Irregular Bodies: Polyhedral Geometry and Material Culture in Early Modern Germany

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Abstract

The dissertation explores the centrality of the Platonic Solids, and polyhedral geometry generally, to the artistic and mixed-mathematical cultures of Renaissance Germany. Beginning with Albrecht Dürer's groundbreaking treatise on geometry, the *Underweyung der Messung* (1525), the dissertation redefines sites of early modern experimentation to include the graphical spaces in which new geometrical knowledge was practiced, invented, contested, manipulated, discarded, and presented. The research describes the historical contexts and development of the practice of polyhedral geometry over the course of the 16th century, expanding from Dürer to the lesser-known textbooks for practical geometry that his work inspired in Germany, and continuing with epitomes of the polyhedral genre, namely Wenzel Jamnitzer's *Perspectiva corporum regularium* (1568) and the drawings of the Augsburg artisan Lorentz Stöer. The dissertation then follows the migration of polyhedra into intarsia and turned-ivory artifacts used for teaching applied geometry to European aristocracy, and concludes by addressing the polyhedral cosmology of the astronomer Johannes Kepler. By tracing the lifespan of polyhedra from their use as perspectival tools and pedagogical devices in Renaissance workshops into courtly *Kunstkammern* and onto the precious surfaces of domestic objects, the dissertation uncovers the influence that the decorative arts had on the conceptualization of geometrical knowledge and its new engagement with materials and concepts of materiality.

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Introduction

Archimedes appears to hesitate, transfixed by the rhombicuboctahedron [**Figure I**]. It rests on the edge of the page in the foreground, like a strange species that the mathematician has only just encountered. In contrast to his body, which is fluid and expressive of textures and energy, there is nothing human about the polyhedron. It is pure, perfect, outside taste, already existent. Archimedes grips a tabula in his left hand, as if debating whether he might risk turning away to try and capture it, and if he did so, whether the polyhedron might vanish beyond the reaches of the Renaissance imagination.

Polyhedra in the Renaissance included two major groups—the defined set of the so-called Platonic Solids (*corpora regulata/regularia* or "regular bodies"), which Plato (ca. 428–348 BC) first describes in *Timaeus* (ca. 360 BC.) and which Euclid reconstructs geometrically at the end of his *Elements* (ca. 300 BC.), and the *corpora irregulata*—the endless potential *corpora* directly derived by slicing up, truncating, or embellishing the Solids—like the rhombicuboctahedron encountered by Archimedes. While the *corpora irregulata* maintained a connection to their Platonic progenitors in that they were created by variably operating upon the *corpora regulata*, their number limited only by the imagination of their inventors, initial Renaissance interest in polyhedra stemmed from the Platonic Solids' uniqueness. Since their first description in Book XIII of Euclid's *Elements* there have only ever been five completely regular and symmetrical polyhedra.[1] These five states of exception in a 16th century swamped in irregularity, only beginning to

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[1] The geometrical definition entails that there are only five regular polyhedra where the faces of each polyhedron are made from equal, regular polygons meeting in the same way at every vertex of the Solid so that the entire polyhedron is inscribable within a sphere.

grapple with the measurement and definition of newly discovered global territories and rumors of monstrous foreign creatures, consumed by the verification of vast astronomical data sets and the purging of translation errors from classical texts, made the *corpora regulata* objects of curiosity, if not desire. The tetrahedron (four triangular faces), the hexahedron/cube (six square faces), octahedron (eight triangular faces), dodecahedron (twelve pentagonal faces), and the icosahedron (twenty triangular faces) radiated a tantalizing promise of consistency and order, perhaps even of perfection, although learning their geometrical construction and properties from Euclid, let alone their precise and measured creation, fell far beyond the boundaries of the typical *quadrivium* curricula.[2]

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[2] The unique thirteen convex and uniform geometries composed by two or more polygons, sometimes known as the semi-regular solids or the Archimedean Solids as per Pappus' designation, belonged within the category of *corpora irregulata*, based as they were upon transformations of the Platonic Solids. The pursuit of defining the semi-regular solids constituted a subset of "irregular" geometrical experimentation on the five regular bodies, though the term "*irregulata*" may be misleading to a modern reader given that the Archimedean Solids are highly symmetrical entities, created by consistent modification of the Platonic Solids. Piero della Francesca refers to the six Archimedean Solids contained in *Libellus de quinque corporibus regularibus* as "*corporibus irregularibus*." Albrecht Dürer designates the seven Archimedean Solids in his *Underweysung der Messung* (1528) as "*ungeregulirten corporen*." And Daniele Barbaro, author of *La pratica della perspettiva* (1568) entitles the section following his discussion of the Platonic Solids as the "*descrittione de i corpi irregulari, che nasceno dai corpi regulari*"—description of the irregular bodies, which were borne from the regular bodies (Barbaro 56). The Archimedean Solids were only collectively referred to in print in the first Latin collection of Pappus' writings in 1588 (Field 1997, ft. 1, 242), nearly 80 years after the publication of *De divina proportione*, and are included piecemeal in some 16th century books on geometry alongside other geometrical constructs that do not fit the definition of semi-regular solids. As such, as much as they constitute a unique thirteen member set in their own right, the Archimedean Solids seem to have been uniquely symmetrical "irregular" products derived from the Platonic Solids. It would not be until the astronomer Johannes Kepler's *Harmonices mundi libri V* (1619) that all of the Archimedean Solids would be published and illustrated together for the first time. On the Renaissance history of the Archimedean Solids, see Field 1997. Their modern designations are truncated cube, truncated tetrahedron, truncated dodecahedron, truncated icosahedron, truncated octahedron, truncated cuboctahedron, truncated icosidodecahedron, cuboctahedron, icosidodecahedron, rhombicuboctahedron, rhombicosidodecahedron, snub cube, and snub dodecahedron.

The most popular edition of *Elements* in the Middle Ages, and consequently the first edition, and book of mathematics, to appear in print, was the medieval mathematician Campanus of Novara's (ca. 1220-1296) reworking of earlier versions of *Elements*, some of which had been translated into Latin from Arabic in the 12th century by Robert of Chester. Written and compiled around 1250, Campanus' *Elements* was published in 1482 by Erhard Ratdolt (1442-1528) (Cromwell 106), a publisher from Augsburg working in Venice.[3] In the preface to the 1482 edition, dedicated to his patron the Doge Mocenigo of Venice, Ratdolt apologizes for the book's long gestation period and blames the complexity behind creating and integrating the new diagrams he was laboring to include alongside Euclid's text; novel mathematical visualizations that would amount to the first ever geometrical diagrams in a printed book. Despite, or because of, their initial difficulty, the more than four hundred woodblock prints from Ratdolt's 1482 edition persisted in being remarkably stable graphic entities and were subsequently recycled over and over again, including in the mathematician Luca Pacioli's (1445-1517) 1509 translation of *Elements*, which was also financed by Ratdolt (Mackinnon 147), as well as in the Venetian humanist and translator Bartolomeo Zamberti's (ca. 1473-1543) new, if mathematically suspect, translation of *Elements* from the Greek into Latin without an Arabic intermediary from 1505. Later editions of *Elements* also hewed to copies or near copies of the mathematical diagrams contained in the 1482 edition. As a representative range of examples, a later 1516 Zamberti edition of *Elements* attempts to duplicate the

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[3] Zamberti's poor grasp of mathematics prompted Luca Pacioli to revise the Campanus edition and republish it in 1509 (Cromwell 108). The 1572 translation of *Elements* by Federigo Commandino (1509-15575) became the definitive edition and remained in use until the nineteenth century (Cromwell 109). On the centrality of Euclid in the Middle Ages, see "Euclid in Medieval Europe" by M. Folkerts in *The Development of Mathematics in Medieval Europe*, Ashgate: Hampshire, UK and Burlington, VT, 2006, Chapter III (1-64).

original Ratdolt diagrams. An *Elements* from 1546 in Basel, *Euclidis Megarensis mathematici clarissimi Elementorum geometricorum*, includes scaled-down versions of diagrams from the Ratdolt edition, though the diagrams are more fully integrated into the text than in the 1482 original in which they are all located in the marginalia.[4] Another *Elements* published in Venice in 1565 also includes identical copies of the Ratdolt diagrams.[5]

Though Ratdolt had added new graphic media to Euclid's text with the aim of balancing its technical terminology and concepts with new explanatory mathematical diagrams, the geometrical diagrams were for the most part two-dimensional constructs, with little aspiration of rendering geometry as three-dimensional figures, let alone representing them in the nascent art of perspective. The exceptions were the diagrams to Book 13, Propositions 13-17 of *Elements* covering the construction of the Platonic Solids and their situation in individual spheres [**Figure II**]. These diagrams attempted a level of three-dimensionality, although Ratdolt's lack of experience in representing three-dimensional geometry resulted in a quasi-incomprehensibility. While the pyramid and the cube are shown in legible axonometric (parallel projection) views, the octahedron, dodecahedron, and icosahedron are less successful and point towards the extremely high level of geometrical and spatial competence required from readers and interpreters of the later books of *Elements*.

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[4] The Basel edition includes an introduction by the German pedagogue Philipp Melanchthon.

[5] For further reference on the spread of Euclid editions, see Anderson and Bos ft. 24, 710-711.

Polyhedra had been experimented with in Italy before, most notably in Paolo Uccello's (1397-1475) perspectival experiments, such as his mazzocchio in *The Battle of San Romano* (ca. 1455) and his marble stellated dodecahedron in the floor of the *Basilica di San Marco* in Venice [**Figure III**]. But the first printed text to theorize their construction and manipulation in perspective, and also to include perspectival images of regular and irregular bodies, was Luca Pacioli's influential *De divina proportione* (1509), which included an unattributed Italian translation of Piero della Francesca's (ca. 1415-1492) manuscript on the Platonic Solids, *Libellus de quinque corporibus regularibus*.[6] The first book in *De divina proportione*, which covers the namesake "divine proportion"—the "golden ratio"—and a study of polyhedra, includes how to derive further polyhedra from the regular solids by means of truncating and augmenting their sides (Cromwell 124).[7] Piero's *Libellus* added a systematic arithmetical and geometrical exploration of the properties of the solids in a stolidly Euclidean vein and was among the first (perhaps indeed the first) text to use the phrase "irregular bodies," or "*de corporibus irregularibus*" (Piero della Francesca 556), to describe his novel truncated adaptations of the Platonic Solids, which then became the subject of textbook-like questions regarding the exact measurement of their sides, surfaces, and square (Davis 58).[8]

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[6] Giorgio Vasari (1511-1574) castigated Pacioli for plagiarism and claimed that Pacioli had come into possession of Piero's manuscripts after his death (Field 2005, 124). For Vasari's comments on Pacioli copying Piero della Francesca, see Vasari 1966-1987 [1971], Vol. 3, Part 1, 258 and 264.

[7] Two quantities are in "golden ratio" if their ratio to each other is the same as the ratio of their sum to the larger of the two quantities.

[8] The *Libellus* was only the latest and perhaps most developed of Piero's writings on polyhedral geometry. In his earlier publication, *De Prospectiva Pingendi* (ca. 1482-1487), Piero had already detailed how to construct regular polyhedra in perspective. See Cromwell 119-120 for an

Though not published in Piero's own lifetime, *De divina proportione* was critical to the dissemination of an approach to learning three-dimensional geometry that used the Platonic Solids as base experimental subjects, not least because of the advancements in the perspectival representation of the Solids the book contained. Though the two extant copies of Pacioli's manuscript of *De divina proportione* in Milan and Geneva are admittedly of significantly higher quality than the woodblock prints in the published edition, rendered as the drawings are in vivid Technicolor by Leonardo da Vinci or his circle, the new printed Solids, all sixty of them, nevertheless were depicted as solids or perforated solid bodies (*solidum* or *vacuum*), unlike the diagrammatic wireframes in contemporary copies of *Elements*.[9] These prints ignited a broad interest in the visualizing of geometry, even if the visualizations themselves would ultimately yield to future refinement. The chiaroscuro print of Archimedes (ca. 1518-1520) in Figure I by Ugo da Carpi (1480-1523) encapsulates the excitement, and even the trepidation, that the newly visualizable Platonic Solids and their irregular cousins inspired post-*De divina proportione*.[10] Knowledge of how polyhedra actually looked, a privilege hitherto reserved for only the most advanced mathematicians was suddenly, if not easy to construct

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elaborated example. And in *Trattato d'Abaco* (ca. 1450), he had approached the regular solids as numerical entities, in contradistinction to Euclid who had not worked with specific numerical values for the solids and their sides (Field, 2005, 120).

[9] The manuscript versions of *De divina proportione* contain one extra image, not present in the printed version, making the count of geometrical bodies sixty in total. In the Geneva codex it is titled "*superflua ex errore*" and in the Biblioteca Ambrosiana codex "*pyramis laterata exagona vacua*" (Biggiogero 12). On the differences between the manuscript and print versions of *De divina proportione*, see Chapter Two.

[10] The print was possibly planned as one of a series of philosopher portraits along with Ugo da Carpi's Diogenes (Gnann 2007a, Vol. 1, 188). See also Gnann 2007a Vol. 1, 187. The Archimedes was printed with five colors and was probably based upon a design by Raphael (Gnann 2014, 64). For further reference on Ugo da Carpi's chiaroscuro prints, see Chapter Four in Gnann 2014.

geometrically, than at least was readily available for consumption through purchase or copying.

Despite the voluminous literature on the history of perspective, there has been little attention devoted to Renaissance polyhedra. Hiding in plain sight within the frame of the new three-dimensional pictorial gaze, polyhedra have mostly been left off of history's stage, never quite aligning with our modern vision of the innovations entailed by Renaissance mathematics and recast, if at all, as geometrical frivolities from the dawn of perspective. And yet to dismiss polyhedra as follies is to ignore the central position they occupied for 16th century artists [**Figure IV**], for whom the studying of perspective through the representation of polyhedral bodies was *de rigueur.* In Italy alone, drawings by Leonardo, Parmigianino, and Carpaccio, prints by Ugo da Carpi and Caraglio, and intarsia in the *studiolo* of the Palazzo Ducale in Urbino by Giuliano de Maiano (1432-1490), the Monastery of Monte Olivetto Maggiore near Sienna, and the Church of Santa Maria in Organo, Verona—both by Fra Giovanni da Verona (1457-1525)—hint at the prominence and utility of polyhedra as both graphic studies and as physical models in the workshop aiding the drawing of perspectival geometry.[11] Studying the precision

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[11] Not everyone was in agreement about the utility of working with polyhedral geometry. When the representation of geometry in perspective was the central purpose of an image and not subservient to the realization of a greater pictorial whole, the effect, for Vasari, of such intense, and even misspent, concentration could drive the artist to melancholy. In discussing the life of Paolo Uccello, Vasari laments Uccello having "labored and lost time over the details of perspective; for although they are ingenious and beautiful, yet if a man pursues them beyond measure he does nothing but waste his time, exhausts his powers, fills his mind with difficulties, and often transforms its fertility and readiness into sterility and constraint…not to mention that very often he becomes solitary, eccentric, melancholy, and poor, as did Paolo Uccello" (Vasari 1912, Vol. 2, 131). Vasari goes further and claims that Donatello seconded his opinion of Uccello's geometrical oeuvre—the "mazzocchi with pointed ornaments, and squares drawn in perspective from diverse aspects; spheres with seventy-two diamond-shaped facets, with wood shavings wound round sticks on each facet." Donatello apparently related to Uccello that "these are things that are only useful to men who work at the inlaying of wood, seeing that they fill their

measurement and construction of geometry, and the Solids in particular, was seen as a necessary step in learning to access higher dimensions of knowledge (Grasselli 196), exhibiting a tacit faith in the relationship between the mathematics of accurate pictorial representation and true understanding of the underlying mathematical rules governing the universe (A. Tormey and J.F. Tormey 1982). Moreover, the coincidence of the golden section with the geometry of regular polygons and polyhedra imbued polyhedra with a harmonic aesthetic principle present in the most perfect of human bodies and architectures, and even in the capital letters of the Latin alphabet as enumerated by Pacioli in the second book of *De divina proportione* (Biggiogero 8).

The high visibility of polyhedra as the premier mathematical artifacts of antiquity, brought into broader cultural consciousness through the use of perspective to represent and experiment upon the Platonic Solids, also benefited from the resurgence of interest in the *Timaeus*. While most medieval philosophers had to work from Calcidus' Latin, translated around 321 AD and which only covered up to section 53C in Plato's *Timaeus* (Allen 238), the edition of the *Timaeus* used by scholars in the Renaissance was the edition translated in its entirety from the original Greek into Latin for the first time by the philosopher Marsilio Ficino (1433-1499) (Allen 239).[12] As head of the Medici-sanctioned

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borders with chips and shavings, with spirals both round and square, and with other similar things" (Vasari 1912, Vol. 2, 132-33). On intarsia as a site of intense polyhedral experimentation see Chapter Four.

[12] The first edition of *Compendium in Timaeum* was published in Ficino's *Platonis Opera Omnia* (1484), while the final version appeared in his *Commentaria In Platonem* (1496). This latter version was republished in the subsequent editions of *Platonis Opera Omnia* of 1561, 1576, and 1641 (Allen 247, ft. 2). Manuscripts of Calcidus' translation and commentary on Plato's *Timaeus* have been documented in most important libraries of Renaissance Italy (Hankins 78). See Appendix I in Hankins for a complete list of these manuscripts.

Accademia Platonica in Florence, whose membership included a number of notable Italian scholars, Ficino was the leading authority and translator of Plato's complete works, and consequently the single most influential force in the reinvigorated interest in Platonism during the Renaissance.[13] His commentary on the *Timaeus, Compendium in Timaeum* (1484/1496), posits a world and World Soul held in perfect balance by geometrical proportion and linked to musical harmonies. For Ficino, there are social significances imbedded within a universe understood in terms of proportion, leading to a network of souls and collections of souls in relation, from that of the individual to those of larger institutional entities such as the state, city, and church.[14] Thus the "creation of Soul is an arithmogony, a flowing forth of numbers in harmonic ratios and proportions" (Allen 240), born out as well in the harmonic sets of relations between the seven known planets.

In Ficino's translation, a wealth of new information from Plato appeared in Latin elaborating upon the geometrical constitution of the four elements (fire, air, water, and earth), a topic that Plato had been in the midst of unveiling at the conclusion of Calcidus' translation. For scholars unable to read Greek, Ficino's *Timaeus* revealed the special significance the regular bodies held for Plato, in that they demonstrated, as per Ficino's *Compendium in Timaeum*, how "natural phenomena are based on the principles of mathematics" (Ficino 93). For Plato, the four elements are the building blocks of all matter and are individually composed of different geometrical "*corpora*," the Latin word

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[13] The *Accademia Platonica* counted Poliziano, Cristoforo Landino, Pico della Mirandola, and Gentile de'Becchi as members.

[14] See also Melanchthon's view of geometry and social order in Chapter Three.

that Ficino himself uses: fire is associated with the tetrahedron, air with the octahedron, water with the icosahedron, the earth with the cube, and the universe or “the whole” with the dodecahedron.[15] In the *Timaeus*, the regular bodies are made from rectangular planes, themselves composed of triangles. Plato insists that the triangles of three of the Solids—fire-tetrahedron, water-octahedron, and air-icosahedron—are interchangeable, in that all three bodies can be created from equilateral component triangles. The separate nature of the earth-cube and the universe-dodecahedron stems from their geometrical incompatibility, as neither can be created from equilateral triangles alone (Kotrč 217-219).

> “The second species of solid [octahedron] is formed out of the same triangles [as the pyramid], which unite as eight equilateral triangles and form one solid angle out of four plane angles, and out of six such angles the second body is completed. And the third body is made up of 120 triangular elements, forming twelve solid angles, each of them included in five plane equilateral triangles, having altogether twenty bases, each of which is an equilateral triangle...The isosceles triangle produced the fourth elementary figure, which is compounded of four such triangles, joining their right angles in a center, and forming an equilateral quadrangle. Six of these united form eight solid angles, each of which is made by the combination of three plane right angles; the figure of the body thus composed is a cube, having six plane quadrangular equilateral bases” (Plato 1892 Vol. 3, 475).

Once the triangles are combined into the faces of each of the regular bodies, the bodies gain elemental affinities that are based upon their formal characteristics. The further variability within each of the four elements is due to differences in the size of the elementary triangles that compose the sides of the bodies.[16] Ficino summarizes:

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[15] Following Plato, Pacioli in *De divina proportione* similarly associated the tetrahedron, octahedron, cube, and icosahedron with the four elements, and the dodecahedron as the symbol of the universe (Biggiogero 8).

[16] Plato 1892 Vol. 3, 478, commentary by Jewett.

> “And he [Plato] considers the pyramidal shape to harmonize with fire because it is slender and better at cutting than the others are, being made of fewer triangles and therefore being the lightest of all shapes. The cube, by contrast, harmonizes with earth, being very solid and stable. The remaining shapes harmonize with the intermediate elements, for they stand at points midway between fire and earth in their relationship to motion and stillness. The dodecahedron, which has twelve faces, harmonizes with the cosmos, in his view, on account of the twelve spheres and the twelve signs of the Zodiac” (Ficino 94).

Though Plato does not specify the mechanics of how the triangles would assemble and reassemble themselves, he does offer a geometrical explanation for the transformation of one element into another, or the mixing of two or more elements, as elements may be disassembled back into their elementary triangular surfaces.[17] When “greater bodies are broken up, many small bodies will spring up out of them and take their own proper figures; or, again, when many small bodies are dissolved into their triangles, if they become one, they will form one large mass of another kind” (Plato 1892 Vol. 3, 474-475).[18]

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[17] On the nature of the composition of the triangles which make up the elements in the Timaeus, see also “On Plato’s ‘Fairest Triangles’ (*Timaeus* 54a),” by B. Artmann and L. Schäfer; and the commentary on the *Timaeus* in *Plato’s Cosmology*, F. M. Cornford, New York: Routledge, 2010.

[18] Ficino further enlisted Plato’s elemental theory against the prevailing views of Aristotelian physics (Hankins 87). Ficino’s harmonious geometry structures continuity between terrestrial bodies situated in the sublunary sphere and the constitution of the heavens, borne out as well even at the scale of the “atomic” structure of the elements, and was in stark opposition to the tenets of Aristotelian physics. Aristotle himself had castigated Plato’s theory of the elements in *De Caelo* (350 BC), claiming that Plato’s dogmatism did not fit the empirical evidence and behavior of the elements in nature. “The thinkers that put forth this view turn out to suggest a theory about the phenomena which, though, conflicts with the phenomena. The reason for this is that they do not start out from the right principles but wish to make everything conform to certain preconceived notions. The principles of sensible things must perhaps be sensible, of eternal things eternal, and of perishable things perishable; in general, the principles must be of the same kind as the things they are principles of. However, the advocates of the reduction to planes are so attached to their preconceived notions that they behave like discussants defending a thesis, for they accept all consequences of their ideas on the belief that they start out from right principles, as if some principles ought not to be judged on the basis of their consequences, and principally of their ultimate goal: the ultimate goal of practical knowledge is the appropriate action, of a physical

The recently translated portions of the *Timaeus* contained geometrical theories of the elements just as the geometrical-perspectival culture of Renaissance Italy was coming into its own. The new geometricality of Plato's description contributed to the Solids becoming sought after subjects of this new perspectival culture and imbued with prestige the practical investment in learning to construct and deconstruct polyhedral geometry. For Plato, the Solids were never, or at least only temporarily, solid, and were rather composed of triangular surfaces in flux, as one element might dissolve, disintegrate, or evaporate into another by shedding or gaining triangles; somewhat opposed, in their immediacy and tactility, to Ficino's efforts to formulate a transcendent, harmonic universe of vast spatial orders that spanned from the cosmos to the individual (Cosgrove 22). By describing the elements as composite geometries, Plato had implied that it was possible to represent the elements geometrically, or rather to draw the component parts of the elements at a size scaled to the size of the paper and the human hand. To augment and combine these geometries graphically, associated as they were with Plato and Euclid, was to simulate on paper the irregular and unexpected creations produced by combining the base forms of matter.[19] The solidity, then, of the illustrations of polyhedra in *De divina proportione*, their attempts to be seen as *a priori* objects *on* surfaces, not objects composed *of* surfaces with their own histories of making and unmaking, cannot necessarily be located in the revival of an ancient text, but in a desire for depicting

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theory to fit those phenomena that present themselves to the senses consistently indisputably" (Aristotle 30).

[19] For the widespread influence of Platonism on the arts of the Renaissance, see *Platonic Architectonics: Platonic Philosophies & the Visual Arts*, J. Hendrix, New York: Peter Lang, 2004; and *Neoplatonic Aesthetics: Music, Literature, & the Visual Arts*, L. de Girolami Cheney and J. Hendrix (Eds.), New York: Peter Lang, 2004.

substance, and substantiveness, born out of the unique priorities of perspectival representation in the Italian Renaissance.

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Creative investment in the visualization of polyhedral geometry was not limited only to Italy, and from the early years of the 16th century onwards flowed freely and reciprocally across the Alps, folded into the fruitful exchanges of skills and styles, motifs and prints that bound Italy and Germany to each other. Jacopo de'Barbari, the Italian painter of the famous portrait of Luca Pacioli with a glass rhombicuboctahedron, first moved to Nuremberg to work for Emperor Maximilian I and then stayed on in Germany in the employ of Frederick the Wise of Saxony and Elector Joachim of Brandenburg. Albrecht Dürer himself copied six interlaced geometrical designs of knots after Leonardo (or from the Academia Leonardi Vinci), exchanged drawings with Raphael, and traveled to Venice to protest the copies of his own work by Marcantonio Raimondi (Phagan 7).[20] Daniele Barbaro, translator and commentator on Vitruvius, transmits the influence of Dürer's techniques for manipulating geometry in his *La pratica della perspettiva* (1568) (Kemp 77). The graphic revival and perspectival rebirth of the Platonic Solids in Italy had begun a transnational, intellectual ferment rising out of the collective realization that serious contributions to geometrical knowledge could be made by artists and mixed-mathematical professionals, linked together as they were by the commonality of geometrical knowledge and in addition to the arts encompassing the fields of architecture, goldsmithery, ballistics, engineering, mechanics, fortification design, instrument-making,

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[20] On the knots of Leonardo and Dürer see C.B. Cappel, "Leonardo, Tagliente, and Dürer:'La scienza del far di groppi,'" *Achademia Leonardi Vinci, IV*, 1991, 72-95.

optics, acoustics, and surveying. As following Euclid's propositions for the accurate construction of a Platonic Solid in three dimensions required deciphering several pages of highly technical Latin text, working in and from perspective to explore the visual properties of polyhedral geometry offered an alternative graphic epistemology to the sequential progression of Euclid's propositions that drew upon the native strengths and talents of artisans, makers, and the newly mobile professional class.[21]

The dissertation proper begins in this crucible of polyhedral effervescence with Dürer's treatise on measurement, the *Underweysung der Messung* (1528)—a critical moment in the history of knowledge tied to the literal unfolding of the Solids into "nets" intended to be copied, cut out, and formed into duplicable, paper models. Though Dürer would surely have been able to claim detailed knowledge of the cosmos given his integral role in creating the first printed celestial map in 1515 for Emperor Maximilian, the *Underweysung* has no pretension of revealing a transcendental and harmonic geometry.[22] It resolutely ignores the platonic associations of the regular solids with the five elements (in fact Plato is not even mentioned); and the human body is wholly absent—split as it is into the later *Vier Bücher von menschlicher Proportion* (1528)—which although its graphic construction has Vitruvian overtones is similarly removed from any explicit cosmology. Tangential to Renaissance Platonism, Dürer's *Underweysung* is framed relative to Euclid's *Elements*, in synthesis with other near-contemporary German geometrical textbooks, such as the *Fialenbüchlein* (1486) by Hans Schmuttermayer and *Püchlein von der filialen Gerechtigkeit* (1486) by Matthes Roriczer. In choosing to limit

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[21] See Appendix for an excerpt from Book 13, Proposition 17 from Euclid's *Elements.*

[22] See Dackerman (Ed.) 90-93.

his exploration of the Platonic Solids to their graphic manipulation and reconceptualization as unfolded surfaces, Dürer partakes neither in the debate over the significance of the Solids as transmutational elemental components nor of the broader causality theorized by 16th century humanists between the universe and the individual (Cheney 237). Rather, the *Underweysung* remains a textbook with a pragmatic material focus that stakes its claim to relevance on its utility in the training of precision measurement, and by extension precision thinking, for artists.

Although he theoretically would have had access to Ficino's new edition of the *Timaeus* via the library of Willibald Pirckheimer, there are other persuasive reasons why Dürer may have chosen to work with solids decomposed into surfaces. Many of the design applications in the *Underweysung* are contextualizable within the interests and professions of craft circles in 16th century Nuremberg, where instruments printed on paper were common and the globe was just beginning to be reworked as flat gores.[23] The sketchbook and pattern book (1560-1572) of Master WG in Frankfurt-am-Main [**Figure V**] evidences that gothic techniques of cutting paper to create architectural elements such as vault designs were still in use in 16th century Germany. But perhaps both Plato and Dürer, independently, grasped something essential about working with geometry. Namely, that a deep understanding of how to manipulate media was critical for the construction of equivalences, whether of the Solids to the elements, or of the Solids to ephemeral qualities like precision and accuracy. To broach entities or topics that could not be seen because they were too vast or too small, or because they were hidden in the

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[23] See Schmidt on printed instruments in Nuremberg. The engineer and instrument maker Georg Hartmann (1489-1564) likely collaborated with Dürer on the illustration of a block sundial in the *Underweysung* (Schmidt 271).

contours of objects or the processes of conceiving of ideas, one needed, perhaps counter-intuitively, to work in the visible, in the graphic, with familiar materials, perhaps even with materials that were close by, that weren't precious, that could be reused and discarded without consequence. In the *Phaedo*, Plato's description of how "the earth, when looked at from above, is in appearance streaked like one of those balls which have leather coverings in twelve pieces" (Plato 1892 Vol. 2, 258), betrays the commonness of the physical model even for the formulation of the most extraterrestrial of viewpoints. Thus while the unfolding of the Solids by Dürer was the result of years of media experimentation and study, Plato too made it to the same conceptual point, albeit much earlier in history and in pursuit of different motives. Even if the commonality with Plato was left purposefully unnoted by Dürer, the reason for their singular convergence upon the surface as the key to unraveling the mysteries of polyhedral geometry was due to their joint appropriation of materials from their immediate environment and the impact these materials had on the development of their regimes of thought.

Polyhedral models, most often made from paper or wood, became commonplace tools and sites of experimentation in the 16th century workshop [**Figure VI**], where they were used both as pedagogical aids for learning to understand Euclid and as tools for learning how to draw (themselves) in perspective. If there was one cultural hub that fully embraced the epistemological potential of polyhedra it was southern Germany, where in opposition to the traditional Euclid-bound curricula of university courses in mathematics, artists and artisans sought much of their geometrical education from a newly emerging genre of popular applied mathematical teaching books, often in part copied from Dürer's *Underweysung*. These *Lehrbücher* promulgated the graphical potential of open-ended geometrical experimentation throughout European workshops, which in turn contributed to the instantiation of polyhedra as the premier representational vehicle for the training and display of geometrical knowledge. Through their remnants in print, drawing, painting, and their traces in texts, the dissertation broaches these models and their images as tools for reasoning embedded with theoretical and empirical knowledge (Morgan 5), and bound into evolving practices of seeing and representing the world through geometry.

Though polyhedra were still primarily pedagogical artifacts at the turn of the sixteenth century, by the century's middle their increasing prominence led master craftsmen and artists to produce graphical works that displayed their mastery of geometry and perspective in dazzling feats of skill. This arms race of geometrical one-upmanship culminated in the *Perspectiva corporum regularium* (1568), a splendid array of printed polyhedra by the world-renowned goldsmith Wenzel Jamnitzer (1507/8-1585) that owed its graphic proficiency to the integral role of drawing, and drawing geometry, in the craft workshop; and conversely, the influence of the workshop and knowledge of material performance gained there back on drawing.[24] *Perspectiva* is emblematic of a sea change in the status of the irregular solids, which by the 1560's amounted to a catalogue of unbuildable paper projects steadily accumulating on the back of a generation of experiments by German geometers. As the sum total of this collective work on polyhedra, in which the Platonic Solids acted as a limiting condition on experimentation and polyhedral variations served as litmus tests of innovation, *Perspectiva* solidified the fruits

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[24] For an overview of the early twentieth century origins of the thesis that artisans and artisanal culture impacted and contributed to the development of the "Scientific Revolution," see Long 10-29.

of artisanal "research" on geometry for new courtly audiences, even at the expense of arguably pushing the *irregulata* to the edge of depictability and into new media forms.

Though polyhedra had initially been reborn as Euclidean artifacts of the Renaissance print revolution, print was not to remain their destination medium. The last third of the dissertation tracks the migration of the Solids into intarsia and ivory, the media into which they most often were translated, firstly through the work of perhaps the most prolific and obsessive drawer of polyhedra, the southern German artist and ornamental printmaker Lorentz Stöer. Using Stöer's work as an exemplar, I argue that the surfaces of decorative art objects constituted a well-received area of innovation and experimentation for the presentation of geometrical experimentation and research. The geometry depicted by Stöer and compendia like *Perspectiva* in turn found expression in the art of lathe-turning, a popular form of mixed-mathematical instruction in courts across Europe and the only field to consistently produce tangible, three-dimensional polyhedra. In investigating the practices and epistemologies of turning polyhedral geometry, with particular reference to the spatial arrangement of Elector August's *Kunstkammer* in Dresden, the dissertation seeks to explicate the significance of ivory turning in the evolving category of Renaissance mathematical knowledge.

Renaissance polyhedra were covetable objects of wonder whose saturation of 16th century visual culture and unstable significances seemed to invite their own repurposing. Emerging as early visualizations to Euclid, they were adopted as pedagogical objects for a generation of makers eager to hone precision design skills. And after transitioning into an established category of graphic *Meisterstücke*, they became tangible emblems of this very precision as diplomatic gifts or as ornamental surfaces on furniture. These identities all existed concomitantly. The geometrical *Lehrbücher*, filled with elementary polyhedra, remained in circulation even as salons across Europe imported cabinetry with virtuosic polyhedral motifs from Augsburg. *Elements* was assiduously taught at the same time as the demand for books of geometrical curiosities flourished. Polyhedra then did not serve solely pedagogical purposes nor were they opportunities for invention alone. Although the dissertation has uncovered a historical arc that charts the century-long expansion of geometrical knowledge of polyhedra as a trajectory of increasing formal and material complexification, there was also a remarkable consistency in the appreciation of polyhedra, regardless of the level of their proficiency and embellishment or the preciousness of their base materials. Whether as workshop geometries or *Kunstkammer* artifacts, polyhedra were always intended to decelerate the experience of seeing—used as much to teach artisans to see with accuracy and to translate their visions onto paper with precision as to entrap viewers and patrons in highly condensed displays of almost otherworldly skill.

Chapter One: Albrecht Dürer's Material Geometry

> Euclid, the man with the sharpest mind, has assembled the foundation of geometry. Those that understand him by themselves may disregard what is written afterwards, because it is written for the young and to those whom otherwise have no one who is a devoted instructor (Dürer 1525).[25]

Albrecht Dürer's (1471-1528) most important contribution to 16th century geometry remains the first of his two major treatises on measurement and proportion, the *Underweysung der Messung* (1525), which synthesized the internal structure, principles, and precision measurement of Euclid's *Elements* with select reference to other classical mathematicians (such as Ptolemy) alongside contemporary sources, both German and Italian.[26] Systematically organizing a wealth of mixed-mathematical, or applied-geometrical, knowledge accumulated over a lifetime of artistic practice, it was intended, as Dürer declared in the *Underweysung's* introduction, to be "not only for painters, but also for goldsmiths, sculptors, stonemasons, carpenters, and all those for whom using measurement is useful."[27] Beginning with Dürer's access to these mixed-mathematical resources in his home city of Nuremberg, the chapter seeks to reframe Dürer's lasting engagement with geometry as substantially concerned with polyhedra, separate and distinct from his travels to Italy and the general Renaissance interest in perspective. Dürer's sketches—which span his entire career as an artist —contain numerous instances

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[25] "*Der aller scharff sinnigst Euclides/ hat den grunde der Geometria zusame gesetzt werden selben woll versteht/ der darff diser hernach geschrieben ding gar nit/ dann sie sind alleyn den iungen und denen so sonst niemander haben der sie trewlich underveyst geschryben*."

[26] *Underweysung der Messung* will heretofore be abbreviated as *Underweysung.*

[27] "*unnd auch nicht alleyn den malern/ sonder Goldschmiden Bildhaweren Steynmeßen Schreyneren und allen den so sich das maß gebrauchen dienstlich seyn mag*."

of polyhedra in all manner of permutations. There are various instances of polyhedra inscribed in cubes or floating in space and preparatory drawings for the polyhedral tiling patterns in the *Underweysung*. Dürer's own personal copy of the *Underweysung* at the Bayerische Staatsbibliothek is replete with sketches of the geometrical diagrams included in the second 1538 edition, including the last unfolded polyhedral geometries to be added to the text. There are even illustrations of Euclid's theorems accompanied by German translations at the British Museum, some of which have been made in Pirckheimer's handwriting (Dürer and Strauss 1974, Vol. 6, 2812). Sketches in Nuremberg show Dürer and his workshop or circle experimenting with various types of polyhedral representations.[28]

Central to the pedagogical philosophy of the *Underweysung* was the transformation of polyhedra into disposable, "unfolded" polyhedral nets that could be printed, cut out, and glued together to create quick copies of the Platonic Solids, alongside Dürer's own geometrical inventions. The materiality at the heart of Dürer's mathematical practice destabilized the Solids and formed the base of the radically tactile and playful relationship to classical geometry characteristic of later 16th century German artists. Heralding a new era of geometric experimentation by Erhard Schön, Augustin Hirschvogel, Wenzel Jamnitzer, Lorenz Stöer, and others, Dürer's pioneering efforts made the domain of geometry attractive to the multifarious manipulations of artists by splicing together the generative possibilities of geometrical invention. Migrating back and forth from the graphic surface to painstaking composites of intarsia or the prodigious

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[28] See Dürer and Strauss 1974, Vol. 6, 2855 and 2856. The handwritten note says, as per Strauss' translation, "This is foreshortened/ this one is raised" (Dürer and Strauss 1974, Vol. 6, 2856).

curvilinear turrets of the turned ivory towers in European *Kunstkammern*, the production of polyhedra, in all their irregular permutations, came to encompass a powerful new site of early modern experimentation in which geometry was practiced, invented, contested, manipulated, discarded, and presented.

