{d}}\left[\left(\boldsymbol{1'\varphi^{u,d}}\right)b(d)\right],\end{aligned}$$

and then set

$$t' = \begin{cases} t+\Delta_t(\hat{t}(r,\bar{s},t)-t) \text{ if } |\hat{t}(r,\bar{s},t)-t|>\epsilon_t, \\ t \end{cases}.$$

6. If $\{r,\bar{s},t\}=\{r',\bar{s}',t'\}$, stop. Else, return to step 2 with $\{r',\bar{s}',t'\}$.

In view of the equilibrium conditions used throughout the algorithm, at the end we have found an allocation which (approximately) satisfies asset market clearing, consistency of tightness with vacancies and aggregate search, and budget balance for workers and the government. Under these conditions, the goods market must also (approximately) clear as in (B.15) by Walras' Law.

## B.3 Solving for the transitional dynamics

Here I outline my approach to solving for the transitional dynamics in section 1.3.4 of the main text. I begin by discussing the algorithm when prices are flexible, defining the natural allocation, and then when prices are fully sticky.

### B.3.1 Flexible price equilibrium

Starting from the stationary RCE, I focus on the case of a temporary and unanticipated but subsequently perfect foresight shock at date 1. While in the main text I study only a shock to the discount factor $\beta_t$, for the present purposes I outline an algorithm which also accommodates a shock to TFP $a_t$ (as in the real business cycle tradition), the borrowing limit $\bar{z}_t$ (as in recent work on credit crises such as Guerrieri and Lorenzoni (2015)), or the separation rate $\delta_t$ (as in recent work studying the Beveridge curve such as Ravn and Sterk (2014)). I allow the generosity $b_t^{UI}$ and duration $\bar{d}_t^{UI}$ of UI benefits to vary over time (say, in response to the macroeconomic shock), and further allow for the exogenous real wage $w_t$ and endowment $\omega_t$ to vary over time (say, in response to the conduct of policy). For all other parameters outlined in Tables 1.1 and 1.2, I assume for expositional simplicity that these are time-invariant, but of course the algorithm easily generalizes.

The goal now is to characterize a fixed point in the sequence

$$\{\{r_1, \bar{s}_1, t_1\}, \ldots, \{r_T, \bar{s}_T, t_T\}\}$$

for $T$ very large, at which point it is assumed that the initial stationary RCE is approximately reached. I denote equilibrium objects in the stationary RCE, solved as outlined in the prior section, with superscript "ss".

Again, two insights simplify the computation: Carroll (2006)'s endogenous gridpoints method, and the assumed functional forms for UI benefits and match efficiencies which limit the size of workers' state space which needs tracking. Now with time variation in the duration of UI benefits, however, it proves useful to define

$$\bar{\bar{d}}^{UI} \equiv \max_t \bar{d}_t^{UI},$$

the maximum duration of UI benefits across the entire simulation path. Then, generalizing the argument made for the stationary RCE, all workers with duration

$$d \geq \bar{\bar{d}} \equiv \max\{\bar{\bar{d}}^{UI}, \bar{d}^{\bar{m}}\}$$

will face identical optimization problems at each date in the simulation, so we need only characterize $\bar{\bar{d}} + 1$ distinct search effort, consumption, and asset policy functions for unemployed agents at each date, each as a function of assets chosen the prior period.

With this in mind, the algorithm to solve for the transitional dynamics in response to a shock to $\{\beta_t, a_t, \bar{z}_t, \delta_t\}$ and time-variation in $\{b_t^{UI}, \bar{d}_t^{UI}, w_t, \omega_t\}$ is as follows:

1. Use the algorithm in section B.2 to solve for the stationary RCE. Denote the discretized policy functions and distributions with superscript "ss".

2. Initialize small, positive tolerance levels $\{\epsilon_{\bar{z}_{+1}}, \epsilon_{\bar{s}}, \epsilon_t\}$ and step lengths $\{\Delta_r, \Delta_{\bar{s}}, \Delta_t\}$.

3. Conjecture a path $\{r_t, \bar{s}_t, t_t\}_{t=1}^T$ in which $\{r_T, \bar{s}_T, t_T\} = \{r^{ss}, \bar{s}^{ss}, t^{ss}\}$ for $T$ large.

4. The "supply-side a la DMP" block:

   (a) Iterate backwards on the equilibrium condition for interiority in vacancy posting (B.8), given optimal retailer price-setting (B.9) and $\theta_T = \theta^{ss}$:

$$a_t \left(1 - \frac{k}{q(\theta_t)}\right) - w_t + \frac{1}{1+r_t}(1-\delta_t) a_{t+1} \frac{k}{q(\theta_{t+1})} = 0 \Rightarrow$$

$$\theta_t = q^{-1}\left[\frac{k}{1 - \frac{w_t}{a_t} + \frac{1}{1+r_t}(1-\delta_t)\frac{a_{t+1}}{a_t}\frac{k}{q(\theta_{t+1})}}\right].$$

(b) Iterate forwards on incumbent employment using $\{\theta_t\}_{t=1}^T$, the consistency of tightness with vacancies and aggregate search (B.12), and $n_1 = n^{ss}$:

$$n_{t+1} = (1-\delta_t)(n_t + p(\theta_t;0)\bar{s}_t).$$

(c) Use $\{\theta_t\}_{t=1}^T$ and $\{n_t\}_{t=1}^T$ to characterize workers' real incomes according to (B.13) and (B.14):

$$y_t^e = w_t - t_t + [a_t(n_t + p(\theta_t;0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t;0)\bar{s}_t)] + \omega_t,$$
$$y_t^u(d) = b_t(d) + [a_t(n_t + p(\theta_t;0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t;0)\bar{s}_t)] + \omega_t,$$

where, following the earlier discussion, there are only $\bar{\bar{d}}+1$ distinct values of $y_t^u(d)$ we need to store in memory: $\{y_t^u(0), \dots, y_t^u(\bar{\bar{d}})\}$.

5. The "search-effort-augmented BHA" block:

(a) Iterate value functions backward using worker optimality conditions (B.1)-(B.7) and resource constraints (B.10)-(B.11) to compute workers' policy functions $s_t(z,d)$, $c_t^e(z)$, $z_{t+1}^e(z)$, $c_t^u(z,d)$, $z_{t+1}^u(z,d)$.

This proceeds as in the stationary RCE, except that we now start from

$$\{\boldsymbol{v}_T^e = \boldsymbol{v}^{e;ss}, \boldsymbol{v}_T^{u,d} = \boldsymbol{v}^{u,d;ss}\},$$
$$\{\boldsymbol{s}_T^{u,d} = \boldsymbol{s}^{u,d;ss}, \boldsymbol{c}_T^e = \boldsymbol{c}^{e;ss}, \boldsymbol{z}_{T+1}^e = \boldsymbol{z}_{+1}^{e;ss}, \boldsymbol{c}_T^{u,d} = \boldsymbol{c}^{u,d;ss}, \boldsymbol{z}_{T+1}^{u,d} = \boldsymbol{z}_{+1}^{u,d;ss}\}$$

and iterate backwards to $t=1$. The key iterative step is again outlined in section B.4.

(b) Use these policy functions to iterate the distribution of agents forward.

This proceeds as in the stationary RCE, except that we now start from

$$\{\tilde{\lambda}_1^e = \tilde{\lambda}^{e;ss}, \tilde{\boldsymbol{\varphi}}_1^e = \tilde{\boldsymbol{\varphi}}^{e;ss}, \{\tilde{\boldsymbol{\varphi}}_1^{u,d}\}_{d=0}^{\bar{\bar{d}}} = \{\tilde{\boldsymbol{\varphi}}^{u,d;ss}\}_{d=0}^{\bar{\bar{d}}}\}$$

and iterate forwards to $t=T$, at which point the distribution of agents should have approximately converged to the invariant one. The key iterative step is again outlined in section B.5.

6. Assess market clearing and consistency conditions at each date using the above policy functions and distributions, and compute $\{r'_t, \bar{s}'_t, t'_t\}_{t=1}^T$ accordingly.

   Consistent with the formulas from the stationary RCE, each date's steady-state net asset demand, aggregate search intensity, and lump-sum tax balancing the government's budget are approximated using

$$\hat{\bar{z}}_{t+1} = \lambda^e_t \boldsymbol{z^{e\prime}_{t+1}} \boldsymbol{\varphi^e_t} + (1-\lambda^e_t) \sum_{d=0}^{\bar{\bar{d}}} \boldsymbol{z^{u,d\prime}_{t+1}} \boldsymbol{\varphi^{u,d}_t},$$

$$\hat{\bar{s}} = (1-\tilde{\lambda}^e_t) \sum_{d=0}^{\bar{\bar{d}}} \frac{\bar{m}(d)}{\bar{m}(0)} \boldsymbol{s^{d\prime}_t} \boldsymbol{\tilde{\varphi}^{u,d}_t},$$

$$\hat{t} = \frac{1-\lambda^e_t}{\lambda^e_t} \sum_{d=0}^{\bar{\bar{d}}} \left[ \left( \boldsymbol{1'} \boldsymbol{\varphi^{u,d}_t} \right) b_t(d) \right].$$

   We can then compute $\{r'_t, \bar{s}'_t, t'_t\}_{t=1}^T$ using

$$r'_t = \begin{cases} r_t - \phi(t)\Delta_r \bar{z}_{t+1} \text{ if } \max_{t\in\{1,\ldots,T\}} |\hat{\bar{z}}_{t+1}| > \epsilon_{\bar{z}_{+1}}, \\ r_t \end{cases},$$

$$\bar{s}'_t = \begin{cases} \bar{s}_t + \phi(t)\Delta_{\bar{s}}(\hat{\bar{s}}_t - \bar{s}_t) \text{ if } \max_{t\in\{1,\ldots,T\}} |\hat{\bar{s}}_t - \bar{s}_t| > \epsilon_{\bar{s}}, \\ \bar{s}_t \end{cases},$$

$$t'_t = \begin{cases} t_t + \phi(t)\Delta_t(\hat{t}_t - t_t) \text{ if } \max_{t\in\{1,\ldots,T\}} |\hat{t}_t - t_t| > \epsilon_t, \\ t_t \end{cases},$$

   where the weighting function $\phi(t)$ is chosen to sum to one over $\{1,\ldots,T\}$ and smoothly approach zero as $t$ approaches $T$. I follow Guerrieri and Lorenzoni (2015) in using a weighting function $\phi(t)$ as it aids in the process of convergence.

7. If $\{r_t, \bar{s}_t, t_t\}_{t=1}^T = \{r'_t, \bar{s}'_t, t'_t\}_{t=1}^T$, stop. Else, return to step 3 with $\{r'_t, \bar{s}'_t, t'_t\}_{t=1}^T$.