Dürer's status as Nuremberg's most celebrated artist would have been sufficient to provide him with access to the geometrical textbooks in the Regiomontanus-Walther library, irrespective of his close friendship with the humanist Willibald Pirckheimer (1470-1530) as well as other Nuremberg scholars. Upon the death of the great mathematician and astronomer Johannes Regiomontanus (1436-1476), who had lived in Nuremberg from 1471-1475, the working library he had amassed there was turned over to Bernhard Walther (ca. 1430-1504), a local merchant and proficient astronomer in his own right, who had collaborated with Regiomontanus on astronomical observations and in the establishment of a printing press tasked with the purpose of revising astronomical texts with new observational data.[29] The collection remained remarkably intact over the next fifteen years, though not likely in situ, and under the auspices of the city of Nuremberg.

During this time, the "*Regiomontan-Waltersche Bücherei*" became an intellectual resource and reference library for a new generation of Nuremberg-based scholars including Pirckheimer; Johannes Werner (1468-1522), who worked on spherical

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[29] Attempts by King Matthias I of Hungary and Croatia (1443-1490) to convince Walther to part with Regiomontanus' books and scientific instruments so that they could be housed in the royal library in Buda [Budapest] were unsuccessful (Petz 238), and the collection remained in Regiomontanus' house, which Walther himself had purchased. Shortly before his own death in 1504, Walther decreed that the entire collection of books and instruments was only to be sold together, with the exception of several books which were sent to Krakow and Italy in 1512 and a selection of the brass instruments which were stolen in 1514.

trigonometry and conic sections; Joachim Camerarius (1500-1574), the classical scholar, Dürer's biographer and the first translator of Dürer's *Underweysung der Messung* (1525) into Latin; the influential globe-maker and cosmographer Johannes Schöner (1477-1547), and others (Rupprich 1966, Vol. 2, 9).[30]

That Dürer was intimately familiar with the house itself is attested to by the fact that in 1509, five years after Walther's death, he purchased the house from Walther's heirs and moved into it with his wife Agnes (1475-1539), living there until his death in 1528 when he was said to have been found with books from his library scattered near him (Rupprich 1966, Vol. 2, 10). Perhaps some of these books were those that he had bought in 1523 from the library because of their "usefulness to painters," when he had paid 10 florins for a selection of ten.[31] Given his direct connections to Walther, Dürer may even have had access to the library while Walther was alive. His parents had known Walther personally, and Walther's wife had been the godmother to Dürer's sister Christina, who was born in 1488.

Due to the library's unique status as a substantial mixed-mathematical resource in Nuremberg, it is extremely likely that Dürer would have used the library for the purpose

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[30] Johann Schöner's introduction to *Joannis de Regiomonta de triangulis, etc,* Nuremberg 1533 also confirms that Pirckheimer had purchased many manuscripts pertaining to ancient mathematics from the libraries of Regiomontanus and Walther (Dürer and Strauss 1977, 14-15). It was only in 1519, after a failed attempt to sell the books to Elector Friedrich the Wise of Saxony, that a portion of the books were sold for the sum of 150 gulden, most of which went to Pirckheimer who recorded the purchase in his personal manuscripts (Petz 239).

[31] (4r: 1523 Januar) "*ad 13 ditto verkauft wir dem Albrecht Dürer 10 pücher von des Berenharts Walthers püchern, so den malleren dienstlich sein vnd durch Wilbolt Pirchamer geschetz worden vnd zalt an münz fl. 10*." Brechnung des "Gemeinen Almosen." (Nr. 42) für die Zeit vom 2. IX. 1522 bis 14. VI. 1523 im Nürnberger Stadtarchiv. Reprinted in Rupprich 1956, Vol. 1, 221.

of deepening his understanding of geometry and perspective and that he could have easily gained entry, either as a family friend or as an inquisitive artist, from very early on in his career, and most certainly should he have desired prior to his second extended 1505-1507 trip to Italy. In Nuremberg, Dürer also would have had contact with all of the above-mentioned mathematicians and could have approached them for help in solving the more complex geometrical issues found in his *Underweysung*. Thus, given Dürer's intimate familiarity with Pirckheimer and his extended circle of humanist scholars and in combination with Dürer's probable access to the library, it is difficult not to defend the characterization of Dürer as having been socially embedded within a learned community capable of answering any or all of the questions he might have had on the mathematical issues that pertained to his work on geometry and measurement.

Despite having access to this mathematical literature, Dürer might very well have needed help reading the ancient texts, such as the newly published translation into Latin, from the original Greek, of Euclid's *Opera* (1505) by Bartolomeo Zamberti, which Dürer acquired in Venice for one ducat.[32] His contemporary biographer Camerarius claims that although Dürer had acquired skills in "*naturalium et mathematicarum rerum scientiae*," he had not undertaken literary studies, which meant in this context "the analysis and also the productive imitation of classical Latin writers" (Price 10). Camerarius does admit however that the natural and mathematical sciences were predicated upon ancient texts which Dürer had "almost learned" (*fere didicerat*), and that Dürer's geometrical treatises evidenced his understanding of scientific principles and his ability to set them down into

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[32] Dürer's copy of *Elements*, which he bought in 1507, is in the Herzog August Bibliothek in Wolfenbüttel. The inscription in Dürer's hand reads "*Daz puch hab ich zw Venedich vm ein Dugatn kawft im 1507 jor. Albrecht Durer*."

words.[33] Regardless of the state of Dürer's literacy, Pirckheimer would have been an inimitable asset here. There are numerous examples of Pirckheimer's writing in Dürer's voluminous literary remains that attest to their close collaboration on mathematical issues and translations, including corrections made by Pirckheimer to drafts of Dürer's *Underweysung* (Rupprich 1969, Vol. 3, 328).[34] Pirckheimer had studied law in Italy from 1489-1495, was fluent in Greek and Latin, and routinely acted as an intermediary between Dürer and Emperor Maximilian I (1459-1519), even going so far as to act as an advisor of antique iconography for some of Dürer's highest profile commissions (Price 14) such as the *Triumphal Arch* or *Ehrenpforte Maximilians I* (first edition printed 1517-1518)—a monumental composite image printed on 36 large sheets of paper from 195

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[33] "*Litterarum quidem studia non attigerat, sed quae illis tamen traduntur, maxime naturalium et mathematicarum rerum scientiae, fere didicerat. Aeque is praecipua ut intelligebat et re explicare noverat, ita et oration sciebat declarare. Testantur hoc scripta eius geometrica, in quibus quid de illa scientia possit require, quantenus quidem tractandam sibi iudicavit, non video.*" Reprinted in Rupprich 1956, Vol. 1, 307-308. For the whole of Camerarius' introduction to the first Latin translation of the *Underweysung*, see Rupprich 1956, Vol. 1, 307-311. Dürer in his *Familienchronik*, compiled in 1524, states that he excelled at his lessons and was consequently brought to school by his father in order to learn how to read and write, though his father also removed him after a few years in order to apprentice in his native goldsmith trade. "*Und sonderlich hate mein vater an mir ein gefallen, da er sahe, daß ich fleisig in der übung zu lernen was. Darumb ließ mein vater in die schull gehen, und da ich schreiben und lessen gelernet, namb er mich wider auß der schull und lernet mich das goltschmid handtwerckh*" (Dürers Familienchronik, Rupprich 1956, Vol. 1, 30). This could mean that Dürer never learned or did not learn enough Latin to be able to comprehend Ptolemy's *Almagest* or Sacrobosco's *De sphaera materiali* on his own, both books that, should he have wanted to read them, were in the Regiomontanus-Walther collection. On the other hand, Dürer himself specified that young artists should learn to "read and write well and be taught Latin in order to really understand writings" (Dürer in Rupprich, Vol. 2, 92), which does imply that Dürer at the very least possessed these skills himself, or believed that young artists would benefit from acquiring skills he himself lacked. See Leder 29-34 on the typical curricula in Nuremberg in the period of Dürer's childhood. Other scholars disagree with Leder and claim that Dürer may well have been able to read Latin (Price 10).

[34] The contemporary historian Johann Neudörffer (1497-1563) records that when the artist Sebald Beham gave his son the fifteen books of Euclid translated into German by the mathematician Johannes Werner, Pirckheimer was the negotiator (Neudörffer 48).

individual wood blocks imbued with trans-historical references.[35] Pirckheimer also possessed his own massive library which, he claimed in a 1503 letter, held a copy of almost every text printed in Italy (Rostenberg 23).[36] Dürer likely had access to this library as well. The solution to the Delic Problem contained in the *Underweysung* appears to be based upon the Eutokius manuscript found in Pirckheimer's library, and Dürer had personally illuminated a selection of Pirckheimer's books.[37] Dürer was likely familiar with the fruits of Italian geometrical-perspectival culture from the Italian volumes in Pirckheimer's private library—such as Pacioli's *Somma di Aritmetica, Geometrica, Proportioni e Proportionalità* (1494).

Two inventories of the Regiomontanus-Walther collection were compiled by Pirckheimer in 1512 and 1522 (Rupprich 1960, 237), the second inventory listing 145 remaining books mainly comprised of classics from antiquity and the middle ages on geometry, perspective, astronomy, and astrology (i.e. Ptolemy, Sacrobosco, Ibn Al-Haytham, Archimedes) as well as contemporary works of relevance including calendars and astronomical tables.[38] Of note in the context of Dürer's *Underweysung* are the many copies of Euclid that were still in the collection, and to which Dürer would presumably

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[35] For a survey of Dürer's projects for Maximilian I, see Rebel 305-323.

[36] Presumably most of these books printed in Italy were in Latin and Greek. The original letter was sent to Konrad Celtis in 1503 and is reprinted in Hartmann 66-67.

[37] The Delic or Delian problem, also known as "doubling the cube," describes a situation whereby the edge of one cube is given and using only a compass and straightedge, it is necessary to construct a second cube of double the volume of the first cube. On the Delic problem and Dürer's use of the Eutokius see Dürer and Strauss 1977, 24 and footnotes 50-54, 34. On Dürer's illuminations for Pirckheimer see Rosenthal, and Eckert 84-88.

[38] The 1522 inventory has been reprinted in Petz 247-262. A later 1563 inventory is published in Zinner 161-168.

have had access. These included the *Preclarissimu[m] opus Elementor[um] Euclidis Megaare[n]sis* (1482), referred to as “Euclides. Impressus. (Geometria)” in the inventory, which was based on the medieval Italian mathematician Campanus of Novara’s (ca. 1220-1296) popular Latin edition of Euclid and had been printed for the first time with accompanying geometrical diagrams in Venice in 1482 by the Augsburg printer Erhard Ratdolt (1442-1528). Also present was a rare manuscript of the first translation of Euclid from Arabic into Latin by the English monk and natural philosopher Adelard of Bath (ca. 1080 – ca. 1152) and a manuscript of Leon Battista Alberti’s *De pittura*, “Liber de picture L. Baptiste de Albertis. (Geometria),” referred to as “De picture babtis” in the 1512 inventory.[39]

Although Dürer might very well have bought Alberti’s *De pittura* as one of his ten “artistically useful” purchases in 1523, the only book from the Regiomontanus-Walther collection definitively identified as having been subsequently in Dürer’s possession was Regiomontanus’ own manuscript copy of Euclid’s *Elements* (Rupprich 1960, 236, and Rupprich 1956 Vol. 1, 222). In fact, it is not surprising that Dürer would have purchased yet another *Elements* to add to the copy he bought in Venice, even if it was more of a collector’s edition at that point. Dürer’s letters contain what I suspect to be among the first translations into German of eleven suppositions and forty theorems from Euclid’s *Perspectiva Naturalis*, some of which are in Pirckheimer’s hand, alongside numerous measured geometrical sketches, notes and illustrations copied from the printed diagrams

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[39] The Latin translations of Arabic texts of Euclid’s *Elements* by Adelard of Bath and Campanus of Novara were the main references for Euclid in the middle ages. The first edition of Euclid to be printed in Greek was published in 1533 on the basis of recovered manuscripts that dated back to Theon, a 4th century Greek mathematician and astronomer (Anderson and Bos 710).

in Zamberti's Euclid [**Figure 1.1**].[40] And although he owned at least two Euclids by 1523 and had continual access to the precious Adelard of Bath edition as well as Ratdolt's 1482 first edition based on Campanus in the Regiomontanus-Walther library, Dürer evidently maintained a lifelong fascination with Euclid and consequently remained interested in the most up-to-date translations. In a letter dated December 5th, 1524 to the German mathematician Nikolaus Kratzer, who was living in London at the time, Dürer inquires how long it will take until Kratzer's planned new German translation of Euclid would be ready.[41]

As we have seen, Dürer was steeped in the wealth of mathematical knowledge locally available to him in Germany, not least of which were several versions of Euclid and other ancient and contemporary German texts from the Regiomontanus-Walther library, which would form the basis of his *Underweysung.*[42] Yet Dürer's engagement with Euclidean geometry has often been coopted into the historical story of his "discovery" of perspective in Italy as if the pursuit of perspective was the only motivation for an artist to

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[40] See Rupprich 1966, Vol. 2, 374-377 and Dürer and Strauss 1974, Vol. 6, 2817-2823. Figure 1.1 is a reproduction of a drawing found in the Sloane Collection at the British Museum—Sloane 5228/213r—and reprinted in Dürer and Strauss 1974, Vol. 6, 2819. Dürer also translated portions of a text on perspectival theory copied from a Piero della Francesca's *De Prospectiva Pingendi*. "*Item prospectiua ist ein lateinisch wort, pedewt ein durchsehung*" (Rupprich 1966, Vol. 2, 373). See Panofsky's *Dürers Kunsttheorie* 1915, 42-43 and papers at the British Museum, Sloane 5228/202.

[41] "*Item als jr mir zw sagett, so jr weill möcht haben, wolt jr den Ewklide in tewczsch bringen, wolt jch geren wissen, ob jr etwas doran gemacht het*." Dürer also briefly covers the religious climate in Nuremberg in the letter and confirms Pirckheimer's planned procurement of a measurement instrument for Kratzer (Rupprich 1956, Vol. 1, 113).

[42] In addition to the Regiomontanus-Walther and Pirckheimer libraries, Dürer was surrounded by the thriving book trade in Nuremberg and might well have seen books on geometry and mixed-mathematics from the publisher Anton Koberger—who was responsible for the Nuremberg Chronicle and was Dürer's godfather (Chipps Smith 2011, 17).

develop their understanding of geometry in the early 16th century.[43] While it may be true that Dürer came back from his second trip to Venice (1505-1507) acquainted with Italian geometrical-perspectival techniques and an additional copy of Zamberti's 1505 edition of *Elements*, and that Dürer may have met Luca Pacioli (ca. 1445 – ca. 1514) in Bologna, who would have been working on his own annotated edition of Euclid at the time (Dürer and Strauss 1977, 13), Dürer had an abiding interest in and access to Euclid in Nuremberg that predated his engagement with Italian perspective.[44] Aside from the copies of Euclid by Adelard of Bath and Regiomontanus' own manuscript, the popular Euclid printed by Ratdolt had been in circulation since 1482 and had probably made its way to Nuremberg long before Dürer's first trip to Italy (1494-1495).

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Dürer's *Underweysung der Messung* (1525), which was published approximately twenty years after the first printed edition of Pacioli's *De divina proportione* (1509), and in the chronology of Renaissance treatises is the next major publication to tackle polyhedra, is a Euclidean treatise on geometry that aims to be a textbook for artists and craftsmen, indeed "everyone desirous of learning about art," and a definitive "source for learning about measurement [*Messung*] with ruler and compass" (Dürer 1525, introduction).[45] Measurement here was understood as basic to geometry, or *geometria*—literally "earth

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[43] See for instance Conway 208-209 and Kemp 55.

[44] Panofsky claims that on his trip to Italy, Dürer learned Piero della Francesca's method of using foreshortening to create perspectival figures (Panofsky 1955, 251).

[45] Text contained in the *Underweysung* is cited with reference to the Diagram on which the text is contained.

measurement"—and the treatise encompassed a synthesis of Euclid with various geometrical texts available in the Regiomontanus-Walther or Pirckheimer libraries and with the mixed-mathematical knowledge Dürer had accumulated over a lifetime of art practice.[46] Dürer even invented a new German scientific prose of descriptive-graphical terms for his geometrical constructions. For instance, "*'Fischblase'* ('fish's bladder') and '*der neue Mondschein'* ('crescent') for the figures resulting from the intersection of two circles," and new terms like "*'Gabellinie'* ('fork line') for hyperbola...'*Schnecken linie*' ('snail line') for spiral" (Panofsky 1955, 245).[47]

The treatise is divided into four books that straddle a range of geometrical topics exceeding the scope of *Elements* both in terms of applied geometry and in the

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[46] When Camerarius translated the *Underweysung* into Latin in 1538, he changed the word *Messung* in the title to *Geometria.* It appears to be Dürer's invention to use *Messung* for what would have been understood as *Geometria.* See Dürer and Strauss 1977, 10 and Rupprich 1969, Vol. 3, 310. As Walter L. Strauss and others have demonstrated, in addition to Euclid the geometry in the *Underweysung* bears the imprint of several multiple mixed-mathematical publications local to Nuremberg, namely the *Fialenbüchlein* (1486) by Hans Schmuttermayer – who was an acquaintance of Dürer's father (Kavaler 43), *Püchlein von der filialen Gerechtigkeit*—Booklet Concerning the Correctitude of Pinnacles (1486) by Matthes Roriczer, and the anonymous *Geometria deutsch, aus der geometry etliche nutzparliche stuck* (ca 1472-1484), possibly attributable to Roriczer (Dürer and Strauss 1977, 16). Dürer directly copied certain diagrams from these 15th century books. As an example, Book II, Figure 16 in Dürer's *Underweysung* is a copy of Figure 27 in Roriczer's *Geometria deutsch.* The Roriczer is reprinted in Kavaler 117. Strauss notes that Dürer adopted the construction of a right angle and the method of locating the center of a circle from Roriczer and the construction of a pentagon from Ptolemy (Dürer and Strauss 1977, 16-17). For further information on Roriczer and Schmuttermayer, see Kavaler 41-43, Shelby et. al 7-28. See also Rupprich 1969, Vol. 3, 309-310. For a good summary and explanation of the additional mathematical texts imbedded in the *Underweysung* see *Albrecht Durer 1471 1971*, Ausstellung des Germanischen Nationalmuseums. Prestel-Verlag, Munich, 341-354.

[47] Panofksy claims this is the first time that anyone had gone to such length to describe "complicated geometrical constructions" in German, and that the "ancient technical language," one might say literal language, of German artisans was borrowed from words invented in order to accurately describe the graphic figures that Dürer was drawing. Although there is only a cursory treatment of the nets, for Panofsky on the *Underweysung* see Panofsky 1955, 254-260.

visualization and invention of geometrical figures, though emphasizing the construction of complex shapes/forms from basic elements.[48] Book I covers the definition and construction of lines—ranging from parallel lines to multiple ways of constructing spirals, spiral projections, hyperbolic lines, conic sections, and parabola to name a few. Book II discusses plane surfaces, the construction of polygons, tile patterns, and the Pythagorean theorem. Book III steps sideways to cover a variety of topics, including column and monument design and construction, sundials, and the construction of the alphabet. Book IV returns to the Platonic and Archimedean Solids, which we will be concentrating on, and follows them by tackling the Delic problem, proportional lines, and various perspectival theories and apparatuses.

Book I (page A II) begins by Dürer describing a point (*Punck*) as "such a thing with neither size, length, width, or thickness. And still it is the beginning and end of all corporeal things we may want to construct, or which we may conceive in our minds."[49] He continues by noting that points are the basic building blocks of geometrical construction "to the extent that no point takes up any space, for it is indivisible and can in our senses or thinking [*synnen oder gedanken*] be placed anywhere. I may use my mind to throw a point high up in the air, or drop it into the depths where I cannot reach it with

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[48] Euclid provides three primitive constructions: A [unique] straight line can be drawn from any point to any other point; A straight line segment that can be continuously extended by a finite amount to produce another straight line segment; A [unique] circle may be drawn centered on any point with any radius. All the geometrical propositions contained in the 13 books of the Elements are derived using only the ability to construct line and circles (Cromwell 61).

[49] "*Aber eyn punckt ist ein solch ding/ das weder Groß Leng Breyt oder Dicken hat/ Und ist doch ein anfang und ende/ aller leiblichen ding/ die man machen mag/ oder die wir in unsem synnen erdencken mugen*."

my body."[50] Given that points are essentially thought-constructs, Dürer says that if one wants to mentally connect them to each other it is possible to do so with a line, but this unrepresentable and impalpable line is also invisible [*unsichtig*]. Visualizing geometry through drawing is necessary to be able to understand the connections between points, and thus, he says, "I want all matters that I have described in this treatise by way of a sketch, drawn in a very succinct way described therein, so that the young, through the help of the imagination, will be able to see with their own eyes and grasp what has been described here" (Dürer 1525, introduction to Book 1).[51]

Dürer has laid out his case for including the visual realm as essential to the learning of geometry in this extraordinary paragraph. His use of the word *Einbildung* (*ein* = inward, *bildung* = image/imagination) points not just towards what was understood as the faculty of the imagination at the time, but to the closely related meaning of the image of an object created and mentally manipulated by the faculty of imagination.[52] Another implicit understanding of Dürer's *Einbildung*—already inching toward Kant's *Einbildungskraft*—is the importance of images (*Bilde*) in the production of a fully formed and educated person (*Bildung* and *Ausbildung*). To draw precise geometry served to train readers in a way that could yield a new capacity for vision, developing a mutual reciprocity between

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[50] "*Wie dan das die hochuerstendigen/ diser kunst woll wyssen/ und darumb erfullt keyn punckt keyn stat/ dann er ist untzerteylich/ unnd er mag doch auß unnsern synnen order gedancken/ an alle end oder ort gesetzt werden/ Dan ich mag mit dem synn ein punckten hoch in lufft werffen/ oder in die tyffen fellen/ da hyn ich doch mit dem leib nit reichen kan.*"

[51] "*Darumb will ich alle ding/ die ich in diesem buchlin beschreib/ auch darneben auffreissen/ auff das meyn darthon/ die iunge zu einer einbildung vor augen sehen/ Unnd dest baß begreiffen*."

[52] With sincere thanks to Katharine Park for her comments on *Einbildung.*

the tactile manipulation of drawing instruments to create exact representations of things (columns, polyhedra, spirals) in a graphic space, and the imagining of these things in a mental space.[53] To follow Dürer's logic was to conceive, perhaps for the first time for many German craftsmen, that virtual images and designs could either be plucked from the *Synnen* or even completely fabricated without visual referent and plotted onto the page via geometrically precise drawing.[54] The proper and exact use of geometry *made visual* and the centrality of visualization in geometry ties together all the geometrical applications in the *Underweysung*, whether they are an unfolded dodecahedron or the constructing of convex columns. Dürer exhibits here a striking clarity of thought and use of contemporary understandings of sensation that underscores just how important it must have been for him to communicate Euclidean concepts to a milieu of creative professionals that might very well have not been familiar with them.

Book IV is dedicated to polyhedra and approaches them in a way that is radically and fundamentally different from any of the other images of polyhedra in geometrical treatises up until that point—namely in Euclid, Piero della Francesca, and Pacioli.[55]

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[53] See Chapter Three where the notion of making/drawing geometry as a form of disciplinary or pedagogical training is developed.

[54] On the capacity of *Einbildungen* to be generated from the imaginative faculty as one of the "internal" senses—which also included cognition, memory, fantasy, and common sense—or from the five "external" senses—vision, hearing, smell, taste, and touch—see Park 1988, 465-473. For an introduction to the relation between observation from nature and invention in Dürer's work, see Parshall 2013, 393-395.

[55] In the second book of the *Underweysung*, Dürer also deals with surfaces of polyhedra in a section on tiled floor patterns. See Dürer 1525, Book 2, Diagrams 22-27. "*Item die sechs eck will ich dreyerley weyß zusamen sehen. Erstlich steck ich sie in ein ander/ das all seyetn eck und winckel an einander an niren/ und nichts lers von feldt darzwischen bleybt*" (Dürer 1525, Book 2, Diagram 24a). "And so I want to show three ways to combine hexagons (*sechs eck*). Firstly, I

Dürer enumerates his logic for providing unfolded surfaces (*ganz offen*) of the Platonic Solids rather than adhering to the more traditional Euclidean diagrams or the perspectival stylings of Pacioli's *De divina proportione.*[56]

> Should one wish to create even more beautiful bodies, which touch a hollow sphere with all of their corners but have uneven [*ungleyche*] surfaces, I will lay them out in their parts, totally accessible in the following section so that each of their surfaces touches the other. Whoever wants to replicate these surfaces should trace the larger (i.e. unfolded) figure on a duplicate piece of paper attached to the original and cut this paper along the figure with a sharp knife so that all the lines on the original paper remain intact. Then place the body together along the lines of the drawing. One should pay attention to this technique because it will be useful for the following figures.[57]

While Pacioli covers the five Platonic Solids and three of the thirteen Archimedean Solids, all illustrated in perspectival images, Dürer unfolds all five Platonic Solids as well as seven Archimedean Solids in the first edition of the *Underweysung* from 1525 and nine in the second 1538 edition in addition to several bodies of his own invention—as

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insert them into each other, so that all sides, corners, and angles are attached to each other without leaving any free space between them." Here Dürer depicts two alternative groupings of hexagons, neither of which will result in a polyhedron when folded up. Dürer claims to only be showing some basic options given the infinite potential permutations—and leaves it up to the reader or artist-reader to conceive of more complex or ornate surfaces. As he says, "If I were to show all these [combinations of figures] now, the book would become much too long. Therefore, one should think about them oneself" (Dürer 1525, Book 2, Diagram 28). "*Solt ich das nun alles hy anzeygen/ so wurd das buchlein vil zulang/ darumb denck im ein ytlicher selbs nach*."

[56] Dürer's use of the word "*ganz*" is itself a loaded proposition. "*Ganz*" can also mean exposed; undone; and accessible—implying that the act of opening up the polyhedra makes them more accessible as geometrical forms.

[57] "*Auch sind noch vill hubscher corpora zumachen/ die auch in einer holen kugel mit all jren ecken an ruren/aber sie haben ungleyche felder/ der selben wil ich eins teyls hernach auf reyssen/ und gantz aufgethan/ auf das sie ein yetlicher selbs zamen mug legen/ welicher sie aber machen will der reyß sie grosser auf ein zwifach gepabt papier/ un schneyd mit einem scharpfen messer auf der einen seyten all ryß durch den einen pogen papiers/ und so dan all ding auß dem ubrige papier geledigt wirt/ als dan lege man das corpus zusamen/ so lest es sich geren in den risen piegen/ darumb nym des nachfolgeten auf reyssens acht/ dan soliche ding sind zu vill sachen nutz*" (Dürer 1525, Book 4, Diagram 34a).

Panofsky notes, one body composed of eight dodecagons, one from twenty four isosceles triangles and one from eight equilateral triangles (Panofsky 1955, 259).[58] Rather than relying upon perspectival images of geometry to train knowledge of the representation of reality, Dürer has the Platonic Solids printed "*in grund gelegt*" (laid out as a plan) and as bodies that have been "*aufgerissen*" (both outlined and unfolded as per the double German meaning) as developable surfaces [**Figure 1.2 + 1.3**].

These planar images would have been produced using woodblocks of the unfolded solids, much like those from the workshop of Hieronymus Andreae (died 1556), the printer of both Dürer's Triumphal Arch for Emperor Maximilian and the *Underweysung*, at the Albertina in Vienna [**Figure 1.4 + 1.5**].[59] To create the woodblocks, the carver

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[58] On the differences between the 1525 and the 1538 editions of the *Underweysung*, see Rupprich 1969 Vol. 3, 314 as well as a forthcoming article "Albrecht Dürer's Personal *Underweysung*" by Andrews in *Word & Image*.

[59] The blocks do not show extensive signs of use and are so tight to the image because they would have been composed for use with type on a page. This is a reason for their compactness. While some of the blocks preserve ink on their surfaces, this is probably just evidence of a test print having been completed, not a final print. Still, it is a mystery why most of the thirty blocks would have been created and not finished or used. Three of the woodblocks have Andreae's monogram (Allgemeines Kunstlerlexicon). On Andreae see Timann 56-63. There is also speculation that some of the blocks may have belonged to Johann Tscherte (1480-1552)—a Vienna-based mathematician and master builder who was also a friend of Dürer but traveled back and forth to Nuremberg (Schreiber, Fischer, and Sternath 467). Although there is no specific provenance information on them, they may have come from the Ambras Castle in Innsbruck, which still has in its possession the woodblocks belonging to Maximilian for Dürer's *Ehrenporte.* It was customary for blocks printed for projects sponsored by Maximilian to stay in his possession. Most of the woodblock nets are not identical to the prints in the *Underweysung*, as strips of geometrical faces radiate outwards from an arbitrary central face giving the overall nets a star-like association (Schreiber, Fischer, and Sternath 467), though several of the blocks do appear to be copies from the *Underweysung.* HO 2006/718 is unique in that it is a one of the only completely finished woodblocks in the set. It matches Diagram 37 from the *Underweysung*. HO 2006/722 has been copied from Diagram 40, and HO 2006/701 from Diagram 38. In this last example, the image on the block has been rotated in comparison to the same image in the *Underweysung*. If a block such as this one had been used on the *Underweysung*, it would have to be positioned at an angle on paper to generate the orientation of Dürer's net. If the block cutter was working backwards from the *Underweysung*, it is likely that the artist did not heed the original orientation of the Dürer

(*Formschneider*), ostensibly Andreae, would have placed the unfolded geometry provided to him by Dürer and laid it upon the woodblock in order to trace it. Andreae would then have excavated an area slightly larger than the traced image and then whittled the remaining material away. Getting a precisely printed straight line on the woodblock—particularly when a print is essentially a composite of multiple straight outlines like Dürer's unfolded polyhedral surfaces—is an art that attests to the deceptive complexity of translating a geometrical drawing to a print. The drawn line can be swiftly executed with the use of a straightedge, while the woodblock-printed line must be constructed and excavated mechanically, digging the wood out from two opposite sides of a line-as-boundary until the boundary becomes thinner and thinner, and at some point transforms into a limit condition that is thin enough to approximate the cut that will eventually be made with scissors, which in itself has no width at all but rather divides a paper segment into two portions. The lines are inferences—infinitely thin edge conditions, extracted from the morass of wood to represent the precisely plotted distance between the vertices of the polyhedral net. [60]

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geometry and oriented it as he pleased; a decision which has no impact on the ability to cut out the solid.

[60] Most of the Albertina blocks contain ornamental polygonal shapes composed of concentric lines covering portions of the nets' faces—none of which is present in Dürer's prints. Some of the blocks use this geometrical ornament to show where to cut the regular solids to produce the semi-regular solids (including examples of prisms and antiprisms). In essence, this means "the relations between the Platonic and Archimedean solids, and different processes for constructing semiregular solids by cutting off vertices of edges from the regular solids" (Schreiber, Fischer, and Sternath 458). These polygonal forms also would have allowed someone to cut out the internal polygons in order to make a skeletal polyhedral model out of paper. Prints from blocks like these very well might have been used as perspectival aids for the complicated skeletal drawings in *De divina proportione*, drawings which otherwise would have required an uncanny anticipation of the parts of the polyhedra visible through the holes in its faces. In *Dürers Gestaltlehre Der Mathematik Und Der Bildenden Künste*, Max Steck includes a small reproduction of a drawing of an unfolded net of a dodecahedron with ornamental polygons, which

The layout of the unfolded nets in the 1525 edition of the *Underweysung* is evidence that the interplay between Dürer's new diagrams of geometry and their descriptions was anything but simple for Andrea's workshop to decipher. The description of the pyramid – the first Solid—has been appended to the bottom of the introductory paragraph to Book IV, without the customary capital letter signifying the beginning of a definition. The description of the next Solid in the sequence—the octahedron—has been printed beneath the image of the pyramid. The placement of two definitions on one page creates a staggering effect that continues through the subsequent pages as the textual and visual description of the Solids fall out of alignment almost immediately. For instance, the description of a cube is included below the images pertaining to an icosahedron, the description of a dodecahedron is beneath the image of the cube, the description of the sphere is beneath the image of the dodecahedron, etc.[61]

Text and image are then briefly realigned on an important intermediary page, in which Dürer describes ways to manipulate the Platonic Solids (*corpora*) graphically in order to transform them into irregular solids (*ungeregulirten corporen*).

> And one is capable of drawing these bodies intersecting one another in a larger duplicate [image], so that one corner sticks into/protrudes to the other plane [surface]...Also one can place a point above or below each plane surface to which

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he has attributed to Leonardo da Vinci's work for Pacioli's *De divina proportione* (1509). No substantiating evidence is provided by Steck as to the location of this drawing, and I have been unable to track down any more information about it. However, it is highly unlikely that the drawing was executed for Pacioli as this type of unfolded geometry was not prevalent until later in the 16th century. If it exists at all, the drawing was more likely drawn or inserted into the margins of a copy of *De divina proportione* sometime after the publishing of the *Underweysung.*

[61] See Figure 1.3.

> the surface can connect should one want to create new sides [angles] for the surface.[62]

Though not represented visually, Dürer is hinting at the sorts of graphic techniques to create geometry that his followers would use to great effect in their later *Lehrbücher.* For instance, Dürer here entails how geometrical bodies may be constructed through intersecting two regular bodies, and taking the outer perimeter as the outline of the new form, or even by stretching the outline of a geometry to accommodate for a new vertex point [*spitz*].

Satisfied that the reader now understands that the nets are to be folded up to create regular and irregular solids, Dürer continues with several truncated solids of his own design (some of which would later be classified as Archimedean Solids), though these too skew out of alignment with their textual definitions.[63] The culprit here is Dürer's truncated tetrahedron, which Dürer describes as being the first body he has represented without planes that are all equal to each other—"*das nit gantz mit seinen planen gleych an einander ist*" (Dürer, Book 4, Diagram 35) [**Figure 1.6**]. The description of the truncated tetrahedron lies above its unfolded net while the description for the following solid (the snub or truncated cube) lies beneath the net, creating on each subsequent page a

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[62] "*...und dise corpora magst du von einer groß zwifach durch einander reyssen/ als so das alweg das ein eck durch des anderen planum sticht...Du magst auch aud side corper auf einen jeden plano einen spitzen punct setzen/ nider oder hoch erhaben so von vill ecken als der planus hat darauf er stet*" (Dürer 1525, Book 4, Diagram 34a).

[63] The thirteen Archimedean Solids are defined as highly symmetric, semi-regular convex polyhedra composed of two or more types of regular polygons meeting in identical vertices. The five Platonic Solids are composed of only one type of regular polygon meeting in identical vertices. Archimedean Solids are generated by evenly manipulating the faces of Platonic Solids, through truncation, expansion, and cantellation. The 1538 edition includes the first presentation in the Renaissance of the truncated cuboctahdron and the snub cube. See "Dürer's Polyhedra," Hart.

staggered pairing consisting of the image of an unfolded net above the textual definition pertaining to the image of the next net in the sequence.

The Platonic Solids were at the forefront of geometrical research by artists and mixed mathematicians in the 16th century, and using unfolded polyhedral nets as their visual definitions would have been a completely new technique of representation. While it is possible that the printer had intentionally set the woodblocks that printed each of these pages, the difficulty of juggling mathematical definitions with their visual counterpoints resulted in misleading if not outright mistaken captioning and an egregious misalignment of text and image. And yet these errors also are comprehensible as they highlight the revolutionary strangeness of Dürer's reformulation of the Solids as surfaces. The *Underweysung* was the first publication to strip the Platonic Solids of their Renaissance solidity—a solidity that Pacioli, Caraglio, Ugo da Carpi and others had struggled so assiduously to achieve in perspective—in favor of decomposing the Solids into their composite surfaces.[64] By disdaining three-dimensional *images* of polyhedra for techniques of *making* polyhedra, Dürer had reinvented geometry as something disposable that could be traced, cut out, and folded together. The Platonic Solids were no longer static, preexistent forms—they had become prototypes, available for use and reuse, opening and closing, their surfaces even stretched and compressed to achieve irregular effects.[65] After the *Underweysung*, it would have been impossible to imagine the Platonic

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[64] With that said, the decomposition of the Solids in the text of Plato's *Timaeus* forms a provocative precursor to Dürer's innovations. See introduction.

[65] The use of the word "irregular" to describe a category of polyhedral forms based on the Platonic Solids is not present in Dürer, but rather in later 16th century artists. For instance, see Lorentz Stöer in Chapter Five.

Solids—perhaps even *geometria* in general—as a transcendently stable, antique entity. Like the series of pillows in his famous drawing at the Metropolitan Museum [**Figure 1.7**], the concept of the Platonic Solid started to imply its own future manipulation into surfaces (and back again).[66]

The absence of a reference image must have made it extremely difficult for readers of *Underweysung* to visualize the products of the unfolded Solids once assembled. Perhaps Dürer suspected that perspectivally representing the Solids in his treatise would only lead to the copying of the images alone and might have overshadowed the more difficult work of tracing, cutting, and folding the nets. Dürer's innovative way of making polyhedra—of seeing them not just as graphic objects but as unfolded paper surfaces—was in service of the basic idea that a precise knowledge of geometry was essential to the training of young artists, not just the copying of a perspectival image. The required precision of cutting out and folding perfect polyhedra cannily inculcated habits and cognitive practices attuned to

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[66] If one looks for precedents concerning surface folding in Dürer's oeuvre, one need look no further than his early drawings of pillows from 1493, which depict the metamorphosis of the surface of a pillow—a singular object that bears the imprint of the prior pressure exerted on its surface and is the first of Dürer's drawing to exhibit this level of detail and extremely delicate crosshatching (*The Robert Lehman Collection* 37). The pillows are frozen in motion, a perpetual state of tension that implies prior and future movement, further disturbances and smoothings. Although Dürer could have outlined the pillows and then filled them up with cross hatching, I believe that Dürer would have drawn the hatching or the folds first, and then drawn the outline of the pillows after, or in sections clustering around the folds. Here the final shape of the pillow emerges from the interplay of texture and shadow that Dürer improvises and to an extent fixed within the frame of the pillow. While it is tempting to see a continuity of interest in folding between Dürer's folded, crumpled pillows, his depictions of fabrics and his later unfolded geometries, the rigorous assembly implied in the unfolded solids sets them apart from the intense, "naturalistic" folds characteristic of his treatment of fabrics, which could be crumpled and smoothed and arranged *ad infinitum*. See Heuer 2011 for an extended meditation on the fold and folding in Dürer's work.

exact observation and measurement.[67] Simply put, if the net of a polyhedron was not perfectly measured and drawn on paper and then just as precisely cut out, the facets of its geometry would not exactly touch when folded up. Unlike perspectival drawing, which then as now, can fool the eye into believing that an image is the exact and realistic rendering of an object from a specific angle, the precision required from the unfolded polyhedron meant that the slightest inaccuracy would be immediately noticed should the form not perfectly fit together. Folding was the standard against which the precision drawing (or copying) of a polyhedron could be used to test the accuracy of its baseline drawing and assembly. Unless the edges of the surface were connected seamlessly to each other, the illusion of solidity, and indeed the very identity of the polyhedron, made up as it is by identical polygonal faces, would be destroyed.

Furthermore, transforming polyhedra into easily reproducible surfaces made them more accessible as pedagogical/stereometric practice objects. In so doing, polyhedra must have become even more prevalent in 16th century artist workshops, particularly for craftsmen who might not have been able to afford having more expensive and time-consuming wooden polyhedra manufactured for them. Although there is almost no reference made to the type of paper used to construct them, the 1640 inventory in the *kurfürstlich-sächsische Kunstkammer* in the Grünes Gewölbe, Dresden, mentions the existence of several polyhedral models "*von türckischen papir*," "*turkischen*" being a common

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[67] This issue was resolved in later textbooks like Hirschvogel's *Geometria* (1543) in which three dimensional renders are included next to the unfolded nets from which they were to be constituted.

blanket adjective used to describe products thought to originate from the Middle East.[68] The type of high quality and heavyweight paper conducive to making models fit for the Dresden court may have been Samarkand paper or Abadi paper, though one assumes that there would have been a wide range of less-precious papers available used for the models constructed in workshops, dependent also upon whether the models were intended to be studies or final forms intended for further translation into perspective.[69]

Geometrical knowledge became portable, easy to traffic, circulate and copy on paper (of sufficient strength) and, in its portability, less mysterious, more associated with making and practice, less abstract, completely ephemeral—useful for testing, graphic experimentation, and geometrical play. Although copying a perspectival image of a Platonic Solid is a procedure that produces a certain type of knowledge—in which the body learns about the Solids by tracing their outlines and, possibly, internalizing their form—this type of knowledge is restricted to one perspective of a particular geometry. Dürer's polyhedral nets were intended to ensure that a student/artist would be forced to become familiar with a geometry from all its physical sides—not just from the image depicted by one perspectival angle. Thus, just after describing the unfolded polyhedral solids, Dürer does not demonstrate their construction and manipulation in perspective but rather uses the cube as a prototypical perspectival subject and rather curtly leaves the rest up to the reader's imagination.

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[68] Though there was little domestic paper production in the Ottoman Empire, fine paper from India, Persia, and the Levant was available in Istanbul. With thanks to Meredith Quinn (Harvard) for her insight and references.

[69] On the relation between physical models of polyhedra and their drawing in perspective, see Chapter Two. For a deeper discussion of the *Kunstkammer* in Dresden, including the polyhedral models and objects listed in the inventory, see Chapter Six.

> Since I have shown how to make several solids, I want to teach how other solids look like and how to bring them into a painting. Therefore, I will choose the simplest body, the cube, and through this show how one may also treat all of the other bodies (Dürer Book 4, Diagram 51a) [**Figure 1.8**].[70]

The relative lack of emphasis on teaching perspectival construction of more difficult objects and the absence of elaboration of any Solids in perspective, bar the cube, suggests that the geometrical components of Dürer's treatises had a different aim than its precursor, Luca Pacioli's *De divina proportione* or later Italian geometry books such as Daniele Barbaro's (1513-1570) *La practica della perspettiva* (1568), both of which relied on polyhedra to demonstrate and teach the study of perspective.[71] By not including perspectival images of polyhedra, Dürer, it seems, was able to separate his pedagogical interest in polyhedral geometry from the teaching of perspective. A polyhedron's most representative visual qualities had to do not with its image but with the number and length of its surfaces and the way its surfaces fit together. As Dürer's dedication to Pirckheimer in the *Underweysung*'s introduction states, without having learned the art of measurement (*die Kunst der Messung*) one could not become a true artisan *(rechter werckman*) (Dürer 1525, introduction).

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[70] "*So dich daf en manicherley corpora wie man die mach anzeigt hab/wil ich auch leren so man soliche gemecht ansicht wie man die in ein gemel mug pringe/se solichem wil ich das schlechtest corpus furnemen/ also den wurffel/ darpey anzeyge das man mit allen copern also handelen mag*."

[71] Barbaro was taught perspectival geometry by Giovanni Zamberti, brother of Bartolomeo Zamberti—who published the retranslated Euclid in 1505 (Cromwell 132). For further reading on the influence of Dürer's *Underweysung* in Italy see Fara 2009. See also Andersen 2007, 194-206 on Dürer's multiple perspectival strategies in the *Underweysung.*

Dürer's unfolded polyhedra push at the uncertain boundary between geometry as an abstract body of textually described knowledge and the representation of this knowledge on paper. They are representations of polyhedra without purporting to *be* polyhedra. In the *Underweysung*, the lines that make up these polyhedral nets are agents of their own erasure; edge conditions proscribing the future action of cutting and folding. By introducing media into the study of polyhedra, geometry that does not readily exist in observable nature, the *Underweysung* thematizes the possibility of making abstract concepts palpable as matter (through the coordination of paper, printed lines, and scissors). And yet, to situate a method of constructing ephemeral models of polyhedral geometry as the geometrical definition of this geometry is to tacitly acknowledge the contingency and locality of geometrical knowledge. Unlike Pacioli, who purports to define polyhedra through their images, for Dürer the translation of an abstract geometrical definition into the graphic visualization of this definition can be nothing more than another iteration of the geometrical definition. The images of polyhedral nets are not "final," nor are they a visual substitution for the textual definition. Rather, the real/physical world of drawing, printing, and making acts as a limiting condition into which the infinite possibilities of geometrical knowledge are made concrete as objects. Dürer has not created visualized definitions of geometry; he has naturalized them, as strange and wondrous products of the human hand. The *Underweysung* is a generator of paper objects that may be observed, studied, and represented as if one were drawing live from nature or while viewing the exotic collection of a *Wunderkammer*. The treatise creates and perpetuates the conditions under which the knowledge that comes from studying objects may be developed, not the knowledge that comes from stitching together a three-dimensional image of a geometry with its textual definition. Unlike the perspectival image, which mimics physicality, geometry for Dürer becomes an actual physical entity with its own innate properties—its own weight, shadow, and spatiality. Polyhedra were by extension no longer idealized abstraction; they had been naturalized as *artificialia* of the human hand.

The new polyhedral geometries and the various forms of their representation were utilized to raise the level of all German craftsmanship in order to create a new kind of craftsman who was an intellectual with his hands through the visualization of what were considered to have been the most advanced sections of Euclid's *Elements*. While Luca Pacioli's perspectival polyhedra were watershed examples of new and innovative attempts to use the power of the image to augment Euclid's textual description and make polyhedra accessible to broader audiences of interested artists, patrons, and mathematicians, Dürer went one step further in the *Underweysung*, transforming the Platonic Solids from abstract philosophical concepts, forbiddingly rendered in *De divina proportione*, into cheap, easily copiable and manipulatable artifacts.[72] In so doing, Dürer changed the nature of geometrical knowledge into something intimate and approachable, something that could be touched, handled, manipulated and experimented with. Although Dürer saw the measurement and construction of the Solids as an intermediary step in building proficiency with geometry that would ultimately aid the depiction of non-geometrical bodies, modeling variations of the Solids become a convention in its own right—first by defusing into southern German artistic circles via widely distributed

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[72] On *De divina proportione* see Chapter Two.

Lehrbücher, and then by becoming the premier representational vehicle for showing off the creation of new geometrical inventions. His treatise was the conceptual and procedural base upon which the following generation of German artists built their approach to geometrical research and experimentation. Later artists and mixed-mathematicians adopted the physicality and tactility of Dürer's geometry, developing proficiencies in drawing and printing solids, collecting polyhedra together on the page, and tweaking them into an infinite array of increasingly irregular permutations.[73] These perspectival geometries were not foreign models of Euclidean knowledge, inherited from Greek times and carefully transmitted from one generation to the next. A new strand of graphical infinity had emerged from experimentation with geometry.

From the mid-16th century onwards, the unfolded "net," signifier of material constructability, transitioned into being a constituent property of mathematical textbooks and the *Underweysung* went through multiple editions, reissued in 1538, 1603, 1604, 1606, and 1618, with copies dispersed in most major central European cities. We see Dürer's influence in books such as Peter Ramus' geometrical teachings, which moved in the direction of integrating "pure" geometrical knowledge with the knowledge gained from the mixed-mathematical experiments with geometry. Ramus' *Arithmeicae Libri duo* (1569) collected visual cues of the existing mathematical knowledge on the Platonic Solids, displaying shaded, skeletal, and unfolded polyhedra—further evidencing that

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[73] The degree of irregularity in the context of geometrical experimentation with the Platonic Solids may be considered the extent of the graphical "distance" or transmutation from the originals.

Dürer's unfolding techniques had become "standard" knowledge [**Figure 1.9**].[74] The grand *Livre de Perspective* (1560) of Jean Cousin (1490-1560/61) contains a section dealing explicitly with producing "*les cinq corps Reguliers de Geometrie*," in perspective and from a surface, as does the section "*De Solidorum Constructione*" in Christian Wolff's (1679-1754) wide-ranging *Elementa Matheseos Universae* (1742) two hundred years later still describe how to construct the Platonic Solids out of nets [**Figure 1.10 + 1.11**].[75] The constructability of the Platonic Solids as a function of their unfolded surfaces seems to have persisted as an integral part of their identity.[76]

Dürer wasn't the only Nuremberger making use of novel representational techniques to attack the standardized visualizations of geometries heretofore presumed defined. Martin Waldseemüller (1470-1520) and Johannes Schöner's (1477-1547) innovative deconstruction of a sphere or globe into twelve paper gores [**Figure 1.12**], which could

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[74] As an example, the page for the geometry of a dodecahedron in Ramus' *Arithmeticae libri duo: Geometriae septem et viginti* (1569) shows three ways of visualizing the polyhedron, including an unfolded net, and reads: 1. Dodecahedra have 30 sides, 60 angled planes, and 20 solids. 2. If twelve equal pentagons with solid angles are orderly combined, they will create (comprehend) a dodecahedron. As you see here. 1. *Dodecaedi latera sunt 30*, *anguli plani 60, solidi 20.* 2. *Si duodecim quinquangula ordinate aequalia solidis angulis componantur, comprehendent dodecaedrum. Ut hic vides* (Ramus 1569, 178).

[75] Employing labeled diagrams of the solids shown in multiple viewpoints (plan, elevation, and the unfolded "*corps developpé*"), the graphic information is collapsed together on the same page in *Livre de Perspective* and linked by construction lines, making cross-referencing different representations of the same solid easy to discern.

[76] "*Rete describere, ex quo cubus construi possit*"—"to describe the net, from which the cube can be constructed" (Wolff 233), or "*Rete pro prisimate describere*"—"to describe the net for a prism" (Wolff 234). The Elementa Geometriae section is divided into several chapters—*Principiis Geometriae; Propositionibus Quibusdam Fundamentalibus; Linearum Rectarum et Triangulorum Symptomatis; De Circuli Symptomatis; De Figuraram Descriptione; De Figurarum Dimensione Ac Divisione.* A special *(Pars Posterior)* section entitled *Elementa Geometriae Solidae Proponit contains three chapters on Principiis Geometriae Solidae, De Sectione et Situ Planorum, and De Solidorum Constructione.*

be cut out and reassembled onto a wooden substructure to form a representation of the earth or celestial sphere, may have influenced Dürer's own initiative to unfold the Platonic Solids. In the *Underweysung* as in Schöner's stretching, flattening and rolling up of the maps of the earth and stars, geometrical form became more fluid, inclusive of implied conceptual movement and manipulation in which the media-graphic form was itself in flux without being considered to have changed the essential identity of the geometry-object. Here the image is a temporary resting point—the fixing into a singular representation of a potentially endless process of reinvention, of folding, of transforming solids to surfaces and back again.

The epistemological implications of unfolding the Platonic Solids were many and various. Now mutable and even mutatable graphically, the Solid was no longer an actual guarantee of solidity. The seeming reality of the perspectival image had been traded for the flexibility and indeed the fragility of a paper model. What were the properties and significations of this interior territory that had eluded the hegemony of the perspectival gaze and had been released by the transmutation of geometry from solid to surface? In their unfolding, as both event and material technique, Dürer's polyhedra emptied out their allegorical content—the crabs, tridents, guns, winds, compasses and all the other items associated with the five elements—and reduced the Solids to their formal properties. This was an act of violence disguised beneath the inoffensive manual operations of folding and unfolding, opening and closing. The Platonic Solids had been ripped apart and glued back together in order to reveal a secret space of possibility and artistic experimentation, substantively free from the elemental and cosmological significations associated with geometry since Plato, free from the weight of antiquity. Geometry was changing from laborious image to disposable and covetable thing—easily substituted, easily reinvented. By eliminating the eternalness of the Solids, Dürer had discovered a new kind of infinity bound to their perpetual destruction. Even as the freedom to invent and distort, to rotate and handle, to crush and dispose that so many later German artists and mixed-mathematicians made use of in their geometrical investigations signified the disintegration of the stability of inherited geometrical knowledge, the Platonic Solids remained both point of departure and inescapable end.

Chapter Two: Beyond the Platonic Model

Within the use of geometry by 16th century mixed-mathematicians and artists, the Platonic Solids constituted a unique genre of investigation and preserved a privileged status as objects of fascination. That each of the Solids was constituted by exactly duplicate faces meant that they were deemed to be models of a quintessentially measurable and quantifiable knowledge of geometry; an antidote to the flux of Aristotelian "natural particulars" and the consequent classificatory impulse of natural philosophy.[77] The Solids could in theory be exactly constructed on paper as per Euclid's instructions, or unfolded and glued together as per Dürer's, and thus the dimensions of a drawing of a Solid could be measured to determine the deviation from the dimensions of a precisely regular, if idealized, Solid. To precisely draw a Platonic Solid was heralded as a critical first step in the formation of the ability to be able to successfully translate other more irregular objects, designs, or phenomena onto paper with accuracy—a skill that was necessary for any profession invested in the practice of precise description and measurement. To master the Solids' three-dimensional representation was to translate the epistemological certainty of their Euclidean definitions into the visual realm, and in turn to communicate the potential of images to be exactly duplicative of that which had been imaged. With the rise of interest in and value of mixed-mathematical knowledge in the 16th century, physical models of Platonic Solids were adopted as commonplace objects in artist's workshops, particularly for the training of abilities in the geometrically modulated study of perspective. These objects served a pedagogical function as "perspective tools;"

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[77] See A. Blair, "The *Problemata* as a Natural Philosophical Genre," *Natural Particulars: Nature and the Disciplines in Renaissance Europe,* MIT Press, 1999, 171-204.

drawing aids that allowed geometers to set out the base lines of a Platonic Solid in perspective, which could then be embellished by hand; tools for drawing precise representations of themselves.

The practice of mathematics, which in the classical tradition of the *quadrivium* had been composed of astronomy, music, arithmetic, and geometry, had already begun in the middle ages to expand into new branches of mathematical knowledge that sought to apply theory to practice—such as in the science of vision (*perspectiva*), the science of weights, and the science of "engines" (*ingeniis*) (Park 2011, 361).[78] Nevertheless, it was the 16th century that witnessed the greatest explosion of interest in what would come to be known as mixed-mathematics or "the middle sciences." This cluster of disciplines all explicitly concerned with the application of geometry to the definition, prediction, and representation of the physical world emerged as the premier driver of upward professional and social mobility in the sciences, vastly increasing the number of working practitioners and multiplying the sites, and texts, in which geometry was considered indispensible.[79]

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[78] For reference see K. Park, "Allegories of Knowledge," *Prints and the Pursuit of Knowledge in Early Modern Europe*, S. Dackerman (Ed.), Cambridge, MA: Harvard Art Museums, 2011, 358-365; V. R. Remmert, *Picturing the Scientific Revolution: Title Engravings in Early Modern Scientific Publications*, Philadelphia: St. Joseph's University Press, 2011; A. Mosley, "Objects of Knowledge: Mathematics and Models in Sixteenth-Century Cosmology and Astronomy," *Transmitting Knowledge: Words, Images, and Instruments in Early Modern Europe*, Oxford: Oxford University Press, 2006, 193-216; S. Hauschke, "The Mathematical Instruments of Wenzel Jamnitzer (1508-1585), *European Collections of Scientific Instruments, 1550*-1750, Leiden and Boston: Brill, 2009, 1-13; J. Bennett, "The Mechanical Arts," in K. Park and L. Daston (Eds.) *The Cambridge History of Science*, Vol. 3: *Early Modern Science,* Cambridge: Cambridge University Press, 2006, 673-695; and A. Payne, *The Telescope and the Compass*, Leo S. Olschki: Florence, 2012.

[79] See Introduction for a list of mixed-mathmatical fields.

In its continual and uninterrupted classificatory use dating back to the system of cataloging used by August the Younger, Duke of Brunswick-Lüneburg (1604-1666), the large and expansive subject heading entitled *Geometrie* at the famed Herzog August Bibliothek in Wolfenbüttel preserves the breadth of 16th and 17th century mixed-mathematical knowledge. Included are numerous translations of Euclid alongside books on embroidery, architecture, goldsmithery, weaponry, optics, astronomy, cartography, surveying, machine design, mathematical *Lehrbücher* for teaching geometry, arithmetic, and perspective. The depth of Duke August's collection of *Geometrie* is firstly indicative of the overarching importance placed by sovereigns on the authority of measurement and consequently the tools and texts used for measuring (Korey 17). But moreover, the transferability of geometrical skills from one application to another enabled similar sets of graphical techniques and tools to map, define, and predict on paper the performance of real world phenomena as geometrical entities whose scales ranged from the micro to the macro. Whether in the exacting construction of alphabetical letters from Christoph Stimmer's *Alphabet* (1549) or the determination of the position of celestial bodies as the basis for constructing linear perspective, as in Caesario Caesariano's edition of Vitruvius' *De architectura* (1521), the ability to conceive and accurately construct geometry in three dimensions was responsible for the dramatic shift between the painterly approximations of reality lambasted by Albrecht Dürer in his *Vier Bücher der menschlicher Proportion* (1528) and the mastery of geometry required to create exact and measurable representations. Mixed mathematical treatises and manuscripts from this period invariably began with common geometrical exercises along the lines of Euclid's *Elements* and progressed towards the elaboration of bodies of specialized knowledge bound together by geometry's ubiquitous applicability. To learn geometry was to learn the basic tools to operate effectively in multiple professional contexts. One was not solely an architect or a goldsmith or a surveyor. To be a practitioner of geometry was to work in diverse, yet related, fields defined by differences in locally situated, and situational, opportunities.

Knowledge of geometry was the basis of all mixed-mathematical knowledge in this period, partially explaining the collection of seemingly disparate *geometria* typically covered by authors in a singular book. Mechanical parts and ornamental motifs could be drawn on pages preceding calculations for fortification designs; astronomical diagraming could be prefaced by perspectival construction. Function remained fluid, united by geometrical principles. As the pedagogue Petrus Ramus (1515-1572) put it, "the description and measuring of the Starres, Countries, Lands, Engines, Seas, Buildings, Pictures, and Statues or Images… [needs] the help of no other art but of Geometry" (Ramus 1636, 1-2).[80]

Over the length of the 16th century, polyhedral variations on the Platonic Solids began to migrate into a diffuse range of media, from intarsia to ivory towers turned on a lathe. The distance between the rigid, text-heavy strictures of Euclid's *Elements* and the geometrical permutations that appeared in Germany, or even the legacy of the representation of geometrical bodies in Europe—such as the stacked polyhedral geometries in Christoph

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[80] The quotation is from Beadvvell's translation of Petrus Ramus' *Via regia ad geometriam*. "But this end of Geometry will appeare much more beautifull and glorius in the use and geometricall works and practice then by precepts, when thou shalt observe Astronomers, Geographers, Land-meaters, Sea-men, Enginers, Architects, Carpenters, Painters, and Carvers, in the description and measuring of the Starres, Countries, Lands, Engines, Seas, Buildings, Pictures, and Statues or Images to use the help of no other art but of Geometry."

Andreas Nilson's *Anleitung zur Linearperspective* (1812)—is *so* vast that to account for their difference *must*, at the outset, account for how the Platonic Solids changed from being accepted as the most complex of the text-bound geometries outlined in Euclid at the beginning of the century into distributed geometrical forms adopted and multiplied by artisans and mixed-mathematicians across Europe by the century's end. The development of these forms and approaches to form originates in post-Reformation Germany, emerging from hubs of precision manufacturing and artisanal craftsmanship in German courts and free cities such as Nuremberg, where geometrical knowledge was not only developed and cultivated as a form of proto-scientific and artistic practice, but also became a prominent product within the circulation of goods and knowledge among courts and cities at the time. In fact, the wealth and intellectual wealth of a court or city came to some extent be measured according to its ability to map itself onto the Platonic Solids. Carpeting lavish decorative art objects, adorning the surfaces of domestic interiors and courtly *Kunstkammern*, and collected together into geometrical compendia, polyhedra are records of a unique category of Renaissance mathematical knowledge, in which novel fruits of geometry were wondered at and collected, much in the same way that exotic objects from the New World adorned the salons of those wealthy enough to procure them.

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Physical models of the Platonic Solids were popular 16th century workshop tools for professionals working with geometry on either side of the Alps. But it was in southern Germany that the Platonic Solids began to morph into a broader range of polyhedral shapes. The new German geometries resembled, without necessarily being, classic

Platonic or Archimedean Solids, and were invariably represented as physical objects that borrowed the spatial characteristics of the workshop models to substantiate the conceit that they were images drawn after real objects [**Figure 2.1**]. Some of the earliest examples come from the artist and satirist Erhard Schön and his fellow Nuremberger, Peter Flötner (1485-1546), who produced a small perspectival drawing of a geometrical solid balanced atop an ornamental column base, substituting the classical sculpture one might expect for a modern paradigm of beauty and proportionality; the juxtaposition of the classical aesthetic of antiquity with the fruits of the new research into perspective and geometry. Capturing the spirit of the times, Schön's so-called "A nude man measuring a block of stone" from 1530 [**Figure 2.2**] depicts a naked man using a compass to measure the proportions of the alien body before him. A play on the German word *Körper* (body), in reference both to the human body and to the common term for the Platonic Solids or "regular bodies" (*corpora regulata* or *regularia*), the image shows how the same tools for precision drawing and measurement were being used to invent new forms on paper, geometrical as well as human.

Prior to the emergence of "irregular" solids, "*irregulata*" in the later terms of Lorentz Stöer or "*ungeregulirt*" as per Dürer, like Flötner's within German workshops, there would have been no reason to distinguish *corpora* from each other in terms of their regularity. There *were* only regular *corpora* for the very reason that geometrical regularity was the definition of what it meant to be a *corpora.* The category of *corpora irregulata* introduced a new measure of artistic freedom into the representation and creation of polyhedral geometry. These solids were no longer limited to being composed of equal polygonal faces (as per the definition of the five Platonic Solids), but could be graphically augmented and tweaked to generate new geometrical inventions. And thus it was newly necessary to distinguish them, as Dürer did in the *Underweysung*, from "those called regular bodies [*corpora regularia*] by Euclid."[81]

Still the *corpora irregulata* maintained a connection to their regular origins. The artist Lorentz Stöer's bound collection of graphic geometrical models in Munich begins with the representative title phrase "*Die Funff Corpora Regularia, auff Viel und Mancherley Arth und Weis Zerschnitten*"—"The five regular bodies, cut apart in a variety of ways"—referring to the multitudinous transformations undergone by the Platonic Solids under Stöer's hand. For Stöer, as for the other German artists and mixed-mathematicians working in the long shadow of Dürer, the Platonic Solids were not unimpeachable Euclidean proofs to be followed, but rather served as the conceptual framework for organizing open-ended practices of graphic experimentation. The Augsburg stone engraver Peter Halt's *Perspectivische Reiss Kunst* (1625), a compendium of geometrical models on ornamental bases, is also grouped together into sets of variations based upon the Platonic Solids.[82] More unusually, *Perspectiva corporum regularium* (1568) by the

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[81] "*die der Euclides corpora regularia nennet.*" Introduction to Book IV, *Underweysung der Messung.*

[82] Several of Halt's unfolded polyhedra are copied from Dürer's *Underweysung der Messung*, and there are comparable geometrical forms to those found in Danieli Barbaro's *Perspettiva* (1568), Lorenzo Sirigatti's *Prospettiva* (1596) as well as Jamnitzer's *Perspectiva*, implying that Halt was aware of all of these works. For example, No. 84 in Halt = diagram 37 in Dürer; no. 91 = D. 38; no. 102 = D. 36; no. 109 = D. 39, but rotated vertically; no. 120 = D. 41; No. 102 = D. 42; no. 131 = the mirrored image of D. 40. All of the Halt diagrams are, however, smaller than the Dürer drawings. The unfolded form of 140b has been copied from Barbaro, no. 27 on page 82, but rotated slightly to the left. Halt's *mazzachio* (No. 161) resembles Barbaro's on page 61; Halt's spiky sphere (No. 170) is similar to Sirigatti's on page 36 of the second book of *Prospettiva.* Halt's conical variants resemble similar forms in Jamnitzer's *Perspectiva.* Halt also includes series of unfolded geometries not found in Dürer—testament to the geometrical advancements since the publication of the *Underweysung.*

goldsmith Wenzel Jamnitzer, a compendia of polyhedral geometry in which the Platonic Solids similarly served as the inspiration for pages of explorative iterations, insists on the title *regularium* applied to the new solids, though this has likely to do with Jamnitzer's desire to associate his invented geometries with their elemental properties as per Plato rather than with the geometrical form of the new solids themselves.[83] The implication of the organizational strategies employed by Stöer, Halt, Jamnitzer, and others, is that the same polyhedron was capable of generating an infinite number of irregular variations, united only by their relationship to the Solids.[84]

Even if the definition of what exactly irregular geometry entailed was necessarily never fixed, *corpora irregulata* maintained their relation to the Platonic Solids, though the nature and strength of the relationship was determined by the individual artist. A notebook by Stöer at the *Staats- und Stadtbibliothek* in Augsburg includes thirteen

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[83] In the introductions to each of his five sections of variations on the Platonic Solids, Jamnitzer does not refer to his new solids as irregular. Rather, he consistently stresses their transformation from the original solid and the extent of their difference. On the tetrahedron, he claims that "from this triangular body or pyramid are further generated in drawing twenty three other bodies of various different kinds." "*Auß disem drianglichten Corpora oder Kegel sind verner drey und Zwaintzig ander Corpora geursacht/ und of mancherley unterschidliche Art zu werckh gezogen*" (Jamnitzer AI). He describes the subsequent variations on the octahedron as "forming 23 other different bodies." "*Werden hernach 23. Andere unterschidliche Corpora formiret*" (Jamnitzer AVI). The dodecahedron has "produced twenty three different bodies of various kinds and shapes [or models/figures/patterns] which have been brought into perspective." "*auß disem warden auch hernach dreyundzweintzig unterschiedliche Corpora mancherley art und form furgerissen und in die Perspectif gebracht*" (Jamnitzer DIII).

[84] Halt also provides his readers with the capacity to graphically reproduce his geometrical models in views other than the one in which he chose to draw it. In Halt, the page facing each geometrical model includes a planimetric and sectional representation in shorthand. For instance, the image of the model on No. 78 has been drawn in wireframe on No. 77 with only one side of the skeletal polyhedra represented. This implies that the pentagonal "cutouts" on the model should be drawn on all sides of the base polyhedra in order to represent No. 78. The five-pronged base and the pyramidal points holding up the polyhedra are also cursorily represented.

drawings depicting shaded mathematical models, the first of which is dated 1595.[85] All the model pages include the phrase "*Geometria et Perspectiva* (*Corpora*)" while several have the added subcategory "*Hexahedron*" (five drawings in total) and geometrical equivalent "*Cubus*" (one drawing) [**Figure 2.3**].[86] These six models, all of which are classified by Stöer as six-sided polyhedra, are superficially so different that without their labeling it would be challenging to intuit a direct relationship between them, let alone to locate them as belonging to the same geometrical category —hexahedron/cubus. For Stöer however, it is apparent that the limits of early modern geometrical morphology—what counted as the same or same enough—was an adequate concept to accommodate a wide degree of visual variation, even if variation did not imply the capability to mathematically transform one object into the other. Rather, it is likely that these geometrical typologies helped to organize Stöer's creative process, enabling a method to locate the drawings within groups, and providing a conceptual framework to produce further graphic experiments on the typology of his choice.

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The use of polyhedra by Renaissance artists divides into two parallel tracks that together expose a fundamental tension between geometry as a stereometric aid in the representation of the human body and nature, and the emergence of an interest in the representation of polyhedral geometry "in and for itself," as demonstrated by Stöer.[87]

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[85] Staats- und Stadtbibliothek Augsburg, 4 Cod. Aug. 247.

[86] The hexahedron and the cube are both six-sided polyhedra.

[87] Kant's "*Ding an sich*" or "thing-in-itself" (*nuomenon*) is what is not given to the senses by way of sense perception (what Kant calls *phenomenonon* but rather, in a similar manner to the Solids,

The first strand is typified by Dürer's *Vier Bücher von menschlicher Proportion* (1528), as well as the work of Erhard Schön (1491-1592) and Heinrich Lautensack (1522-1590), though perhaps their earliest precursor is the French craftsman and architect Villard de Honnecourt (born ca. 1175), who worked with basic stereometric geometries overlaid on natural forms [**Figure 2.4**].[88] *De divina proportione* by Luca Pacioli and the work of Ugo da Carpi (ca. 1480 – 1520/1532) and Jacopo Carraglio (ca. 1500/1505 – 1565), as well as Dürer's *Underweysung der Messung* (1525), are representative of the second mode of polyhedral drawing –in which the polyhedron is the explicit subject of the image.

The *Vier Bücher* was conceptualized by Dürer as a more advanced treatise on drawing to be studied after the *Underweysung*.[89] Learning how to represent human bodies as

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exists in the world without, or prior to, sense perception. Kant's theory of transcendence and transcendental judgment would ensue from this clandestine *ding* that is impossible to locate and yet must be taken as existing. See Kant, Critique of Pure Reason 161-162 [A29/B45-A31/B46].

[88] The work of Villard de Honnecourt survives on 33 sheets of parchment in the *Bibliothèque Nationale* in Paris. Villard includes several stereometric figures composed of triangles as well as simple linear geometry superimposed on top of animals and human heads. Villard claims in this document that "the art of the lines/methods of drawing as taught by the discipline of geometry facilitates work" (Bucher 112), though, it needs to be said, his figures do not seem to be well integrated with the geometry. It is hard to see that the geometry enabled the figures to be drawn over and around it. The sheep on Figure 2.1 is a good example—the geometry is too basic to give rise to such a complicated form. It may well be that the geometry was added after the sheep was drawn as a form of analysis that showed the shapes that could be found in objects, rather than being the result of a specific geometrically-inflected process of drawing. It has been argued that the superimposition of geometry and, in certain cases, grids, over figures is indicative of an approach to figural geometry that carries over from architectural design methods—such as that of Hans Hösch's *Geometria Deutsch* (1472), Erhard Schön's *Unterweisung* (1538-42), and Juan de Arfe z Villafane's *Commensuration* (1558), all of whom pursued a certain kind of reciprocity between the geometry of bodies and buildings (Bucher 116). For further reference on Villard de Honnecourt, see the introduction (15-39) in Bucher, which also includes a reprint of his drawings with descriptions per page, as well as the multiple introductory essays in *Carnet de Villard de Honnecourt.* A. Erlande-Brandenburg, R. Pernoud, J. Gimpel, and R. Bechmann, Paris: Stock, 1986.

[89] In his dedication to Pirckheimer in the *Vier Bücher*, Dürer declares: "In order that these instructions be better understood, I have issued a book about measurements which describes lines,

polyhedral volumes was essential to Dürer's system of drawing, and his early stereometrical studies and sketches for the *Vier Bücher* depict bodies as semi-abstract figures composed partially, or entirely, of geometric solids in various poses and states of animation, revealing Dürer's developing and experimental interest in the use of proportional ratios as a means of determining the human body.[90] These drawings show Dürer grappling with generating geometrical ratios of body proportions using a compass, though he eventually moves towards systems of ratios that balanced the dimensions of body parts to each other, sometimes even surveying the ratios of the body's parts together in elaborate detail.[91]

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surfaces, solid bodies, etc. [the *Underweysung*] Without this book my instructions may not be fully understood. It is therefore necessary for anyone who wishes to engage in this art to first be well acquainted with measurement. He should know how to draw the ground plan and elevation of an object in the matter employed by skillful stonemasons for daily use. Lacking this, he might not comprehend my teachings in every respect" (Dürer and Strauss 1974, Vol. 4, 2384, translated by Strauss).

[90] Stereonomy refers to the use of readily measureable solids or polyhedra. The adjective "stereometric" is a common art historical term used in the description of figures, such as those Dürer composed from smaller volumes. Contemporary notes contained with Dürer's papers at the British Museum, which Conway claims are not in Dürer's own hand, though in my opinion they can be ascribed to him, acknowledge the difficulties of representing the human body in opposition to the regularity of the geometry taught in Euclid. "Seeing also that the 'measure' of a human figure is specially hard to comprehend, amongst other reasons because the human figure is composed neither by rule nor compass but is contained within irregular curved outlines, it is specially hard to write and treat of it. Those persons acquainted with Geometry well understand, because Euclid's books of Geometry deal only with straight lines and circles and teach how to measure the bodies contained within them, such as Plato's five regular bodies, cone and cylinder, and no others; and that measure, too, is employed not to describe or paint them but to reason about what they contain or encompass" (Dürer, translated in Conway 175). With reference to the Dürer manuscripts at the British Museum, Vol. 4, 132a and b. Many of Dürer's geometrical studies are contained in the so-called *Dürer Skizzenbuch* in the *Sächsische Landesbibliothek* in Dresden.

[91] For instance, "the foot is one seventh of the entire height. Its height equals one third of its length. The ankle is at the midpoint of the height of the foot. The toes measure a third part of the length of the foot" (Dürer 1972, 60). In these later proportional systems, Dürer used elements from Vitruvius which he nevertheless selectively modified with drawing instruments to suit his own aesthetic sense, though as Pollmer-Schmidt claims, "the principle of achieving harmony

The bodies in the *Vier Bücher* are plastic, mutable entities, composed of graphic shapes that provide little resistance to visual dissection, and may be manipulated, sliced, or stretched to achieve the desired aesthetic effect. In connection with an illustration of a stereometric body in elevation and plan in *Vier Bücher*, Dürer himself remarks that the stereometric method "may be useful for sculptors beginning to learn this craft who intend to cut a figure from wood or stone. In order to copy a figure exactly they can chop away from the square surfaces what is necessary, without cutting off too much or leaving too much."[92] Two heavily faceted, stereometric heads from Dürer's *Skizzenbuch* [**Figure 2.5**] dated 1519 may even illustrate this intermediary step in the production of lifelike sculptural figures.[93]

In order to be reformed into workable parts, the body is disassembled into more easily manageable sections and set within polyhedral geometries that confine and define it volumetrically [**Figure 2.6**]. These geometries, which are easier to manipulate in three dimensions than irregular body parts, are then rotated to reveal multiple sides of the body part on the graphic space of the page, such as in the example of the different sides of the

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through relationships and analogies and defining distances through fractions of body length...can also be traced to Vitruvius" (Pollmer-Schmidt 123). Dürer mentions Vitruvius in a 1523 letter to Pirckheimer, in the context of complaining about how Jacobus (ostensibly Jacobo de' Barbari) (ca. 1460/70 – before 1516) "showed me how to construct a man and a woman based on measurements...But Jacobus, I noticed, did not wish to give me a clear explanation." Thus Dürer took matters into his own hands and "read Vitruvius, who has written a bit about human limb proportions."— "*Doch nam ich mein eygen ding für mych vnd las den Fitrufium, der beschreibt ein wenig van der glidmas eines mans*" (Rupprich 1956, Vol. 1, 102).

[92] See Rupprich 1969, Vol. 3, 261 for original German quote. Translated in Dürer 1972, 208.

[93] Pollmer-Schmidt points out that some of Dürer's stereometric figures may have been drawings of mannequins. One such figure in the *Skizzenbuch* appears to have ball joints. See Pollmer-Schmidt 124-125, Munro 15-16. See also A. Weixlgartner, "Dürer und die Gliederpuppe," *Beiträge zur Kunstgeschichte. Franz Wickhoff gewidmet*, Vienna, 1903, 80-90.

upper part of a man's torso.[94] The anatomical studies of Lautensack's *Des Circkels unnd Richtscheyts* (1564), much of which was based upon the *Vier Bücher*, extend Dürer's stereonomy by diagrammatically exploring the relationship between the invisible linear elements that support and articulate the graphic representations of the human body [**Figure 2.7**]. The central figure in Lautensack's stereometric drawing is flanked by two ephemeral linear constructions. On the left, Lautensack has included a skeleton—that which lies beneath the skin and supports the overall form of the body. On the right stand the geometrical outlines into which the body's form can be graphically inscribed and then graphically dissected in parts, as Dürer says, "because in each polyhedron (*eckigen Corpus*) all parts, points, and lines of the body will be easily displayed."[95]

Even as stereometric experimentation with geometry began to infiltrate studies of the human body and graphically flirt with the techniques required to physically produce sculptural bodies, polyhedral geometry was materializing into a distinct genre of representation, separate from any use it had as a stereometric drawing aid. The newly visualizable Platonic Solids inspired perhaps the most prominent collection of early modern polyhedra—Luca Pacioli's *De divina proportione* (manuscript ca. 1496-1499, first printed edition 1509), which in its detail, coloration, and morphology, far exceeded the visual impact of Ratdolt's Euclid. Though the drawings are not integrated into the

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[94] Drawing a singular figure from multiple viewpoints may have been a way for Dürer to compete with the experiential possibilities of sculpture. See for instance Dürer's *Four Nude Women (The Four Witches)* (1497) and the print's analysis in Schoch, Mende, and Scherbaum, Vol. 1, 61, and *Albrecht Dürer: His Art in Context*, (edited by Jochen Sander) 130-131, as well as the drawings by Master PM and Jacopo de' Barbari on 126-127 and 132-133.