### B.3.2 Fully sticky price equilibrium

When final goods prices are fully sticky at $\bar{P}$, the real/nominal dichotomy is broken and policymakers can directly affect real interest rates through the path of nominal interest rates $\{i_t\}_{t=1}^{\infty}$:

$$r_t = i_t \ \forall t.$$

I outline an algorithm below for any given path of nominal rates which ensures that the long-run stationary RCE is realized. In my analysis in the main text, I focus in particular on a path which targets the natural rate of interest before it hits the zero lower bound and once it is permanently away from it, and is at the bound otherwise:

$$i_t = \begin{cases} r_t^n \text{ if } \min\{r_\tau^n\}_{\tau=1}^{t} \geq 0 \text{ or } \min\{r_\tau^n\}_{\tau=t}^{\infty} \geq 0 \\ 0 \text{ otherwise} \end{cases}, \tag{B.16}$$

where $r_t^n$ is solved using the algorithm of the prior subsection.

Computationally, the key difference from the flexible price case is that we will now search for the path of tightness $\{\theta\}_{t=1}^{T}$ which equilibrates goods markets rather than real interest rates $\{r_t\}_{t=1}^{\infty}$ which equilibrate asset markets. That is, for $T$ large we will seek a fixed point in

$$\{\{\theta_1, \bar{s}_1, t_1\}, \ldots, \{\theta_T, \bar{s}_T, t_T\}\}.$$

This reflects the standard Keynesian logic that when prices are sticky, production will adjust to meet desired consumption demand.

With this in mind, the algorithm to solve for the transitional dynamics in response to a shock to $\{\beta_t, a_t, \bar{z}_t, w_t, \delta_t\}$ and time-variation in $\{b_t^{UI}, \bar{d}_t^{UI}, w_t, \omega_t\}$, taking as given monetary policy $\{i_t\}$, is as follows:

1. Use the algorithm in section B.2 to solve for the stationary RCE. Denote the discretized policy functions and distributions with superscript "ss".

2. Set real interest rates $\{r_t = i_t\}_{t=1}^{T}$, which for policy paths of the form (B.16) (as are studied in the main text) requires using the algorithm in section B.3.1 to solve for

$\{r_t^n\}_{t=1}^T$.

3. Initialize small, positive tolerance levels $\{\epsilon_x, \epsilon_{\bar{s}}, \epsilon_t\}$ and step lengths $\{\Delta_\theta, \Delta_{\bar{s}}, \Delta_t\}$.

4. Conjecture a path $\{\theta_t, \bar{s}_t, t_t\}_{t=1}^T$ in which $\{\theta_T, \bar{s}_T, t_T\} = \{\theta^{ss}, \bar{s}^{ss}, t^{ss}\}$.

5. The "supply-side a la DMP" block:

   (a) Iterate forwards on incumbent employment using the *conjectured* $\{\theta_t\}_{t=1}^T$, the consistency of tightness with vacancies and aggregate search (B.12), and $n_1 = n^{ss}$:

$$n_{t+1} = (1 - \delta_t)(n_t + p(\theta_t; 0)\bar{s}_t).$$

   (b) Use $\{\theta_t\}_{t=1}^T$ and $\{n_t\}_{t=1}^T$ to characterize workers' real incomes according to (B.13) and (B.14):

$$y_t^e = w_t - t_t + [a_t(n_t + p(\theta_t; 0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t; 0)\bar{s}_t)] + \omega_t,$$
$$y_t^u(d) = b_t(d) + [a_t(n_t + p(\theta_t; 0)\bar{s}_t - k\theta_t\bar{s}_t) - w_t(n_t + p(\theta_t; 0)\bar{s}_t)] + \omega_t.$$

6. The "search-effort-augmented BHA" block, identical to that in the flexible price algorithm:

   (a) Iterate value functions backward using worker optimality conditions (B.1)-(B.7) and resource constraints (B.10)-(B.11) to compute workers' policy functions $s_t(z, d)$, $c_t^e(z)$, $z_{t+1}^e(z)$, $c_t^u(z, d)$, $z_{t+1}^u(z, d)$.

   (b) Use these policy functions to iterate the distribution of agents forward.

7. Assess market clearing and consistency conditions at each date using the above policy functions and distributions, and compute $\{\theta_t', \bar{s}_t', t_t'\}_{t=1}^T$ accordingly.

In particular, following (B.15), approximate excess demand for goods with

$$\begin{aligned}\bar{x}_t &\equiv \left[\lambda_t^e \left(\int_z c_t^e(z)\varphi_t^e(z)dz\right) + (1-\lambda_t^e)\left(\sum_{d=0}^{\infty}\int_z c_t^u(z,d)\varphi_t^u(z,d)dz\right)\right] \\ &\quad - \left[a_t(n_t + p(\theta_t;0)\bar{s}_t - k\theta_t\bar{s}_t) + \omega_t\right], \\ &\approx \left[\lambda_t^e \boldsymbol{c_t^{e\prime}}\boldsymbol{\varphi_t^e} + (1-\lambda_t^e)\sum_{d=0}^{\bar{\bar{d}}} \boldsymbol{c_t^{u,d\prime}}\boldsymbol{\varphi_t^{u,d}}\right] - \left[a_t(n_t + p(\theta_t;0)\bar{s}_t - k\theta_t\bar{s}_t) + \omega_t\right].\end{aligned}$$

We can then compute $\{\theta_t'\}_{t=1}^T$ using

$$\theta_t' = \begin{cases} \theta_t + \phi(t)\Delta_\theta \bar{x}_t \text{ if } \max_{t\in\{1,\ldots,T\}} |\hat{\bar{x}}_t| > \epsilon_x \\ \theta_t \end{cases},$$

where $\phi(t)$ is a weighting function as described in the flexible price case, so that positive excess demand for goods implies an increase in vacancy-posting and thus the conjectured $\theta_t$.

We can then compute $\hat{\bar{s}}$, $\hat{t}$, $\bar{s}'$, and $t'$ using the same approach as in the flexible price case.

8. If $\{\theta_t, \bar{s}_t, t_t\}_{t=1}^T = \{\theta_t', \bar{s}_t', t_t'\}_{t=1}^T$, stop. Else, return to step 4 with $\{\theta_t', \bar{s}_t', t_t'\}_{t=1}^T$.

Note that in step 7 we could have updated $\theta_t$ according to market clearing in the asset market (as I do in the stationary and flexible price cases) rather than the goods market, since in equilibrium one will imply the other according to Walras' Law. I use the goods market in this case to more transparently reflect the Keynesian intuition that production is adjusting to meet desired demand.

At the conclusion of the algorithm, we must check to ensure that retailers will indeed be willing to satisfy aggregate consumption demand according to (1.66) in the main text, which requires that their mark-up

$$\mu_t = \frac{\bar{P}}{P_t^I} \geq (1+\tau^r) = \frac{\varepsilon - 1}{\varepsilon} \tag{B.17}$$

at each $t$, where the last equality follows from the maintained assumption that $\tau^r = -\frac{1}{\varepsilon}$ throughout the analysis. Using the assumption $\mu_T = \mu^{ss}$ and fact that in the stationary

RCE $\mu^{ss} = 1$, we can compute the equilibrium mark-ups implied by the above allocation by iterating backwards on (B.8), reproduced here:

$$\mu_t^{-1} a_t \left(1 - \frac{k}{q(\theta_t)}\right) - w_t + \frac{1}{1+r_t}(1-\delta_t)\mu_{t+1}^{-1} a_{t+1} \frac{k}{q(\theta_{t+1})} = 0.$$

Provided that the implied $\{\mu_t\}_{t=1}^T$ are positive, there is an $\varepsilon$ close enough to one which ensures that (B.17) is satisfied at all dates. I verify that this is the case in all of the simulations presented in the main text. An alternative approach would be to set $\varepsilon$ as a calibrated parameter (say, using empirical evidence on mark-ups), in which case the above algorithm would need to be modified to ensure that the constraint (B.17) is satisfied at all dates.

Finally, note that as discussed in footnote 55, under simulations when the path of UI is not very generous in response to the macroeconomic shock, I find that the above algorithm leads to $\theta_1 \to 0$, suggesting that an interior equilibrium with positive vacancy posting does not exist in view of the collapse in aggregate demand. To obtain a well-defined interior equilibrium, I enrich the model to allow firms to costlessly lay off workers as described in Appendix A.8, which I find that they only do in the initial period when the shock is realized. Computationally, this simply means that we are seeking a fixed point in the initial layoff rate $\zeta_1$ (characterized in (A.49) of the Appendix) to equilibrate the goods market, instead of $\theta_1$ (which will be zero). The mark-up $\mu_1$ consistent with producers' optimal choice of layoffs $l_1$ in (A.47) solves

$$\mu_1^{-1} a_1 - w_1 + \frac{1}{1+r_1}(1-\delta_1)\mu_2^{-1} a_2 \frac{k}{q(\theta_2)} = 0,$$

replacing (B.8) at this initial date. Since $\theta_1 = 0$, workers will optimally exert no search effort at the initial date: $s_1(z,d) = 0 \ \forall z, d$. The algorithm otherwise proceeds identically to that above.

## B.4 Iterating backward on workers' value functions

Here I outline the sub-algorithm used to iterate backward on workers' value functions, which is a fairly standard application of Carroll (2006)'s endogenous gridpoint method to the present setting. This sub-algorithm is used both in solving for the stationary RCE and for the transitional dynamics in response to a macroeconomic shock.

I take as given the values

$$\{\boldsymbol{v}^e_{t+1}, \{\boldsymbol{v}^{u,d}_{t+1}\}^{\bar{d}}_{d=0}\}$$

and (if available) policies

$$\{\{\boldsymbol{s}^{u,d}_{t+1}\}^{\bar{d}}_{d=0}, \boldsymbol{c}^e_{t+1}, \boldsymbol{z}^e_{t+2}, \{\boldsymbol{c}^{u,d}_{t+1}\}^{\bar{d}}_{d=0}, \{\boldsymbol{z}^{u,d}_{t+2}\}^{\bar{d}}_{d=0}\}$$

defined over a $N_{t+1} \times 1$ grid $\boldsymbol{z}_{t+1} = (z^1_{t+1}, \ldots, z^{N_{t+1}}_{t+1})'$ where $z^1_{t+1} = \underline{z}_t$. The goal is to iterate backward on workers' optimal programs in (1.52)-(1.55) to obtain

$$\{\boldsymbol{v}^e_t, \{\boldsymbol{v}^{u,d}_t\}^{\bar{d}}_{d=0}\},$$
$$\{\{\boldsymbol{s}^{u,d}_t\}^{\bar{d}}_{d=0}, \boldsymbol{c}^e_t, \boldsymbol{z}^e_{t+1}, \{\boldsymbol{c}^{u,d}_t\}^{\bar{d}}_{d=0}, \{\boldsymbol{z}^{u,d}_{t+1}\}^{\bar{d}}_{d=0}\},$$

where these vectors are defined over a $N_t \times 1$ grid $\boldsymbol{z}_t = (z^1_t, \ldots, z^{N_t}_t)'$ where $z^1_t = \underline{z}_{t-1}$. It may be that we seek to preserve the original grid (that is, $\boldsymbol{z}_t = \boldsymbol{z}_{t+1}$), as will be the case when this sub-algorithm is used to characterize the stationary RCE.