[95] "*Dan in einem yedlichen eckete corpus mogen alle teyl/ puncten un linien des leybs gar leychtlich angezeygt werden*." Dürer 1528, Book 4, fol. V3r.

main text of *De divina proportione*, the size and visual attention commanded by the Pacioli's polyhedra communicates the artist's clear intention to visually embody the graphic potential of the accurate perspectival construction of polyhedral geometry [**Figure 2.8 + 2.9**].[96]

None of the other visualizations of the Platonic Solids would have prepared Italian audiences for the proficiency and color-saturated elegance of the *De divina proportione* manuscript. Compared to the simplistic geometrical diagrams contained in 15th and 16th century editions of Euclid's *Elements*, these polyhedra would have been experienced as dazzlingly novel. The manuscript included three-dimensional skeletal solids adjacent to the renderings of the regular solids, a conceit that would have required the artist to accurately visualize the view through the polyhedra. As if to accent their realness, all the polyhedra in *De divina proportione* are suspended in space from strings tied to rings hooked to the underside of nameplates at the top of each page. The hanging strings are taught and twisted, connected via rings drilled into the polyhedra's top facets or via elaborate knots tied to several linear segments for stability [**Figure 2.10**], as if the

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[96] It remains a point of contention whether the manuscript copies, which are in Milan and Geneva, can be attributed to Leonardo, who Pacioli had met at the court of Ludovico Sforza in Milan where Pacioli was appointed to teach mathematics from 1496-1499 (Cromwell 123). The order of the polyhedra in the Geneva manuscript is different than that of the printed volume. I would contend, as in the case of Lorentz Stöer, that there was an original order which more closely approximated the order of the printed edition—in which there is a coherent and gradual move towards complexity, from one polyhedral category to the next. In the Geneva edition, for instance, "*icosohedron elevatvs solidvs*" and its companion "*ycosohedron elevatvs vacvvs*" appear out of sequence between a tetrahedron and a hexahedron—which is the order in the book. Thus, although I would need to see the book into which the Geneva *De divina proportione* was bound, it could very well be that the volume was compiled later in a way that did not accurately reflect the intellectual and conceptual conceit of grouping similar polyhedra together. On Pacioli and patronage for *De divina proportione* see Azzolini 116-117. Field mentions that actual 3d models would have accompanied presentation copies of the manuscript versions of *Divina proportione* and that Leonardo's drawings could have been substitutes for these models, though no references are provided (Field 2005, 125).

polyhedra were depictions of physical models gently rotating in space, while the excess string has been tied into an ornamental bow gently framing the polyhedron's nameplate.[97]

Pacioli was still in residence at the Sforza court in Milan, completing work on *De divina proportione* (Cromwell 124), when Jacopo de' Barbari (1460/70 – 1516) painted his famous *Portrait of Luca Pacioli* (ca. 1495-1500) [**Figure 2.11**]. Like the manuscript drawings, the painting prominently features a dangling polyhedron, in this case a hanging glass rhombicuboctahedron, half-filled with water in order to further dramatize the volume of its container. On the table below, Pacioli is shown drawing a geometrical diagram (possibly a tetrahedron inscribed in a circle) using a copy of Ratdolt's Euclid as reference, (Mackinnon 134) which is itself reflected in the lower right and upper left facets of the glass rhombicuboctahedron, and a wooden model of a dodecahedron.

Similarly, an engraving depicting the philosopher Diogenes by Giovanni Jacopo Caraglio (ca. 1500/1505-1565) after Parmigianino, shows Diogenes pointing towards a reproduction of a dodecahedron from the 1509 printed edition of Pacioli's *De divina proportione* while staring at another book, which is not identified [**Figure 2.12 + 2.13**].[98]

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[97] There are numerous examples of polyhedra by Leonardo. See Foglio 518r, 849r, 930r, and 1040r in the *Codex Atlanticus* in the *Bibliothek Ambrosiana* in Milan and MsE 56r in the Leonardo Manuscripts, located in the *Bibliothèque de l'Institut de France*, Paris. 930r in the Codex even shows an icosahedron drawn by Leonardo with a hole drilled into its upper triangular facet through which a string to hang it from has been threaded.

[98] There is an earlier edition of the *Diogenes* engraving that omits the dodecahedron—see Gnann 2007b 125-128. A drawing of *Diogenes* in private collection in New York also depicts the main figure pointing at a blank book (Ekserdjian 2008, 369-372). Though Diogenes had a harsh opinion of mathematicians, he had written a biography of Plato contained in his *Lives and Opinions of Eminent Philosophers* that was commonly reproduced at the beginning of 16th century editions of Ficino's *Opera*. It could well be that Caraglio is referencing Diogenes as a biographer of Plato rather than any innate connection between Diogenes philosophy and mathematics or geometry.

Given that this edition of *De divina proportione* was the only printed book to represent shaded and skeletal polyhedra side by side in three dimensions, the dodecahedron would have been recognized by knowledgeable contemporary audiences as a direct reference to Pacioli and indeed Plato. That Caraglio referred to a real edition is substantiated by the matching four segments of diagrammatic lines that frame the dodecahedron. They correspond in location to the four sets of text on each of the Pacioli pages—which title the dodecahedron in Greek, Latin, and Italian—the last being the page number. Ostensibly, Caraglio is implying that Diogenes is consulting a copy of Euclid's *Elements*, the book propped open in front of him, using the polyhedral illustrations in *De divina proportione* to explicate the content of Euclid's text. The relative sizes of the physical books available to Caraglio at the time seem to substantiate this theory. The 1482 edition of *Elements* published by Ratdolt was 32 cm (12.6 in) in height and both Pacioli's 1509 edition of *Elements* and Zamberti's 1509 edition of *Opera* by Euclid were 30 cm (11.8 in) high. By comparison, *De divina proportione* was a slightly smaller 27 cm (10.6 in), a size differential born out by the scale of the books depicted in *Diogenes.* Directly confirming the relationship between the (mathematical) text he was reading and its accurate, perspectival representation, the stick in Diogenes' hand points directly to the center of the dodecahedron, the image that allowed him to more fully grasp, and visualize, the contents of the text.[99]

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[99] See Popham Vol. 2, plate 109, no. 107 for sketches at the Uffizi relating to the Diogenes print. Ekserdjian 2006, 213 identifies the studies on the same sheet as including Diogenes' right hand, arm and left leg. On a sheet of paper, within the image, is scrawled the word *giometria* standing in for a future dodecahedron that will be added in later. It is a textual placeholder. There is also another contemporary chiaroscuro woodcut print of Diogenes by Ugo da Carpi after Parmigianino (1503-1540)—Date 1520-1530. Metropolitan Museum of Art, Acc. Num. 17.50.1—from four blocks of grey-green ink. This Diogenes is much more expressive, swathed in color that

The models in the Pacioli portrait aid the understanding of Euclid's text, much in the same way that the impetus to invent and print diagrams alongside Euclid's suppositions were intended to help readers master the difficult material. Both Barbari and Caraglio depict the act of learning to relate the visual properties of three-dimensional polyhedra to their description in Euclid as a method of decoding and working through Euclid's construction of polyhedra. But if polyhedra served the purpose of clarifying Euclid, they did so by being present and available for consultation in the study or workshop of an artist/mathematician by virtue of their inclusion in printed editions of Euclid or in *De divina proportione*. However, did they also exist as physical objects, as the physicality of the renderings in *De divina proportione* and the portrait of Pacioli seem to imply?

After having widely scoured depictions and descriptions of geometrical craftsmanship in the Renaissance, the evidence appears to suggest that polyhedra were indeed commonplace objects in 16th century workshops, even though the remaining visual examples are limited. This may attest to their relative invisibility, or ubiquity, except in those instances when an individual's working knowledge of geometry was intentionally foregrounded. In the foreword to his *Geometriae practicae novae et auctae tractatus* [I-

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accentuates the visceralness of Diogenes' muscular body and the billowing clothe that is draped over him. The book he is pointing to in the image is generic, with a beige block and lines standing in for the text of the book. We have no idea which texts he may be consulting. The stick that he is using may be pointing at the text, or he may even be using it to prop open the book, though certainly for reference. Ekserdjian mentions that if the chiaroscuro woodcut had been completed first, it would likely have been Parmigianino who would have been responsible for the additional elaborate elements in the engraving—i.e. the adding of the background landscape behind the featherless chicken (a reference to Diogenes' mocking of Plato's definition of Man as a "featherless biped"), as well as the dodecahedron in the book, the rocks and plants near his right hand, the mouse on the edge of the barrel, and the standing lamp. Conversely, if the engraving was done first, then the chiaroscuro involved a process of simplification—not meant to deal with "minute particulars of the sort engraving thrives on" (Ekserdjian 2006, 220). For Ekserdjian's discussion of the two prints and the relationship between Parmigianino, Caraglio, and Ugo da Carpi see Ekserdjian 2006, 219-220.

IV], printed in Nuremberg in 1641, Daniel Schwenter (1585-1636) confessed to having created regular bodies from paper, wood, and stone in his youth.[100] Vittore Carpaccio's (1460-1520) sketch of a scholar at work in Moscow's Pushkin Museum displays polyhedral shaped objects hanging from strings above an artist/scholar's workspace, possibly astrolabes, armillary spheres, or even geometrical models [**Figure 2.14**].[101] The mathematician Johannes Faulhaber's (1580-1635) *Neue Geometrische und Perspektivische Inventiones* (1610) includes an image of the five Platonic Solids depicted as tangible objects on hooks above the door of a studio, in which a man is working on a perspectival drawing of a cube [**Figure 2.15**].[102] Damiano Zambelli's (1480-1549) *Tool of Intarsia* (1538) on the choir door of the Basilica di San Domenico in Bologna shows a simple wooden polyhedra alongside several measuring and chiseling instruments [**Figure 2.16**].[103] The painting *Nürnberger Schreibmeisters Johann Neudörffer und eines Schülers* (1561) by Nicolas de Neufchâtel (1539-1573) depicts Neudörffer pointing at a wooden dodecahedron and also includes a wooden cube hanging behind him with its vertices pointing up [**Figure 2.17**]. Neudörffer's student carefully follows his instruction, seemingly attempting to draw the dodecahedron in a notebook.

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[100] "*Als ich in meiner Jugend durch wunderliche mittel/ un fast von mir selbsten/ ohne einig Geometrisch Fundament in wissenschaft der funff corporum regularium (derer halben Euclides seine Elementa Geometrica und Arothmetica geschriben) gelanget/ dieselbe lusts halbe/ auß Papier/ Holtz/ un Stein schnidte…*" (Schwenter, *Vorredean den leser*)

[101] Two other works by Carpaccio—the first on the verso of the aforementioned sketch and the other being the painting *Saint Augustine in his Study* (1502) in Venice, both feature hanging mixed-mathematical instruments, though no polyhedra.

[102] The caption to the Faulhaber image reads: "*Cubus NB. wann der ligende Grund off das Ligende Täffelin gemacht wirdt/ so kompt das Corpus off den Tisch darunder/ darunder/ onn hinwiderumb/ u. Gerbrauch zur Perspectiv*."

[103] On the Choir of the *Basilica di San Domenico*, see Iotti and Zavarra 221-243.

After close inspection of the painting at the *Germanisches Nationalmuseum* in Nuremberg, the depicted cube reveals itself to have been hung on a painted nail protruding from a wall above the heads of Neudörffer and his student. The cube rests in mid-air, supported by nearly invisible hardware. Presumably Neudörffer would hang his new dodecahedron next to the cube once he had finished working on it.

Perhaps not so precious as to be stored out of view, but fragile enough so as not to be kept on a low surface where the risk of being damaged would have been greater, polyhedral models could have been put to use either as stereometric drawing aids or as tools for helping to visualize ancient Greek geometry. The detailed attention to the fastening in the Pacioli manuscript, which strives for realism in the Milan copy and is blatantly notional and represented by a single line in the Geneva copy [**Figure 2.18**], leads one to believe that the artist intended to represent polyhedra as real and physical objects, possessed of weight and capable of creating and casting shadows, even based upon real models that would have been dangling from hooks and twisting in the breeze blowing through an open window.[104] The taut strings, tangled around each other by the models' gentle rotation, are held in tension by the weight of the polyhedra, even as the length of string after the fastening knot frames the page in a physically impossible, ornamental flourish. This excess string is not at all excessive to the image, for it is its doubling as realistic support and graphic enhancement communicating the ambivalent if hyper-charged naturalism here, as if the artist was committed to making the polyhedra seem as real and even as model-like as possible, without reducing their meaning to that of

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[104] Though there is little textural reference made to the actual material of the polyhedra, one assumes they would have been made from wood as in the intarsia by Fra Giovanni da Verona (ca. 1457-1525) or as can be seen in the Neufchâtel painting.

a workshop tool alone. As reconsecrated by Pacioli in his manuscript, these "workshop geometries" fluctuate between their depiction as graphic artifacts made stunningly realistic through the mathematics of perspective and their identity as objects commonplace to the Renaissance *studiolo*, elevated through the force of artistry into substantial emblems of geometrical knowledge.

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The *corpora irregulata* had no textual description and could not be drawn, plotted, or modeled outright since unlike the preexisting *corpora regulata* they first had to be invented. As such, *corpora irregulata* comprised a genre of form-making that had more in common with design practice, in which a form is developed through graphic experimentation, than they did with the strictures of precisely translating a geometrical definition from text into the realm of the visual. *Corpora irregulata* were designed directly in perspective, as graphic forms that would terminate on paper, making use of the workshop models of polyhedra to set out the base lines of a shape that could be "irregularly" augmented. As we know from Halt's *Perspectivische Reiss Kunst*, the first step was to capture the outline of the physical workshop model in a graphic space. While this could have been done theoretically by sight alone, Renaissance workshops used a variety of tools to help direct the eye to the correct point on the object to be brought into perspective. For instance, Halt shows a polyhedron placed upon a simple measuring device with a rotating hinge, which enables the polyhedron to be traced by a pencil held vertically by an adjustable bracket [**Figure 2.19**]. "Its use [the drawing instrument] is clear from number 8, where it is seen how on the hinge (*Regel*), the Octahedron, which is lying on one of its sides, can be pushed back and forth and moved onto the drawn line of the affixed paper...” (Halt 18).[105] The octahedron to which Halt refers originated as a physical body. It would have been translated into a perspective drawing, and then graphically operated upon, chiseling into its surfaces, adding protuberances, or embellishing with further geometrical subdivisions to create Halt’s irregular solids.

Halt’s description is only one of many 16th century representations of experimental setups and tools for drawing objects, presumably also polyhedral models, in perspective. In Augustin Hirschvogel’s etching *Perspectiva* [**Figure 2.20**], several geometrical forms lean against each other or are balanced at an angle—the only point of contact with the tiled grid that makes up the ground being the perimeter line stretching between two of their vertices.[106] A compass is propped up in the corner against a stack of solids, drawing attention to the practical origins of producing perspectival geometry and foregrounding the geometrical objects that form the basis of learning to draw in perspective. There are also more explicitly reconstructive scenes such as the famous images of Dürer’s perspectival apparatus capturing a lute or Jost Amman’s (1539-1591) portrait of Wenzel Jamnitzer at work.[107] Amman’s Jamnitzer shows Jamnitzer measuring out a geometry on paper with reference to an elevation pinned onto a board [**Figure 2.21**]. Behind him in an

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[105] “*Sein Gebrauch ist auß No. 8 klar/ da gesehen wirt/ wie auf der Regel/das Octöedro auf seiner flachen einer ligend/ hin und wider geruckt/ und auf die gerissene lini deß aufgekleistersten Papirs/ kan geführt werden...*”

[106] Though it has been abstracted and reduced to its base geometrical form, the cross in *Perspectiva* remains something of a mystery. It also matches a similar form in Lautensack’s *Lehrbuch*, and in a print in Hirschvogel’s *Concordia*—both of which include crosses with ladders leaning up against them. It may well be that the ladder leaning against the cross refers to the *Kreuzabnahme* (Descent from the Cross or Deposition of Christ).

[107] On the afterlife of Dürer’s perspective apparatus see Hauschke 2009, 176-177.

alcove, several cubes are precariously balanced on each other, the bottom two resting upon their vertices in opposition to the laws of gravity and the top perched nimbly on its corner point.[108] One can conceive of Jamnitzer drawing his elaborate hexahedral variants from *Perspectiva* using the cubes as props, which he would have placed in front of him on the desk, adjusting the sightline of the viewing apparatus to correspond with the cubes' vertices and then working from the depicted elevation to whittle away and alter the profile of the drawing, in perspective, to construct layers of geometrical complexity and embellishment.

In his *Ein Schöner kurzer Extract der Geometriae und Perspectivae* (1599), the Nuremberger Paul Pfinzing von Henfenfeld (1554 – 1599), mixed-mathematician and chronicler of perspectival technology, elaborates on the rooms within which Dürer and Jamnitzer worked [**Figure 2.22**]. In particular, Pfinzing claims that Jamnitzer made use of a "*Perspectiv Tisch*" in composing his book on the "*funff Regulierten Corporibus*," though as much can be gleaned from Amman's depiction of Jamnitzer.[109] Pfinzing's

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[108] For further information on Jamnitzer's perspectival setup, see S. A. Bedini, "The Perspective Machine of Wenzel Jamnitzer," *Technology and Culture,* Vol. 9, No. 2 April 1968, 197-202.

[109] Pfinzing relates a history of the perspectival systems and technology of artists working in Nuremberg—including Albrecht Dürer, Heinrich Lautensack, Hans Lencker, and Hans Haiden. For each artist, Pfinzing details the perspectival apparatus they used to draw their graphical-geometrical work, and the technological improvements they made upon their predecessors. In regards to Jamnitzer, he claims that "…He then let a book begin at the five regular bodies, and the infinite number of bodies that originate from it, which he then brought out through this way of perspective. He had his perspective desk in a special room in his house, screwed in so that it could not move; standing so that he could attach to it strings from a screw on the room's wall. And thus in such a place of his house, his perspective had been realized in his works." (Pfinzing 1598, 9)—"*Wie er dann ein Buch außgehen lassen von den funff Regulierten Corporibus, unnd derselben unauffhörlichen darauß volgenden Corporen/ Die er also durch diese Art der Perspectiv außbringt. Er hat seinen Perspectiv Tisch inn einer sondern Stuben inn seinem Hauß/ so angeschraubt unnd sich nicht bewegen können/ stehendt gehabt/ daß er die Saiden dargegen an einer schrauben der Stubenwendt anlagen können/ und also an solchen Ort seines Hauses/ seine Perspectiv zu werken verbunden gewesen ist*."

detailing of Jamnitzer's *Perspectiv Tisch* shows an experimental apparatus that transformed Jamnitzer's living room into a giant "perspective machine" for making drawings through the attachment of a counter-weighted string to the living room wall for stability, speaking to the importance placed upon technologies for drawing in perspective and how integrally they were considered in relation to the geometries they helped generate.[110] Further to this point, on the page following his imagining of Jamnitzer's experimental setup, Pfinzing has depicted a blue-colored, pyramidal model made from slices of wood, each rotated and of decreasing size, copied from page H-I of Jamnitzer's *Perspectiva*. On top of the copy, Pfinzing has balanced the perspective apparatus apparently used to draw the model [**Figure 2.23**], in so doing bestowing upon the technology equal status to the geometry it had engendered, and highlighting the importance of perspectival and measurement tools in learning how to accurately translate and thus define reality on paper.

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[110] It should be said that while Pfinzing's rendering of Jamnitzer's perspectival installation seems to be superficially similar to Amman's image, the component parts of the apparatus have been rearranged in a jumbled manner that I believe would not have been conducive to perspectival drawing. The position of the armatures on the table relative to the blank paper and the way that there is no room for someone to work given that the table is placed up against the wall, leads me to believe that the image is not a realistic or even particularly careful depiction of Jamnitzer's living room. Though the artist of Pfinzing's *Ein Schöner kurzer Extract der Geometriae und Perspectivae* is not identified by Pfinzing, Hauschke attributes the drawings to Lorentz Stöer, who had a preexisting relationship with Pfinzing having worked together with him on the Pfinzing-Atlas (ca. 1594) (Hauschke 2009, 182). On the Pfinzing Atlas see the exhibition catalogue *Der Pfinzing-Atlas von 1594*, Staatlichen Archive Bayerns, Munich, 1994. However, in my opinion it is hard to reconcile Stöer's mastery of geometry and his intimate knowledge of Jamnitzer's oeuvre in particular, as outlined in Chapter Five, with such an inaccurate drawing.

The winged figure in *Melencolia I* (1514), one of Dürer's three so-called *Meisterstücke* engravings [**Figure 2.24**] alongside *Knight, Death and the Devil* (1513) and *St. Jerome in his Study* (1514), has garnered many readings over the history of its interpretation.[111] Many of her accouterments are borrowed from the allegory of geometry, who is traditionally personified as a woman engaged in acts of measurement, usually working at a table surrounded by tools, sometimes set into the very landscapes that she is purporting to measure.[112] Perhaps most notably, a truncated rhombohedron looms large in the middle distance, dividing the foreground—a scene replete with the detritus of geometrical measurement—from the limitless background of a placid sea that vanishes into the

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[111] The literature on *Melencolia I* is voluminous. In addition to Panofsky's *The Life and Art of Albrecht Dürer* and *Saturn and Melancholy*, see K. Giehlow's "Dürers Stich 'Melencolia I' und der maximilianische Humanistenkreis," *Mitteilungen der Geselschaft für vervielfältigende Kunst*, 1903, 29-41; P. L. Sohm, "Dürer's 'Melencolia I': The Limits of Knowledge," *Studies in the History of Art*, Vol. 9 (1980), 13-32; G. Agamben's *Stanzas: Word and Phantasm in Western Culture*, University of Minnesota Press, 1992; "Dürer's 'Melencolia I': Melancholy and the Undecidable," by Wojciech Bałus, *Artibus et Historiae*, Vol. 15, No. 30 (1994), 9-21; H. Böhme, *Albrecht Dürer, Melencolia I im Labyrinth der Deutung,* Frankfurt am Main: Fischer Taschenbuch Verlag, 1989; P. Schuster, *Melencolia I - Dürers Denkbild*, Berlin: Gebr. Mann, 1991; B. Schulte, *Melancholie : von der Entstehung des Begriffs bis Dürers Melencolia I*, Würzburg: ERGON, 1996; P. Doorly, "Dürer's Melencolia I: Plato's Abandoned Search for the Beautiful," *The Art Bulletin*, 86:2 (2004), 255-276; M. Büchsel, *Albrecht Dürers Stich Melencolia, I. : Zeichen und Emotion : Logik einer kunsthistorischen Debatte*, München: Wilhelm Fink, 2010; R. Hoffmann, "Im Zwielicht: zu Albrecht Dürers Meisterstich Melencolia I," Köln: Böhlau Verlag, 2014. While philosophy and literature have not shied away from *Melencholia I*, of particular note is Thomas Mann's figure of Adrian Leverkühn, the main protagonist in *Doctor Faustus*, who owns a copy of the print. Walter Benjamin, and more recently Giorgio Agamben, have taken up the melancholic angel in order to frame questions of *trauerspiel* and tragedy, symbol and allegory. See Benjamin: *Der Urspurung das Trauerspiels* and Giorgio Agamben *Stanzas.*

[112] The primary objects or tools in the allegory are a sphere or globe (the "geo" in geometria) and a compass (the essential tools of the navigator and cartographer). See Panofsky 1955, 161 on the relation of geometrical tools in the *Melencolia* to the geometrical paraphernalia that surround allegories of geometry, such as Gregor Reisch's *Margarita Philosophica.* On allegories of geometry see Park 2011, 358-365.

horizon, ostensibly the type of natural territory the geometer purports to measure.[113] The presence of the polyhedron, globe, and measuring tools suggest that at the very least the representation of geometry was central to Dürer's intention, even if these objects form only one critical part of this multi-layered and by now intertextual image.[114]

The symbolic centrality of geometry and geometrical knowledge in *Melencolia I* is further upheld by geometry's supporting role in all forms of mixed (or applied) mathematics, such as surveying, cosmology, astronomy, geography, and astrology—knowledge practices all present in the engraving, even if the introspection and listless self-reflection of the figure differs greatly from the industriousness and certainty of the more common types of geometrical allegory like Johann Sadeler's (I) (1560-1600)

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[113] There has been much debate about the exact definition and form of the polyhedron. As per Klibansky, Panofksy, and Saxl, the truncated rhomboid in *Melencolia I* refers to "a figure formed of six rhomboids which, owing to the cutting off of two opposite points (those in which the sharp angles of the rhomboid meet), has been transformed into an octahedron" (Klibansky, Panofksy, and Saxl 400). A summary of some of the disagreements over the polyhedron's precise shape are summarized on 400-402. For further reading see P. J. Federico, "The Melancholy Octahedron," *Mathematics Magazine,* pp. 30-36, 1972; T. Lynch, "The geometric body in Durer's engraving *Melancholia I,*" *Journal of the Warburg and Courtauld Inst.*, pp. 226-232, 1982; C. H. MacGillavry, "The Polyhedron in A. Durer's 'Melencolia I': An Over 450 Years Old Puzzle Solved?" *Netherland Akad Wetensch. Proc.,* 1981; P. Schreiber, "A New Hypothesis on Durer's Enigmatic Polyhedron in His Copper Engraving 'Melencolia I'," *Historia Mathematica,* 26, pp. 369-377, 1999; J. Sharp, "Durer's Melancholy Octahedron," *Mathematics in School,* Sept. 1994, pp. 18-20; H. Weitzel, "A further hypothesis on the polyhedron of A. Dürer's engraving Melencolia I," *Historia Mathematica* 31 (2004), 11-14. For a good description of the tools in *Melencolia I*, see Klibansky, Panofksy, and Saxl 327-329. The Sächsische Landesbibliothek in Dresden possesses the preparatory drawing of the polyhedron in *Melencolia I*, which is depicted with a floating eye to signify the point of sight used to generate the solid's perspectival angle.

[114] As if anticipating the interest it would generate by including the number "I" in its title, *Melencolia I* was also recognized as an extraordinary work in its own time. See Jan Wierix's (1549-1620) faithful rendition after Dürer from 1602, in which Wierix replaces Dürer's name with his own, rendering the resultant print a conspicuous copy of the original but not a forgery. In turn, Dürer himself may have been looking at Martin Schongauer's (1440-1491) *St. John the Evangelist* as a partial inspiration for *Melencolia I.* From Nadine Orenstein's presentation on the 500th anniversary of *Melencolia I,* Metropolitan Museum of Art, Spring 2014.

Geometrie [**Figure 2.25**].[115] Distinct from Dürer's morose angel-geometrician, *Geometrie* sits on the ruins of classical architecture, demonstrating the reciprocity between techniques of measuring the models and drawings she has assembled and the surveying of the terrestrial territories they purport to represent. It is as if the perils of circumnavigating the world were no more difficult than the act of sweeping her compass around the globe. The unflappability of Sadeler's *Geometrie* comes from her belief that by seeing the world in her image, that is in geometrical terms through a representational device like the globe, she will be able to impact the reality the globe purports to represent. Creating a direct relation between subject and the model of a subject that may be directly operated upon, the lines *Geometrie* measures pierce the veil of corporeal phenomena, radiating an ordering effect on the world beyond the model's corresponding confines. The 16th century print by Marin Bonnemer of *Geometry and Astronomy* similarly depicts the title figures proudly standing in classical pose, brandishing their measurement tools while teams of surveyors and sailors valiantly demonstrate the universal utility of measuring land and navigating via reference to the stars [**Figure 2.26**]. Although there is a demonstration of thinking, the Bonnemer and Sadeler figures are anything but self-reflective; they do not seem to doubt anything, let alone the utility of their endeavors. They are unabashed personifications of the limitless possibility of using geometry to conclusively represent, and thus to know, the world, and the consequent importance of the mixed-mathematical professions.

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[115] Later allegories of geometry tend to include visual references to weaponry and the geometry of ballistics, another form of mixed-mathematics, through the inclusion of cannons or other weapons. See for instance *Allegory of Geometry* by Jacob Herreyens (I) (1671-1732). Object number RP-P-OB-55.388, Rijksmuseum.

Yet even within an allegory as seemingly unconflicted as Bonnemer's, there are elements that cause the epistemological edifice upon which the allegory is constructed—the universal certainty of geometrical knowledge applied to the real world—to collapse unintentionally in on itself. Leaning against the rocky outcropping next to Bonnemer's *Geometry* figure are several Platonic Solids, which are nearly camouflaged by the poor quality and hesitancy of their lines and shading. Bonnemer felt confident enough to render exotic vegetation, scientific instruments, and columns covered in grotesqueries, but to construct, or fake, convincing solids in perspective was evidently still too challenging of a task. A disconnect between the enthusiasm for polyhedra and the capacity to visualize polyhedra in perspective palpably haunted Bonnemer and other artists, such as the artist responsible for the printed edition of Pacioli's *De divina proportione* (1509). Unlike the manuscript originals, these later printed polyhedra are roughly rendered with clumsy and imprecise crosshatching. The awkward shading betrays a lack of understanding of how to graphically communicate three-dimensional forms on paper [**Figure 2.27 + 2.28**].[116] As representative examples, the dashed lines representing indirect light on the leftmost face of the "*Octaedron Planum Solidum*" do not read against the uneven distribution of shading intended to portray the solids two darker faces. And rather than articulate the edges of the solid, the dark shading patterns printed on the solid's faces do not accent the seam between them but rather, in their seemingly haphazard orientations, efface the very three-dimensionality the artist was trying to achieve. Similarly confused is the "*Octaedron Elevatum Solidum*," where the

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[116] For a comparison of the various manifestations of, and precursors to, *De divina proportione* by Pacioli, Piero, and Leonardo see Pacioli et. al 2010. See Pacioli 2009 for reproductions of the Milan and Geneva manuscripts.

orientating of contour lines fights incoherently with the edges of the solid. The artist was operating with three stock textures of dashed, solid, and empty patterns and that while he conceived of a virtual light source located to the upper left above the solid, he deployed his patterns with only the most basic knowledge of how to bring the solid vividly to life—as theoretical icons of the distribution of light rather then with an attention to the way shadows cast their presence onto a subject.

These melancholy polyhedra hint at a subtext of artistic frustration. Even as Bonnemer and *De divina proportione* conjure up a world in which the benefits of geometry and mixed-mathematical practice are universal, and the certainty of the Euclidean proof may be secured through images, both artists are limited by their own abilities—unable to represent the most basic building blocks of the study of geometry, if not matter itself, as per Plato's *Timaeus.*

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Polyhedra became both indispensible components of stereometric drawing techniques and geometric study by artists in the 16th century as well as physical props or intense graphic entities as in *De divina proportione*, and began to migrate into a diffuse range of media. Fra Giovanni da Verona's (1457-1525) intarsia in the Monastery of Monte Olivetto Maggiore near Siena and in the Church of Santa Maria in Organo, Verona, both completed around 1520, contain a 72-sided sphere, an icosahedron, a truncated icosahedron, two elevated icosidodecahedra, a cuboctahedron, and a cube with equilateral pyramids affixed to each face, all derived from *De divina proportione.*[117] Michelangelo replaced the customary sphere surmounted by a cross on the lantern commissioned by Pope Leo X in 1520 for the New Sacristy in the Basilica of San Lorenzo in Florence with a gilded copper "elevated duodecahedron" (*duodecedron elevates solidus*).[118] The later polyhedral manifestations exploited the novelty of these new variations on the Platonic Solids, which were used to demystify and extend Euclid and had the double benefit of being strange and beautiful visual artifacts.

The same regularity that would make the Platonic Solids "unfoldable" for Dürer also made them eminently teachable constructs that united the practice of drawing with the making of three-dimensional objects. The coming century's *Lehrbücher* seized on the potential of polyhedra to train the visual imagination and turned the invention of graphic geometries into a tenet of artisanal education, beginning a trend of learning geometry through graphic forms that would continue through the 19th century, as can be seen in Jean-Francois Niceron's *La perspective curieuse* (1652) or the geometrical sculptures populating the pastoral landscapes of Nilson's *Anleitung zur Linearperspektive* (1812) [**Figure 2.29**]. Though there was no overarching directive, no singular institution that sponsored all the transformations of geometry in the 16th century, the *corpora irregulata*

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[117] See "Fra Giovanni's Intarsia Polyhedra," Hart.

[118] In January 1525, Michelangelo wrote Pope Clement VII that "Stefano has finished erecting the lantern for this chapel in said church of San Lorenzo, and when he revealed it, everybody was pleased with it, and Your Holiness will be too. Let's have the ball made, it will be about a yard tall, and to make it different from the others I have thought of making it faceted, which I think will make it look graceful, so that is how we are going to make it." Quote as reprinted from a press release for the exhibition "Medici Splendour: Pope Leo X and Florence" (2013). Vasari also upholds that installed in the sacristy is "a ball with seventy-two faces made by the goldsmith Piloto, which is very beautiful" (Vasari 1915, Vol. 9, 43).

in accumulation constituted the origins of a coherent body of geometrical research into the morphological possibilities of geometry, specifically defined by their irregular sets of relation to the Platonic Solids.

And yet to the Renaissance mixed-mathematician, the degree to which geometrical precision could be fatally compromised by the variable skill of its practitioners mirrored the reality of an untamable world made from imperfect objects increasingly in need of measure. Even as *corpora irregulata* were symbolic of a newly limitless potential for geometrical invention, they also served as ambitious attempts to extend geometry to the more ragged parts of the world. In this sense, the struggles of the French pedagogue Petrus Ramus (1515-1572) to connect his geometry lessons with real world phenomena and situations is indicative of the conscious effort of mixed-mathematicians to weld the geometric to the real.[119] Unlike the original 1569 edition which had no such capstone image, the 17th century re-edition of Ramus' *Via regia ad geometriam* (1636) replicates an image demonstrating the measurement of the depth of a well from an earlier chapter on surveying, inserting it behind the last chapter, which dealt with the measurement of cylinders [**Figure 2.30**]. [120] The real world, with all its variation, was vastly dissimilar to geometrical representation, and students, as they arrived to the end of their course of study, needed to be reminded that their knowledge of geometry was integrally related, however much it might appear to be an abstraction.

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[119] The only set of images illustrating "real world" application in Ramus' original *Arithmeticae libri duo: Geometriae septem et viginti* (1569) concern the use of the Jacob's staff and surveying practices in the section dealing with the measuring of right angles by right-angled triangles.

[120] The full title is *Via regia ad geometriam. The vvay to geometry. Being necessary and usefull, for astronomers. Geographers. Landmeaters. Sea-men. Engineres. Architecks. Carpenters. Paynters. Carvers.*

Perhaps it is resignation weighing down the figure in *Melencolia I*, whose posture hints that the act of mediating the world of abstract geometrical knowledge with the concrete reality of objects, models and visualizations was anything but a simple task. The limits of geometry to describe the world and the limitless possibilities of geometrical invention are centrally thematized in *Melencolia I*, becoming the subject and title of the engraving and repeated again in a host of imitators inspired by Dürer's reinterpretation of the allegory of geometry such as in Hans Sebald Beham's (1500-1550) *Melencolia* (1539) [**Figure 2.31**], which further substantiated the uniting of melancholy with the study of geometry and mixed-mathematics.[121] Though many of the staples remain in the Beham image, such as the sphere, the workshop tools, and the resigned expression on the face of the main figure as she distractedly toys with a compass, the enigmatic *corpora irregulata* from *Melencolia I*—a surrogate for all the three-dimensional geometry artists were newly struggling to invent, understand, and represent—has been conspicuously avoided.

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[121] Many depictions of melancholia after Dürer included geometrical instruments, a good selection of which are reproduced in the appendix to *Saturn and Melancholy*, and in addition to Beham include Virgil Solis, Abraham Bloemaert, Giovanni Benedetto Castiglione, and several anonymous others. The melancholic influence of Saturn may also account for the presence of a globe, or celestial globe, in allegories of melancholy. For instance, three figures in *Saturn and his Children*, a print by Maarten van Heemskerck (1498-1574), appear to be taking astronomical measurements to figure out astrological influence as a fearsome Saturn hovers in the sky above them, devouring a child with his left hand and grasping a scythe with his right.

Chapter Three: Training in Abstraction

The combination of intense competition for commissions, strict standards of quality and an unprecedented concentration of talent made Nuremberg the site of intense mixed-mathematical innovation. It particular, it seems that expertise in representing polyhedra had become recognized widely enough to serve as an emblem of the city's world-renown precision industry. By 1608, Hieronymus Braun, chancellery clerk of Nuremberg, presented to the Nuremberg council his *Prospekt der Reichsstadt Nürnberg*, the first map of the city to accurately represent all of Nuremberg's existing buildings, crowned with two polyhedral models [**Figure 3.1**].[122] And like Braun's *Prospekt*, the 1623 engraving of the goldsmith Wenzel Jamnitzer and the mathematician and "writing master" Johann Neudörffer (1497-1563) foregrounds two central figures in the mixed-mathematical research which had been conducted in Nuremberg, complete with a polyhedron tucked in beneath Jamnitzer's feet [**Figure 3.2**].[123]

While the study of geometry in university was just one level in a pyramid of knowledge topped by theology, for mixed-mathematical practitioners geometry *was* the central theoretical focus of their craft. The groundswell of geometrical *Lehrbücher* emerging from Nuremberg is emblematic of this centrality and the extent to which practice was seen to generate geometrical knowledge deserving of reflection. *Lehrbücher* written by

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[122] Braun's *Prospekt* was also the first map to show Nuremberg's buildings three-dimensionally in an oblique view that preserved their accurate dimensions in plan.

[123] The engraving was produced by Eberhard Kieser and is contained within *Politisches Schatzkästlein* by Daniel Meisner. The image of Jamnitzer is based upon the portrait by Jost Amman. The text reads "Nothing is better than art on earth; Nothing more useful can be found than art; Art is a true friend; Therefore all artists should be revered."

working mixed-mathematical practitioners like Augustin Hirschvogel and Heinrich Lautensack, filled a gap in the acquisition of geometrical knowledge that was deemed increasingly critical to the development of skills in measurement and representation. It would not be until later in the 17th and 18th centuries that professional training (*Berufsausbildung*) in Germany would become the domain of specialized schools, *Handwerksschule*, partially through the educational reforms of Wolfgang Ratke (1571-1635) and John Amos Comenius (1590-1670) (Beck 408). Among Ratke's pedagogical principles were several approaches seemingly cribbed straight from the workshop environment and the *Lehrbücher* they had produced, including the importance of learning through experience and experiment, rather than from rote memorization, and a focus on acquiring knowledge by proceeding from the concrete to the abstract (Britannica Biography of Wolfgang Ratke).[124]

In order to fully grasp the influence of the mixed-mathematical workshop on the practice of geometry in Germany, it is necessary to unfold a broader historical perspective on the place of geometry within the development of the German higher educational system. Phillip Melanchthon (1497-1560), the influential German educational reformer and Martin Luther's (1483-1546) colleague at the University of Wittenberg, was troubled by the incitement to revolution contained within the radical Protestant writings of evangelical Lutherans.[125] Flare ups of violence and disobedience by Protestants in Swabia, Thuringia, and the Black Forest threatened the stability of the German states even as the brutal defeat of the revolutionaries in the German Peasants' War (1524-1525),

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[124] http://www.britannica.com/biography/Wolfgang-Ratke

[125] See the writings of Thomas Müntzer (ca. 1489-1525), Andreas Karlstadt (1486-1541), and Felix Manz (ca. 1498-1527).

Europe's largest popular uprising prior to the French Revolution, resulted in casualties numbering in the hundreds of thousands.[126] These current events convinced Melanchthon that Luther's messages were being transformed into a broad, and unintended, anti-establishmentarianism that surpassed and overshadowed Luther's grievances with the Catholic Church.[127]

As Sachiko Kusukawa has shown, education played a key role in Melanchthon's method of attempting to instantiate a respect for order, civility, and the law.[128] Melanchthon's new humanist curriculum, which was based upon interpretations and selective emphases of traditional Aristotelian philosophy, actively sought to internalize within students a world-view in which civil disobedience was understood to be antithetical to the divine

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[126] The war is known in German as *Deutscher Bauernkrieg* or *Revolution des gemeinen Mannes.* The burning of the peasant leader Jacklein Rohrback in 1525 is hauntingly depicted in Peter Haarer's *Beschreibung des Bauernkriegs* (1571). The defeat of the peasants at the Battle of Frankenhausen, a bloody skirmish fought on May 15th 1525, signified the end of the German Peasants' War. Müntzer was also caught and executed at Mühlhausen on May 27th, 1525.

[127] Melanchthon was appointed a Greek professorship in 1518, while Luther was already the professor of theology, and would later come to be known as the *Praeceptor Germaniae*. See *The Radical Reformation*, M. G. Baylor (Ed.), for writings by the "radical reformers." In light of the risk posed by the radical reformers, Melanchthon and Luther turned towards the so-called "magisterial reformation"—a re-estimation of Reform in which initiative would come not from the populace but from the ruling princes and governments (Kusukawa 1999, xiv). And although future political crises would test the theories of non-resistance upheld by the leaders of the German Reformation, the lawfulness of opposing the emperor (and state) would remain a fraught philosophical-religious issue for the Lutherans, requiring serious debate and differing forms of justification (Skinner 199). One such political crisis was the *Refutation* of the Augsburg Confession by Emperor Charles V, which had been drawn up by Melanchthon in 1530 with the hope of reaching a compromise with the Catholic princes. See Skinner 1999, particularly 189-238; Tracy's *Europe's Reformations 1450-1650*; and Brady Jr.'s *German Histories in The Age of Reformations, 1400-1650.*

[128] S. Kusukawa, *The Transformation of Natural Philosophy: The Case of Philip Melanchthon* (Ideas in context), Cambridge: Cambridge University Press, 1995; P. Melanchthon and S. Kusukawa, *Philip Melanchthon: Orations on Philosophy and Education* (Cambridge texts in the history of philosophy). Cambridge, U.K.; New York: Cambridge University Press, 1999.

order of nature and society (Kusukawa 1999, xxii).[129] To study was to recognize the balance and proportion underlying the natural and divine order of all things. From man's moral behavior to the proper regulation of cities, the values of just societies and the apprehension of celestial movements, the rule of law and authority was intentionally naturalized, a socialization intended to flatten out dissent and revolt while reinforcing respect and deference for institutional establishments.

Melanchthon's educational philosophy was aided by the legacy of Scholasticism, which prided itself on transmitting a well-trodden hierarchy of knowledge.[130] In *Margarita Philosophica* (1503), a Latin encyclopedia by the Carthusian prior Gregor Reisch (ca. 1467-1525), historical figures peak meekly from behind the walls of a tiered tower representing an allegory of education [**Figure 3.3**]. Cicero for Rhetoric, Euclid for geometry, Ptolemy for astronomy, Aristotle for logic, and others. Grammar leads a pupil into the building, where he will have to make his way up through the hierarchy of subjects before reaching theology (represented by Peter Lombard) at the top. Education

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[129] Melanchthon's pedagogical and curricular reforms of the German school and university systems radiated out from his new arts curriculum at the University of Wittenberg and were tremendously influential in Lutheran Germany and beyond. In England, the Reformation swept across Oxford and Cambridge, purging their curriculums of Scholastic authors. The syllabus, reconceptualized in 1549, made mathematics a much more prominent component of English education, which was now to be taught four days a week, for one hour, from 12pm, and was based on Euclid and Ptolemy (Hannam 117). Of geometry, the administrator and royal advisor Cuthbart Tunstall (1474-1559) expressed sentiments similar to those of Melanchthon: "God, architect of all things gave their form to the fabric of the world and every created things in it so that all would reveal symmetry among themselves...The power of proportions...witnesses that God has arranged all things." Quoted in Hannam 118, from Cuthbart Tunstall's *De arte supputandi*, Strasbourg 1538, 178.

[130] The new arts curriculum was the result of Melanchthon's direct consultation with the universities of Tübingen, Leipzig, and Heidelberg, and numerous lectures, publications, and visitations which often resulted in rules established in churches and schools throughout Germany. Melanchthon was personally involved only in establishing three schools: Magdeburg (1524), Eisleben (1525), and Nuremberg (1526) (Scheible 36).

was to be as structured, stable, and unquestioning as the design for the building was sound and everlasting; a buffer against the social fractures that threatened to tear apart the fabric of German society.

University requirements for the study of mathematics did not for the most part reflect the growing mixed-mathematical expertise that characterized southern Germany's thriving precision industries, nor did it adequately address the demand for increased training in geometry and it's applications that the rise of mixed-mathematical professions required.[131] The heavy reliance of the humanist curriculum upon the classics—Ptolemy's *Almagest*, and Pliny's *Natural History*, alongside time-honored medieval staples such as Sacrobosco's *De Sphaera*, meant that university courses in mathematics remained steadfastly committed to the worthwhileness of teaching geometry through the theoretical exercises and proofs contained in Euclid's *Elements*. The most notable changes to occur in mathematical instruction had less to do with the content of the curriculum than with the recommendation of new textbooks, such as using the modern translation of *Elements* from the purified Greek text or substituting Sacrobosco for the work of the Paris professor Oronce Finé (1494-1555) (Brockliss 590).[132] Moreover, though mathematical instruction since the Middle Ages had bridged into optics, music theory, and astrology, most universities only demanded a minimal competence with the first few chapters from *Elements* (Brockliss 589), creating a vast gap between the average student's academic

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[131] The discovery of the New World lent a particular urgency to certain mixed-mathematical fields for which practical knowledge was necessary, such as navigation and cartography, resulting in the establishment of specialized schools without connection to universities in Spain (Pederson 466).

[132] Euclid's *Elements* was not substantially addressed in the Sacrobosco commentaries, let alone the construction of the Platonic Solids in Book XIII of *Elements*.

knowledge of Euclid and all the possible real world and professional scenarios to which geometry might be usefully applied.[133]

In contradistinction, craft knowledge was a highly guarded commodity in the 16th century, particularly in a city like Nuremberg which was rife with competition. Apprentices did not expect a codified course of study and had to learn skills through observation and assistance in the workshop (Hanschmidt 39). As few apprentices were literate, mastery of a trade was not dependent upon reading and writing skills, though literacy and knowledge of contract law were useful for successfully navigating through guilds' citizen-run administrations (Schmitt 73). Practical instructions for the obligatory apprenticeship required by master craftsmen were usually for this reason not written down, though standards of conduct and the quality of workmanship were enforced by the ordnances of each city's guilds or council bodies. In most cases, apprentices were required to spend anywhere from two to five years in training, and could range in age from twelve to eighteen years old (Wesoly 110-11). Few if any had formal or theoretical training in mathematics, art, or mechanics prior to commencing their apprenticeship (Prass 149).

Humanist textbooks invariably were in Latin, which excluded the vast majority of object-makers and their apprentices. For most makers then, the only way to gain mathematical knowledge was from their peers or master craftsmen, or through seeing the work of artists and artisans at the forefront of integrating mathematics and art, like Albrecht

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[133] For instance, the statutes of the faculty of arts at the *università e dei collegii dello studio di Bologna* required that only the first three books of Euclid were to be taught one per year, in the first three years. The 1389 statutes from the *Universität zu Wien* state that only the first book of Euclid was to be taught (Thorndike 42-43).

Dürer. In Germany, image-heavy treatises on geometry, or geometrical *Lehrbücher*, published locally in the south, were essential to the communication and distribution of cutting-edge scientific and geometrical knowledge to artisans and mixed-mathematicians. These *Lehrbücher* were traded, collected, copied, and circulated in tandem with hands-on practice, forming an informal network of training in geometry and perspective targeted to German object-makers irrespective of their level of literacy, in place of an official university system or institutional training. *Lehrbücher* were often also pitched as clarifications or refinements of Dürer's writings, which were considered difficult or opaque (Seidenfuβ 135), and consequently enjoyed a great deal of popularity throughout Germany.[134] While in surveying and measuring disciplines innovation might occur in the tools used for measurement, in the trades that produced objects, graphic strategies circulating by means of the numerous *Lehrbücher* emerged to serve as the conceptual bases for learning how to design in the abstract, apart from material, cost, and any other practical considerations that might impact a design during development. Though perhaps initially spurred by Dürer's urging in the *Underweysung* that increased attention to precision measurement was necessary for German artists to better compete with their Italian counterparts, the popularity and wide distribution of the *Lehrbücher* cannot be explained by regional insecurity alone. Indeed these books surged to fill a gap in artisanal education by purporting to specifically target and develop the ability of object-makers to think and to design complex form in three dimensions.

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[134] First editions remain today in the libraries of Berlin, Heidelberg, Mainz, Rostock, Munich, Dresden, Freiberg, Gotha, Wolfenbüttel, Halle, Hannover, Bamberg, Augsburg, Hesse, Nuremberg, and Tübingen.

Dürer's pioneering use of geometry in the *Underweysung der Messung* (1525) was immensely influential in Germany (and beyond), introducing tactility, invention, and media to the Renaissance conception of *geometria*, particularly through his revolutionary sundering of the Platonic Solids into unfolded surfaces.[135] Central to the pedagogical philosophy of the *Underweysung* was the transformation of polyhedra into paper polyhedral "nets" that could be printed, cut out, and glued together to create quick copies of the Platonic Solids, alongside Dürer's own polyhedral inventions. These copies enabled a radically tactile and playful relationship to geometry, heralding a new era of abstract experimentation by later 16th century German artists such as Augustin Hirschvogel (1503-1553) and Heinrich Lautensack (1522-1590), as well as Wenzel Jamnitzer (1507-1585), Hans Lencker (1523-1585), Paul Pfinzing, (1554-1599), Hieronymus Rodler (d 1539), Erhard Schön (1491- 1592), Lorenz Stöer (1537-1621) and others, which in turn completed the transformation of polyhedra from neo-classical stereometric drawing aids in the 15th century Italian studiolo into the premier objects of northern European geometrical curiosity in the 16th century.

The precedent of Dürer's unique engagement with the Platonic Solids created a substantial research track in southern Germany, of which the *Lehrbücher* remain a record, divergent from the priorities of Melanchthon's humanist curriculum and suffusing the study, measurement, and production of polyhedra and polyhedral-ish geometries with a spirit of mathematical inventiveness, newness, and erudition. The improvement and development of techniques for constructing new geometrical forms sharply diverged from

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[135] See Chapter One.

a humanist mathematics predicated upon learning the basics of *Elements*. Though not mutually excusive, at stake was the identity of geometrical knowledge. Geometry both described a preexisting classical order, eternal, timeless and distinct from the messiness of everyday life, and defined a site for the exploration of new shapes, forms, and utilities which had never before been seen.

Still there was a degree of porosity between practice-based mathematicians and academic life. Makers could be asked to participate in academia or collaborate on projects with local sovereigns. Protestant universities, often emulating Melanchthon's Wittenberg model, provided necessary expertise in applied mathematics, particularly surveying and practical geography, when required by local princes. The University of Helmstedt offered students geographical instruction in *mathematicus inferior*, although the instructional texts were the standard *Arithmeticae practicae methodus facilis* (1576) by Gemma Frisius, Johannes Honter's *Rudimenta cosmographica* (1546), Pliny's *Historia naturalis*, and Ptolemy's *Geographia* (Moran 1981, 260). The school St. Egidien of Nürnberg, also guided by Melanchthon and Germany's first *gymnasium*, hired Johannes Schöner (1477-1547), one of the most prominent map-makers and builder of globes in Nuremberg, as professor of mathematics (Scheible 37).[136] Nevertheless Schöner's textbook, *Solidi ac sphaerici corporis* (1517), is consciously text-heavy theory, for which the authority of text was deemed to better uphold the dignity of the discipline of astronomy than drawings.[137] Bereft of images, it makes no visual reference to the geometrical techniques

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[136] Schöner was also an astronomer, astrologer, cosmographer, cartographer, scientific instrument maker, and priest.

[137] The entire quote reads: "*astronomicae disciplinae dignitatem, tantu alijs scietice praeeminere censeo: quantum coelestia humanis rebus sunt praestantiora*."

inherent in Schöner's map-making and globe-building practice. And while some German universities like Helmstedt employed practical mathematicians who taught an "inferior" strain of mathematics, the pedagogical trajectory still led to the actor's category of *mathematicus superior*, which remained firmly based upon Euclid and Ptolemy.

Melanchthon too promoted a mathematical pedagogy that delved deeper into text as opposed to developing skills in the graphical manipulation of geometry. Far from taking advantage of the blossoming knowledge and techniques for the visualizing geometry happening in the artist and mixed-mathematical workshops of the newly Protestant southern German cities, the primarily non-visual textbooks shied away from the worldly and the material, away from physical practice and making. The political agenda in keeping geometry (in Germany) contained within a newly Protestant framework, separate from pressing geopolitical concerns and removed from practical application, emblematized a schism between the mathematical study of classical texts occurring within the university/secondary school system and that occurring within environments predicated upon the production and selling of goods and services, for which the theorems of Euclid were less useful than the practice of learning how to graphically construct and manipulate geometry.

Neither the higher nor lower mathematical curricula adopted the mixed-mathematical *Lehrbücher* widely circulating around Germany at the time. Even as the educational reforms of Melanchthon encouraged an investment in mathematics which was not necessarily common throughout other humanist-inspired school systems (Methuen 387), and the Protestant university proved not inhospitable to providing mathematical expertise to courts, the newest developments in the swiftly accumulating knowledge on the visual possibilities of geometry, the *Lehrbücher* for teaching geometry and perspective, were neither destined for nor products of university education, but emerged in tandem with the priorities and conceptual innovations contained within mixed mathematical practices.

Arising out of the vibrant artisanal culture of southern Germany, two books published in the vernacular, *Geometria* (1543) by Augustin Hirschvogel and *Des Circkels unnd Richtscheyts* (1564) by Heinrich Lautensack, were popular *Lehrbücher* for teaching methods of conceptualizing and inventing geometry through constructing polyhedra.[138] In contradistinction to the textbooks of the humanist curricula, both *Lehrbücher* contained little explanatory text and were rather filled with exuberant prints intended to train the geometrical imagination of mixed-mathematicians/artists for whom knowledge of geometry and the ability to manipulate three-dimensional form on paper had become essential. The *Lehrbücher* are records of prior experimentation with geometrical form by their authors as much as they are also experimental spaces encouraging explorative form-making and drawn epistemologies. As such, unlike *Elements*, they are concerned with a unique type of utility or practicality—what *geometria* was and could be in and of itself—aside from geometry's fundamental role in measurement and description, and apart from the repeatable theorems of Euclid. Taken together, Hirschvogel and Lautensack's books demonstrate some of the unique contributions made by artisans/mixed-mathematicians to

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[138] Though *Geometria* only went through one edition, existent copies in Germany remain in Berlin, Göttingen, Mainz, Munich, Nuremberg, Wien, and Wolfenbüttel. Later authors mention Hirshvogel and/or utilize his diagrams, in particular *Geometriae practicae novae et auctae tractatus* (1641) by Daniel Schwenter and Juan de Arfe's (1535-1603) *De Varia commensuración para la escultura y arquitectura* (1585). *Des Circkels unnd Richtscheyts* was republished in 1618 and is available in Berlin, Dresden, Freiberg, Gotha, Göttingen, Heidelberg, and Munich. Originals of both *Lehrbücher* are also widely available in American collections.

the way that geometry was understood, taught and used in Germany, and testify to how the invention and manipulation of three-dimensional polyhedral geometry had become integral to 16th century artisanal training.[139]

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A famous artisan in his time, Augustin Hirschvogel is described by the historian J. G. Doppelmayr in his *Historische Nachricht von den nürnbergischen Mathematicis und Künstlern…*(1730) as a glass-painter with great skill in design, etching, painting, and enamel work, and who, later in life, also engaged in stone cutting, and mathematics (Doppelmayr 199).[140] By mathematics, Doppelmayr is referring to Hirschvogel's *Geometria* (1543) [**Figure 3.4**], a *Lehrbuch* published in Nuremberg shortly before Hirschvogel set off into Europe where he apparently applied himself to astronomy and geography, designing maps of Moscow and of Austria dedicated to Ferdinand I, the Holy Roman Emperor (Doppelmayr 156).[141] Hirschvogel produced over three hundred etchings of gold vessels; ornament for goldsmiths; book illustrations; religious, historical,

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[139] For summary overviews of a group of 16th century German artists, including Jamnitzer, Lencker, Hirschvogel and others—all of whom wrote textbooks on perspective and geometry—see Richter 54-92 and Seidenfuß 125-207.

[140] The distinction Doppelmayr makes between the type of work counted as mathematics and art retroactively compartmentalizes the more fluid, mixed-mathematical practices of Renaissance Nuremberg. For references to Hirschvogel see *Katalog der Gedenkschau Augustin Hirschvogel (1503-1553)*, Historisches Museum der Stadt Wien, Vienna, 1953; K. Schwarz, *Augustin Hirschvogel. I. Lebensbeschreibung und Zeichnungen*, Heidelberg: Rößler & Herbert, 1915; *Augustin Hirschvogel – Ein deutscher Meister der Renaissance.* Berlin: Julius Bard, 1917; Schaper, C. *Die Hirschvogel von Nürnberg und ihr Handelshaus.* Nuremberg: *Selbstverlag des Vereins für Geschichte der Stadt Nürnberg*, 1973.

[141] See Hirschvogel's *Moscouia Sigmunds Freyherns zu Herberstain Neyperg und Guetenhag & c vertteütsct* (1557) and his undated maps of Austria—*Das in dem Ertzhertzogtumb Vnter Osterreich*.

and mythological scenes; coats of arms; and representations of hunting and animals, though he is only known to have produced one etching that included polyhedra, *Perspectiva* (1543), which was produced in the same year as *Geometria*, his only *Lehrbuch* (*Katalog der Gedenkschau* 6). Nevertheless, *Geometria* remained popular and was still in use at least one hundred years after its first printing.[142] In the preface to Schwenter's *Geometriae practicae novae…*, Schwenter acknowledges his debt to Hirschvogel, as well as to Dürer and Vitruvius. "Hereupon I was lent by a good acquaintance of mine the well-known Geometria of Augustin Hirschvogel. I have studied it with great zeal, and because it is easy, not complicated, and well-argued, also with great pleasure."[143]

Hirschvogel's *Geometria* consists of two volumes, in which the corresponding reference text and visualizations are cannily separated; a strategy likely intended to maximize the use of *Geometria* among the high majority of apprentices and craftsmen that were not literate. Though *Geometria* in itself is not lengthy, its extended title indicates the breadth of Hirschvogel's ambition: *Ein aigentliche vnd grundtliche anweysung, in die Geometria, sonderlich aber, wie alle regulierte, vnd vnregulierte Corpora, in den grundt gelegt, vnd*

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[142] I have not found evidence of further editions, though *Geometria* was widely dispersed across Germany as previously noted. It is not surprising that Schwenter was familiar with Hirschvogel given that they had both lived and published geometrical treatises in Nuremberg.

[143] The entire quote reads "*Hierauff ist mir von einem gutem Bekandten/ Augustin Hirschvogels/ weyland Burgers in Nürnberg/Geometria geliehen worde/ hinter dise habe ich mich mit grossem eyser gemacht/ und weil sie sein leicht/ schlecht und gerecht/ sie mit lust durch studiert/ biß mir unter deß auch Wolff Schmids von Bamberg Geometria Anno 1539 zu Nürnberg gedruckt/ ungefehr unter die hand komen/ ein sonderlich sein und wolgegründet Büchlein für die anfahenden/ darauß ich dann gelernet/ was proportio und proportionalitas, wie etliche zureden pflegen/ seye/ und kan mit warheit sagen/ daßich aus grund gedachter beeder Werklein/ hernach auch Albertum Dürerum, Vitruvium und andere Auctores mit nutz lessen könen…"*

in das Perspecktiff gebracht, auch mit jren Linien auffzogen sollen warden—which translates as "An authentic, thorough instruction in geometry, especially how all the regular and irregular bodies are inscribed in the ground, and brought into perspective, also with their lines illustrated." There is a teacherly rhyming phrase inscribed under the title of *Geometria*, which itself lies above an owl balanced on top of a polyhedra bearing the Latin words "Spero Fortuna Regressum," which says "*Das Buch Geometria ist mein Namen. All freye Kunst aus Mir zum ersten kamen. Ich bring Architectura und Perspectiva zusamen*"—"The book Geometria is my name. All free art came first from me. I bring architecture and perspective together."[144] The main text in *Ein aigentliche vnd grundtliche anweysung* is divided into a number of chapters, each subdivided into numbered points corresponding to an explanatory image in *Geometria*. The length of text is unusual for a German *Lehrbuch*, and points to what Hirschvogel must have felt was the necessity of giving additional proscriptive instruction on drawing and construction and for his desire for the text to be taken seriously on its own as "a noble and useful art of measurement (known in Latin as Perspectiva) which has been kept hidden in the German language, and which for the common man is hard to acquire and learn, given that the better part of it was written in Latin and Greek (Hirschvogel 1543, introduction)."[145]

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[144] The phrase "*Spero Fortuna Regressum*" was the motto of the Dutch publisher Jasper Tournay and was accompanied by an image of Fortune on a globe. With thanks to Lorraine Daston for this reference. It translates as "I hope for a return of fortune," and is a reference to the *Aeneid* 11.413: "*neque habet Fortuna regressum*"—Fortune cannot reverse her course (Blackburn 222).

[145] "*Gunstiger Herr/und fürderer/Nach dem Bißher/durch unsere vorfordern/ ein langezeit dise edle/ am nützliche kunst des messens (Perspectiva in Latein genant) in Deutscher sprach ganz verborgen gehalten/ und den gemeinen man/ zulernen schwerlich zubetomen/ auch den meren thail in Grietischer und Lataeinischer sprach verfast.*"

Hirschvogel covers simple geometries (lines and circles) before concentrating on the main subject of *Geometria,* “Der Ander teyl de corporibus”—“The other part concerning the bodies (Platonic Solids).” Here, on double page spreads, are found rendered images of polyhedra floating alongside their planimetric and unfolded views [**Figure 3.5**]. Using a technique in the chapter on pyramids, which he repeats for all the rest of the figures in the book, Hirschvogel describes how to represent a pyramid in a circle (in perspective) and how to draw four adjacent equal triangles so that they can be folded up into a pyramid. Then Hirschvogel continues to proscribe a method of construction. “So you should, cut and fold this glued/stuck-together paper, and thereafter you will have a material body.”[146] Later in chapter 6, he explicitly mentions the use of scissors and bending (*biegen*) to create geometrical bodies, much as Dürer did in his *Underweysung*.

The extended title for *Geometria* includes several phrases which reoccur among the various *Lehrbuch* authors, and which describe the graphical techniques that underpin the geometry taught or demonstrated in each book.[147] While the aim of the *Lehrbuch* was often to explicate principles by which objects could be “*in das Perspecktiff gebracht*” (brought into perspective), these principles varied from author to author, as the graphical techniques for constructing and inventing objects in perspective depended, to an extent, on the preferences of a particular author. Setting aside the historical importance of perspective as a goal in and of itself, these personal preferences used to construct geometry in perspective form the core of each author’s graphic epistemology. An

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[146] “*Solche magst du von diesem züsamen gebapten Papir auß schneyden unnd züsamaenlegen/ das dir ein materlich Corpus daraauf wirt*.”

[147] “*…und durchreiß auff halben teyl m it einem Schnitzer, so du es züsamen legen wilt, auff das es sich h(n)ester? Lieber biegen (to bend) mag, so hast du das dritt Corpus*.”

archaeology of the geometrical operations used to produce form uncovers each of their unique contributions to the development of geometrical knowledge.

For Hirschvogel, perspective was a new state of being into which geometries and other objects, understood as physical entities, could be carried. The first clue from Hirschvogel's title is the word "*gebracht*," from "*in das Perspecktiff gebracht*," which both implies a directional vector whereby something is moved from one place to the next and a traceable stationary state of a tangible thing. Thus perspective is also a space, albeit a graphic one, where haptic objects can be determinedly laid down or transported from our own experiential environment. "*In den grundt gelegt*"—meaning to situate/locate a geometrical body in/on the ground (or in plan)—refers to this act of "setting down" an object as an unfolded graphic net to prepare it for translation into perspective, first through building it as a paper model and then by rendering the model perspectivally back on paper.[148] Unlike in the upcoming discussion of Heinrich Lautensack, Hirschvogel plots his "regular bodies," the Platonic Solids, in circles, using the circle as an equidistant boundary to construct the Solids in perspective.[149] Even when he describes his irregular bodies—further articulated polyhedra, covered in increasingly more facets then the standard Platonic Solids [**Figure 3.6**]—these geometries are also capable of being constructed perspectivally within a circle. Thus the development of geometrical

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[148] The graphic techniques and supplementary technologies for drawing physical objects in perspective was a site of great variation and investment in the 16th century. Some of these examples have been covered in Chapter One (Dürer) and Chapter Two (Halt and Jamnitzer).

[149] We are aware that his regular bodies (the pyramid previously mentioned for example) have been "brought into perspective" by the way their sides are shaded, but there is no appreciable development between the outlines of the pyramid, which have been inscribed in the circle, and the finished perspectival pyramid.

complexity, for Hirschvogel, is equated with further subdivision of surfaces, until his forms begin to approach their limit condition as spheres.[150]

Finally, “*mit ihren Linien aufzogen*”—“with their lines illustrated”—refers to the unfolded sides of his solids laid flat against the surface of the page. Similar to Dürer’s earlier proscriptions in the *Underweysung der Messung*, Hirschvogel’s instructions make it explicit that these graphic “nets” were intended to be folded up to construct solids made from paper. The Platonic Solid was to be constructed from the “ground” of the paper, or the space of the page. In the case of the aforementioned pyramid, the shaded, perspectival pyramid also functions as a scale model of the foldable paper pyramid. The lines double as tracks guiding the path of scissors, the unfolded pyramid most probably having been transferred over to another page by pricking the edges of the figure with a needle, through an additional loose leaf of paper, which could then be cut up to form the paper pyramid without ruining the original book. Hirschvogel’s geometrical imaginary, at least as it was represented in *Geometria*, was actually dependent upon these specific techniques of making. The terms of his inventiveness were defined by the technologies he used to draw. Unlike Lautensack, who represented geometries that would be very difficult to construct from paper or by folding, the logic guiding Hirschvogel’s geometry is the development of a more expanded repertoire of shapes predicated specifically upon their constructability from paper.

Geometria is evidence that by the mid 16th century, geometry had become a self-conscious topic for inventive mixed-mathematical research by German artists, and that

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[150] Hirschvogel’s use of the sphere as a limit condition is telling, as a property of each Platonic Solid is that each possesses a concentric sphere that passes through all their vertices.

this research was informed by practice, making, the physicality of materials and technologies, and the work of Albrecht Dürer, rather than by the time-honored tradition of repeating Euclidean constructions, as in Melanchthon's university curriculum. While Dürer's new investigations into the constructability of the Platonic Solids from an unfolded paper surface had further popularized the act of unfolding to the broader artistic community, it was the *Lehrbücher* that were responsible for popularizing what might otherwise have been a novel graphic technique into a constituent property of geometrical form. The popularity of the *Lehrbücher* confirms the utility that the construction and representation of the Platonic Solids were seen to have as well as their familiarity in mixed-mathematical circles. These polyhedra were economical to make and to experiment with, and could then serve as objects to be drawn in perspective or as the base forms for more complicated geometrical confections.[151] That Hirschvogel uses the Platonic Solids for learning geometry reinforces the polyhedra's status as the pedagogical objects of choice for experiments in the new science of perspective. Geometry was increasingly an area of invention, not for regurgitation, and it was the *corpora regulata* (*regularia*) that were the most common experimental subjects to be operated upon.

Much like Hirschvogel, Heinrich Lautensack, author of *Des Circkels unnd Richtscheyts* (1564) and "enthusiast of geometry and perspective" (Doppelmayr 161), belonged to a prominent family of Nuremberg artists.[152] His father, Paul Lautensack, a German painter

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[151] For instance, see Wenzel Jamnitzer's *Perspectiva corporum regularium* (1568) and Peter Halt's *Perspektivische Reißkunst* (1625).

[152] The entire title is *Des Circkels unnd Richtscheyts auch der Perspectiva, und Proportion der Menschen und Rosse kurtze, doch gründtliche Underweisung dess rechten gebrauchs: Mit vil schonen Figuren, aller anfahenden Jugent, vnd andern liebhabern dieser Kunst, als Goldschmiden, Malern, Bildhauwern, Steinmetzen, Schreinern.*

and organist from Bamberg, had moved to Nuremberg in 1525 after embracing the Reformation, and Heinrich Lautensack himself later settled in Frankfurt where he made a living as a goldsmith, painter, and engraver (Bryan 186). As with Hirschvogel's *Geometria*, *Des Circkels unnd Richtscheyts* is a *Lehrbuch* self-avowedly modeled on Dürer's treatises. It is similarly pitched to goldsmiths, painters, stonecutters, carpenters, and builders (Lautensack ii) and also covers a spread of topics running from basic geometry through the construction of the polyhedral solids in plan and perspective.[153] In its latter sections, *Des Circkels unnd Richtscheyts* deals with drawing architecture and human bodies in perspective as well as human proportion, much of which is copied directly from Dürer's *Vier Bücher von Menschlicher Proportion* (1528).

Lautensack gives instructions on how to draw polyhedra in a planimetric view, which he repeats throughout the book and illustrates with a series of diagrams, as in his rendition of a hexagonal polyhedron shown in plan and as a skewed plan in perspective [**Figure 3.7**].[154] Yet unlike Hirschvogel, Lautensack's conceptual ground is perpendicular to the surface of the page; it sits within the space of the page. The sentence preceding the

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[153] Dürer made a similar declaration in the introduction to his *Underweysung.* See Chapter One.

[154] The construction of the hexagonal polyhedron, labeled as nr. 25 in the accompanying Figure, is described as follows. "For the 4th, if you want to make the tetragon into a hexagon, so you need to discard from it a seventh part on one side, like with the triangle [from an earlier example], that is correct. Afterwards, make a cross inside it, and also a straight cross. So the straight cross gives lengthwise two segments on both ends. [So] if you then divide each half of the original tetragon widthwise into two parts, you would have on the top and at the bottom two segments on both sides, as it is drawn." "*Zum Vierdten/ wenn du die vierung zum sechs eck wilt machen/ so mustu ein siebendtheil an der ein seiten darvon thun/ wie am dreieck; so ist sie recht/ darnach mach ein Creutz uber ort darinn/ und auch ein gerads Creutz/ so gibt das gerad Creutz der leng nach an beyden enden zwei eck/ so du denn der breyten nach uber zwerg ein jeglichs halb theil der vierung in zwei theil theilest/ so hastu unden und oben an benden seiten zwei eck/ wie es hie ist aussgerissen*" (Lautensack 20).

hexagonal figure translates as "Here I have, in a poor manner, extracted it [the hexagon] out from the ground/plan into perspective" ("*aus dem grundt aussgezogen*").[155] For Lautensack, form is created through an extrusion that stretches out from the ground plane and is then selectively modified by secondary graphic techniques. In this instance, the extruded hexagon has been filled in, with triangular cuts excavated from its interior and diamond shapes ringing its sides. Lautensack is not concerned with the buildability of his forms as paper models, and provides no instructions about how to unfold them, assemble them from paper or how to draw them. Instead, the hexagon is the geometrical inspiration for the rendered perspectival figure, clearly hexagonal in origin, which lies beneath it on the page.

Because for Lautensack perspectival geometry was seen to be "extracted from the ground/plane/base" ("*aus dem Ground auffziehen*"*)*—out from the plan and into perspective, he depicts his geometrical inventions as models situated on an abstract ground and casting printed shadows onto the page [**Figure 3.8**]. Using this graphic technique in the reverse direction in order to construct two-dimensional geometries, a pattern could be traced in the ground-plane ("*in dem Grund gelegt*"), projected down from any number of sectional "cuts" through a three-dimensional solid. Although these cuts would not allow one to fold a solid out of paper, the implication is that if one took enough horizontal section cuts through a solid, it would be possible to join the slices together, one on top of the other, to reconstruct the solid graphically, much like gluing together sequential slices through the trunk of a tree might allow one to reconstruct the

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[155] "*Hie hab ich es auff ein schlechte art auß dem grundt aussgezogen in die Perspectiff*" (Lautensack, 20).

profile of the entire trunk. Thus one solid object (perforated with triangular holes for example) could theoretically generate an infinite number of potential Swiss-cheese-like two-dimensional slices, much as one slice (or plan) could be used to generate solids through extrusion, which could then be infinitively modified through cutting, filling, and appliqué.

Given the myriad potential ways to invent form, Lautensack's commitment to extrusion cut through the morass of creative possibility and contributed to a clearly defined area of graphical experimentation. From the same polygonal base, a limitless number of geometrical permutations are possible. The geometrical body is no longer a stable, singular solid, nor even a stable if foldable surface, but contains within itself the latent techniques of its own deconstruction and transformation. This is a shift in emphasis from the intended strictures of Euclid to a mathematical text predicated upon developing its user's capability to generate geometrical variation himself; variation that unlike in Dürer's *Underweysung* is more than just implied but is also palpably irregular, overtly *not* in *Elements* nor capable of being described in text alone. Lautensack proscribes bounded geometrical play based on related difference, in which a repertoire of geometrical form is experimentally developed through two main techniques: projecting a planimetric "cut" of a polyhedra onto a "ground" and extruding a solid polyhedral object out of a plan that could then be tweaked, incised, and developed to achieve a range of related geometries. Once modeled graphically in three dimensions, "physicalized" movement animates Lautensack's geometrical constructions, like many of the geometrical *Lehrbücher*; a twisting and stretching; stars teetering on their vertices; the properties of materials translated into a graphic world.[156]

Still the onus was on the student, or apprentice, of geometry to bridge the large gaps between "theory" and practice, though this pedagogical approach was not made explicit by Lautensack unlike in Dürer's *Underweysung*. While Lautensack teaches planimetric construction (of his hexagon for example), he does not provide enough information to draw the geometry he depicts in perspective. They stand as aspirational models of skill – products of his research into the invention of forms in perspective. Lautensack probably found it to be just as important to train the eye to recognize correct (or nearly correct) perspectival constructions; to have readers be able to leap agilely between the plan and the third dimension—and thus also to develop the skill to think three-dimensionally from a two dimensional drawing. There is an implied space of imaginative development between the ground plan and the 3d object. The commitment lies in developing the link between the second and third dimensions and in offering up to his readers the fruits of his professional labor with and study of geometry.

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[156] The use of physical operations as the inspiration for equivalent graphic-mathematical operations has been discussed by Daston. In particular, Daston relates how the mathematician Theodore Olivier (1793 – 1853) helped to develop and popularize the physicalist tradition of French synthetic geometry, a tradition that conspicuously straddled the boundaries between mathematics and mechanics (Daston 1986, 269). For further reading on the 19th and 20th century legacy of using physical models in the teaching of mathematics, particularly in Germany, see D. E. Rowe, "Mathematical models as artefacts for research: Felix Klein and the case of Kummer surfaces," *Mathematische Semesterberichte 60*, 2013, 1–24; H. Burmann, H. et al, "Die Sammlung Mathematischer Modelle und Instrumente des Mathematischen Instituts." *Ganz für das Studium angelegt: Die Museen, Sammlungen und Gärten der Universität Göttingen.* D. Hoffmann and K. Maack-Rheinländer (Ed.), Göttingen: Wallstein, 2001, 175–181; A. Sattelmacher, A, "Geordnete Verhältnisse. Mathematische Anschauungsmodelle im frühen 20. Jahrhundert," *Berichte zur Wissenschaftsgeschichte* 36.4 (2013): 294-312; and A. Sattelmacher, "Zwischen Ästhetisierung und Historisierung: Die Sammlung geometrischer Modelle des Göttinger mathematischen Instituts,"*Mathematische Semesterberichte* 61.2 (2014): 131-143.