The steps are as follows:

1. Compute the continuation values $\boldsymbol{\mathfrak{v}}^e_t$ and $\{\boldsymbol{\mathfrak{v}}^{u,d}_t\}^{\bar{d}}_{d=0}$ defined over $\boldsymbol{z}_{t+1}$:

   (a) If the date-$(t+1)$ policies are unavailable, we first compute $\{\boldsymbol{s}^d_{t+1}\}^{\bar{d}}_{d=0}$ by inverting (B.1):

$$\boldsymbol{s}^d_{t+1} = \min\left\{(\psi')^{-1}\left[p(\theta_{t+1}; d)\left(\boldsymbol{v}^e_{t+1} - \boldsymbol{v}^{u,d}_{t+1}\right)\right], \frac{1}{p(\theta_{t+1}; d)}\mathbf{1}\right\}.$$

(b) Then use the definitions in (B.4) and (B.5) to compute

$$\begin{aligned}
\mathfrak{v}_{\boldsymbol{t}}^{\boldsymbol{e}} =& (1-\delta_t)\boldsymbol{v}_{\boldsymbol{t+1}}^{\boldsymbol{e}} + \delta_t \mathfrak{v}_{\boldsymbol{t}}^{\boldsymbol{u,0}}, \\
\mathfrak{v}_{\boldsymbol{t}}^{\boldsymbol{u,d}} =& (p(\theta_{t+1}; d_{+1}(d))\boldsymbol{s}_{\boldsymbol{t+1}}^{\boldsymbol{d_{+1}(d)}})\boldsymbol{v}_{\boldsymbol{t+1}}^{\boldsymbol{e}} + (1 - p(\theta_{t+1}; d_{+1}(d))\boldsymbol{s}_{\boldsymbol{t+1}}^{\boldsymbol{d_{+1}(d)}})\boldsymbol{v}_{\boldsymbol{t+1}}^{\boldsymbol{u,d_{+1}(d)}} \\
& - \psi(\boldsymbol{s}_{\boldsymbol{t+1}}^{\boldsymbol{d_{+1}(d)}}),
\end{aligned}$$

where

$$d_{+1}(d) \equiv \min\{d+1, \bar{d}\}.$$

2. Compute the derivatives $\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{e}}$ and $\{\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{u,d}}\}_{d=0}^{\bar{d}}$ defined over $\boldsymbol{z_{t+1}}$:

   (a) If the date-$(t+1)$ policies are unavailable, we can use the average of slopes between adjacent points in $\mathfrak{v}_{\boldsymbol{t}}^{\boldsymbol{e}}$ and $\{\mathfrak{v}_{\boldsymbol{t}}^{\boldsymbol{u,d}}\}_{d=0}^{\bar{d}}$.

   (b) If the date-$(t+1)$ policies are available, and assuming that the conditions justifying the use of the Envelope Theorem hold, we can directly compute $\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{e}}$ and $\{\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{u,d}}\}_{d=0}^{\bar{d}}$ using (B.6) and (B.7):

$$\begin{aligned}
\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{e}} =& (1-\delta_t)u'(\boldsymbol{c}_{\boldsymbol{t+1}}^{\boldsymbol{e}}) + \delta_t \mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{u,0}}, \\
\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{u,d}} =& (p(\theta_{t+1}; d_{+1}(d))\boldsymbol{s}_{\boldsymbol{t+1}}^{\boldsymbol{d_{+1}(d)}})u'(\boldsymbol{c}_{\boldsymbol{t+1}}^{\boldsymbol{e}}) + (1 - p(\theta_{t+1}; d_{+1}(d))\boldsymbol{s}_{\boldsymbol{t+1}}^{\boldsymbol{d_{+1}(d)}})u'(\boldsymbol{c}_{\boldsymbol{t+1}}^{\boldsymbol{u,d_{+1}(d)}}).
\end{aligned}$$

3. Invert the Euler equations in (B.2) and (B.3) to define the $N_{t+1} \times 1$ consumption policies $\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{e}}$ and $\{\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{u,d}}\}_{d=0}^{\bar{d}}$ defined over the endogenous gridpoints solved in the next step:

$$\begin{aligned}
\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{e}} =& (u')^{-1}\left(\beta_t(1+r_t)\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{e}}\right), \\
\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{u,d}} =& (u')^{-1}\left(\beta_t(1+r_t)\mathfrak{v}_{\boldsymbol{t,z}}^{\boldsymbol{u,d}}\right).
\end{aligned}$$

4. Compute the $N_{t+1} \times 1$ endogenous gridpoints $\boldsymbol{z}_{\boldsymbol{t}}^{\boldsymbol{e}}$ and $\{\boldsymbol{z}_{\boldsymbol{t}}^{\boldsymbol{u,d}}\}_{d=0}^{\bar{d}}$ using the resource constraints (B.10) and (B.11):

$$\begin{aligned}
\boldsymbol{z}_{\boldsymbol{t}}^{\boldsymbol{e}} =& \hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{e}} + \frac{1}{1+r_t}\boldsymbol{z_{t+1}} - y_t^e, \\
\boldsymbol{z}_{\boldsymbol{t}}^{\boldsymbol{u,d}} =& \hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{u,d}} + \frac{1}{1+r_t}\boldsymbol{z_{t+1}} - y_t^{u,d}.
\end{aligned}$$

5. Compute the $N_{t+1} \times 1$ value functions $\hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{e}}$ and $\{\hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}$ defined over these endogenous gridpoints:

$$\begin{aligned}\hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{e}} &= u(\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{e}}) + \beta_t \boldsymbol{\mathfrak{v}}_{\boldsymbol{t}}^{\boldsymbol{e}}, \\ \hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}} &= u(\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}) + \beta_t \boldsymbol{\mathfrak{v}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}.\end{aligned}$$

6. Interpolate $\{\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}, \hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$ over the $N_t \times 1$ grid $\boldsymbol{z}_t$, defining the desired consumption policies and values $\{\boldsymbol{c}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\boldsymbol{c}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}, \boldsymbol{v}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\boldsymbol{v}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$.

   Note that for each $i \in \{e, (u,0), \ldots, (u,\bar{d})\}$, we can use the monotonicity of the equilibrium asset policy function to conclude that

$$z_{t+1}^i(z) = \underline{z}_t \ \forall z \leq z_t^{i,1}$$

   where $z_t^{i,1}$ is the first element of $\boldsymbol{z}_{\boldsymbol{t}}^{\boldsymbol{i}}$ since, by construction, $z_{t+1}(z_t^{i,1}) = z_{t+1}^1 = \underline{z}_t$. Hence, in the course of the interpolation of $\hat{\boldsymbol{c}}_{\boldsymbol{t}}^{\boldsymbol{i}}$ and $\hat{\boldsymbol{v}}_{\boldsymbol{t}}^{\boldsymbol{i}}$ over $\boldsymbol{z}_t$, if $\underline{z}_{t-1} < z_t^{i,1}$, I append

$$\begin{aligned}\hat{c}_t^{i,0} &= y_t^i + \underline{z}_{t-1} - \frac{1}{1+r_t}\underline{z}_t, \\ \hat{v}_t^{i,0} &= u(c_t^{i,0}) + \beta_t \mathfrak{v}_t^{i,1}\end{aligned}$$

   to the set of points over which I interpolate. This simple way of accounting for the borrowing constraint is one of the key advantages of Carroll (2006)'s endogeneous gridpoint method.

7. Compute the $N_t \times 1$ saving policies $\boldsymbol{z}_{\boldsymbol{t+1}}^{\boldsymbol{e}}$ and $\{\boldsymbol{z}_{\boldsymbol{t+1}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}$ using the resource constraints (B.10) and (B.11):

$$\begin{aligned}\boldsymbol{z}_{\boldsymbol{t+1}}^{\boldsymbol{e}} &= (1+r_t)\left(y_t^e + \boldsymbol{z}_t - \boldsymbol{c}_{\boldsymbol{t}}^{\boldsymbol{e}}\right), \\ \boldsymbol{z}_{\boldsymbol{t+1}}^{\boldsymbol{u},\boldsymbol{d}} &= (1+r_t)\left(y_t^{u,d} + \boldsymbol{z}_t - \boldsymbol{c}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\right).\end{aligned}$$

8. Compute the $N_t \times 1$ search effort policies $\{\boldsymbol{s}_{\boldsymbol{t}}^{\boldsymbol{d}}\}_{d=0}^{\bar{d}}$ by inverting (B.1):

$$\boldsymbol{s}_{\boldsymbol{t}}^{\boldsymbol{d}} = \min\left\{(\psi')^{-1}\left[p(\theta_t; d)\left(\boldsymbol{v}_{\boldsymbol{t}}^{\boldsymbol{e}} - \boldsymbol{v}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\right)\right], \frac{1}{p(\theta_t; d)}\mathbf{1}\right\}.$$

## B.5 Iterating forward on worker distributions

Here I outline the sub-algorithm used to iterate forward on the distribution of workers across time given a particular set of policy functions. This sub-algorithm is used both in solving for the stationary RCE and for the transitional dynamics in response to a macroeconomic shock.

I take as given the policies

$$\{\{\boldsymbol{s}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}, \boldsymbol{c}_{\boldsymbol{t}}^{\boldsymbol{e}}, \boldsymbol{z}_{\boldsymbol{t+1}}^{\boldsymbol{e}}, \{\boldsymbol{c}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}, \{\boldsymbol{z}_{\boldsymbol{t+1}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$$

defined over a $M \times 1$ grid $\boldsymbol{z} = (z^1, \ldots, z^M)'$. The goal is to use the transition probabilities implied by these policies to advance the distribution of agents at the beginning of period $t$

$$\{\tilde{\lambda}_t^e, \tilde{\boldsymbol{\varphi}}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\tilde{\boldsymbol{\varphi}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$$

and obtain the distribution of agents in the middle of period $t$

$$\{\lambda_t^e, \boldsymbol{\varphi}_{\boldsymbol{t}}^{\boldsymbol{e}}, \{\boldsymbol{\varphi}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$$

and then at the beginning of period $t+1$

$$\{\tilde{\lambda}_{t+1}^e, \tilde{\boldsymbol{\varphi}}_{\boldsymbol{t+1}}^{\boldsymbol{e}}, \{\tilde{\boldsymbol{\varphi}}_{\boldsymbol{t+1}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}.$$

Each of these marginal distributions will be defined over the same $M \times 1$ grid $\boldsymbol{z}$.

The steps are as follows:

1. Transform the policy functions into transition probabilities in $\{\{\tilde{\boldsymbol{T}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}, \boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{z},\boldsymbol{z}+1;\boldsymbol{e}}$, $\{\boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{z},\boldsymbol{z}+1;\boldsymbol{u},\boldsymbol{d}}\}_{d=0}^{\bar{d}}\}$:

   (a) Let $\tilde{\boldsymbol{T}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}}$ be the $M \times 1$ vector of probabilities of becoming employed within the period for initially unemployed agents of duration $d$, so

   $$\tilde{\boldsymbol{T}}_{\boldsymbol{t}}^{\boldsymbol{u},\boldsymbol{d}} = p(\theta_t; d)\boldsymbol{s}_{\boldsymbol{t}}^{\boldsymbol{d}}.$$

   (b) For each $i \in \{e, (u,0), \ldots, (u,\bar{d})\}$, let $\boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{z},\boldsymbol{z}+1;\boldsymbol{i}}$ be the $M \times M$ transition matrix from

$z$ to $z_{+1}$ for agents with employment status $i$ in the middle of the period, where

$$T_t^{z,z_{+1};i}(z^{(1)}, z^{(2)}) = \begin{cases} 1 - \frac{(z^i_{t+1}(z^{(1)}) - z^{(2)})}{z^{(2)+1} - z^{(2)}} \text{ if } z^i_{t+1}(z^{(1)}) \in [z^{(2)}, z^{(2)+1}) \\ 1 - \frac{(z^{(2)} - z^i_{t+1}(z^{(1)}))}{z^{(2)} - z^{(2)-1}} \text{ if } z^i_{t+1}(z^{(1)}) \in (z^{(2)-1}, z^{(2)}) \\ 0 \text{ otherwise} \end{cases}$$

for each $(z^{(1)}, z^{(2)}) \in \{z^1, \ldots, z^M\} \times \{z^1, \ldots, z^M\}$. Following Kopecky (2007), this matrix is constructed as if agents with assets $z^{(1)}$ are forced to choose their next period assets by playing a lottery between the nearest two available gridpoints to $z_{t+1}(z^{(1)})$, with weights proportional to the distance from each gridpoint.

Note that when applying these steps to find the invariant distribution in the stationary RCE, the transition probabilities are all time-invariant, so this step need not be repeated every time the distribution of agents is simulated forward one period.

2. Compute $\{\lambda_t^e, \boldsymbol{\varphi_t^e}, \{\boldsymbol{\varphi_t^{u,d}}\}_{d=0}^{\bar{d}}\}$ using $\{\boldsymbol{\tilde{T}_t^{u,d}}\}_{d=0}^{\bar{d}}$:

$$\lambda_t^e = \tilde{\lambda}_t^e + (1 - \tilde{\lambda}_t^e) \left( \sum_{d=0}^{\bar{d}} \boldsymbol{\tilde{T}_t^{u,d\prime}} \boldsymbol{\tilde{\varphi}_t^{u,d}} \right),$$

$$\boldsymbol{\varphi_t^e} = \frac{1}{\lambda_t^e} \left[ \tilde{\lambda}_t^e \boldsymbol{\tilde{\varphi}_t^e} + (1 - \tilde{\lambda}_t^e) \left( \sum_{d=0}^{\bar{d}} (\boldsymbol{\tilde{T}_t^{u,d}} \cdot \boldsymbol{\tilde{\varphi}_t^{u,d}}) \right) \right],$$

$$\boldsymbol{\varphi_t^{u,d}} = \frac{1}{1 - \lambda_t^e} \left[ (\mathbf{1} - \boldsymbol{\tilde{T}_t^{u,d}}) \cdot \boldsymbol{\tilde{\varphi}_t^{u,d}} \right],$$

where $\cdot$ denotes element-wise multiplication and these equations reflect the fact that in the baseline model, all employed agents at the beginning of the period remain employed through the middle of the period. When simulating agents in the sticky price transitional dynamics which accommodate layoffs (as described in footnote 55 and Appendix A.8), the above formulas are generalized to account for the layoff rate $\zeta_t$.

3. Compute $\{\tilde{\lambda}_{t+1}^e, \boldsymbol{\tilde{\varphi}_{t+1}^e}, \{\boldsymbol{\tilde{\varphi}_{t+1}^{u,d}}\}_{d=0}^{\bar{d}}\}$ using $\{\boldsymbol{T_t^{z,z_{+1};e}}, \{\boldsymbol{T_t^{z,z_{+1};u,d}}\}_{d=0}^{\bar{d}}\}$:

(a) Compute the fraction of employed agents at the beginning of next period:

$$\tilde{\lambda}^e_{t+1} = (1-\delta_t)\lambda^e_t$$

(b) Approximate $\hat{\varphi}^e_t(z_{+1})$ and $\hat{\varphi}^u_t(z_{+1}, d)$, the distributions of *next period's debt* given the employment status in this period:

$$\hat{\boldsymbol{\varphi}}^e_t = \boldsymbol{T}^{z,z_{+1};e\prime}_t \boldsymbol{\varphi}^e_t,$$
$$\hat{\boldsymbol{\varphi}}^{u,d}_t = \boldsymbol{T}^{z,z_{+1};u,d\prime}_t \boldsymbol{\varphi}^{u,d},$$

(c) Use Bayes' Rule to compute $\{\tilde{\boldsymbol{\varphi}}^e_{t+1}, \{\tilde{\boldsymbol{\varphi}}^{u,d}_{t+1}\}^{\bar{d}}_{d=0}\}$:

$$\tilde{\boldsymbol{\varphi}}^e_{t+1} = \hat{\boldsymbol{\varphi}}^e_t,$$
$$\tilde{\boldsymbol{\varphi}}^{u,d}_{t+1} = \begin{cases} \delta \hat{\boldsymbol{\varphi}}^e_t \text{ if } d = 0, \\ \hat{\boldsymbol{\varphi}}^{u,d-1}_t \text{ if } d \in \{1, \bar{d}-1\}, \\ \frac{\mathbf{1}'\boldsymbol{\varphi}^{u,\bar{d}-1}_t}{\mathbf{1}'\boldsymbol{\varphi}^{u,\bar{d}-1}_t + \mathbf{1}'\boldsymbol{\varphi}^{u,\bar{d}}_t} \hat{\boldsymbol{\varphi}}^{u,\bar{d}-1}_t + \frac{\mathbf{1}'\boldsymbol{\varphi}^{u,\bar{d}}_t}{\mathbf{1}'\boldsymbol{\varphi}^{u,\bar{d}-1}_t + \mathbf{1}'\boldsymbol{\varphi}^{u,\bar{d}}_t} \hat{\boldsymbol{\varphi}}^{u,\bar{d}}_t \text{ if } d = \bar{d}, \end{cases}$$

Note that the last equation reflects my definition of $\tilde{\boldsymbol{\varphi}}^{u,\bar{d}}_\tau$ and $\boldsymbol{\varphi}^{u,\bar{d}}_\tau$ as approximating

$$\tilde{\varphi}^u_\tau(z, d \geq \bar{d}) \equiv \sum_{d=\bar{d}}^{\infty} \tilde{\varphi}^u_\tau(z, d), \quad \varphi^u_\tau(z, d \geq \bar{d}) \equiv \sum_{d=\bar{d}}^{\infty} \varphi^u_\tau(z, d),$$

respectively, for all $\tau$.

# Appendix C

# Appendix to Chapter 2

## C.1 Proofs of results in the main text[1]

### Proposition 2.1: three properties of the natural allocation

*Proof.* Properties 1 and 2 are clear by examination of the equilibrium conditions defining the natural allocation in Lemma 2.1. Here I prove property 3 regarding constrained efficiency.

The constrained planner with a utilitarian objective faces

$$\begin{aligned} \max_{\substack{c_H, c_F, c_H^*, c_F^*, \\ u \in [0, 1-(1-\delta)n_0], u^* \in [0, 1-(1-\delta^*)n_0^*], \theta, \theta^*, n, n^*}} \quad & u(c,n) + u^*(c^*, n^*) \text{ s.t.} \\ (Evol): n &= (1-\delta)n_0 + p(\theta)u, \\ (Evol)^*: n^* &= (1-\delta^*)n_0^* + p^*(\theta^*)u^*, \\ (RC)_H: c_H + c_H^* &= a[n - ku\theta], \\ (RC)_F: c_F + c_F^* &= a^*[n^* - k^*u^*\theta^*], \end{aligned} \tag{C.1}$$

where I already account for the fact that the planner will choose identical production levels across varieties from a particular country, and identical consumption of those varieties from residents of a particular country, given the symmetry assumptions made on tastes and

[1] I provide proofs of results for which the steps are not fully obvious from the main text. Formal proofs of results not provided here are available on request.

technologies.

It is straightforward to use $FOC(u)$, $FOC(u^*)$, $FOC(\theta)$, and $FOC(\theta^*)$ to show that

$$u = 1 - (1-\delta)n_0, \quad u^* = 1 - (1-\delta^*)n_0^*, \tag{C.2}$$

since raising participation in the labor force will reduce the social cost of employing the same number of workers in each country.

It is also straightforward to use $FOC(c_H)$, $FOC(c_H^*)$, $FOC(c_F)$, and $FOC(c_F^*)$ to show that

$$c_H^* = \left(\frac{1-\gamma}{\gamma}\right) c_H, \quad c_F = \left(\frac{1-\gamma}{\gamma}\right) c_F^* \tag{C.3}$$

given the Cole and Obstfeld (1991) parameterization of preferences.

Finally, we can use $FOC(c_H)$, $FOC(c_F^*)$, $FOC(\theta)$, $FOC(\theta^*)$, $FOC(n)$, and $FOC(n^*)$ to obtain

$$a = \frac{\chi}{\gamma} c_H n^{\varphi} + \frac{1}{\eta} \frac{ka}{q(\theta)}, \tag{C.4}$$

$$a^* = \frac{\chi}{\gamma} c_F^* n^{*\varphi} + \frac{1}{\eta^*} \frac{k^* a^*}{q^*(\theta^*)}. \tag{C.5}$$

(C.2)-(C.5), coupled with the constraints of (C.1), define the constrained efficient allocation. Comparing these to the conditions defining the natural allocation in Lemma 2.1, it is clear that inducing the efficient levels of domestic consumption and production at arbitrary $a$ in Home requires

$$\tau^r = -\frac{1}{\varepsilon}, \quad \beta = 1 - \eta,$$

and inducing the efficient levels of domestic consumption and production at arbitrary $a^*$ in Foreign requires

$$\tau^{r*} = -\frac{1}{\varepsilon}, \quad \beta^* = 1 - \eta^*.$$

Then, imported consumption levels $\{c_F, c_H^*\}$ will be efficient according to (C.3) at the relative price $s = \frac{c_F^*}{c_H}$, where the right-hand side is evaluated at the constrained efficient allocation. □

**Lemma 2.2: steady-state employment in sclerotic vs. fluid labor markets**

*Proof.* Following Lemma 2.1 and Assumption 2.1, the steady-state $\{\bar{c}_H, \bar{n}, \bar{\theta}\}$ are characterized by

$$a = \frac{\chi}{\gamma}\bar{c}_H\bar{n}^{\varphi} + \frac{1}{1-\beta}\frac{ka}{q(\bar{\theta})},$$

$$\bar{n} = (1-\delta)\bar{n} + p(\bar{\theta})(1-(1-\delta)\bar{n}),$$

$$\bar{c}_H = \gamma a\left[\bar{n} - k(1-(1-\delta)\bar{n})\bar{\theta}\right].$$