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Free imperial cities like Augsburg and Nuremberg, the latter having nearly half of its craft workshops filled with metalworkers, were situated most advantageously to capitalize on the fervent interest in the decorative arts and mixed-mathematics.[157] Guilds and craft organizations swelled to include new members, increasingly regulating standards of craftsmanship, the number of employees allowed in workshops, and taking on responsibility for checking the purity of metals.[158] In Nuremberg, where guilds had been outlawed following an abortive guild rebellion in 1349, it was the powerful city council (*Rat*) which, intimately connected both to the city's government and craft economy, acted in a guild-like manner, setting prices and working hours, apprenticeship

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[157] Since the 14th century, when the oldest surviving listing of Nuremberg craftsmen, from 1363, denoted fifty trades and 1,227 masters, at least half of the craftsmen in Nuremberg were metalworkers, engaged in the production of goods for export, with iron imported from the neighboring Upper Palatinate, and copper and tin from Saxony and Bohemia (Wendehorst 20-21).

[158] Before guild members finished items, their work had to be inspected for the requisite standard of purity. If the work of a goldsmith was deemed of sufficiently high quality, the work was struck with the town mark and sometimes the year's or warden's mark, and returned to the goldsmith for finishing. The records of the London Goldsmith Company are full of references to the struggle required to maintain standards of workmanship and the faith in the company's ability to regulate itself. Examples abound. In 1574, one Charles Pierson was "openly punished in the stocks within the Hall, in the sight and presence of the whole Company there present, for graving certain articles, made of silver, as gold—the same being only over-laid with gold and soldered with gold solder, to the manifest injury of the Queen's people" (17 Eliz.-1574, Memorials 78). Others were put on trial for "deceitful workmanship," such as producing counterfeit marks (Memorials 90) or placing marks on substandard work (Memorials 92). To avoid a scandal, the Wardens even paid a client themselves when a company goldsmith refused to return the money for a cup that had been commissioned from him and "which was found to be under standard" (Memorials 98). For further reference on standards, see Forbes 72-73 and Beasley and Dove, 2013.

and procurement rules, establishing production specifications, and assessing and upholding the quality of workmanship (Strauss 39).[159]

The 17th century reputation of the Nuremberg workshops as sites in which the prodigious application of geometry was practiced is mirrored by its renown in the two previous centuries. The famous astronomer and mathematician Johannes Regiomonatus (1426-1476) was impressed enough with the scientific/mixed-mathematical innovations streaming out of Nuremberg's workshops to abandon his patronage under the King of Hungary and resettle in Nuremberg in 1471 to establish a small observatory and scientific printing press. But perhaps it was Petrus Ramus (1515-1572), the French humanist and educational reformer, who most passionately espoused the benefits of the practical application of and tactile relation to geometry as utilized in southern German workshops (Smith 2004, 66).[160] In his *Prooemium mathematicum* (1567), Ramus defends geometry against charges of uselessness by emphasizing its importance in mixed-mathematics and the arts (Goulding 36). In Ramus' view, France could above all learn from the Germans about how to unify theory and practice, a goal he considered integral in mathematical pedagogy.[161] Passing through Nuremberg in 1568, when he apparently visited the

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[159] In Nuremberg, the Order of 1572 stated that goldsmiths could only employ four journeymen and two apprentices in their workshop. They could however engage other journeymen for model and pattern making. (Hayward 42).

[160] Ramus' writings, sometimes called "Ramism," were influential in Germany and on the Germans' self-perception of their own mixed-mathematical practices. Ramism was further disseminated in Germany through the writings of Henning Rennemannus and Frédéric Beurhusius (Hooykaas 113) and sometimes became combined with the logic of Melanchthon into what is known as "Philippo-Ramism."

[161] Ramus' *Arithmeicae Libri duo* (1569) collected visual cues of the existing mathematical knowledge on the Platonic Solids, displaying shaded, skeletal, and unfolded polyhedra—further evidencing that Dürer's unfolding techniques had become "standard" knowledge. But despite his enthusiasm for practice and artisanal epistemologies, the reduced sizes of the polyhedral images

workshops of the artists Wenzel Jamnitzer and Hans Lencker, substantiated his belief that the combination of mixed-mathematics with the artisanal activity occurring there was evidence of a new method of practical reasoning (Smith 2004, 67) that was producing exciting and innovative results. According to Ramus' contemporary biographer Thomas Freigius, Jamnitzer and Lencker had constructed a machine of such rarity that they wanted to charge money from Ramus and his companion, Frédéric Reisner, to see it.[162] Apparently, Ramus was so intrigued that he offered in return to have one of his treatises on geometry translated into German (by Reisner) if they would let him approach the machine (Waddington 210).

The integration of a material and practice-based focus into the geometrical research and teaching within mixed mathematics is emblematic of a shift in priorities away from the transcendent proportions of a universal order a là Melanchthon towards a set of "physicalized" geometrical operations inspired by the tactile properties of generating geometrical objects, albeit visualized in two dimensions. The resultant approach substituted the theoretical, university-based geometrical exercises and definitions of Euclid (an abstraction in which geometry was divorced from real world application) for a theoretical geometry used to teach perspectival construction (the precise plotting of imaginary geometry on paper), making use of novel graphic manipulations of information. The assembly strategy of geometrical forms in Hirschvogel, out of paper and

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in *Arithmeticae libri duo* imply that there was no expectation that students were supposed to generate their own paper models by tracing and copying the unfolded shape. And the overwrought complexity of Ramus' geometrical definitions, which could have been concisely expressed in an image, situates him in the more traditional Euclidean vein of non-visual, mathematical exposition.

[162] See also the entry on Jamnitzer in Doppelmayr 160-161.

in and out of planimetric and perspectival views, further promoted physical constructability as a key property of geometry. Lautensack's *Lehrbuch* showed how one base geometry could produce limitless, related others. Operating on shapes, and inventing shapes to operate on, made newly possible a critical differentiation between representation and the geometrical definition/identity of form. An object could remain newly itself in many different graphic ways, whether rotated, excavated, adorned, unfolded, or extruded.

The self-reflexiveness with which geometry was used and made the explicit subject of images in 16th century Germany underscores the visual conventions that define the presentation of geometrical material in this period and elevated to prominence the importance of precision drawing skills, much like Dürer had hoped. Just as geometry formed the core knowledge essential to all forms of mixed-mathematics, so too was its understanding and application fundamental to learning how to construct drawings with precision. What had begun as training exercises in Dürer, further developed in Hirschvogel, began in Lautensack to have already explicitly shifted into the realm of graphic invention; a further loosening of the already permeable boundary between theory and practice, the teaching of knowledge and the selling of prowess. A market evolved for visual form that was first and foremost geometrical in character rather than figural or imitative of nature, separate and apart from the utilization of the Platonic Solids as pedagogical tools and workshop geometries. Consequent repertoires of irregular geometrical motifs would start to radiate out from Germany inclusive of the syntaxes and cadences of geometrical abstraction, such as the ceiling designs of Georg Hass published in Vienna in 1583 [**Figure 3.9**]. The Platonic Solids changed also, not in their graphic constitution but in *what* they constituted. Their unique study proportionally marginalized by the increasing development of irregular polyhedral variations, the Solids would find stable purchase as signifiers of professional aptitude in geometry on the frontispieces and portraits of mixed-mathematicians [**Figure 3.10**].

Mixed-mathematicians of the Renaissance harnessed geometry in all areas of their practices, pioneering a new range of practical/geometrical techniques intended to make the world more comprehensible and accurately mappable. They aimed to demonstrate that adding geometry to a practice that had not previously made use of it would bring rigor, precision, and increased utility (Bennett and Johnston 9). Yet the opposite is also true. The geometries of practices (and objects) were extracted out and made constituent visual properties, and typologies of geometrical play developed that were accompanied by a new, abstract geometrical and spatial aesthetic.[163] This process of externalizing geometry and making it perceivable and teachable, was bound up with the intricacies of a pervasive, drawing-based epistemology, for it was in drawing that geometry could be recovered in objects and made manifest in further designed objects. The acceptance of, or even concentration on, the materiality of the drawing medium allowed perceptual attention to remain concerned with surfaces—of pages and of unfolded shapes—without entirely bowing to the hermeneutics of the depicted perspectival-geometrical content. Medial awareness, what Friedrich Kittler has referred to as the perception of "principles of image storage, transmission, and processing above their various realizations" (Kittler

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[163] On the function of play and playfulness in the Renaissance, see Rothstein 106-114 and P. Findlen, "Jokes of Nature and Jokes of Knowledge: The Playfulness of Scientific Discourse in Early Modern Europe," *Renaissance Quarterly, 43*(2), 1990, 292-331.

2010, 26), seems to have emanated outwards from the life-experiences and careers of mixed mathematicians whose livelihood depended upon understanding the properties and potentialities of material, whether gold or the time-telling shadows cast by the sun.

Chapter Four: The Aesthetic of Complexity

The vast shifts in the social and cultural landscape of Reformation-era Germany had far-reaching implications for artists and other craft practitioners. The less explicitly emotive images advocated by Luther had their roots in the devotional habits widespread throughout the Middle Ages, and were dependent upon activating the intellectual cognition of a viewer "to recognize the *idea* behind the image" (Morrall, 1998, 83 and 86), rather than relish in the aesthetic pleasure of the painted surface. Although Protestant iconoclasm ultimately may have renewed the power of the image through its negation as Joseph Koerner has claimed, at the most prosaic level the following generation did not produce a Dürer, a Cranach, an Altdorfer, or a Holbein, to name a few of the great Northern European artists whose careers ended at the turn of the 16th century (Koerner 2004, 13).[164] Consequently the post 1500 period in Germany has been traditionally viewed as an era in which painting and sculpture fell into decline without even offering up the exuberant loosening of morals and styles, of lines and subjects, least of all the frisson produced by the dramatization of decadence (Koerner 2004, 27).

Still as the spread of appropriate subject matter contracted away from the depiction of expressive biblical themes, the decorative arts, replete with secular, geometrical motifs, surged in virtuosity and popularity.[165] Johann Isaak Ehe's (1586-1632) *Design for a Chandelier with Sixteen Candles* (1632) embodies the shift towards a repertoire of

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[164] On the function of iconoclasm within Reformation art and the changing significations of the image, see Koerner 2004. On shifts in artistic style during the Reformation, see A. Morrall, *Jörg Breu the Elder: Art, Culture and Belief in Reformation Augsburg*, Aldershot, England: Burlington, VT: Ashgate, 2001.

[165] On the prevalence of geometrical patterns in textile design see Speelberg 10-15, 26.

acceptable subject matter that had expanded to explicitly feature and highlight polyhedral geometry as objects of contemplation and pleasure in the domestic interior [**Figure 4.1**].[166] As a further case in point, the Hirschvogelsaal, a small pavilion in Nuremberg built in 1534 by Peter Flötner (ca. 1490-1546) for the patrician Leinhard Hirschvogel, contains a group of little-known ornamental columns covered in emblems of contemporary knowledge done in wainscoting (wood relief).[167] Cascading down two columns dealing with measurement, surveying, architecture, astronomy, and geometry are the technological instruments used for mixed-mathematical data—the compass, the sundial, the globe, the clock, and the astrolabe. At the base of the columns lie polyhedral models [**Figure 4.2**].

Geometrical subjects were also not limited to singular objects. The measurement of real world phenomena using geometrical principles having become fundamental to mixed-mathematics, for instance in geodesy, fortification, navigation, and painting (Andersen and Bos 698), spawned increased investment in and production of tools for working with geometry. In addition to intricately embellished, decorative art objects, 16th century Nuremberg was known for manufacturing ornate precision instruments such as state-of-

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[166] The drawing is a design in watercolor for a brass, or gilded wood, chandelier constructed from stellated polyhedra. Not only would the polyhedra have picked up the light thrown off by the candles, the flickering would also have been intended to animate the polyhedra themselves, the interplay of cast shadows causing the polyhedra to appear to quickly rotate. The text says *Dieser hangente Leuchter ist von Meister Isaac Ehe Trompetten macher in Nurnberg/ A° 1632. also gefertigt wordon"*—"This hanging chandelier was made by Master Isaac Ehe, trumpet maker in Nuremberg, 1632." The chandelier would have been of substantial size, measuring four feet across when built as based on the Nuremberg *Schuh* marker indicated on the drawing (Alsteens and Spira 188).

[167] The original Hirschvogelsaal was destroyed in 1945 during WWII and has since been reconstructed.

the-art terrestrial and celestial globes, sundials, and surveying instruments that were exported across Europe.[168] Printed ornament books of geometrical patterns became commonplace in workshops, offering up quotable repertoires of motifs in series that found their way onto all types of designed surfaces; from building facades to fabrics and precious metals—cultivating an aesthetic of complexity in which ornamentation was integrated into all subjects including even its functional operation [**Figure 4.3**].

The slippage between highly precise, scientific objects, which were often lavishly decorated, and decorative art objects that displayed precise geometrical and technical expertise, mutually-reinforced the reliability of Nuremberg artisans to potential buyers. The result was a vibrant, local culture of hyperactively articulate luxury objects, instruments, and surfaces, often embellished to the extreme; an aesthetic that changed everyday furniture, compasses, lock mechanisms or even suits of armor into base models for exuberant, hypnotic ornamentation. In turn, patrons both developed a taste for and drove the market for these luxury objects, which served also as flamboyant showpieces of the skill and time required for their creation. Though a robust culture of consumption for luxury goods had existed since the 15th century in Europe (Goldthwaite 212), patronage in 16th century Germany came to equate beauty and desirability with complexity, and

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[168] Johannes Regiomontanus' astronomical observation was completed with instruments from Nuremberg, while the city council purchased portable clocks by the locksmith Peter Henlein to give as honorary presents to Philipp Melanchthon and Emperor Charles V in 1530 and 1541 respectively. On scientific instruments in Nuremberg and central Europe generally see *Treasures of Astronomy*, Nuremberg: Germanisches Nationalmuseum, 1983; G. Strano (Ed.) et al., *European Collections of Scientific Instruments, 1550-1750*, Vol. 10, Leiden; Boston: Brill, 2009; and B. Stephenson, M. Bolt, and A. F. Friedman, *The Universe Unveiled: Instruments and Images through History*, Cambridge, UK: Cambridge University Press, 2000.

thus positioned artisans in competition with each other to produce ever more intricate items.

In order to satisfy these hyperarticulated commissions, lead workshops formed networks of professional associations with groups of highly skilled, specialized makers. As a representative example, Neudörffer details the collaboration required to produce a silver altar panel for the King of Poland. While Melchior Bayr, Nuremberg goldsmith, was commissioned to make the piece, he worked with local craftsmen Peter Flötner, who provided the wooden models and figures (*die Patron und Figuren von Holz*), and Pancraz Labenwolf, who cast these figures in brass. Using the brass figures as models, Bayr hammered and engraved the silver on top of them to create the final panel itself (Neudörffer 125). In addition, Hans Dürer, Albrecht Dürer's brother, provided the original designs for the silver altarpiece, Georg Pencz painted the wings of the altar, and Georg Herten produced the ornamental woodcarving (Brandl 59).[169]

As object-based commissions very often required multiple components or diverse skill sets that could not be found in the same workshop, specialized makers emerged to corner niche markets in the 15th and 16th centuries, further contributing to the ability of the Nuremberg artisans to render objects in prodigious levels of detail. Competition for specialized parts of commissions led craftsmen to attempt to corner niche markets in order to be known for one specific dimension of craft production. This is vividly

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[169] In his letters, Philipp Hainhofer (1578-1647), merchant and art collector, details the immense coordination required to produce the *Pommerscher Kunstschrank* (Pomeranian cabinet of curiosities), from 1615-1617, for Duke Philip II of Pomerania. Wenzel Jamnitzer's correspondence with Archduke Ferdinand of Austria, between 1556 and 1562, similarly elucidates the collaborative nature of artisanal practice (Smith 2009, 47).

illustrated in *Das Hausbuch der Mendelschen Zwölfbrüderstiftung zu Nürnberg*, a collection of images continuously compiled from the 15th century [**Figure 4.4**].[170] In the *Hausbuch*, one can see the extent of the professional specialization that was evidently both entirely normal and profitable as a business model. For instance, Thomas Erngast, depicted in 1535 with his profession listed as a *Ringkelschmid* (buckle-maker), is shown producing reams of buckles with a hammer and chisel. The image of Frycz Zorn from 1482 identifies him as a *Nagler* (nail-maker).[171] While Ving…N.N. (full name unknown) from as early as 1414, was a *Vingerhuter* (thimble-maker).[172] While some exceptional artisans could take on multiple commissions and coordinate networks of collaborators, the average craftsman in Nuremberg defined himself by such kinds of highly specialized activity. Together these craftsmen contributed to a distributed network of assembly-line-like skilled labor that could be activated *en masse* to produce the ornamental masterpieces for which the city was renowned.[173]

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[170] Established in 1388 by a wealthy Merchant, Konrad Mendel, the foundation provided food and housing to twelve elder craftsmen at a time, each of whom had their presence recorded in the *Hausbuch* as a portrait, in which they were identified by profession and depicted at work in their chosen craft alongside their names, birthdate, and membership number in the *Zwölfbrüderstiftung*. It was only discontinued in the 19th century.

[171] Hausbuch, Amb. 279.2° Folio 24 recto (Landauer I); Amb. 317.2° Folio 101 recto (Mendel I).

[172] Hausbuch, Amb. 317.2° Folio 5 verso (Mendel I).

[173] The embrace of specialization was not universal. Published in the England in 1606, *The Gouldesmythes Storehowse*, possibly written by two Hannibal Gamons, father and son, was presented to the Goldsmith's Company, London, in 1606 (Jenstad 40) and laments the inability of contemporary goldsmiths to partake in all parts of the trade. "Secondly, to be a golde worker, it is as rare to have in one man, those partes which belonges to sutche a workman. For wheras his skill oughte to do anye thing pertinent to a golde worker, it is divided into general mens skils, As one to make Jewels onlye, an other Ringes, others Borders, others chaines braslette, others wyer worke, and so for everyone generall works, to be made of golde A general golde worker. And so no generall golde worker, but a parte of one. Therefore it is very expedient, that everye workman, which wilbe acconnted a perfit worke mayster, to labor with all his industrie, and diligence, to

German decorative arts and craft knowledge reached its peak in this period, achieving levels of visual complexity never again to be attained. Nuremberg in particular became a world-renowned center of precision manufacturing during the 16th century, benefiting from the changing socio-economic circumstances engendered by the Reformation and acting strategically to capitalize on and promote patronage of the self-consciously complex.[174] Guiding the prevailing aesthetics towards products which only Nuremberg artisans could produce, visual complexity conveying craft knowledge became the guiding principle for much decorative artwork.[175] By flooding the market with luxury items unable to be produced either by nature, or more practically by guilds, crafts organizations, or cities bereft of Nuremberg's skilled network of collaborators and technological expertise, Nuremberg was able to respond to and invent new parameters for

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gaine ?? to be syngular in the arte, which he professethe: And so sutche a parson, is to be acconnted a hole workeman and not a part of one" (19th chapter, Folio 32).

[174] In the 16th century, the political instability of the Reformation was used by many guilds to exert popular will for protestant reform on city councils, in those instances where incentives for preserving a guild's social standing and economic base seemed to hinge on religious reform (Broadhead 1979, 80). In the free imperial cities, introduction of the Reformation usually occurred with direct participation of the community, with councils organizing votes in the guilds, or in a communal assembly (Moeller 64), and the city as a whole converting to Protestantism by proxy. Yet the evolving function of guilds as "democratic" bodies, bent on preserving their economic advantages and serving as forums for political discourse, made their status uniquely vulnerable to the success of the protestant battles being waged across Germany. For a persuasive case study on the Weavers and Butchers guilds in Augsburg and their differing rationales for supporting or opposing the Reformation, see P. J. Broadhead, "Guildsmen, Religious Reform and the Search for the Common Good: The Role of the Guilds in the Early Reformation in Augsburg," *The Historical Journal*, Vol. 39, No. 3 (Sept., 1996), 577-597.

[175] Self-conscious complexity was not solely the domain of the visual arts. As dramatized in Richard Wagner's comic opera *Die Meistersinger von Nürnberg*, first performed in Munich in 1868, David, the fictional apprentice to Hans Sachs, details the complicated rules for singing in a *Meistersinger* contest. "Every word and note must sound clearly, wherever the voice rises and where it falls; don't begin too high or too low, so that the voice can reach all the notes. Be sparing with the breath, so that it doesn't pinch, and give out on you at the end; don't start humming with the voice, also, at the end of the word, don't buzz with the lips. Don't make changes in ornament and coloratura, every embellishment strictly following in the Masters' footsteps" (Wagner 35).

taste which were self-beneficial and, for the most part, unattainable by competitors jockeying for the same courtly commissions. Thus the strategy of developing an aesthetic of complexity, and marketing it as the prevailing taste, sought to engrain in buyers the impression that Nuremberg makers were at the forefront of mixed-mathematical research and technical prowess.

The capabilities of Nuremberg workshops were a result, in the most practical sense, of the role of drawing and measurement skills. Skill in the accurate and measureable representation of objects or components of objects, a skill cultivated through the use of geometrical *Lehrbücher*, was critical to the successful manufacture of the *Meisterstücke* produced in Nuremberg. Goldsmiths, intarsia-specialists, and other craftsmen who dealt with ornament and ornamentation worked through drawing to train their subordinates, develop their own thoughts, coordinate design components, perfect their proposals for commissions, and present progress or presentation designs to clients. The practicalities of producing secular, decorative art, in which biblical themes were often replaced by geometrical or naturalistic motifs, led to typologies of drawings, and even typologies of approaches to drawing, that were uniquely informed by artisanal experience. Drawings by goldsmiths were explorative, iterative, concerned with surface transformations and seriality.[176] Bringing to bear the wealth of their experience as makers, goldsmiths distinguished themselves as innovators in the graphic realm, communicating their intimate familiarity with the performance of matter in flux and their eminently pragmatic ideas about the generation of form.

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[176] On goldsmith drawings see Farmer; and Warncke Vol. 2.

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The *Jeweller's Pocket Book* (ca. 1550, south Germany) at the British Museum, a small calfskin bound book intended for ease of portability, contains several drawings in pen and arrived at the museum in 1978 stuffed with prints and drawings.[177] While a rare item, it was not unusual for precious metal workers in the 16th century to keep sketchbooks of observations on human and animal figures alongside future design ideas and graphic experiments in the representation of ornament and geometry [**Figure 4.5**]. A drawing of a circular plate divided up into seven sections from the *Jeweller's Pocket Book* has two portions of ornament filled in [**Figure 4.6**]; a shorthand reference to what the completed plate would look like with either of two design options. The sketched variations in ornament [**Figure 4.7**] show different permutations of the same idea through drawing, each iteration only very slightly modified from its predecessor. All of these designs were potentially useful, as they could find application, in whole or in part, in multiple commissions or remain as a source of future inspiration.

Perhaps the greatest collections of 16th century goldsmith drawings belonged to Basilius Amerbach (1533-1591), a prominent lawyer and art collector from Basel, Switzerland, who amassed over his lifetime a substantial collection of drawings and objects; the so-

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[177] It is difficult to say definitively whether the inserted drawings are connected with the early history of the book. (i.e. they could have been assembled by a more recent owner), nevertheless, it would not be impossible that the original owner might have kept several prints and drawings by other artists as inspiration within his pocket book. Several of the inserted drawings in the book evidence the interchange of information (and motifs) between different artists, specifically with reference to Hans Holbein the Younger and Virgil Solis. See appendix in J. Rowlands, *Drawings by German artists and artists from German-speaking regions of Europe in the Department of Prints and Drawings in the British Museum,* London: British Museum Press, 1993.

called *Amerbach-Kabinett*.[178] Among its many treasures, the *Kabinett* contains items from different goldsmith workshops contemporary to Amerbach, including molds and design drawings from Wenzel Jamnitzer's workshop, which Amerbach apparently bought after Jamnitzer died from the plague in 1585. The *Historisches Museum* in Basel is in possession of a number of lead patterns used by Jamnitzer that have been matched to parts of the splendid coffer he was commissioned by the city of Nuremberg to make for Maximilian II in 1570 (Söll-Tauchert et al. 249).[179] These patterns are small strips of ornament, the largest is 9.4 cm by 2.5 cm, depicting *Rollwerk*, a miniature row of columns, and interlocking rope-like motifs.

To build up the visual complexity of Maximilian's coffer, or indeed any highly ornamental metalwork, the goldsmith Wenzel Jamnitzer would have had to have relied upon multiple patterns, which could be used as a kit of interchangeable parts.[180] These patterns would have been based on repeatable strips of ornament from *Musterbücher*, or pattern books, as in the following print from the workshop of Peter Flötner in 1546

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[178] The *Amerbach-Kabinett* was posthumously purchased by city of Basel in 1661 and its contents held jointly at the *Kunstmuseum* and the *Historisches Museum* in Basel.

[179] Maximilian II gave the coffer to his son, Phillip II, who passed it on to his sister who lived in the *Monasterio de las Descalzas Reales* in Madrid. The coffer remains there today as a reliquary. While it is likely that the Kabinett contains more items from Jamnitzer's workshop, the Kabinett's various inventories only mention Jamnitzer by name in regards to one item—a mold of an ornamental saddlebow for King Maximilian II. Inventar D, 1585-87 says "*Item ein Sattebogen mit einer Caritas vnd hinder theil eines satels mit gÿbs (von keiser Maximilian sattel, so Gamützer Ze Nornberg in silber gemacht) abgossen*" (reprinted in Landolt 148); "Item, a saddlebow with a caritas and the back part of the saddle cast with plaster (of King Maximilian's saddle, so made in silver by Gamützer of Nuremberg). In Inventory G (July 1662 and corrected in 1664), the same saddlebow is referred to again as "*item Zweÿ stukh von Gÿps von Kaÿs. Maximiliani reitsattel*" (Beiträge 187)—"Item two, a plaster piece of Emperor Maximilian's riding saddle." See also Hayward, "The Erlangen Saddle Plate Designs."

[180] See Hayward 59-60.

(Warncke Vol. 2, 57) [**Figure 4.8**]. Masterpieces like the coffer were composite objects made up of numerous patterns cast onto its surface in individual fragments, utilizing ornamentation that would have been compiled over years of commissions and could be repeated in series.[181] For this reason, lead patterns were among the most precious and important commodities that a goldsmith could own, as they could be reused, recombined, and repurposed for future objects.[182] To covet the patterns of another goldsmith meant that one could essentially copy the ornamental motifs that defined a goldsmith's style, a problem that the Nuremberg city council took very seriously. Council records show that on the 19th of December, 1549, one goldsmith, Petern Kuster, was threatened with being thrown in jail if he would not report how he had come into possession of Jamnitzer's mold (*mödel*) and patterns (*kunsteisen*) and those who had given the objects to him.[183]

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[181] Jamnitzer was also the first to utilize revolving stamps in order to produces strips of continuous ornament without the inevitable, if subtle, breaks when pieces of ornament are joined together (Hayward 53).

[182] Casting from patterns was only one dimension of the 16th century goldsmith workshop, as can be seen in Etienne Delaunne's (1519-1583) engraving from 1576. Assistants were employed to melt down existing gold items for reuse or to make the lead patterns in the first place—beginning with the construction of wax casts that would be subsequently covered in terracotta to make base molds for the lead patterns. The molds themselves had first to be carved, often from a soft wood like lime or pear, by *Bildschnitzer* or *Formscheider* (model makers) as they were called. Others specialized in soldering tiny metal parts together or onto the main commission, forming new molds for special elements, flattening metal, and gilding, embossing, and enameling. There were even a collection of goldsmith workshops that concentrated solely on a singular aspect of the manufacturing process, a further degree of sub-specialization in line with the professional atomization seen in *Das Hausbuch der Mendelschen Zwölfbrüderstiftung*. Overseen by a master goldsmith, designs could be outsourced to a close-knit group of specialized goldsmiths in order to produce highly complex objects, almost through a distributed form of skilled assembly-like production. For further elaboration on the art of the goldsmith, and how it changed or persisted from the medieval practice, see *On Divers Arts* (ca. 1122) by Theophilus, a comprehensive manual of 12th century craft knowledge produced by a working artist, and which includes an extensive section on metal-working.

[183] "*Petern Kuster, dem goldschmid, auflegen, die warheit anzuzeigen, woher ime der Wenzls Gamitzers model und kunsteisen komen und wers ime geben hab. Im fal dan, dass ers sich*

Perhaps the most overlooked products of goldsmith workshops were the designs and design studies produced by masters and their assistants [**Figure 4.9**]. Much less expensive than gold or silver and easily used for experimentation, drawings were the primary medium the goldsmith used to work out their ideas, train the aesthetic sensibilities of their studio staff through theoretical exercises, and convey design intentions to clients. As evidenced from the three hundred plus anonymous drawings from 16th century goldsmith workshops belonging to the *Amerbach-Kabinett*, termed the "*Basler Goldschmiedrisse*," skill in drawing was an essential prerequisite to goldsmith training, and would have been tightly bound up both with the creative process and with the selection of a design to ultimately pursue in precious metal.[184] In some cases, drawings could even be used as "photographic" records of precious artworks, for instance to augment the inventory of an estate. The design for a covered goblet from Hans Holbein the Younger's (1497-1543) workshop [**Figure 4.10**] may have been one such example, given that the exactness of the lines and the hyper-reality of the rending was above and beyond what would have been required from a working drawing.[185]

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widersetzen wirt, sol er ins loch geschafft warden." From the Ratserlässe der Reichsstadt Nürnberg (Frankenburger No. 22, 6).

[184] For further information on the *Baslerrisse* see Shelby 58-59; W. Ueberwasser, "Spätgotische Bau-geometrie: Untersuchungen an den 'Basler Goldschmiedrissen,'" *Jahresberichte der Oeffentliche Kunstsammlung Basel*, N. F., 25-27 (1928-30), 79-123; and R. Mager, "Zur Technik der 'Basler Goldschmiedrisse," *Jahresberichte der Oeffentliche Kunstsammlung Basel* (1963), 35-37.

[185] See C. Müller, "Holbein oder Holbein-Werkstatt? Zu einem Pokalentwurf der Gottfried Keller-Stiftung im Kupferstichkabinett Basel," *Zeitschrift für Schweizerische Archäologie und Kunstgeschichte*, Band 47, 1990, 33-42.

The great variety of uses to which drawing could be put in the goldsmith workshop resulted in a vast array of drawing types and graphic epistemologies—a knowledge provided through and by the graphic image without further textual elaboration. In contradistinction to the Holbein, the following two drawings from the *Basler Goldschmiedrisse* are clearly amateur in quality [**Figure 4.11 + 4.12**]. The first, a goblet (*Deckelpokal*) drawn in red chalk, is incomplete, revealing an uncertain pencil outline in its midsection—quite likely the result of an unfinished design experiment. The artist can be seen to have struggled with representing the elongated, vertical ovals in perspective and has added a gestural line beneath the top of the goblet, as if standing in for a future more proficient rendering of ornament. The protruding candelabra-like cover of the goblet is awkwardly similar to the base, as if the artist had not been able to come up with a unique capstone element for the piece and had opted instead for simply scaling down the goblet's base for reuse at the top. The second drawing, a clumsy sketch of a pitcher drawn with a dull pencil, is clearly the work of someone just learning how to design on paper. The lack of precision makes it difficult to imagine that this drawing could have been used as a final design for an object, while the absence of ornament likely attests to the author's deliberate concentration on the pitcher's outline. The ponderous, inelegant proportions of the pitcher, with its thin spout appended to the thick body of its container, and the more proficient if still amateur goblet in chalk, are examples of invisible, "internal" documents created within the workshop by its personnel; abortive design studies, not intended for public consumption, generated by the pedagogical roles that workshops played in using drawing to develop and refine the design aesthetic of its assistants.

Besides beginner sketches, the *Basler Goldschmiedrisse* includes also a number of proficient, abstract geometrical investigations in series. Perhaps this was envisaged as a solution to a pressing technical challenge faced by the 16th century decorative arts in the 16th century, namely the picturing of transformation and the capturing of the movement of a form in an immobile genre (Jeanneret 108). There are also pages of linear design permutations for late gothic-style baldachins and wimpergs (gothic ornamental gable with tracery), inked in hardline, showing elevation options and geometric plans [**Figure 4.13**].[186] It could be that these drawings were design options representing sequences of possibilities for a new work.[187] It is also likely that the plans and elevation pairings were drawn one after another on the same page. The artist would have started with one scheme and then altered it in the next iteration. The design series displays an interest in sequential transformation, as the linear elements of one scheme after the next are graphically tweaked to create new options; first the outermost spires rotate inwards, while the central element is extended up; next the two curvilinear spires rotate inside of the outer elements, and so on. That they were drawn in series would have permitted them to be seen relative to each other, allowing the master goldsmith to choose one particular option to develop

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[186] These drawing typologies have precedents in ornamental drawings produced for Gothic architecture. During the 15th and 16th centuries in Germany, several popular construction manuals were written. Appearing in the 1480s, *Büchlein von der Fialen Gerechtigkeit* (Booklet Concerning the Correctitude of Pinnacles) by Matthes Roriczer and a second booklet by Hanns Schmuttermayer, a goldsmith who had dealings with Dürer's father, both deployed graphic techniques similar to those in evidence in the *Baseler Risse*. Both Roriczer and Schmuttermayer's treatises are concerned with laying out how to draw the plan and elevation of a pinnacle. The technique of rotating squares, a graphic strategy at the heart of Gothic design practice, was used. As Kavaler notes, "the derivation of the ground plane through rotated and inscribed squares was one of the basic operations for establishing the form of numerous elements of a Gothic building and an introduction to the geometrical armature of Gothic design" (Kavaler 43). For further discussion of the wimperg and baldachine drawings in the *Baseler Risse* see Seeliger-Zeiss 49-57.

[187] There are several basic stereometric figures, like a *mazzachio* or a perspectival cube with holes (drawn in pen with a gray wash).

through object making. While a client might not have been able to tell the difference between any of the baldachin design options once built—they all contain multiple spires, curved symmetrically with crosses—and any one of the options might well have satisfied a commission for producing a baldachin, this type of a drawing would have allowed the goldsmith to decide, himself, which option he liked best. Lining the schemes up next to each other was a graphic conceit that allowed him to assess their merits and take forward the iteration that he felt best represented his design intention.

As can be seen from the *Basler Goldschmiedrisse*, goldsmithery was intimately tied to graphic practices that enabled the development of sophisticated surfaces to be conceptualized prior to the production of objects.[188] Epistemological knowledge given through drawing and site specific to the goldsmith workshop emerged to enable designs to be created, and assessed, in sequence and relative to each other. These designs, whether they were base motifs or fragments of ornament were then applied to the variety of precious objects the goldsmith was commissioned to produce, resulting in the base form of the commission defining how the ornament was to be deployed. A plate design required ornament that could work on a circular shape; a vase, ornament that could wrap

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[188] The production of accurate drawings conveying the sense of a future design, prior to receiving funding or a commitment from a potential patron, was also a laborious enterprise. On December 22, 1556, Italian artist Jakob Strada reported to Archduke Ferdinand II on Jamnitzer's desire to produce for him a vessel modeled after the creation of Adam and Eve, and Jamnitzer's response to Ferdinand's request for a design drawing. Jamnitzer apparently replied, via Strada, that he could make the drawing as detailed as Ferdinand would want, but that he first needed to know Ferdinand's opinion as to the final size of the piece. Furthermore, that it would also be difficult to understand the work solely from a drawing as the final piece would take up more space (would be at a larger scale) than the drawing, and thus it would be prudent to make a model out of plaster, as was the common practice (Translated by author from Schönherr's paraphrasing of Strada's letter to Ferdinand, the original quote not having been included by Schönherr, Schönherr 291).

around its sides.[189] In practice, it appears that this ornament-driven thinking led to designs, and design thinking, possessed of strong metamorphic dimensions, although metamorphosis was not restricted to the figural and might well be depicted through the interplay, transformation, and packing of abstract, geometrical motifs, as in the wimperg designs in the *Basler Goldschmiedrisse*. Taking into account the roving movement of the eye, goldsmiths conjured up sumptuous displays of talent intended to entertain the viewer; a further iteration of the taste for the aesthetic of complexity that swept across southern Germany, made possible by the intense, creative concentration on the surface of things.

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Wenzel Jamnitzer (1508-1585), master goldsmith of Nuremberg, was born in Vienna and became a citizen of Nuremberg and its goldsmith association in 1534. In the historian Johann Neudörffer's *Nachrichten von Künstlern und Werkleuten* (Nuremberg 1547), Wenzel and his brother Albrecht are described as working both with silver and gold, having a great understanding of perspective and proportion, and having cut coats-of-arms and seal-dies in silver, stone, and iron.[190] By the 1730 publication of Johann Gabriel

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[189] The "Jasparware Plaque: The Apotheosis of Homer" (1786)—MLA 1909, 12-1, 186, designed by J. Flaxman Jr. and its accompanying "Pegasus Vase" (1786)—MLA pottery cat. I 712 by the famous potter Josiah Wedgewood (1730-1795), both in the British Museum, attest to the continued issue of wrapping ornament around base objects. In this case, the Apotheosis of Homer, itself "unrolled" from a Greek vase belonging to Sir William Hamilton in the museum's collection, served as the inspiration for Flaxman's relief. The relief was then translated by Wedgewood back around the unique curvature of his new Pegasus Vase.

[190] "*Sie arbeiten beide von Silber und Gold, haben der Perspectiv und Messwerk eine grossen Verstand, schneiden beide Wappen und Siegel in Silber, Stein und Eisen*" (Neudörffer 126). Reprinted in an 1875 edition published in Vienna.

Doppelmayr's (1677-1750) *Historische Nachricht von den nürnbergischen Mathematicis und Künstlern…*, Jamnitzer's reputation as the most renowned of 16th century goldsmiths had long been established, not least because of the work he had produced for four Habsburg emperors—Charles V, Ferdinand I, Maximilian II, and Rudolf II. His prodigious skill and success had also been recognized by his contemporaries in the city's influential community of goldsmiths. After new rules governing the submission of masterpieces in Nuremberg were issued in 1572, all applicants for the title of master goldsmith were to produce masterpieces working off of a pattern cup by Jamnitzer as a base model, although variation in ornament was allowed (Hayward 38-39).[191]

On the basis of his graphic work with geometry, Jamnitzer appears twice in *Historische Nachricht*, as both a goldsmith as well as an accomplished mathematician; a "lover of the art of perspective," who had represented more than 160 regular and irregular bodies in his luxuriously illustrated meditation on the Platonic Solids, *Perspectiva corporum regularium* (1568) (Doppelmayr 160).[192] *Perspectiva* is divided into six sections, the first five including twenty three floating geometrical variants based upon the five Platonic Solids, and the last section including twenty three pages of freestanding geometrical models [**Figure 4.14**]. Perhaps the quintessential expression of the German fascination with polyhedra, *Perspectiva* stands as a testament to the particular contributions of artisanal practices to the study of geometry. Page after page of increasingly elaborate engravings of polyhedra unfurl in *Perspectiva*, unencumbered by textual explanation, and

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[191] In order to become a master goldsmith in Nuremberg, an applicant needed to produce a columbine-cup, a gold ring set with a precious stone, and a steel seal-die.