It will prove useful later (in the proof of Proposition 2.3) to begin by working with

$$\kappa \equiv \frac{k}{q(\theta)},$$

which has an economic interpretation as the equilibrium hiring costs per hired worker. Combining the above steady-state conditions, $\bar{\kappa}$ solves

$$0 = v(\bar{\kappa}; k, \delta) \equiv 1 - \chi\left(1-\delta\bar{\kappa}\right)\bar{n}(\bar{\kappa}; k, \delta)^{\varphi+1} - \frac{1}{1-\beta}\bar{\kappa},$$

where

$$n = \bar{n}(\bar{\kappa}; k, \delta) = \frac{p(q^{-1}(k/\bar{\kappa}))}{\delta + (1-\delta)p(q^{-1}(k/\bar{\kappa}))}.$$

in steady-state. We can then show that

$$v_{\bar{\kappa}} = \frac{1}{\bar{\kappa}}\chi\left(1-\delta\bar{\kappa}\right)\bar{n}^{\varphi+1}\left[\frac{\delta\bar{\kappa}}{1-\delta\bar{\kappa}} - (\varphi+1)\frac{\bar{\kappa}}{\bar{n}}\frac{\partial\bar{n}}{\partial\bar{\kappa}} - \frac{\frac{1}{1-\beta}\bar{\kappa}}{1-\frac{1}{1-\beta}\bar{\kappa}}\right] < 0,$$

$$v_k = \frac{1}{k}\chi\left(1-\delta\bar{\kappa}\right)\bar{n}^{\varphi+1}\left[-(\varphi+1)\frac{k}{\bar{n}}\frac{\partial\bar{n}}{\partial k}\right] > 0,$$

$$v_{\delta} = \frac{1}{\delta}\chi\left(1-\delta\bar{\kappa}\right)\bar{n}^{\varphi+1}\left[\frac{\delta\bar{\kappa}}{1-\delta\bar{\kappa}} - (\varphi+1)\frac{\delta}{\bar{n}}\frac{\partial\bar{n}}{\partial\delta}\right] > 0,$$

where

$$\frac{\partial \bar{n}}{\partial \bar{\kappa}} = \frac{\bar{n}}{\bar{\kappa}} \left( \frac{\delta}{\delta + (1-\delta)p(\bar{\theta})} \right) \frac{\eta}{1-\eta} > 0,$$
$$\frac{\partial \bar{n}}{\partial k} = -\frac{\bar{n}}{k} \left( \frac{\delta}{\delta + (1-\delta)p(\bar{\theta})} \right) \frac{\eta}{1-\eta} < 0,$$
$$\frac{\partial \bar{n}}{\partial \delta} = -\frac{\bar{n}}{\delta} \left( \frac{\delta(1-p(\bar{\theta}))}{\delta + (1-\delta)p(\bar{\theta})} \right) < 0.$$

By the implicit function theorem, we then have

$$\frac{d\bar{\kappa}}{dk} = -\frac{v_k}{v_{\bar{\kappa}}} > 0,$$
$$\frac{d\bar{\kappa}}{d\delta} = -\frac{v_\delta}{v_{\bar{\kappa}}} > 0.$$

Now consider the comparative statics of $\bar{n}$ with respect to $k$ and $\delta$. First we have that

$$\begin{aligned}\frac{d\bar{n}}{dk} &= \frac{\partial \bar{n}}{\partial k} + \frac{\partial \bar{n}}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{dk}, \\ &= \frac{\chi\,(1-\delta\bar{\kappa})\,\bar{n}^{\varphi+1}}{v_{\bar{\kappa}}\bar{\kappa}} \left[ \frac{\delta\bar{\kappa}}{1-\delta\bar{\kappa}} - \frac{\frac{1}{1-\beta}\bar{\kappa}}{1-\frac{1}{1-\beta}\bar{\kappa}} \right] \frac{\partial \bar{n}}{\partial k}, \\ &< 0\end{aligned}$$

as claimed. Then we have that

$$\begin{aligned}\frac{d\bar{n}}{d\delta} &= \frac{\partial \bar{n}}{\partial \delta} + \frac{\partial \bar{n}}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{d\delta}, \\ &= \frac{\chi\,(1-\delta\bar{\kappa})\,\bar{n}^{\varphi+1}}{v_{\bar{\kappa}}\bar{\kappa}} \left[ \left(1 + \frac{1}{1-p(\bar{\theta})} \frac{\eta}{1-\eta}\right) \frac{\delta\bar{\kappa}}{1-\delta\bar{\kappa}} - \frac{\frac{1}{1-\beta}\bar{\kappa}}{1-\frac{1}{1-\beta}\bar{\kappa}} \right] \frac{\partial \bar{n}}{\partial \delta}.\end{aligned}$$

The term in brackets here is of ambiguous sign. However, when $\bar{n}$ is close to 1 and thus $p(\bar{\theta})$ is also close to 1, we have that

$$\frac{d\bar{n}}{d\delta} > 0$$

as claimed. □

**Lemma 2.4: second-order approximation to welfare**

*Proof.* Plugging in the implementability constraints

$$c_F = \frac{1-\gamma}{\gamma} s c_H, \quad c_H^* = \frac{1-\gamma}{\gamma}\frac{1}{s} c_F^*$$

into utilitarian social welfare defined in (2.46), the terms of trade drop out and

$$U = \left(\log c_H - \chi \frac{n^{1+\varphi}}{1+\varphi}\right) + \left(\log c_F^* - \chi^* \frac{n^{*1+\varphi^*}}{1+\varphi^*}\right) + const.$$

Further plugging in the implementability constraints

$$c_H = \gamma \frac{1}{D_H} x, \quad c_F^* = \gamma \frac{1}{D_F^*} x^*,$$
$$n = f^{-1}(x/a), \quad n^* = f^{*-1}(x^*/a^*)$$

implied by the definitions of output $\{x, x^*\}$ and technologies $\{f, f^*\}$ in section 2.4.2, we can write social welfare as

$$U(D_H, x, D_F^*, x^*, a, a^*) = \left(-\log D_H + \log \gamma x - \chi \frac{(f^{-1}(x/a))^{1+\varphi}}{1+\varphi}\right) + \left(-\log D_F^* + \log \gamma x^* - \chi^* \frac{(f^{*-1}(x^*/a^*))^{1+\varphi^*}}{1+\varphi^*}\right) + const.$$

In view of the separability between terms given the assumed preferences and technologies, up to second order we then have

$$U = \bar{U} + \bar{U}_{D_H}\bar{D}_H\hat{D}_H + \frac{1}{2}\bar{U}_{D_H D_H}\bar{D}_H^2\hat{D}_H^2 + \bar{U}_x\bar{x}\hat{x} + \frac{1}{2}\bar{U}_{xx}\bar{x}^2\hat{x}^2 + \bar{U}_{xa}\bar{x}\bar{a}\hat{x}\hat{a}$$
$$\bar{U}_{D_F^*}\bar{D}_F^*\hat{D}_F^* + \frac{1}{2}\bar{U}_{D_F^* D_F^*}\bar{D}_F^{*2}\hat{D}_F^{*2} + \bar{U}_{x^*}\bar{x}^*\hat{x}^* + \frac{1}{2}\bar{U}_{x^*x^*}\bar{x}^{*2}\hat{x}^{*2} + \bar{U}_{x^*a^*}\bar{x}^*\bar{a}^*\hat{x}^*\hat{a}^* + tips, \quad \text{(C.6)}$$

where a bar over the partial derivative means that it is evaluated at the zero-inflation steady-state, and *tips* denotes terms independent of policy.

First consider the price dispersion terms. Plugging in the price index $P_H$ into the dispersion index $D_H$ and expanding $D_H$ up to second order in $\mathcal{P}_{\mathcal{H}}$ around the zero-inflation steady-state yields

$$\hat{D}_H = \frac{1}{2}\iota(1-\iota)\varepsilon\hat{\mathcal{P}}_H^2,$$

so that $\hat{D}_H^2$ will be zero up to second order terms. Moreover, using (C.19) to plug in for domestic (producer-price) inflation, we obtain

$$\hat{D}_H = \frac{1}{2}\left(\frac{1-\iota}{\iota}\right)\varepsilon\pi_H^2. \tag{C.7}$$

Finally, it is straightforward to show that

$$\bar{U}_{D_H}\bar{D}_H = -1. \tag{C.8}$$

Analogous steps in Foreign then imply

$$\hat{D}_F^* = \frac{1}{2}\left(\frac{1-\iota^*}{\iota^*}\right)\varepsilon^*\pi_F^{*2}, \tag{C.9}$$

$$\bar{U}_{D_F^*}\bar{D}_F^* = -1. \tag{C.10}$$

Now consider the terms involving output. It is straightforward to differentiate the objective and obtain

$$U_x = \frac{1}{x} - \frac{1}{af'(f^{-1}(x/a))}\chi(f^{-1}(x/a))^{\varphi},$$
$$U_{x^*} = \frac{1}{x^*} - \frac{1}{a^*f^{*\prime}(f^{*-1}(x^*/a^*))}\chi(f^{*-1}(x^*/a^*))^{\varphi^*},$$

using the inverse function theorem. Since

$$f'(n) = 1 - \frac{1}{\eta}\frac{k}{q(\theta)},$$
$$f^{*\prime}(n) = 1 - \frac{1}{\eta^*}\frac{k^*}{q^*(\theta^*)},$$

and the steady-state is constrained efficient per Assumption 2.2, the efficiency conditions (C.4) and (C.5) mean that

$$\bar{U}_x = 0, \tag{C.11}$$

$$\bar{U}_{x^*} = 0, \tag{C.12}$$

so the first-order terms involving output in (C.6) drop out (justifying linear approximations of the implementability constraints as sufficient to characterize the Ramsey optimal allocation up to first order, following Woodford (2003)). Again differentiating the objective, making use of the inverse function theorem, and evaluating it in steady-state yields

$$\bar{U}_{xx}\bar{x}^2 = -\left[1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f}\right], \tag{C.13}$$

$$\bar{U}_{x^*x^*}\bar{x}^{*2} = -\left[1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}}\right], \tag{C.14}$$

given the definitions of the elasticities $\epsilon$ in the statement of the lemma. Finally, since the natural allocation is constrained efficient per Assumption 2.2, standard arguments mean that we can complete the square and collect second-order terms so that

$$\frac{1}{2}\bar{U}_{xx}\bar{x}^2\hat{x}^2 + \bar{U}_{xa}\bar{x}\bar{a}\hat{x}\hat{a} = \frac{1}{2}\bar{U}_{xx}\bar{x}^2\tilde{x}^2 + tips, \tag{C.15}$$

$$\frac{1}{2}\bar{U}_{x^*x^*}\bar{x}^{*2}\hat{x}^{*2} + \bar{U}_{x^*a^*}\bar{x}^*\bar{a}^*\hat{x}^*\hat{a}^* = \frac{1}{2}U_{x^*x^*}\bar{x}^{*2}\tilde{x}^{*2} + tips. \tag{C.16}$$

Plugging in (C.7)-(C.16) into (C.6), we obtain the desired approximation (2.47) given the definitions of $\lambda_\pi$, $\lambda_\pi^*$, $\lambda_x$, and $\lambda_x^*$. □

### Lemma 2.5: first-order approximation to implementability

*Proof.* First note that the implementability constraints (2.40) can be combined to give

$$\begin{aligned}
\frac{\mathcal{P}_H}{P_H} &= \frac{\frac{\chi}{\gamma}c_H n^\varphi}{a\left(1 - \frac{1}{1-\beta}\frac{k}{q(\theta)}\right)}, \\
&= \frac{1}{D_H}\frac{\chi a f(n) n^\varphi}{a f'(n)}, \\
&= \frac{1}{D_H}\frac{\chi x (f^{-1}(x/a))^\varphi}{a f'(f^{-1}(x/a))},
\end{aligned}$$

where the second and third lines use the definition of technology $f(n)$, output $x$, and the constrained efficiency of the natural allocation per Assumption 2.2 (implying that the Hosios (1990) condition $1 - \beta = \eta$ is satisfied). Log-linearizing this condition around the zero-inflation steady-state, we obtain

$$(\hat{\mathcal{P}}_H - \hat{P}_H) = -\hat{D}_H + \left[1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f}\right](\hat{x} - \hat{a}), \tag{C.17}$$

for the elasticities $\epsilon$ defined in the statement of Lemma 2.4, while in the natural allocation the same steps imply

$$\hat{x}^n = \hat{a}. \tag{C.18}$$

Log-linearizing (2.43) around the zero-inflation steady-state, we obtain

$$\pi_H = \hat{P}_H = \iota \hat{\mathcal{P}}_H, \tag{C.19}$$

$$\hat{D}_H = 0. \tag{C.20}$$

Combining (C.17)-(C.20), we obtain the Phillips Curve at Home

$$\pi_H = \frac{\iota}{1-\iota}\left[1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f}\right]\tilde{x}.$$

With exactly analogous steps, we can combine and log-linearize the implementability constraints (2.41) and (2.44), making use of Assumption 2.2, to obtain Foreign's Phillips Curve

$$\pi_F^* = \frac{\iota^*}{1-\iota^*}\left[1 + \frac{\varphi^* - \epsilon_{n^*}^{f^{*\prime}}}{\epsilon_{n^*}^{f^*}}\right]\tilde{x}^*.$$

Finally, log-linearizing the implementability constraint

$$s = \frac{c_F^*}{c_H}$$

both in the sticky price equilibrium and in the natural allocation, in gap notation we have

$$\hat{s}^n = \hat{s} + \tilde{c}_H - \tilde{c}_F^*. \tag{C.21}$$

Log-linearizing implementability constraint (2.45) around the zero-inflation steady-state yields

$$\hat{s} = \pi_H - \pi_F^*, \tag{C.22}$$

while log-linearizing the implementability constraints

$$c_H = \frac{\gamma}{D_H}x, \quad c_F^* = \frac{\gamma}{D_F^*}x^*.$$

around the zero-inflation steady-state and using (C.20), the analog for Foreign, and gap notation yields

$$\tilde{c}_H = \tilde{x}, \quad \tilde{c}_F^* = \tilde{x}^*, \tag{C.23}$$

Combining (C.21)-(C.23) yields

$$\hat{s}^n = (\pi_H + \tilde{x}) - (\pi_F^* + \tilde{x}^*)\,,$$

the constraint posed by membership in a monetary union. □

### Proposition 2.2: institutional irrelevance

*Proof.* It is apparent from the implementability constraints in Lemma 2.5 that

$$\pi_H = 0, \quad \tilde{x} = 0, \quad \pi_F^* = 0, \quad \tilde{x}^* = 0$$

can be achieved if and only if $\hat{s}^n = 0$. Since the natural allocation is constrained efficient by Assumption 2.2, it follows that the constrained efficient allocation can be achieved if and only if $\hat{s}^n = 0$. Finally, note that

$$\begin{aligned} \hat{s}^n &= \hat{c}_F^{*n} - \hat{c}_H^n, \\ &= \hat{x}^{*n} - \hat{x}^n, \\ &= \hat{a}^* - \hat{a}, \end{aligned}$$

where the third line uses (C.18) and the analog for Foreign. □

### Lemma 2.8: impact of $\{\phi, \phi^*\}$ on the optimal policy rule

*Proof.* It is immediate that

$$\frac{d\xi}{d\left[\frac{\lambda_\pi(\mu)^2+\lambda_x}{\mu(\mu+1)}\right]} > 0, \quad \frac{d\xi}{d\left[\frac{\lambda_\pi^*(\mu^*)^2+\lambda_x^*}{\mu^*(\mu^*+1)}\right]} < 0.$$

Using the definitions of $\{\lambda_\pi, \lambda_x, \mu\}$ in Lemmas 2.4 and 2.5,

$$\frac{\lambda_\pi(\mu)^2 + \lambda_x}{\mu(\mu+1)} = \left(\frac{1-\iota}{\iota}\right) \frac{\epsilon\left(\frac{\iota}{1-\iota}\right)\phi^2 + \phi}{\left(\frac{\iota}{1-\iota}\right)\phi^2 + \phi}.$$

It is straightforward to then show

$$\frac{d\left[\frac{\lambda_\pi(\mu)^2+\lambda_x}{\mu(\mu+1)}\right]}{d\phi} \propto \epsilon - 1 > 0.$$

Analogous steps imply

$$\frac{d\left[\frac{\lambda_\pi^*(\mu^*)^2+\lambda_x^*}{\mu^*(\mu^*+1)}\right]}{d\phi^*} \propto \epsilon - 1 > 0.$$

The result immediately follows. □

## Lemma 2.9: $\{\phi, \phi^*\}$ in closed form

*Proof.* Given the effective units of labor engaged in production

$$f(n) \equiv \left[n - k(1-(1-\delta)n_0)p^{-1}\left(\frac{n-(1-\delta)n_0}{1-(1-\delta)n_0}\right)\right],$$

it is straightforward to compute that in steady-state

$$\epsilon_n^f = \frac{1 - \frac{1}{\eta}\frac{k}{q(\bar{\theta})}}{1 - \delta\frac{k}{q(\bar{\theta})}},$$

$$\epsilon_n^{f'} = -\frac{\frac{1}{\eta}\frac{k}{q(\bar{\theta})}}{1 - \frac{1}{\eta}\frac{k}{q(\bar{\theta})}}\left(\frac{1-\eta}{\delta\eta}\right),$$

and analogously for $f^*(n^*)$, $\epsilon_{n^*}^{f^*}$, and $\epsilon_{n^*}^{f^{*\prime}}$. □

## Proposition 2.3: relative accommodation

*Proof.* First note that the only endogenous object in

$$\phi \equiv 1 + \frac{\varphi - \epsilon_n^{f'}}{\epsilon_n^f} = 1 + \frac{\varphi + \frac{\frac{1}{\eta}\frac{k}{q(\bar{\theta})}}{1-\frac{1}{\eta}\frac{k}{q(\bar{\theta})}}\left(\frac{1-\eta}{\delta\eta}\right)}{\frac{1-\frac{1}{\eta}\frac{k}{q(\bar{\theta})}}{1-\delta\frac{k}{q(\bar{\theta})}}}$$

is the steady-state hiring costs per hired worker

$$\bar{\kappa} = \frac{k}{q(\bar{\theta})},$$

which was studied extensively in the proof of Lemma 2.2.

So first considering the desired comparative static with respect to hiring costs $k$, we have that

$$\frac{d\phi}{dk} = \frac{d\phi}{d\bar{\kappa}}\frac{d\bar{\kappa}}{dk}.$$

Since

$$\begin{aligned}
\frac{d\phi}{d\bar{\kappa}} &\propto \epsilon_n^f \left(\frac{d-\epsilon_n^{f'}}{d\bar{\kappa}}\right) - (\varphi - \epsilon_n^{f'}) \left(\frac{d\epsilon_n^f}{d\bar{\kappa}}\right), \\
&= \epsilon_n^f \left(\frac{1}{(1-\frac{1}{\eta}\bar{\kappa})^2}\left(\frac{1-\eta}{\delta\eta^2}\right)\right) + (\varphi - \epsilon_n^{f'}) \left(\frac{1}{(1-\delta\bar{\kappa})^2}\left(\frac{1}{\eta} - \delta\right)\right), \\
&> 0,
\end{aligned} \tag{C.24}$$

and we know from the proof of Lemma 2.2 that $\frac{d\bar{\kappa}}{dk} > 0$, we can conclude

$$\frac{d\phi}{dk} > 0$$

as claimed.

Now turn to the comparative static with respect to magnitude of flows $\delta$, where

$$\frac{d\phi}{d\delta} = \frac{\partial\phi}{\partial\delta} + \frac{\partial\phi}{\partial\bar{\kappa}}\frac{d\bar{\kappa}}{d\delta}.$$

The challenge here is that while

$$\begin{aligned}
\frac{\partial\phi}{\partial\delta} &\propto \epsilon_n^f \left(\frac{\partial-\epsilon_n^{f'}}{\partial\delta}\right) - (\varphi - \epsilon_n^{f'}) \left(\frac{\partial\epsilon_n^f}{\partial\delta}\right), \\
&= \epsilon_n^f \left(-\frac{\frac{1}{\eta}\bar{\kappa}}{1-\frac{1}{\eta}\bar{\kappa}}\left(\frac{1-\eta}{\delta^2}\right)\right) - (\varphi - \epsilon_n^{f'}) \left(\frac{1-\frac{1}{\eta}\bar{\kappa}}{(1-\delta\bar{\kappa})^2}\bar{\kappa}\right), \\
&< 0,
\end{aligned}$$

we know from (C.24) and the proof of Lemma 2.2 that

$$\frac{\partial \phi}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{d\delta} > 0.$$

In economic terms, while a lower magnitude of flows directly raises the welfare cost of output fluctuations ($\frac{\partial \phi}{\partial \delta} < 0$), in equilibrium it also reduces hiring costs per hire $\bar{\kappa}$ in the present framework ($\frac{d\bar{\kappa}}{d\delta} > 0$), which has an offsetting effect on the welfare cost of output fluctuations (since $\frac{\partial \phi}{\partial \bar{\kappa}} > 0$). The fact that lower flows reduce hiring costs per hire contradicts the standard result from classic DMP models in which, by raising the continuation value of a match, a lower separation rate incentivizes vacancy posting and thus raise hiring costs per hire. The reason is twofold: the present setting is a one-period model with no continuation value of a match, and (more importantly) unlike classic DMP models I assume risk averse agents with a convex disutility of labor. Lower flows tend to raise the marginal rate of substitution between labor and consumption, and thus *dis*-incentivize vacancy posting in the present model.

In the reasonable benchmark case where $k$ (and thus $\bar{\kappa}$ — see the note below) is close to zero and steady-state unemployment $1 - \bar{n}$ is also close to zero, it is straightforward to show that

$$\frac{\partial \phi}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{d\delta}$$

is an order of magnitude smaller than

$$\frac{\partial \phi}{\partial \delta}$$

and thus

$$\frac{d\phi}{d\delta} < 0$$

as claimed.

Note that the above argument implicitly uses the fact that as $k \to 0$, $\bar{\kappa} \to 0$. This can be seen as follows: as $k \to 0$, the steady-state equilibrium conditions defining $\{\bar{c}_H, \bar{n}, \bar{\theta}\}$

$$a = \frac{\chi}{\gamma}\bar{c}_H \bar{n}^{\varphi} + \frac{1}{\eta}\frac{ka}{q(\bar{\theta})},$$

$$\bar{n} = (1-\delta)\bar{n} + p(\bar{\theta})(1-(1-\delta)\bar{n}),$$

$$\bar{c}_H = \gamma a \left[\bar{n} - k(1-(1-\delta)\bar{n})\bar{\theta}\right],$$

make evident that (there exists a path in which) the equilibrium $\bar{\theta} \to \bar{\bar{\theta}}$, where

$$\bar{\bar{\theta}} := \frac{p(\bar{\bar{\theta}})}{\delta + (1-\delta)p(\bar{\bar{\theta}})} = \bar{\bar{n}},$$

$$\bar{\bar{n}} := \chi \bar{\bar{n}}^{\varphi+1} = 1.$$

That is, the steady-state equilibrium converges to the frictionless benchmark. This means that

$$\bar{\kappa} \equiv \frac{k}{q(\bar{\theta})} \to 0$$

as used above.