[192] I have subsequently abbreviated *Perspectiva corporum regularium* as *Perspectiva*.

culminating in representations of geometrical models that abandon their polyhedral origins. That *Perspectiva* was deeply important to Jamnitzer is evident, not least of all due to the time, patience, and training it would have taken to produce the intricate underlying drawings. Despite the fact that in the introduction to *Perspectiva* Jamnitzer laments the "long and boring path" he had to follow during his forty years of practicing the "very lovely and gracious art (which is called *Optica* by the learned and otherwise commonly *Perspectiva*)," mounted onto the back of his grave in Nuremberg's St. John's cemetery (*Johannisfriedhof*), is a metal plaque based upon the title page of *Perspectiva*, with female personifications of Arithmetic, Geometry, Perspective, and Architecture in relief, two of whom are holding up polyhedra, surrounding his portrait [**Figure 4.15**].[193] The grave presents him as a scientist and theorist of geometry, rather than the prodigious maker of decorative art objects he produced as the most famous goldsmith of his time. Irrespective of his fame and success, Jamnitzer's grave stakes his claim to eternal relevance on his perspective and geometry theory, not on the masterpieces for which he was known, like the *Merckelsche Tafelaufsatz* (centerpiece) commissioned in 1549—a crowning achievement in the decorative arts for its seamless integration of artificial forms and natural bodies cast from nature (Farmer 95).

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[193] Jamnitzer succumbed to the plague in 1585. The inscription on the grave reads "*WENTZEL JAMNITZER ALT 78 IM 1585*," and "*CHRISTUS IST MEIN LEBEN – STERBEN IST MEIN GEWI[NN]."* The figures also represent the four elements. Geometry on the upper left holds a twenty-sided Platonic Solid while Architecture on the upper right holds a dodecahedron. The Metropolitan Museum of Art has in its possession a silver gilt relief (produced after 1568) subsequently mounted as a mirror frame that is also a variation on the title page of *Perspectiva* [Met Acc. Num. 17.190.620] and has been attributed to Jamnitzer. The expense required to produce the frame further substantiates the importance of the work to him. For a wider discussion of Jamnitzer's grave-plate in the context of other posthumous presentations of goldsmiths in St. John's cemetery, see S. Hauschke, "Der soziale Aufstieg eines Handwerkers: die Grabplatte des Goldschmieds Wenzel Jamnitzer," *Künstlergrabmäler: Genese, Typologie, Intention, Metamorphosen*," B. U. Münch (Ed.), Petersberg: Michael Imhof, 2011, 97-109.

Virtuosity in the ability to reproduce nature was the highest level of skill to which Jamnitzer aspired in his commercial practice, and yet his epitaph praised instead his skills in drawing geometry to the highest degree of detail. To understand how it is that a goldsmith, more renowned for life-casting animals, chose to invest the time and energy it would have taken to produce *Perspectiva corporum regularium*, the epitome of the polyhedral genre and, critically, had the drawing skills to pull it off, is to query the new prominence of geometry in Reformation-era decorative arts in Germany as well as the integration of drawing into the goldsmith workshop. The ability to think through and represent forms on paper was a necessary tenet of professions, like goldsmithery, tasked with manipulating expensive material into extremely expensive objects, being that it was much cheaper and more efficient to experiment on paper than with gold and silver. *Perspectiva* implicates the status of the drawing in the workshop and inherently interpolates as to how the realities of the workshop—both commercial and tactile—might have informed the graphic, if not literally in form then in an approach to form-making conditioned by the realities and materialities of artisanal practice.

The drawing records Jamnitzer's experimentation with different designs and his search for an ornamental solution that will work given the parameters of a commission. Ornament thus furnishes the nature of the decorative arts in terms of a relation between a surface that holds the primary meaning and the value of the underlying object. There is a schism between form (structure) and surface; form is something on which "meaning" and value can be applied, engraved, or cast. *Perspectiva* inflects and is inflected by these drawing practices, in which the image also serves as a site for the freezing of the *processes* of material metamorphosis into print, reflecting Jamnitzer's daily engagement with the shaping and manipulation of molten metal—a poetic description, in artistic terms, of what could easily be read today as filmic syntax.

Perspectiva was printed by Jost Amman and engraved by Hans Sachs, who published his own *Ständebuch* (Book of Trades) in Frankfurt that same year, and was included in the princely *Kunstkammern* of Prague, Dresden, and the Ambras castle in Innsbruck alongside commissions in gold by Jamnitzer (Kayser 49).[194] Jamnitzer had begun work on *Perspectiva* at least ten years before it was published, as in 1558 he had already sent a letter to Archduke Ferdinand II (1529-1595) in Innsbruck claiming to have "invented the laudable, useful and ingenious art of perspective, the likes of which have never been seen before" and including as well examples of his work on perspective.[195] Dedicated to Emperor Maximilian II (1527-1576), older brother of Ferdinand II, the full title translates as "a diligent exposition of how the five regular solids of which Plato writes in the *Timaeus* and Euclid in his Elements are artfully brought into perspective using a particularly new, thorough and proper method never before employed. And appended to

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[194] The woodblock prints for Sachs' *Ständebuch* were by Jost Amman (1531-1591), early collaborator with Virgil Solis (1514-1562), and producer of the famous image of Jamnitzer at work in his studio.

[195] "*dass ich die löbliche, nützliche und synreiche kunst der perspective also erfunden, dass dergleichen zuvor nie gesehen worden…"* Reprinted in Schönherr 294. Given that *Perspectiva* would ultimately be dedicated to Emperor Maximillian II, and not to Ferdinand II, it is likely that Jamnitzer was not searching for financial support from Ferdinand, but was rather seeking to keep him informed on the progress of his project on polyhedral geometry. Jamnitzer would not have taken the risk of sending unsolicited material to the Archduke had he not been supremely confident in his artistic abilities and of the work's commercial potential as an artifact in and of itself, as an advertisement of Jamnitzer's skill, and as a method of maintaining a relationship with potential clients during the long interims between commissions or while a work was being completed.

this a fine introduction how out of the same five bodies one can go on endlessly making many other bodies of various kinds and shapes."[196]

Unlike Italians working in similar fields who typically included extensive textual commentary, such as Benvenuto Cellini (1500-1571), Leon Battista Alberti (1404-1472), or Sebastiano Serlio (1475-1554), the relation between text and image in German geometrical and perspectival treatises leaned heavily towards the visual. Still, although Jamnitzer did state in the foreword to *Perspectiva corporum regularium* his intention to publish a second more writerly companion treatise on geometry (Kayser 60), Jamnitzer had aspirations for *Perspectiva* itself in its currently primarily image-based incarnation to bear upon humanist discourse, if not from a texual analysis of ancient works as from his own graphic approach to the Solids and their transformation. Each of the first five sections is prefaced by an elaborate frontispiece that describes one Platonic Solid not just in German, but also in Latin and in descriptive text, linking the Solid explicitly to the element associated with it by Plato in the Timaeus.[197] For instance, the frontispiece for the pyramid [**Figure 4.16**] reads "*A.1; Ignis. Das Feuer* [Fire]*; Tetrahedron; Siue Pyramid trilaterata. Ein trianglichter Kegel* [A triangular cone]." The frontispiece is followed by a description of how to construct a pyramid, similar in tone and content to the many *Lehrbücher* to which Jamnitzer presumably had access.

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[196] *Perspectiva corporum regularium. Das ist/ein fleyssige Fürweysung/wie die Fünff Regulirten Corper/darvon Plato inn Timaeo/Und Enclides inn sein Elementis schreibt/etc. Durch einen sonderlichen/newen/behenden und gerechten Weg/der vor nie in gebrauch ist gesehen worden/gar künstlich inn die Perspectiva gebracht/und darzu ein schöne Anleytung/wie auss denselbigen Fünff Cörpern one Endt gar viel andere Corper/mancherley Art und gestalt/gemacht/unnd gefunden werden mügen...* Nürnberg, 1568. Translated in Veltman 13.

[197] For description of the *perspectiva* see Seidenfuß 214-224; and Richter 80-82.

> "The first of the five regular bodies is a body made from four equilateral triangles, surfaces or intersections of straight lines on which it may be placed. It has six edges or straight lines, twelve flat angles and four corners to its geometry. From this triangular body or cone further originate twenty-three other bodies, made in a variety of different ways. As is shown hereafter."[198]

The content of the five sections consist of twenty-four polyhedral variants apiece, beginning with the generative Platonic Solid, six per individual page, four pages in total [**Figure 4.17**]. The polyhedra are represented as floating objects inside six circular vessels whose rims are connected by a continuous strip of material studded with four ornamental protuberances. Each of the four pages proceeds from the simplest polyhedral variant in the top left position to the solid that Jamnitzer considered to have been most completely transformed in the lower right position. Though not entirely consistent, each collection of polyhedral variants appears to devote the first page to transformations that required a graphic excavation of the solid, the second page adds a progressive chopping up of the solids' surface, while the last two pages explore the intersection of two or more solids and their stellation. Following the exploration of the polyhedral variants is a section consisting of forty freestanding models, two per page, in clear mastery of the genre of unbuilt polyhedral models populating the German *Lehrbücher*, and three final pages depicting tableaus of mazzachios, stars, crosses, pyramids, and supporting structures. Nevertheless *Perspectiva* is no common *Lehrbuch*, published to supplement Jamnitzer's income and to be used in Nuremberg's workshops. In the degree of its graphic aspirations, *Perspectiva* resoundingly surpassed the records of geometrical

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[198] "*Der erst unter den Fünf regulirten Cörpern Ist ein Corpus gemacht/ von vier gleichseitigen triangeln/ Flechen oder Pöden gerader Linien/ darauff es gestalt warden mag/ hat sechs seiten oder gerader Linien Zwölff flache Winckel unnd vier Cörperlicher Eck. Auß disem drianglichten Corpore oder Kegel/ find verner drey und Zwainzig ander Corpora geursacht/ und uf mancherley unterschidliche Art zu werckh gezogen. Wie hernach gesehen wirt*."

investigations contained in the various *Lehrbücher* circulating around Nuremberg's workshops, even as it took advantage of his potential audience's familiarity with the genre. *Perspectiva* strove to demonstrate Jamnitzer's mastery of perspective and his learned aspirations to connect his graphic work to contemporary theory on optics and perspective and the ancient texts of Plato and Euclid.[199]

Interspersed throughout the *Rollwerk* that makes up each section's frontispiece are objects and figures that testify to the mixed-mathematical associations of the Platonic Solids as well as the longstanding connection between the Platonic Solids and the corresponding elements.[200] The frontispiece for the pyramid, which is associated with fire, is decorated with dragons, guns, lanterns, incense, and torches while the

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[199] "Perspective is an art in which the character, type, and nature of the lines and forces will radiate out from our face onto other things here and there is taught, so that everything in the entire world is able to be viewed through our human eyes. The heavenly bodies and the firmament as well as the earthly bodies, such as the mountains, lands, buildings, castles, cities, villages, and landscapes or other bodies, and all in all everything that is conceived and comprehended through the face, near or far, high or low, also consisting of uneven angles, as is likely, can be captured by the art of perspective and can furthermore be discovered and their causes understood in the selfsame manner. Thus there is a beautiful subtlety whereby all of the bodies can be represented [with the use of perspective] on a level plane or place, with all proportions in the proper thickness, width, and length, also similarly defined and outlined, as is delineated in each instance through the face, to describe *unmachen*, following the differences or distances of the position of the human eyes. Thus it is believed that it [the body drawn in perspective] exists in bodily form [on paper] and in essence." "*Die Perspectiva* [*ist*]*... ein Kunst die da lehret/ von eigenschafft/ art und natur/ der Linien und Strom so von unserem gesicht auff andere ding hin und wider geworffen warden/ dann alles das/ so inn der gantzen welt durch unsere Menschliche augen angeschawet wirdt/ es seyen die himlischen Cörper/ und Firmament/ oder aber die irdische/ als Gepyrge/ Gründe/ Gebewe/ Schlösser/ Stett/ dörffer und Landschafft oder andere Corpora/ und in Summa alles das/ so durch das gesicht gefast/ und begriffen warden mag/ nahendt oder ferr/ hoch oder nider/ auch von winckeln undeckhen gestalt wie es wol/ das felt alles in die kunst Perspectiva, und wird auß derselben ferner geursacht und erfund/ so ein schöne subtilitet/ alle dieselben Cörper davon ist melding geschehen/ auff einen ebnen plan order platz/ mit aller Proportz gebürender dicken/ Praiten und Leng/ auch abschneydung und verlierung derselben/ wie es dan ieder zeyt das gesicht gibt zu delinirn/ beschreiben unmachen/ nach unterschied oder ferne des standts und menschlichen augen/ also das menniglich nit anders vermaint/ dan es stehe Cörperlich und wesentlich alda vorhanden*."

[200] Jamnitzer's *Rollwerk* is classified as strapwork ornament in the Floris tradition (Farmer 97).

icosahedron, represented by water, includes crabs, squid, tridents, shells, turtles and sea-serpents.[201] The polyhedra are also linked with vowels depicted above each page of designs. “Tetraedron” with A, “Octaedron” with E, “Hexaedron” with I, “Icosaedron” with O, and “Dodecaedron” with U. The Alpha-Omega coupling (represented by Jamnitzer as the vowels A-U), the first and last letters in the classical Greek alphabet, corresponds to the first section of *Perspectiva*, signified by the most simple polyhedron, the tetrahedron, and the last section, signified by the dodecahedron, the most complex (Luminet 255).[202]

It could well be that Jamnitzer saw his use of vowels as a classificatory mechanism for his polyhedral permutations that attested also to the Solids’ vital importance or perhaps even a humanist reference conveying knowledge of Lucretius’ *De rerum natura* (first printed edition 1473), in which Lucretius refers multiple times to the recombinability of letters, the building blocks of language, as mirroring the changing states of matter.[203] Later artists, such as Peter Halt in the title page to his *Perspectivische Reiss Kunst* (1625), would further explicate the correspondence between the Solids and vowels [**Figure 4.18**].

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[201] The octahedron signifies air; the hexahedron—earth; and the dodecahedron—heaven (the fifth element).

[202] Although in several of the Neoplatonic systems vowels are representative of the immaterial, (Principe 43), the association of the alphabet with geometry has its own parallel trajectory in the Renaissance. Luca Pacioli included letters in the appendix to *De divina proportione* and there is a section covering the drafting of the alphabet in the section proceeding Albrecht Dürer’s unfolding of the Platonic Solids in his *Underweysung der Messung.*

[203] “For the same letters signify sky, sea, earth, rivers, sun, the same too crops, trees, living creatures...but it is by position that things sound different. So in things themselves likewise when meetings of matter, its motions, order, position, shapes are changed, things too are bound to be changed” (Lucretius II.1013-1022). Translation by C. Bailey, 1947. With gratitude to Lorraine Daston for mentioning the connection to Lucretius.

In Halt, the vowels are drawn as three-dimensional objects and are balanced atop the Platonic Solids, the building blocks of perspectival and geometrical knowledge. As the preface to the second posthumous edition of *Perspectiva Literaria* (1595) by Jamnitzer's contemporary, the goldsmith Johannes Lencker (1523-1585), put it, "*wie man ohne Vokale nicht sprechen könne, ohne die regulären Körper nichts in der perspektivischen Reisskunst erreiche*"—"as one cannot speak without the use of vowels, so without the regular bodies no knowledge of the art of perspective is attainable."[204]

In the foreword to *Perspectiva*, Jamnitzer spells out his perspective on the natural order of the universe. "God...is good and true and has everything beautifully and artfully ordered...Heaven and earth and the magnificent lights, the sun, moon, and stars, with which he has adorned the heavens."[205] Within this order, the infinite variety of "earthly bodies" ("*irdisch Cörper*"), including mankind, possess their own individual and distinct dispositions; distinctions that may be revealed through interrelationship, much as combining the primary elements like fire and water reveals their fundamental characteristics.[206] In Jamnitzer's view, nature is defined by these degrees of variation and

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[204] The Lencker quotation is from Veltman 55, footnote 94, in turn quoted from the introduction by A. Flocon in the 1964 reprinted edition of *Perspectiva corporum regularium.*

[205] "*Das Gott...ist gutig un trew/ das es aber alles schon und artlich geordnet… Himel und erden und die Herrlichen liechter Son Mohn/ und Stern/ damit er den Himel gezieret hat…"* (Jamnitzer, Forward).

[206] "How miraculously that with the love of God alone, the four elements and the companion five universal essences are ordered under heaven. All earthly bodies and precisely we men and everyone else are measured according to his complexion and qualities. So do fire and water prefer to be composed and maintained, both likewise in body as to the eye. And still fire is not extinguished by water, and likewise water is not completely consumed through fire." "*Wie wunderlich hat nur der liebe Gott die vier Elementa/ und derselbigen fünffte wesenheyt under dem Himel geordnet/ darauß alle irdische Cörper/und wir menschen selbs/ genaturet und gemessigt werden/ und ein ieder sein Complexion und eigenschafft hat/also/ das das Fewer und Wasser/ bedes zugleich in einem Cörper als im auge verfasset und erhalten warden mügen/ und*

change, a view that dovetailed an Aristotelian natural philosophy with the "keen pleasure...aroused by transmutation in the late Renaissance" (Newman 115).

> "At this moment, I would therefore like, in enthusiastic service of this art, herein to describe with great wonderment the five regular bodies, which Euclid dealt with in book 13 of the Elements and likewise the same five bodies by the revered Greek teacher Plato. In addition, I want to show if at all possible, that there are still more elements in nature than the four natural elements and the selfsame five universal essences. Although there can be no more than five regular bodies, there are other kinds of bodies, which are formed in the same way and can be produced from the same geometrical bases [*Böden*]. These may still be compared to the selfsame five elemental bodies of nature, because, namely the trilateral pyramid or the cone both terminate into points as do fire and flames. Basically, this is a stressful and difficult aim; thus the cube or dice, which has a square base, and the other bodies are more difficult to aim for because their bases are difficult to lay out. And like all the other earthly bodies, all living creatures both men and plants, namely foliage and grass, have been created from a mixture of these four elements of nature placed together, as well as have also other geometrical bodies been unendingly mixed and set together, as is the intention of my work in which 140 different bodies can be seen, both whole and skeletal. I have rather wanted to bring these [geometrical] bodies into my new perspective because they have been transformed [*gekehren*] inwards and outwards and into many kinds of different corners, sides, angles, and points."[207]

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doch also/ das das fewr vom wasser nit erlescht/ desgleiche das wasser durch fewr nit gentzlich verzert. . ." (Jamnitzer, Forward)

[207] "*Damit ich nun begirigen diser Kunst/ hierin dienen möchte/ hab ich mir fürgenumen/ die funff Corpora Regulaira, davon Euclides in seinem 13. Buch Elementorum handtlet/ deßgleichen der hochberümbte Griegisch Lehrer Plato in Timaeo dieselben fünf Cörper/ mit grosser verwunderung beschreybt/ und daneben anzeigt/ so wenig müglich sey/ das uber die vier naturliche Element/ und derselben funfte wesenheyt/ noch andere mehr Elementa in der natur sindt/ so wenig können auch andere mehr Corpora regularia/ das sind andere mehr Corpora von gleich förmigen/ und gleich grossen Böden gemacht warden/ dazu so vergleicht er noch dieselben fünff Cörper den Elementen der natur selbst/ dan gleicher weyß/ wie der Pyramis Trilaterata nemblich/ der kegel oben zu gespitzt ist/ also hat auch das fewr und flammen oben seine spitzen/ Item wie das Erdtrich ein last und schwerlich zubewegen ist/ also ist der Cubus oder würffel/ schwerlicher zubewegen/ also der ander Cörper einer/ dieweil seine Böden/ das quadrat/ gros und hart aufligendt ist/ und wie alle andere irrdische Cörper/ von disen vier Elementen der Natur zusamen gesetzt und miscirt warden/ in den lebendigen Creaturen/ als menschen und sich/ sowol/ als in den Vegitatilibus, Nemblich laub und graß/ sowol warden auch ander Geometrica Corpora auß disen fünf Corporibus ohn endtlich miscirt und zusamen gesetzt/ wie in disem meinem vorhabenden werckh/ in die 140. unterschiedliche Cörper gesehen warden/ gantz unnd durchsichtig/ Ich hab aber sonderlich dise Cörper in meiner newen Perspectif gebrauchen*

Perhaps more than any other Renaissance artist, Jamnitzer's goldsmithery was legendary for broaching the limits of the art-nature divide. Literally making use of plants and animals as the base for his life-cast covered compositions, Jamnitzer's work could be seen as a direct copy from nature, in which nature has imprinted its form onto material, not solely a process whereby an artist had striven to replicate nature as perfectly as possible.[208] But what Jamnitzer's goldsmithery could not do was dramatize the process of gaining the "deductive and propositional" knowledge (Smith 2010, 29) that came from the manipulation of matter. Jamnitzer's foreword to *Perspectiva* makes evident that he intends to use the Platonic Solids as emblems to the transformation and potentiality inherent in nature, and that he sees geometrical recombinability, *ohn endtlich miscirt* (unendingly mixed), as an analogous process of invention and construction. The transmutation of the Platonic Solids into a potentially infinite stream of polyhedral geometry signified, for Jamnitzer, the natural act of creation and the divine order of God. By showing "*wie auss denselbigen Fünff Cörpern one Endt gar viel andere Corper/mancherley Art und gestalt/gemacht"*—"how out of the same five bodies one can go on endlessly making many other bodies of various kinds and shapes"—Jamnitzer raised the intellectual stakes of perspectival work with geometry. Perspective was not just a teachable art for representing objects, and geometry instruction not just a stepping-stone on the way to more advanced forms of applied geometrical knowledge. Accurately and measurably working with perspectival geometry was akin to bringing new earthly bodies into existence; to creating life on paper where before there had been none.

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wöllen/ dieweyl sie mancherley und viel unterschiedliche Eckh/ seythen/ winckel und spitzen/ einwartz und außwartz gekehrt haben..." (Jamnitzer, foreward).

[208] See Newman; and Smith 2004.

By viewing his work as performing the principles of natural variation, Jamnitzer was implicitly laying claim to knowledge of natural philosophy, abetted by the years of practice and observation. But the graphic liquidity of *Perspectiva* could only have been conceptualized by someone versed in the behavior of molten materials. The precedents for these graphic transformations lie not only within the goldsmith workshop drawings but also in the relationship that goldsmithery structures to the manipulation of form.[209] *Perspectiva* performs the state changes of metal beginning from the basic Platonic Solids to a series of highly articulated variants. This is an alchemical transformation of the graphic emblems of perspectival geometry; one that showed that the nature of the Solids as conceived by Plato and Euclid was transitory, part of a continuum of movement.

The use of the adjective "alchemical" is more than just a ruse. As Henrike Haug has recently shown, knowledge of the formation and manipulation of metals affected the working processes of both goldsmith and alchemists alike (Haug 2014, 80).[210] Haug's analysis of the collection of mid 16th century theories compiled in Johannes Mathesius's (1504-1565) "Third Sermon. On the Origin, Growing and Reduction of Metals and Minerals and Ores" glosses not only the overlap of alchemical and metalworking knowledge but also "how strongly early modern natural philosophy and metallurgic research are influenced by the Christian episteme." Moreover, the purification of metal in the forge was an established biblical allusion to the revelation of the obscure and to the

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[209] Florentine treatises from the period award Vulcan, or Hephaestus, god of fire and all craftsmen, the status of being a quintessentially modern artist, one capable of making his imagination tangible with his *fervor ingenii*, or enthusiastic spirit of invention (Göttler 136).

[210] On the relations between art and alchemy see Newman 115-163, 2004; D. Von, B. Wismer, and S. Dupré *Art and Alchemy: The Mystery of Transformation* (2014); J. Wamberg (Ed.) *Art & Alchemy* (2006); Battistini 2007, Smith 2010, Principe 2013.

quality of God's work.[211] "The melting process (passing from a solid state to a fluid one) is a metaphor for the mystical opening of the soul" (Battistini 309).[212] It is into the space created by the intersection of metallurgy, alchemical transformation and Christian theology, that we must insert Jamnitzer the goldsmith and author of *Perspectiva*, who, like God in Mathesius' *Sarepta* "*hat mancherly schmeltzwerck inn seinem laboratorio*" —"had all kinds of melted products in his workshop."[213]

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Jamnitzer illustrates Michel Jeanneret's argument "for a sixteenth century swept up in change and fascinated by genesis and metamorphosis" (Jeanneret 1). *Perspectiva* is, ultimately, the graphic product of an artistic culture seeped in the pragmatics of making decorated objects, albeit elevated to the level of a printed masterpiece; born of the same creative reality as the many drawings that would have been floating around Jamnitzer's workshop and informed by workshop practices and the innate properties of the material handled there.[214] Jamnitzer created his work in a way that exceeds the objecthood-ness of

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[211] See 1 Cor 3:12-15 (cf. H 1:71)" (*Opus Paramirum*, footnote 2, 305).

[212] Base metals, like lead or zinc, were treated by alchemists in the manner that the sick were treated by physicians, by purging them of impurities in the heat of the forge. "For just as gold is tested in fire a seventh time, the physician must be proven by fire a seventh time and more" (Paracelsus, *Opus Paramirum* H 1:69, 305). In his final sermon, Death's Duel, the cleric John Donne (1572-1631) even uses the language of alchemy, and goldsmithery, to describe spiritual rebirth (Keller 490)—from death to resurrection, and man's relationship to the Christ sacrifice (Haug 2014, 83).

[213] Quoted in Haug 2014, 82, footnote 7.

[214] On maker's knowledge and goldsmithery in particular see Smith 2012, Smith 2004, Smith and Beentjes 2010, and H. Haug's "Wunderbarliche Gewechse: Bergbau und Goldschmiedekunst im 16. Jahrhundert," *Kritische Berichte: Mitteilungsorgan des Ulmer Vereins*, Vol. 40, 2012, 49-63.

its final product. This meant that his viewers would consume the images he etched onto the surface of his masterworks in a linear or circular sequence over long periods of time, as conditioned by the shape of the object, and that geometry could be just as engrossing as figures, particularly if viewers understood geometry in terms of its transformation from one thing to another. He understood too that the shape of the underlying object would define the types of narrative he could portray and whether it had a start or an end or whether it ran on a perpetual loop. And that complexity would cause his viewers to slow down and allow themselves to become absorbed in the weight of the detail, growing joyful then fearful as their gaze would have darted across the different scenes etched in gold, or even losing themselves in a rapture of wonderment.

Stumbling upon a stone tableau in the church of the Italian abbey he is visiting, Brother William of Baskerville, the narrator of Umberto Eco's *The Name of the Rose*, describes the ways in which his mood fluctuated as his awareness of the scene before him expanded, bit by bit, to include the entirety of the work in front of him. "When our eyes had finally grown accustomed to the gloom, the silent speech of the carved stone, accessible as it immediately was to the gaze and the imagination of anyone (for images are the literature of the layman), dazzled my eyes and plunged me into a vision that even today my tongue can hardly describe" (Eco 32-33). Evocatively dramatized by Eco, the flow of form and bodies in ornament held within itself a series of psychological or perceptual effects communicating to the 16th century viewer a sequence of meanings in the time designed for their reception by its creator. The goldsmith workshop was not limited to the production of objects; it also produced narratives wrapped around objects, or, in the case of *Perspectiva*, endless transformation reproduced as wondrous filmstrips bound together. Echoing Hephaestus's forging of the shield of Achilles in the Iliad, the *locus classicus* for the narrative skill of the craftsman, these *Meisterstücke* deaccelerated seeing for connoisseurs of the decorative arts in an ekphrasis charged and recharged through the consumption of layers and lines of surface ornament.[215]

The time and labor imbedded in the production of *Perspectiva*'s pages—of Jamnitzer certainly, but also the printer Jost Amman's contributions as well as those of a host of invisible workshop collaborators, technical apparatuses and paper or wood polyhedra—were intended to provoke admiration and wonder in *Perspectiva* as a portable, and purchasable, mathematical *Wunderkammer* that exhibited the "aesthetic of virtuosity" for which *Wunderkammern* were characteristic. To create "difficult" or impossible objects that graphically performed the discipline and labor required to produce them, was to justify to buyers the cultural worth and asking price. If, as Lorraine Daston and Katharine Park claim, "philosophers traditionally measured nature's skill by the elegant economy with which she had fitted form to function" (Daston and Park 277), *Perspectiva* was supernatural, captivating spectators with the extent of its engrossing precision while capitalizing on the taste for the complexification of surfaces, so prized by 16th century patrons, that had enabled Nuremberg's unrivaled precision-manufacturing industry to thrive.

The unique drawing culture that supported the production of decorative art objects in crafts workshops engendered a level of design literacy of which *Perspectiva* is exemplary, directly conditioning Jamnitzer's predilection for transformation. It remains

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[215] With thanks to Lorraine Daston for mentioning the Shield of Achilles in this context.

as an epitome of artisanal contributions to geometry, intimately tied to pervasive cultures and practices of drawing, to an evolving aesthetic that valued abstraction and complexity, and to the influence of ornament-making as an epistemic practice; a new, artisanal investment in the theoretical, inflected by the materialities of practice and the cinematics of viewing ornament. Jamnitzer was understandably proud of his accomplishments in visualizing and inventing geometry, perhaps even believing that his work could rival the humanist contributions to geometrical knowledge. *Perspectiva* thematizes the entire design process, as one polyhedral shape morphs into the next from page to page. The graphic forms are unstable, elastic, prone to change and metamorphoses, not unlike the precious metals that Jamnitzer spent his days reworking. Gold, in whatever form or shape it arrived to the workshop, would necessarily be pounded into gold leaf, subjected to the heat of the forge, and melted down, in order to make it workable and to remove impurities. A lifetime of melting down the work of other goldsmiths and transforming the raw material into new, exquisite forms must surely have impressed in Jamnitzer a bittersweet appreciation for the fragility and temporality of objects. And thus a palpable violence quivers at the edges of even the most fragile of Jamnitzer's physical creations. Only by daring to achieve prodigious heights of complexity might a goldsmith safeguard, as much as possible, the eventual melting down and reuse of his work. To epitomize, aesthetically, a moment in the collective taste, to be representative of all that a particular style had to offer, was as much as he could do to try and guarantee the preservation of his labor.

Chapter Five: Migrating Media

Abetted through precise techniques of making and drawing polyhedra on paper, by the mid 16th century the Platonic Solids had become firmly established as the primary source for generating new geometrical forms by mixed-mathematicians and artists. There seems to have been one sustained moment in the late 16th century roughly coinciding with the publication of Wenzel Jamnitzer's *Perspectiva corporum regularium* (1568) when polyhedra in Germany suddenly made an evolutionary leap from the page into other more precious materials. Best typified by the work of Lorentz Stöer (active 1557- 1620/21), creators and consumers of these newly migratory geometries took advantage of the overlap between the taste for the two dimensional representation of perspectival geometry and the experimental woodwork that would crystallize into the practice of intarsia, later marquetry, which would provide new, cutting-edge techniques and surfaces for the presentation of covetable visual-mathematical knowledge. In this genre of ornamental prints and drawings destined for media transfer, polyhedra were used as representative emblems of geometry and geometricalness, coming to adorn domestic furniture and select interiors of some of Europe's finest residences and collections. As a phenomenon exemplified by Stöer's little known oeuvre, the proliferation of late 16th century irregular bodies were responsive to the acts of making and conceiving of intarsia and productive of a seemingly infinite, geometrical generativity that occurred in the gap, or relation, between medias.

Intarsia in the 16th century was a new genre of experimental woodwork that required the ability to work between media, dissecting and recomposing graphic figures in another material, and consequently constituted if not in name then in practice a new sub-specialty of *Schreiner* in the tightknit German guild system.[216] Accompanying the production of intarsia was an intimate relation to the network of images produced by and floating around workshops in the form of recombinable ornament prints. Within this print genre, Stöer's evocative collection of eleven woodblock prints from 1567, *Geometria et Perspectiva* [**Figure 5.1 + 5.2**], was one of only a few visual references for *Schreiner* looking to integrate irregular perspectival solids into their compositions.

"*Lorenz Stör ist weniger bekannt als er verdient*" (Stetten 283).[217] This statement, from Paul von Stetten's expansive Enlightenment-era history of the applied arts in Augsburg, attests to the lack of awareness about Lorenz Stöer (active 1557 – 1620/21) even in the 18th century.[218] Perhaps, as Stetten states, it was due to the fact that only "design drafts (*Entwürfe*) betraying a bold and lively spirit" remain of his work.[219] Absent are any of the paintings that could have testified to Stöer's avowed profession as a "*Maller Burger Inn*

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[216] The art of intarsia or marquetry extends back to Asia Minor and found popularity in the crafts of the Roman Empire, though the techniques faded after the empire's decline. Resuscitated by Florentine artists in the 14th century, marquetry consists of the reconstruction of a preparatory or reference drawing as a distinct surface composed of small slabs (3-10 mm thick) of veneer wood elements or tesserae. For further reading on the technologies of intarsia see "Der Werkstoff und die Technik" in Flade's *Intarsia*, 379-409; Pierre Ramond's *Masterpieces of Marquetry* Vol. 1-3; and *The Gubbio Studiolo and its Conservation* Vol. I and II (O. Raggio and A. M. Wilmering respectively).

[217] "Lorenz Stör is less well-known than he ought to be."

[218] *Kunst gewerb- und handwerks geschichte der reichs-stadt Augsburg*, 1779-1788

[219] "*Entwürfe, die einen kühnen und lebhaften Geist verrathen.*" (Stetten 283) Stetten continues that he does, however, "know of two stone tablets by him of considerable size, on which biblical and moralistic phrases have been inscribed in a masterly manner, in the most delicate way embossed, etched, and gilded."—"*Ich weis von ihm zwo steinerne Tafeln von ahsehnlicher Größe, auf welche bibliche und moralische Sprüche mit Schreibmeisterischen Zügen eingefaßt, auf das zierlichste erhaben, geäzet und vergoldet sind*" (Stetten 284).

Augsburg" ("painter and citizen in Augsburg"), as he described himself in the title page of his *Geometria et Perspectiva*. Only fragments are known about his life. Stöer gave up his citizenship in Nuremberg to move to Augsburg in 1557, and had already by April 8th 1555 received a royal privilege from Ferdinand I to print a book with the title *Perspectiva a Laurentio Stöero in lucem prodita*, of which no copies are presently existing (Allgemeines Lexikon 91).[220] The earliest known copy of *Geometria et Perspectiva*, possibly, if not unlikely, the *Perspectiva a Laurentio Stöero* referred to in the royal privilege, appeared in 1567. But here too there is room for conjecture. One version was published by Hans Rogel I and another by Michael Manger, both of whom were publishers (and competitors) active in Augsburg in this period, where Stöer himself appears in the tax records from 1562 until 1597.[221] There are two mentions of Stöer in the council records of Nuremberg. The first, dated May 24th 1557, shows that Stöer was known to the tax office. While the second, dated August 13, 1595, is about a dispute over payment (Hampe 246).[222]

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[220] Reprinting text or illustrations from Stöer's work was to be punished by the confiscation of the reprint and fine of ten gold marks. From Wood 2003, 5, with reference to H. von Voltelini (Ed.) "Urkunden und Regested aus dem K. und K. Haus-, Hof- und Staatsarchiv in Wien," *Jahrbuch der kunsthistorischen Sammlungen in Wien* 11 (1890), lxvii, no. 6475.

[221] A later edition from 1617 was published in Augsburg by Steffan Michelspacher. Prints from *Geometria et Perspectiva* are available in the collections of most major museums.

[222] "*Lorennz Stör, mahler, hat sein bürgerrecht auffgesagt, gewöndlich verschreibung geben unnd ist damit inn die losungstuben gewiesen worden*" [1557, II, 8b],May 24, 1557 (No. 3667, Hampe Vol. 1)—Lorennz Stöer, painter, recited his citizen rights, gave his customary proscription (?) and is known for it in the tax office(?)—*losungstuben.* The entire quote from 1595 reads "*Lorentzen Störs, mahlers von Augspurg, forderung der gemachten abriß des eingenommenen augenscheins halben, das klein waidtwerckh betreffendt, weiln er von dem, so er solchem original nachmachen soll, 40. f. und von dem gemachten vorwerckh 8f. fordern thuet, soll man auf die verordtnete herren stellen, wie sie mit ihme abkommen mugen*." Lorentzen Störs, painter and citizen of Augsburg, has demanded the completed inventory of his appearances [in regards to] the small *waidtwerckh* in question, because he demands 8F for the completed work and 40F for the

On the basis of the title page to *Geometria et Perspectiva*, in which Stöer claims that the images of his "*zerbrochne Gebew*" (ruined buildings) would be useful to carpenters (*Schreiner*), it appears that Stöer explicitly courted the copying of his work by other craftsmen who scoured the contemporary art scene for patterns to use in the intarsia and marquetry that graced the most prized southern German furniture.[223] In this context, Stöer's relocation to Augsburg was a savvy professional move, as by the mid 16th century the city had become the leader of luxury German cabinetmaking and was known for a local specialty of intarsiated *Schreibtische* (cabinets that folded down into writing desks).[224] If any consortium of 16th century craftsmen had the skill and professional opportunities to produce elaborate perspectival intarsia it was the Augsburg *Schreiner*, whose skills were highly coveted by royalty across Europe.[225] The Augsburger Lorenz Strohmeir was commissioned by King Charles V in 1554, his fellow citizen Bartholmä Weishaupt made an ornate cabinet for King Philipp II which was delivered to Spain, and in 1600 Hieronymus Fleischer worked for the arch-duchess Maria of Austria and her daughter the Queen of Spain (Stetten 114). A petition made in 1568 to the Augsburg city

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imitating of his originals. One must support the honorable sirs, as he [Stöer] would like to make an agreement with them."