Analogous arguments imply that for Foreign,

$$\frac{d\phi^*}{dk^*} > 0, \quad \frac{d\phi^*}{d\delta^*} < 0,$$

where the latter is true when $k^*$ (and thus $\bar{\kappa}^*$) is close to zero and steady-state unemployment $1 - \bar{n}^*$ is also close to zero.

The result on relative accommodation then is immediately implied by Lemmas 2.8 and 2.9. □

## C.2 Derivation of Nash bargained wages

Following Shimer (2010)'s approach to the Nash bargain, at Home let $\tilde{v}_n(\hat{W})$ denote the marginal value to the representative household of employing an additional worker at wage $\hat{W}$, let $\tilde{\Pi}_n(\hat{W})$ denote the marginal profit for the representative producer of employing an additional worker at wage $\hat{W}$, and let $\tilde{v}^*_{n^*}(\hat{W}^*)$ and $\tilde{\Pi}^*_{n^*}(\hat{W}^*)$ be analogs in Foreign. Then

Nash bargained wages solve

$$W = \arg\max_{\hat{W}} \left[\tilde{v}_n(\hat{W})\right]^{\beta} \left[\tilde{\Pi}_n(\hat{W})\right]^{1-\beta}, \tag{C.25}$$

$$W^* = \arg\max_{\hat{W}^*} \left[\tilde{v}^*_{n^*}(\hat{W}^*)\right]^{\beta^*} \left[\tilde{\Pi}^*_{n^*}(\hat{W}^*)\right]^{1-\beta^*}. \tag{C.26}$$

To compute $\tilde{v}_n(\hat{W})$ at Home, let $\hat{v}(n, W; \epsilon, \hat{W})$ be the value to the representative household of $n$ workers employed at wage $W$ and $\epsilon$ workers employed at wage $\hat{W}$. Building on (2.6),

$$\hat{v}(n, W; \epsilon, \hat{W}) = \max_{\{c_H(j)\}_j, \{c_F(j^*)\}_{j^*}} \log c - \chi \frac{(n+\epsilon)^{1+\varphi}}{1+\varphi} \text{ s.t.}$$
$$\int_0^1 P_H(j) c_H(j) dj + \int_0^1 P_F(j^*) c_F(j^*) dj^* \leq Wn + \hat{W}\epsilon + \Pi + \int_0^1 \Pi^r(j) dj - T.$$

Then we have

$$\tilde{v}_n(\hat{W}) \equiv \frac{\partial}{\partial \epsilon} \hat{v}(n, W; \epsilon, \hat{W})|_{\epsilon=0}. \tag{C.27}$$

To compute $\tilde{\Pi}_n(\hat{W})$ at Home, let $\hat{\Pi}(n, W; \epsilon, \hat{W})$ be the profit for the representative producer of $n$ workers employed at $W$ and $\epsilon$ workers employed at $\hat{W}$ after vacancy posting costs have been sunk. Building on (2.8),

$$\hat{\Pi}(n, W; \epsilon, \hat{W}) = (P^I a - W)n + (P^I a - \hat{W})\epsilon$$

Then we have

$$\tilde{\Pi}_n(\hat{W}) \equiv \frac{\partial}{\partial \epsilon} \hat{\Pi}(n, W; \epsilon, \hat{W})|_{\epsilon=0}. \tag{C.28}$$

In Foreign we similarly have

$$\tilde{v}^*_{n^*}(\hat{W}^*) \equiv \frac{\partial}{\partial \epsilon^*} \hat{v}^*(n^*, W^*; \epsilon^*, \hat{W}^*)|_{\epsilon^*=0}, \tag{C.29}$$

$$\tilde{\Pi}^*_{n^*}(\hat{W}^*) \equiv \frac{\partial}{\partial \epsilon^*} \hat{\Pi}^*(n^*, W^*; \epsilon^*, \hat{W}^*)|_{\epsilon^*=0} \tag{C.30}$$

for $\hat{v}^*(n^*, W^*; \epsilon^*, \hat{W}^*)$ and $\hat{\Pi}^*(n^*, W^*; \epsilon^*, \hat{W}^*)$ defined analogously to those objects in Home.

Now, evaluating the right-hand side of (C.27) and (C.29) using the Envelope Theorem, the marginal value to the representative household of an additional worker employed at $\hat{W}$

or $\hat{W}^*$ is

$$\tilde{v}_n(\hat{W}) = \frac{\hat{W}}{P}u_c - u_n,$$
$$\tilde{v}^*_{n^*}(\hat{W}^*) = \frac{\hat{W}^*}{P^*}u^*_c - u^*_n,$$

Evaluating the right-hand side of (C.28) and (C.30), the marginal value to the representative producer of an additional worker employed at $\hat{W}$ or $\hat{W}^*$ is

$$\tilde{\Pi}_n(\hat{W}) = P^I a - \hat{W},$$
$$\tilde{\Pi}^*_{n^*}(\hat{W}^*) = P^{I*}a^* - \hat{W}^*,$$

It is then straightforward to establish that the solutions of (C.25) and (C.26) yield

$$W - P\frac{u_n}{u_c} = \frac{\beta}{1-\beta}(P^I a - W),$$
$$W^* - P^*\frac{u^*_n}{u^*_c} = \frac{\beta^*}{1-\beta^*}(P^{I*}a^* - W^*).$$

as described in section 2.2.3 of the main text.

# Appendix D

# Appendix to Chapter 3

## D.1 Proofs of results in the main text[1]

### Proposition 3.3: equilibrium in non-financial corporate sector

*Proof.* Demonstrating that the conjectured equilibrium is in fact an equilibrium requires showing that (i) the parametric conditions in Assumption 3.4 ensure that $\{p_2, p_0\}$ solving (3.1) and (3.2) exist, and (ii) there exist off-path beliefs under which all entrepreneurs are indeed acting optimally.

I start with (i). By the implicit function theorem, let $p_2^{(3.1)}(p_0)$ be the upward-sloping schedule defined by (3.1) and let $p_2^{(3.2)}(p_0)$ be the downward-sloping schedule defined by (3.2). Then we have

$$p_{npv} = p_2^{(3.2)}(p_{npv}) < p_2^{(3.1)}(p_{npv}) \in (p_{npv}, p_{max}),$$

where the last relation follows from

$$0 < d$$

[1] I provide proofs of results for which the intuition is not fully obvious from the main text. Formal proofs of results not provided here are available on request.

by Assumption 3.4. And we have

$$\infty = p_2^{(3.2)}(\underline{p}) > p_{max} > p_2^{(3.1)}(\underline{p})$$

for $\underline{p} := v(\underline{p}) = (1+r)c^e$, where the last relation follows from the fact that

$$d < \frac{(1+r)\left(\frac{v(p_{max})}{v\left(E[p|p\in[p_{npv},p^{max}]]\right)} - 1\right)}{\frac{1}{2}(1-p_{max})} < \frac{(1+r)\left(\frac{v(p_{max})}{v\left(E[p|p\in[\underline{p},p^{max}]]\right)} - 1\right)}{\frac{1}{2}(1-p_{max})}$$

by Assumption 3.4. Since we know $p_{npv} \in (p_{min}, p_{max})$ and $\underline{p} \in (p_{min}, p_{npv})$ following

$$\frac{v(p_{min})}{1+r} < c^e < 1 < \frac{v(p_{max})}{1+r}$$

the Intermediate Value Theorem guarantees that there exists a (unique) $\{p_0 \in (\underline{p}, p_{npv}), p_2 \in (p_0, p_{max})\}$ solving

$$p_2^{(3.2)}(p_0) = p_2^{(3.1)}(p_0).$$

Now I turn to (ii). Consider the following off-path beliefs:

- If $z^E + z^D = 1 - c^e$,

$$\mu(p|z^E, z^D) = f(p|p \in [p_0, p_2]).$$

- If $z^E + z^D > 1 - c^e$,

$$\mu(p|z^E, z^D) = \delta\{p = \max\{p_{min}, p^*(z^D)\}\}$$

  where

$$p^*(z^D) := v(p^*) = (1 + r + \frac{1}{2}(1-p^*)d)z^D.$$

Note that here $\delta$ is the Dirac delta function. Under these beliefs, for entrepreneurs who seek $1 - c^e$ in external financing, households' posterior is that they are just like equity issuers in the conjectured equilibrium. But for entrepreneurs who seek more than $1 - c^e$ in external financing, households' posterior is quite pessimistic, consistent with the intuition that "high" types would seek to minimize external finance since frictions render it costly. In particular, their posterior in this case assigns entrepreneurs to be the lowest type who could in fact raise $z^D$ in debt. This latter condition ensures that beliefs are "reasonable" in the present setting where households should be able to backward induce from entrepreneurs' ability to raise informed capital (debt) their minimum type. Assumption 3.4 implies that

$$\frac{1+r+\frac{1}{2}(1-p_{min})d}{2(1+r)+\frac{1}{2}(1-p_{min})d} < c^e \Rightarrow \frac{1+r+\frac{1}{2}(1-\underline{p})d}{2(1+r)+\frac{1}{2}(1-\underline{p})d} < c^e,$$
$$\Rightarrow v(p_0) > (1+r+\frac{1}{2}(1-\underline{p})d)(1-c^e),$$

ensuring that even entrepreneur $p_0$ could raise up to $z^D = 1 - c^e$ in informed capital.

It is straightforward to verify that under these beliefs, all entrepreneurs are acting optimally. □

### Proposition 3.4: countercyclical leverage in non-financial corporate sector

*Proof.* Let $\tilde{F}$ be the CDF of the transformation $\tilde{p} \equiv \frac{p-p_{min}}{p_{max}-p_{min}}$. Then following Definition 3.1, $\tilde{F}(\tilde{p}) = \tilde{p}^{\alpha}$ (the CDF of a $Beta(\alpha, 1)$ random variable), and we are interested in the effect of local changes in $\alpha$ around $\alpha = 1$ (the $U[0,1]$ case).

Since assets and leverage are

$$a^e = 1 - F(p_0) = 1 - \tilde{p}_0^{\alpha},$$
$$l^e = \frac{1-F(p_2)}{1-F(p_0)}(1-c^e) = \frac{1-\tilde{p}_2^{\alpha}}{1-\tilde{p}_0^{\alpha}}(1-c^e),$$

for $\tilde{p}_0 \equiv \frac{p_0-p_{min}}{p_{max}-p_{min}}$ and $\tilde{p}_2 \equiv \frac{p_2-p_{min}}{p_{max}-p_{min}}$, it is straightforward to show

$$\frac{da^e}{d\alpha} \propto -\frac{d\tilde{p}_0}{d\alpha}\frac{\alpha}{\tilde{p}_0} - \ln \tilde{p}_0,$$
$$\frac{dl^e}{d\alpha} \propto (1-\tilde{p}_2^{\alpha})\tilde{p}_0^{\alpha}\left(\frac{d\tilde{p}_0}{d\alpha}\frac{\alpha}{\tilde{p}_0} + \ln \tilde{p}_0\right) - (1-\tilde{p}_0^{\alpha})\tilde{p}_2^{\alpha}\left(\frac{d\tilde{p}_2}{d\alpha}\frac{\alpha}{\tilde{p}_2} + \ln \tilde{p}_2\right).$$

Differentiating the system (3.1) and (3.2), we can show:

- $\frac{dp_2}{d\alpha}\frac{\alpha}{p_2} > 0$, which remains bounded above zero even for $d$ (and $c^e$) on the high end of their respective ranges in Assumption 3.4,
- $\frac{dp_0}{d\alpha}\frac{\alpha}{p_0} < 0$.

The second point immediately implies that $\frac{da^e}{d\alpha} > 0$. And when $d$ is on the high end of its range in Assumption 3.4, so $\tilde{p}_2$ is close to its upper bound 1 and thus initial leverage is low, we have that $\frac{dl^e}{d\alpha}$ takes the sign of $-\frac{d\tilde{p}_2}{d\alpha} \propto -\frac{dp_2}{d\alpha} < 0$. □

### Proposition 3.5: equilibrium in banking

*Proof.* Demonstrating that the conjectured equilibrium is in fact an equilibrium requires showing that (i) the parametric conditions in Assumption 3.5 ensure that $\{p_1, p_2, 1+r^b, l^b\}$ solving (3.3), (3.4), (3.5), and (3.6) exist, and (ii) there exist off-path beliefs under which all bankers are indeed acting optimally.

I start with (i). First note that we can combine (3.5) and (3.6) to obtain (3.7), and then can plug the latter into (3.3) and (3.4), to obtain the system

$$\frac{1}{2}(1-p_2)d = (1+r)\left[(1+m^s)\left(\frac{1+\frac{\frac{1}{2}d}{1+r+\frac{1}{2}d}}{1+\frac{\frac{1}{2}d}{1+r+\frac{1}{2}d}\gamma_d^s}\right)-1\right], \tag{D.1}$$

$$v(p_1) = (1+r)\left(1+(1-c^e)\left[(1+m^s)\left(\frac{1+\frac{\frac{1}{2}d}{1+r+\frac{1}{2}d}}{1+\frac{\frac{1}{2}d}{1+r+\frac{1}{2}d}\gamma_d^s}\right)-1\right]\right), \tag{D.2}$$

$$\gamma_d^s = E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_1, p_2]\right]. \tag{D.3}$$

Hence, if we can prove that $\{p_1, p_2, \gamma_d^s\}$ solving this smaller system exist, we are done.

I will focus on the parametric case of small $m^s$ and $d$, following Assumption 3.5. First note that given $\frac{\partial}{\partial p_2}\gamma_d^s < 1$ by Assumption 3.5, we can use the Implicit Function Theorem to argue that (D.1) implies the well-defined function $p_2(p_1)$ over $p_1 \in [p_{npv}, p_{max})$ which is upward-sloping for $m^s$ and $d$ small. In fact, as $m^s, d \to 0$, the fact that $\chi \equiv \frac{m^s}{d}$ satisfies

$$\frac{1}{2}\frac{1}{1+r}\left(E\left[\frac{p}{\frac{1}{2}+\frac{1}{2}p}|p \in [p_{npv}, p_{max}]\right]-p_{max}\right) < \chi < \frac{1}{2}\frac{1}{1+r}\left(\left(\frac{p_{npv}}{\frac{1}{2}+\frac{1}{2}p_{npv}}\right)-p_{npv}\right)$$

(by Assumption 3.5) implies that $p_2(p_1)$ converges to an upward-sloping function $\overline{p}_2(p_1)$ where

$$\overline{p}_2(p_{npv}) \in (p_{npv}, p_{max}).$$

We can then use the Implicit Function Theorem to argue that (D.2) implies the well-defined function $p_1(p_2)$ over $p_2 \in (p_{npv}, p_{max}]$ which is downward-sloping. And as $m^s, d \to 0$, $p_1(p_2)$ converges to the constant

$$\overline{p}_1 = p_{npv}.$$

It follows that $\{p_1 = p_{npv}, p_2 = \overline{p}_2(p_{npv})\}$ solve the system of limiting equations as $m^s$ and $d$ converge to zero and $\chi$ remains in the range above. Since the conditions (D.1)-(D.3) are all well-behaved, it follows that for small but positive $m^s$ and $d$, the upward-sloping $p_2(p_1)$ and downward-sloping $p_1(p_2)$ intersect near the point $\{p_1 = p_{npv}, p_2 = \overline{p}_2\}$, and an equilibrium exists.

Now I turn to (ii). With beliefs

$$\mu\left(m^s | l^b < \bar{z}^S(r^b; m^s) + (1 - \bar{z}^S(r^b; m^s))\frac{1}{1 + \frac{\frac{1}{2}d}{1+r}}\right) = 0,$$
$$\mu\left(m^s | l^b \geq \bar{z}^S(r^b; m^s) + (1 - \bar{z}^S(r^b; m^s))\frac{1}{1 + \frac{\frac{1}{2}d}{1+r}}\right) = 1,$$

where $\mu$ satisfies as well the conditions of Lemma 3.3. Under these beliefs, a household perceives a low-leverage issuer to be unskilled with probability one, and a sufficiently high-leverage issuer to be skilled with probability one. These beliefs reflect the intuition that only the (more profitable) skilled bankers can afford the costly signal of issuing risky debt.

It is straightforward to verify that under these beliefs, all bankers are acting optimally. □

### Proposition 3.6: procyclical leverage in banking

*Proof.* First differentiate the system (D.1)-(D.3), which given the conditions of Assumption 3.5 can be used to show that for $m^s, d$ small:

- $\frac{d\gamma_d^s}{d\alpha} > 0$,
- $\frac{dp_2}{d\alpha} > 0$, is bounded above zero even as $m^s, d \to 0$, and remains bounded above zero even if $\chi \equiv \frac{m^s}{d}$ is arbitrarily close to the bottom of the range in Assumption 3.5,
- $\frac{dp_1}{d\alpha} < 0$ but converges to zero as $m^s, d \to 0$.

Then with respect to bank leverage,

$$\begin{aligned}\frac{dl^b}{d\alpha} &\propto \frac{d}{d\alpha}\bar{z}^S(r^b; m^s),\\ &\propto \frac{d}{d\alpha}\gamma_d^s,\\ &> 0,\end{aligned}$$

where the first line uses (3.5) and the second line uses (3.7) (which combines (3.5) and (3.6)).

And with respect to bank assets

$$\begin{aligned}a^b &= (F(p_2) - F(p_1))(1 - c^e)(1 + m^s),\\ &= (\tilde{p}_2^\alpha - \tilde{p}_0^\alpha)(1 - c^e)(1 + m^s),\end{aligned}$$

where the second line uses the definition of the transformations used extensively in the proof of Proposition 3.4, we have that

$$\frac{da^b}{d\alpha} \propto \tilde{p}_2^\alpha \left(\frac{d\tilde{p}_2}{d\alpha}\frac{\alpha}{\tilde{p}_2} + \ln \tilde{p}_2\right) - \tilde{p}_1^\alpha \left(\frac{d\tilde{p}_1}{d\alpha}\frac{\alpha}{\tilde{p}_1} + \ln \tilde{p}_1\right).$$

Hence, $\frac{da^b}{d\alpha} > 0$ when $\chi$ is near the bottom of the range in Assumption 3.5, and thus $\tilde{p}_2$ is near the upper bound 1, and thus the initial share of bank loans in credit provision is sufficiently high. □

### Proposition 3.10: constrained efficient equilibria

*Proof.* Relative to the problem facing the constrained planner described in the text, consider a *relaxed* problem in which the planner can directly choose the consumption, investment, and financing decisions of entrepreneurs, banks, and households subject only to the economy's aggregate *resource* constraints and the *technological* constraints posed by the investment technology of each entrepreneur and the deadweight costs of bank loan and bond financing.

It is first straightforward to see that this planner will not have banks issue any risky debt, as this would increase the social costs of financing with no social benefit. With this in mind, and given the linear preferences of each agent, the aggregate resource constraints for the economy at each date mean that the planner with ex-ante utilitarian objective faces

$$\max_{1\{invest\}(p), z^B(p), z^D(p)} \left[c^e + Hc^h - \int_{p_{min}}^{p_{max}} 1\{invest\}(p)[1 + m^s z^B(p)]dp\right] +$$
$$\beta \left[\int_{p_{min}}^{p_{max}} 1\{invest\}(p)[v(p) - \frac{1}{2}(1-p)dz^D(p)]dp\right] \text{ s.t.}$$
$$z^B(p) + z^D(p) \geq 1 - c^e, \quad \forall p.$$

It is straightforward to see that the solution to this program has the bang-bang nature

$$1\{invest\}(p) = 1 \text{ iff } v(p) \geq \frac{1}{\beta} + \min\{\frac{m^s}{\beta}, \frac{1}{2}(1-p)d\}(1-c^e),$$
$$z^B(p) = 1 - c^e \text{ iff } 1\{invest\}(p) = 1 \text{ and } \frac{m^s}{\beta} < \frac{1}{2}(1-p)d,$$
$$z^D(p) = 1 - c^e \text{ iff } 1\{invest\}(p) = 1 \text{ and } \frac{m^s}{\beta} > \frac{1}{2}(1-p)d.$$

Then note that in the competitive equilibria under study, $1 + r = \frac{1}{\beta}$. It follows that the above allocation can be achieved in the feasible set (the set of allocations supported as competitive equilibria following Definition 3.3), as outlined in the claim and depicted in Figure 3.21. Hence, this allocation is in fact *constrained* efficient as well. □

### Proposition 3.11: implementation of constrained efficiency through leverage cap

*Proof.* Recall that by Lemma 3.2 we have $1 + r^b \geq (1 + r)(1 + m^s)$ in any equilibrium, and following Lemma 3.3 I will restrict the analysis only to equilibria with bank diversification.

First suppose there exists an equilibrium with $1 + r^b > (1 + r)(1 + m^s)$ and $l^b \leq \bar{l}^b \equiv \bar{z}^S(r^{b,ce}; m^s)$. If such an equilibrium is a separating one, skilled banks earn positive profit which is inconsistent with free entry. If such an equilibrium is a pooling one, unskilled banks earn positive profit (from a subsidy from equity issuance) which is inconsistent with free entry. Hence, by contradiction no such equilibrium exists.

Next suppose that there exists an equilibrium with $1 + r^b = (1 + r)(1 + m^s)$ and $l^b \leq \bar{l}^b \equiv \bar{z}^S(r^{b,ce}; m^s)$. Given this value of $1 + r^b$, $\{p_1, p_2\}$ are determined by (3.3) and (3.4), and in particular we have $p_1 = p_1^{ce}$ and $p_2 = p_2^{ce}$. Since $l^b \leq \bar{l}^b \equiv \bar{z}^S(r^{b,ce}; m^s)$, it is clear this equilibrium achieves the constrained efficient allocation as described in Proposition 3.10. □