[223] The title reads "*Geometria et Perspectiva Hier Inn Etliche zerbrochne Gebew den Schreiner in eingelegter Arbait dienstlich auch vil andern Liebhabern zu sonder gefallen geordnet unnd gestalt Durch Lorenz Stöer Maller Burger Inn Augspurg. LS. Mit Rö: Käy: Maÿ: u aller genedigste Privilegio nit nach zetruckhen, 1567.*" Here in Geometria et Perspectiva are honorably ruined buildings, especially and pleasingly arranged and designed by Lorenz Stöer, painter and citizen of Augspurg, useful for craftsmen working in intarsia and many other enthusiasts. LS. With Rö: Käy: Maÿ and all royal priviledges, printed after 1567.

[224] On the tension between intarsia and cabinet-makers in 16th century Augsburg, see Hellwag 470-71.

[225] This is not to say that the *Schreiner* were the only craftsmen to create polyhedra. Italy is famous for Fra Giovanni da Verona's (1457-1525) geometrical intarsia in the Monastery of Monte Olivetto Maggiore near Siena and Verona's Church of Santa Maria in Organo.

council enunciates the success that the city had begun to enjoy with its "cabinet making, which earns praise abroad and in this city for how all sorts of things can be so skillfully inlaid, which no painter can match in the colors" (Alfter 52).[226]

The disappearance of Stöer from art historical genealogy at once confirms the existence of a gap between histories of Renaissance print culture and histories of the decorative arts, and provides a much-needed bridge between these two interconnected disciplines that nonetheless remain discreet. In his ornate intarsia-covered doors for the apartment of Phillip II, located in Escorial near Madrid, Bartholmä Weishaupt incorporated motifs found in the printed oeuvre of Vredeman de Vries, Théodore de Bry and Nicolaes de Bruyn, and featured strapwork cartouches and polyhedral geometry inspired by Stöer (Gruber 401). Dr. Virginie Spenlé at Kunstkammer Georg Laue in Munich has noted that a sphere with a diamond poking through its surface holes on the gallery's aforementioned Augsburg cabinet appears to match a similar form on Plate 8 from Stöer's *Geometria et Perspectiva* [**Figure 5.3**], though the geometry on the cabinet is reversed in regards to the print likely indicating direct copying from the reverse of the print. Christopher Wood speculates that several rare Stöer woodcuts sold into private collection by Munich auction house Hartung & Hartung were the direct model for intarsia panels on the lower doors of a cabinet in Ulm, located at the Ulmer Museum (Wood 2003, 15). Additionally, in her book on the *Wrangelschrank*, the epitome of German achievement in intarsia, Lieselotte Möller notes similarities between some of Stöer's geometries and similar forms on a

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[226] "*Es ist eine lange Zeit her diese Statt Augspurg hoch berühmt gewesen und ist es noch wegen ihrer Kistlerarbeit, die man in der Frembde und auch in dieser Statt uns das Lob gibt, wie man so scharpff Dinge allhie einlegt, welches keinem Maler möglich ist von Farben nachzukommen*" (Quoted in Hellwag 457, translation from Alfter 2008).

cabinet in the Victorian & Albert Museum in London.[227] Thus, it is likely that Stöer's *Geometria et Perspectiva*, which went through three editions and was widely distributed across Germany, was appreciated not just as a printed book of fantastical images, but also as a unique source of geometrical motifs that could be combined onto the intarsiated surfaces for which Augsburg was famous.

The collaging together of reference images into post-apocalyptic visions in intarsia populated by strange creatures, overgrown ornament, and the occasional polyhedron was not just an Augsburg phenomenon [**Figure 5.4**], and numerous examples attest to the reach of the southern German *Lehrbücher*, if not Stöer's prints specifically. Nor was "the ruin" solely an artistic form. Thinking through and with ruins became also a prevailing literary form in 16th century Germany and Spain through the development of the *Trauerspiel*.[228] Ruins populated with polyhedra appeared on a grand coin chest for Archduke Ferdinand II by the court carpenter Conrad Gottlieb for the Ambras Castle in Innsbruck [**Figure 5.5**] and on a sumptuous tabletop in *pietre duré* made by the Castrucci workshop in Prague for Prince Karl von Liechtenstein (1569-1627) (Distelberger 32)

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[227] On the *Wrangelschrank* see G. Jászai, *Der Wrangel-Schrank.* Nildhefte des Westfälischen Landesmuseums für Kunst und Kulturgeschichte, Nr. 21, 1984; L. Möller *Der Wrangelschrank und die verwandten süddeutschen Intarsienmöbel des 16. Jahrhunderts*, Berlin: Deutschen Verein für Kunstwissenschaft, 1956. On the London cabinet see Möller 1956, no. 70, fig. 179. In addition to *Geometria et Perspectiva*, *Buchlein von den alten Gebewen* (ca. 1555) a set of prints after Jacques Androuet Du Cerceau's engravings from 1550 by Léonard Thiry and the printmaker Virgil Solis (1514-1562) was also highly influential in Germany as was Hieronymus Cock's *Praecipua aliquot romanae antiquitatis ruinorum monumenta* (1551) (Wood 2003, 12).

[228] See Benjamin's *Der Urspurung das Trauerspiels*. See also Pedro Calderòn de la Barca**,** Andreas Gryphius, and Martin Opitz.

[**Figure 5.6**].[229] A pair of miniature cabinets in finely-gilded leather cases, one in the Metropolitan Museum of Art and the other at the Georg Laue Gallery in Munich, each fold open to reveal several polyhedra on their inside covers [**Figure 5.7**] and inlaid ivory scenes possibly referring to the liberal arts on its drawers.[230] These were pieces that would have been viewed as masterworks appropriate for the wealthy and might have been filled with jewelry or other precious items. The *Museum Angewandte Kunst* in Frankfurt has in its possession a small lectern covered in twenty different polyhedra. There is a chessboard in the Kunstkammer of the Kunsthistorisches Museum in Vienna, originally mentioned in the 1596 inventory of the Ambras castle in Innsbruck, made from various woods, ebony, ivory, and mother-of-pearl, and depicting all manner of polyhedra.[231] Though perhaps the most extravagant showpiece of polyhedral geometry in existence is an early 17th century southern German cabinet in the Museum für Angewandte Kunst in Cologne [**Figure 5.8**].[232] Concealed behind inoffensive scenes of

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[229] The chest is mentioned to the 1596 Ambras inventory of Archduke Ferdinand II. See Boeheim. The tabletop, which remains in the Liechtenstein Museum in Vienna, depicts a tableau of compartmentalized images of pastoral scenes, musical instruments, weaponry, and Prince Karl's monogram in the center. There are four identical geometrical assemblages consisting of a star octahedron nested inside a skeletal octahedron, each of which points in the direction of the centrally-located royal crest, while the outer border is ringed by alternating motifs of polyhedra (two interpenetrating tetrahedron), salamanders, snails, and frogs. For further reading on Prince Karl's tabletop see C. Vincent, "Prince Karl I of Liechtenstein's Pietre Duré Tabletop," *Metropolitan Museum Journal*, nr. 22, 1987, 157-178.

[230] The cabinet in the Metropolitan Museum of Art was originally a gift from the Nuremberg town council to an important visitor, possibly a Hapsburg given the presence of the insignia of the Hapsburg eagle, which may have been added after the cabinet was already complete. The wood used on the interior is made from Hungarian ash; a type of wood with a grain similar to marble in miniature and thus used to create the illusion of stone. [From conversation with Dr. Wolfram Koeppe at the Metropolitan Museum of Art]. It is likely that the two cabinets were completed by the same workshop.

[231] As a reference, see Seipel (Ed.) 160-162.

[232] As a further reference, see Colsman 132-139.

courtly life played out against pastoral settings, the cabinet's doors, once thrown open, reveal an almost sensory overload of polyhedra. Fifteen drawers with three polyhedra each are flanked by two central doors, themselves covered in a grid of polyhedra. A further set of hidden compartments conceals more polyhedra, themselves framed by two doors showing noblemen wandering through a pleasure garden (Colsman 135); a disjunction between the philosophical passivity of contemplating existence and the hive of fermenting mathematical experimentation bubbling within artisanal workshops.[233]

Ornamental prints, like the kind in *Geometria et Perspectiva* or the prints upon which the polyhedral components of the Cologne cabinet were taken, were in high demand as they supplied design vocabularies which could be used to adorn objects in any way that the purchaser of the print desired, from cutting and pasting elements onto the surface of furniture or using them as a base for design in intarsia. Books of perspective and geometry were especially popular models for *Geschrottenwerk*, as intarsia was called by the 16th century Nuremberger Johann Neudorffer, and most of the German perspectivists, as well as the Italians Daniel Barbaro and Lorenzo Sirigatti, were aware that their work could well be used as the basis for wood inlay in high-profile commissions (Jarvis 4). These prints tended to combine self-consciously highbrow subject matter with the tacit acknowledgement that the particular application of the ornamental print would depend upon the size and requirements of the furniture for which it was intended.[234] The Dutch

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[233] The images within the upper compartment depict a woman, possibly an allegory of nature, seated in a fertile landscape, feeding a dog with her right hand and a putto with her left. Andrew Morrall is working on a forthcoming book that will include an analysis of this allegory.

[234] Walter Rivius' *Vitruvius Teutsch* (Nuremberg 1548), Wendel Dietterlin's *Architectura und Ausstheiliung der V Seulen* (Nuremberg 1598), and Sebastiano Serlio's *Tutte l'opere*

artist Hans Vredeman de Vries even published two books on the ornamental orders in 1565 in which he urged readers to sample the classical forms from his books and "with what one finds here one may do with what one sees fit."[235]

In one of the few contemporary articles on Stöer, Wood has discussed how in intarsia, "pattern, not optical experience, becomes the starting point for the artist." Rather than work forwards from an image to intarsia, Wood postulates that techniques of intarsia practice may have encouraged *Schreiner* to work backwards from their knowledge of the properties of wood and wood-cutting to design their "improbable fictional worlds" (Wood 2003, 17). Yet there is a lot of evidence to suggest that the copying of geometrical solids from prints and drawings were integral to the incorporation of fantastical geometry into intarsia. In addition to the similarity between Stöer's printed geometrical forms and those appearing in intarsiated surfaces, irregular solids, many of which were rendered as curvilinear forms in perspective, were highly desirable and given the difficulty in producing them accurately may even have fallen under the purview of specialist geometers, like Stöer or his contemporary the famous Nuremberg goldsmith Hans Lencker (1523-1585).

Christian I of Saxony (1560-1591), Elector of Saxony from 1586 until his death at age thirty, was only sixteen when he finished working "with his own hand" on his *Perspectief*

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d'architettura et prospetiva (volumes published from 1537 onwards) were widely circulated visual references.

[235] Vredeman de Vries, quoted in Heuer 2009, 104. The two books are entitled "*Den Eersten Boeck Ghemaect opde Twee Columnen Dorica En Ionica*"—the first book on the subject of the Doric ans Ionic columns) and "*Das ander buech Gemacht auff die zway Colonnen, Corinthia und Composita*" —the other book treating the two Corinthian and Composite Columns.

Buch (1576), a collection of hand-drawings of architectures and geometrical objects preserved in the Kupferstichkabinett of the Grünes Gewölbe in Dresden.[236] Under the tutelage of Lencker, who himself had published in 1557 his own *Perspectiva Literaria*, a collection of twelve perspectival representations of alphabetical letters and nine evocative mathematical models, the *Perspectief Buch* preserves Christian's graphic studies in perspectival geometry.[237] The drawings are done in hardline with ink, and then carefully shaded with pencil and chalk. All of the geometrical models are Christian's original inventions and with two telling exceptions there is little evidence of copying from other books or images of geometrical models. It is only by looking at the raised texture on the back of pages 29 and 30 of the *Perspectief Buch* [**Figure 5.9**] that it is possible to tell that a different technique was used for these two spherical figures. Christian would have placed his page under the sheet of paper bearing the original image and traced the image (which can be seen through the page) with a sharp instrument or sharpened pencil. This would have transferred the shape from below onto the page above, which Christian could have then redrawn in pen and colored. Perhaps Duke Christian's tracing of his spherical geometry was indicative of their perceived complexity. Might it be that Christian found these geometries difficult to represent and consequently relied upon another drawing or print as a base reference, most likely one of his teacher's drawings? The difficulty that

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[236] The entire introductory inscription reads "*Perspectief Buch Darinnen ordentlich zu befindenn/ die stuck/ welche der Durchlauchtige Hochgeborne Fürst vnndt Herr/ Herr CHRISTIANUS, Hertzogk zu Sachsenn, Landgraf in Thüringen, Markgraf zu Meissen auf Hansenn Lenckers Burgers zu Nürnbergk vnterthenige vnterweisung vonn dem letzten tagk Februarij dess 1576. Jars an vor sich mitt eigener handt gerissen hat*."

[237] It is known that Lencker earned a salary for his perspectival expertise, receiving 117 Gulden and 9 Kreutzern in 1572 for the giving of lessons on "*kunststücke aus der perspevtiua*," or the feats of perspective. SHStA Dresden, 10037, Rentkammer-Rechnungen, Nr. 183, fol. 228r. For further information on Lencker in Dresden see Gluch.

spherical forms presented Christian might have been a reason that there are barely any other types of curvilinear forms in the rest of the book (and certainly no other spherical forms).

A copy of *Geometria et Perspectiva* bound into the collection of Stöer's drawings and prints in the Universitätsbibliothek Erlangen, entitled *Optica* and attributed to "Laurenzo Stöer," also bears striking proof of an attempt to spirit away two spherical forms [**Figure 5.10 + 5.11**].[238] In the ninth print, the outlines of the first sphere, a deathstar-like concoction on a star-shaped base consisting of four spherical segments propped open to reveal an internal sphere, as well as the second, a spiky ball hanging from a rope tied to the top of a truncated arcade ruin, have been scored with a utensil sharp enough to create a raised ridge on the reverse side of the paper. The ridge was then coated in red chalk, only visible on the verso side of the print, enabling a potentially repeatable negative impression of the globes to be made. The technique is remarkably subtle. Only by running ones fingers along the front surface of the image is it possible to sense the slight indentation, indicating the prior presence of a sharp instrument used to create the reverse impression.

While we don't know exactly why only parts of *Geometria et Perspectiva*'s ninth print were copied, nor whether the copies were destined for intarsia or for other media forms, that only the spheres were transferred likely speaks to their specialness, whatever criteria for specialness was held by the copier. A culture that prized the collection of "the new"

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[238] The title and attribution are scrawled on the fore edge of the book.

and “the unusual” guaranteed that these fantastical elements would continue to have a life beyond the confines of their original page.

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By the time Stöer began work on *Geometria et Perspectiva*, the morphology and solidity of the Platonic Solids had long since been made unstable. For later 16th century German artists, it was Jamnitzer’s *Perspectiva corporum regularium*, and not the images accompanying Euclid’s *Elements*, that formed the base reference work for further geometrical invention. There is perhaps no greater testament to the near infinite variability of the Platonic Solids than Stöer’s hand drawings, which are spread between Munich, Erlangen, Augsburg, and have only recently been rediscovered in Houghton Library at Harvard University.[239] Consisting of hundreds of wildly colored drawings of regular and irregular polyhedra, the several dates marked in the Munich and Harvard volumes (1562, 1564, and 1584 for Munich and 1585, 1598 for Harvard) reveal a vast intellectual and artistic project that spanned three decades of drawing at least.

The farthest back that one can trace the Munich volume is to its possession by Johann Franz Ecker von Kapfing (1649-1727), Fürstbishof von Freising from 1695 until his death.[240] The volume was located in the Dombibliothek in Freising until the middle of the 18th century when it was transferred to its present location in Munich. Information on the

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[239] Handschrift Cim 103 in the *Universitätsbibliothek München* has been digitized in its entirety and made available for purchase as a cd by Harald Fischer Verlag, 2006. The Harvard manuscript Typ 520.67.810 was rediscovered by me during the course of my dissertation research and has remained completely unbeknownst to specialists, it’s anonymity perhaps preserved by a less common, if not incorrect, spelling of “Stöer” as “Stör” in Harvard library’s online catalogue.

[240] The book is mentioned in the *ex libris* of Bishop Eckher in 1696.

Harvard volume is similarly scarce, if more evocative. Catalogue notes indicate that the volume was bought by Philip Hofer in 1950, an avid book collector and former curator of printing and graphic arts at Harvard. Hofer bought the volume from a renowned dealer in rare books, H.P. Kraus, who had himself purchased the volume, along with twenty thousand others, from Prince Franz Josef II, Prince of Lichtenstein, in 1948. There is a record of this sale both on the actual reference card (165.2.1—referred to as *Geometria et perspectiva corpora regulata et irregulata)* within the card catalogue of the Lichtenstein Museum in Vienna—it says "verkauft an Krauss 1948"—and more generally in Kraus' autobiography *A Rare Book Saga* (1978).[241] Prior to its sale, the book had belonged to the collection of the Austrian general Frank Ritter von Hauslab (1798-1883), which he donated to Lichtenstein in the 19th century.[242]

There is enough overlap in the format and presentation of the drawings to suggest that the work within the Munich and Harvard volumes would have been created concurrently. Both consist nearly entirely of watercolor drawings outlined in pen or pencil. Most of the time, the colors are fantastical arrays of blues, greens, magentas, and oranges, devoid of

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[241] Under the chapter title "An Old Noble Family Sells," Kraus recounts how the prince, desperate to move his books from Vienna, which was still occupied by the Russians after the end of World War II, smuggled the books to a schoolhouse in the village of Schaan in Lichtenstein and after some negotiations with Kraus and under pressure of discovery, sold the books on the condition that they be removed in three days (Kraus 153-154). Harvard's Stöer volume is described in a letter dated 7/26/48 as one of the books that Kraus wanted to buy and was referred to as *Geometria practica um 1600*.

[242] In the section on "*Physik und Chemie*," which was a subheading of "*Mathematik und Astronomie*," in the handwritten catalogue of the books given to Lichtenstein by Hauslab—"*System der Hauslab-Sammlung*"—there is reference to a book of geometrical drawings of represented geometry "*Geometr. Zeichnen, allg. darstellende Geometrie*" that is likely the Stöer volume. While there is no direct mention of Stöer or geometrical models, the notion of representation (*darstellende*) is highlighted, in contrast to the more common militaristic applications of "practical geometry" (maps, fortification drawings) which formed a large component of Hauslab's collection.

material or weight [**Figure 5.12 + 5.13**].[243] Both of the volumes begin with title pages ringed by geometrical models. While the Harvard volume is unfinished, missing a central text on its title page, the Munich volume is complete and includes the title phrase "*Die Funff Corpora Regularia, auff Viel und Mancherley Arth und Weis Zerschnitten*"—"The five regular bodies, cut apart in many ways" [**Figure 5.14**]—referring to the multitudinous transformations undergone by the Platonic Solids under Stöer's hand.[244] Many of the pages include two models together, as if staged in perpetual dialogue with each other, and are mounted horizontally, with the models' bases facing the spine of the book [**Figure 5.15**]. Their shadows are used as notional, or gestural, proof of the models' three-dimensionality and have not been subjected to any kind of geometrical rigor [**Figure 5.16**]. Occasionally, particularly in the Munich volume, the same model appears again in a different color and is rotated to display another face to the viewer. Nearly all of the models are situated on a long shelf that stretches throughout the volumes, giving the impression that the pages could be removed and the segments of the "shelf" joined together to create one long, continuous presentation of the fruits of Stöer's imagination.

Though the vast majority of Stöer's irregular bodies were different from one another, the act of copying, on several different graphic registers, remained critical to their construction and development. Unlike Winckelmann's proclamation, whereby "there is only one way for the moderns to become great…by imitating the ancients" (Winckelmann 61), the ancients had left behind little if any graphic traces of their

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[243] In a few instances the watercolor wash seems to reference wood, the likely material of the "perspective tools" in Faulhaber's workshop.

[244] Both volumes repeat the phrase "*Geometria et Perspectiva Corpora Regulata et Irregulata*" on multiple pages.

capacity to invent and visualize geometry. The recombinability of the five elements in the *Timaeus* and the infinite atomic matter of *De rerum natura*, let alone Euclid's *Elements*, were bound to imageless originary texts that could thus only describe if not depict the transformational processes that so intrigued the German geometers. Having been an object of visual attention since *De divina proportione* nearly one hundred years earlier, the representation of the Platonic Solids no longer conveyed the modern interpretation of ancient material to which Pacioli, Dürer, and Caraglio had aspired. In order to be modern for Stöer, in order to make a living and appeal to the tastes of his own day, he had to shake off Winckelmann's future Romanticism of ancient Greece and the contemporary humanist echoes of reverence and push into areas that had not been cordoned off by ancient expertise.

Whereas working off a folded up paper or wooden polyhedral model would be helpful for drawing new solids in perspective that took their basic shape from the Platonic Solids, Stöer's solids branched into territories with little or no reference to Plato's iconic forms. His models were designed as unique three-dimensional objects, complete with dashed construction lines enabling a fully "transparent" view [**Figure 5.17 + 5.18**]. Present in his Erlangen drawings are a variety of partial copying techniques and ridges on the back of drawings indicating copying, as well as construction lines, basic attempts at shading in pen, and testing with less proficient lines. One half of a spherical irregular solid has been traced through [**Figure 5.19**]—the minimal amount of inscribing necessary to copy the entire solid, given its symmetry. The high degree of practical use suggests that these pages were working drawings from Stöer's workshop; perhaps even a set of models intended to be copied by Stöer or his apprentices and fleshed out in color on other sheets of paper or in other media.

The Harvard volume is also replete with evidence of copying, although interestingly only in a latter, less proficient section of geometrical models. Although the volume's two sections are continuous with each other, the line quality of the second section is substantially less precise. There are tiny pinpricks on the back of each of these latter drawings, with marks clustered around the edge points of the figures, while almost none of the images in the first section of the book have pinpricks [**Figure 5.20**]. It appears likely that the artist placed the pages over an already drawn graphic model, and used a pin to transfer its outline points onto a new page. The fact that this technique is not used in the beginning of the volume leads one to believe that the latter artist was unable to conceptualize the necessary geometry on his own and that these drawings were likely then made by an imitator or student. The catalogue notes to the Harvard volume in Houghton Library point out that while several of the pages are signed with Stöer's monogram, others in the second less proficient section have the monogram IAB—which might possibly refer to Hans Rogel (died 1592 Augsburg) or a Johannes R (Rogel?). I have identified the main watermark of the first section as a variation of the "arms of Nuremberg," an eagle inside a shield, found on paper produced in Nuremberg from 1585-1591, and also possibly in Augsburg (Reference image 919, Briquet). There is some of this paper in the second section, though also present is a coat of arms represented in the watermark belonging to Freising, Germany (Reference image 2255, Briquet: Hallstadt 1598-99, Neumark 1598, Eichstadt 1604). Analysis of the drawing quality and the watermarks of the paper leads me to believe that these were originally two different bodies of work.

There is also ample evidence to suggest that Stöer was an active participant in a learned community of German mixed mathematicians who were all following each others experiments with geometry, owned each other's books, were aware of humanist texts, had access to royal patronage, and executed copies of geometries they found interesting. The direct influence of Jamnitzer's *Perspectiva* is most palpable in the Stöer notebook at the Staats- und Stadtbibliothek in Augsburg and an anonymous collection of drawn geometrical models in Wolfenbüttel, which also includes copies from Hans Lencker's *Perspectiva Literaria* [**Figure 5.21**], who himself had copied one of his models from Jamnitzer.[245] Both page 15 in Wolfenbüttel, which includes a circle bounding the geometry reminiscent of the same element in *Perspectiva*, and page 25 in Stöer's Augsburg notebook were directly copied from an octahedron in *Perspectiva* (page BI) [**Figure 5.22**], although there are many other examples.[246] Hans Jakob Ebelmann's *Gebäude Bekrönungen Decken* (1609) even sought to incorporate two distorted giant polyhedra from *Perspectiva* into a ceiling design [**Figure 5.23**].

Although there is often little text to be read, no sentence that jars in its incompleteness and reveals that pages have been bound out of sequence, Renaissance collections of

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[245] The inventory number for the Wolfenbüttel manuscript is Cod Guelf. 74.1 Aug. 2°

[246] Cod Guelf. 74.1 Aug. 2°: Page 10 = the upper right hand drawing on p CII in *Perspectiva;* P 9 = middle right hand drawing on p CII in *Perspectiva*; P 7 = upper right drawing on DV of *Perspectiva;* P 12 = upper left on CI in *Perspectiva*; P 25 copied from EIIII in *Perspectiva.* P 27 and 28, which state "*Geometria Perspectiva Corporum Regularium*" and "*Geometria et Perspectiva*" refer explicitly to Stöer's work, although they are not drawings included in the Munich volume.

polyhedra were intended to be viewed in a particular order.[247] While Stöer's volumes appear at first to be the result of collating hundreds of randomly generated drawings, subsequently paginated to give them the veneer of consistency, a closer examination suggests that the drawings were compiled by someone with an awareness of the underlying structure ordering Stöer's geometrical investigations but without the patience to make sure that the bound volume consistently reflected it. The result gives a false impression of manic creativity, blurring together the iterative explorations into a general "irregularity." Still an overarching progression of simpler to more complex forms, often beginning with one of the Platonic Solids as a starting point, appears to emerge, however tentatively, from the mass of geometrical information over prolonged study.[248] For it seems that Stöer, much as Jamnitzer did in *Perspectiva*, grouped his graphic experiments by Platonic Solid and intended them to be apprehended in the page-wise vicinity, and conceptual relation, to the Solid which inspired their generation.[249]

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[247] The page numbers which run continuously through the volumes are almost certainly later interventions.

[248] There are even micro organizational-patterns; in the Harvard volume, for instance, the beginning pages are taken up by the five Platonic Solids, each drawn in six differently rotated views. Prolonged time with the drawings reveals relationships between geometries bound in different sections of the book (or even between the Harvard and Munich volumes). There are pairings, triplets, sequences of incrementally modified models all tantalizingly related to each other, yet spread countless pages apart.

[249] As further proof of the centrality of the Platonic Solids, in Stöer's *Optica* there sits a table of geometrical categories with five columns, one for each Platonic Solid, or "*elementa optica*," divided into three rows called I Planum; II Acies; and III Cvspis—categorizing a different view of the solid rotated in graphic space. Following the table within *Optica* are a series of line drawings of the Platonic Solids, many marked with Stöer's distinctive monogram, followed by one page of a colored "skeletal" rendering of the same solid in color. The colored rendering illustrates the line drawing that immediately precedes it, which itself refers to the categories on the foldout table.

Beneath the bright, if still enigmatic, colors of Stöer's polyhedra lurks a didactic exploration of the limits of a geometrical irregularity that could never have been achieved without the advancements in geometrical visualization and invention pioneered by the century's earlier *Lehrbücher*. Not only did these *Lehrbücher* popularize the *idea* of designing geometry, they also served as precedent and inspiration for the idea that the individual artist or mixed-mathematician could make unique and important contributions to geometrical knowledge. Perhaps this is why irregularity was never conceived in a vacuum, but always kept in bounded relation, however tenuous, to its closest Platonic Solid. Exploring the constraints of these canonical forms and their combinations as opportunities for design externalized the creative potential of polyhedral, or "polyhedralish," geometry. To view one of Stöer's models was to be dazzled by its perspectival proficiency and also to recognize the deformations and augmentations that had transformed it from the antique incarnation familiar to any student of Euclid. To transform and to invent was to be modern; to build upon the old and yet to create an aesthetic of contemporary relevance was surely at the heart of Stöer's mixed mathematical project. Perhaps the drawings are best conceived as elaborate variations on a theme that Stöer held dear. After all, given the spread of dates written in the volumes, Stöer would have had to have saved some of the pages for over thirty years. Records of a geometrical imagination unrivaled among his fellow artists in Augsburg, the volumes are testament to a marvelous master project stretching across, and possibly encompassing, the lessons learned from a whole life's work on the possibilities of geometrical form.

Inscribing, pricking, tracing from under and over; the mathematical models of Jamnitzer, Lencker, Stöer, and others were subject to a range of copying strategies. Despite the fact that the only prints explicitly intended for direct use in intarsia to have survived Stöer are in *Geometria et Perspectiva*, it would appear that the scars of reproduction born by his hand drawings point as well to numerous attempts to translate his wide stock of singular mathematical models into other media, either by apprentices or even possibly by Stöer himself. The symphony of pinpricks is indicative of a need for reproductive certainty; the certainty of an exact process for translating geometrical forms as the artists had first drawn them and the certainty of knowing that the form which had been copied was exactly the same as the original. In this sense, all the copying was as close as another artisan or apprentice could come to a classically mathematical proof—determined not by Euclidean logic but by the concentration and choreography of hand and eye. Exactness, irrevocable certainty, seriousness of purpose, a likely crop of diligent apprentices—copying had become a rigorous epistemological practice of "getting to know" new and important geometry.

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Geometrical models made their way from drawing and print into intarsia because these forms were desirable and marketable as much for their novelty as for their perceived complexity. To present a phalanx of polyhedra on the intarsiated surface of a cabinet in one's living room, as would have been the case with the Cologne cabinet, was intended to signal the owner's appreciation and consequent understanding of work at the forefront of geometrical experimentation. A new irregular body possessed a certain measure of significance because it would have constituted an incremental expansion of the domain of perspectival geometry, one solid at a time. To display a new geometrical form was to have brought into existence a new "body" that did not previously exist, and to possess such a body was to own the ontological method of its generation.

If we accept that these irregular bodies were widely considered to be strange and wondrous objects of desire by the middle of the 16th century, it stands to reason that the artists and mixed-mathematicians who produced them did not do so solely out of personal interest, but that they were considered marketable artifacts that could attract further business to their creators. Master German geometers never intended to supplement their incomes by publishing pedagogical *Lehrbücher*. *Perspectiva corporum regularium* and *Geometria et Perspectiva* are showpieces of talent and knowledge, the epitome of years of mixed mathematical experimentation, and both were widely purchased and copied, as we have seen.

But in contradistinction, Stöer's drawings of geometrical models were not made to be sold, at least not individually, and did not just imply infinite transformation. They were, in their multitude, nearly infinite in quantity. They were neither preparatory models for works to be rendered later in a more precious material nor were they intermediary steps integrated into the production of more accurate drawings or stereometric working drawings, though working drawings leading up to the final rendered models do exist, in Erlangen for example. They were also not manipulatable, as were Michelangelo's wax models which were designed to be dipped into hot water and bent as required, and could not be easily altered later to suit the desire for a different perspectival angle, whether for

Stöer or his patrons (Goldscheider, text to Fig. 21-23, unpaginated).[250] It is hard to imagine that late Renaissance audiences would have encountered Stöer's models and believed that they were drawings of real objects that could be manufactured.

And yet in their totality, the drawings must have been a source of personal pride for Stöer, as well as a transpositional conduit of his geometry to other media. It may be provocative to consider whether Stöer's hundreds of geometrical models were not conceptualized as an array of potential options for inclusion in intarsia. A spread of work intended for appropriation, clients would have been able to compare and contrast solids one after the other. Perhaps their representation on stands was to make them more tangible, nodding as well to Jamnitzer's standing geometrical models in final section of *Perspectiva,* as if the geometrical abstraction would be difficult to understand and consequently select if they could not be understood as physical models. Assuming that Stöer, like other artists, was aggressively competing for commissions in Augsburg and Nuremberg, becoming known as *the* artist specializing in perspectival geometry would have been a smart and accepted strategy, especially given the popularity of irregular bodies and the microspecialities of southern German artisans. Even if the geometry were intended for larger compositions, having them all lined up permutation after permutation would have made it easier to compare all of the relevant geometrical options together.

As with many 16th century artists explicitly working with images that signified geometrical knowledge, Stöer sought to use drawing both to explore geometrical

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[250] Recent scholarship has attributed the Michelangelo models formerly in the LeBrooy Collection, some of which are referenced by Goldscheider, to the artist Johann Gregor Van der Schardt (1530/31- after 1581). See Adams 2015.

invention of geometry in iteration and also to provide options to his clients, though the series was not his only method. Stöer's exquisite large-scale pen drawing in the *Germanisches Nationalmuseum* [**Figure 5.24**] necessitates decelerated viewing across its broad surface and depicts a façade, likely a wall painting, bursting with embellishment and ornamentation, including one of his signature geometries at the base. Here Stöer does not choose to render one proposed façade, but incorporates multiple possible permutations, legible by the sharp disconnects between segments of the drawing. In order to make use of the drawing, a client would have had the option to choose elements from each of the segments to come up with a design to fit their tastes; a feat of evidently common mental acrobatics that would require the client to imaginatively connect disparate pieces of the drawing together and complete the entire image in their head.[251] As in the Stöer notebook in Erlangen, where several copies displayed an economy of means in their reproduction of only half of symmetrical designs, the façade drawing also prides efficiency over redundant labor. The result is a hyper-condensed vision of geometrical possibility that borrows more from the graphic conventions of late 16th century ornament prints, which often included multiple versions of a design in the same composition, than it does the stereotypical architectural elevation.[252]

As has been noted, the Early Modern period did not observe a stable distinction between "discovery and invention, making and finding" (Fleming 2). The discovering of new geometrical forms and their graphic invention constituted the same epistemological

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[251] Stöer was not the only artist to make use of this technique for presenting multiple design options on one drawing. See also Gabriel Krammer's (active 1598 to 1606) collection of prints in *Architectura* (1600).

[252] See Griffiths 116-117.

structure, accounting for the blossoming of innovation in 16th century geometry. Copying was integral to the circulation of geometrical knowledge within and between workshops and the transference, or implication of transference, from ornament print to wood or building surface yielded a productive frisson of geometricality that was thematized over and over again by decorative artists. Stöer's prodigious output was a consequence of a newly tactile relation to geometry, pioneered by Dürer and his followers and perfected by Jamnitzer, whom we know Stöer copied closely. Exploiting the natural similarities between drawing and intarsia-making and acknowledging the material constraints of wood-cutting resulted in a stripped-down, some would say modern, aesthetic. But his work was also representative of the power of transference between and within media, as well as between artists, to generate a limitless experimentation with geometry. If the established route of Euclidean geometry led ultimately to the Platonic Solids, the nature of geometrical innovation pioneered by mixed-mathematical practitioners was the "incremental newness" (Masse 169) of exploring the Solids through serial transformation.[253]

Stöer's fantastical repertoire of polyhedra were in understandably high demand and were picked up by practitioners (and possibly his own apprentices) who tried, both crudely and with sophistication, to transfer them out of his pages and into their own. These techniques of information transference or transmission [*Übertragung*], while not confined to Stöer's work, are representative of a certain selectivity of gaze; a roving set of geometrical predilections that fixated on certain aspects and aesthetics of Stöer's work, ignoring

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[253] Though Masse is discussing the serialization of periodicals in Early Modern France, I hold that the term is transferable here.

others entirely. Perhaps Stöer's irregular bodies were able to migrate particularly well from their original print sources into intarsia because they were instantly recognizable emblems of mathematic-ness and new-ness—coveted by patrons. While the prevailing taste for highly complex visual imagery was a determining factor in the creation of the imaginary architectures, ruined landscapes, and intricate polyhedral objects that characterized the surfaces of Augsburg's luxury furniture exports, there were other available pattern options that would have conveyed a similar level of prodigious skill and technical prowess. The aspiration for designers and patrons must have been to present the state of modern mixed-mathematical knowledge, for which Southern Germany was becoming a wellspring of talent, on its most precious surfaces and through its equally unrivalled abilities in carpentry.

Stöer's work in particular conveys a savvy understanding of its own potential for future translation, perhaps because it was inspired by other southern German geometrical research and was, at least in part, self-consciously pitched to another medium. His drawings gesture towards a self-conscious exploration of the connections between print and drawing; or perhaps rather the aesthetics of reiteration and translation. The twelve ink drawings of floating geometrical figures in Stöer's Augsburg notebook (some of which were drawn very unusually in colored ink) speak to the impulse to imitate methods of mechanical production, such as were responsible for the exquisite quality of Jamnitzer's etchings, and also seem to inspire their own appropriation as etchings or woodcuts. Again copied from *Perspectiva*, the two icosahedron variants on page 32 and 34 (CV in *Perspectiva*) [**Figure 5.25**], and the fabulous tetrahedron variant on page 33 (AII) [**Figure 5.26**] of the Augsburg notebook manage somehow to outshine the Jamnitzer originals. The line-weight and crosshatched penmanship show incredible command of line, light, shadow, as well as an intimate familiarity with producing three-dimensional geometrical forms on paper.[254] Having adopted the abstracted shading characteristic of the etching, in which shadow is shown by hatched line and light is indicated by the absence of line, Stöer emulates Jamnitzer through a technological mimicry that returns the human hand to the forbiddingly perfect renderings in *Perspectiva.*

The power of Stöer's drawn models lie in their exquisite rendering of the corresponding models' absence; a representation that purports to depict the presence of a real object where there never was one, where perhaps there was only ever a graphic precursor, copied and reinvented through the ages.[255] Stöer's models perpetually substantiate the importance of the Platonic Solids, precisely through their obscuration. By redrawing the boundaries of what it meant for geometry to be associated with the Platonic Solids—in a sense extending geometry beyond the geometrical definition of the Platonic Solids—Stöer's models renewed faith in the Solids' eternity even as they reinvented them. Wood claims that premodern "images...were understood...as links to an originary reference point...Artifacts within such a chain could be substituted for one another without impairment of reference" (Wood 2008, 15). Perhaps for educated Renaissance audiences Stöer's models would never have been able to escape their origins in ancient Greece, and neither would they have wanted to, since they gained their imaginative power precisely

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[254] The crosshatching also attempts to replicate the pattern of crosshatching in Jamnitzer, although the original is not exactly duplicated.

[255] On presence in art see "What is 'presence'?" by A. Harrison in *Presence: The Inherence of the Prototype within Images and Other Objects.* R. Maniura and R. Shepherd (Ed.), Hants, UK and Burlington, VT: Ashgate, 2006, 161-172.

from their antique reference, no matter how tenuous the connection. Stöer's models attempt to accommodate the Euclidean definitions of the five Solids, even as the definition is no longer able to adequately describe the shapes that have come to be understood as "Platonic." In place of *actual* Platonic Solids, Stöer conjures up an aesthetic of Platonic geometricality.

Given the endlessness of Stöer's geometrical experiments and the ascribed purpose of his prints, and possibly drawings, as destined for ornamental application in intarsia, the infrequent placement of his monogram raises the question of when it exactly was that incremental invention actually resulted in a form which Stöer deemed to have required an explicit statement of authorship. Occasionally, we find a simple watercolor drawing, such as one depicting stacked geometrical solids in the *Universistätsbibliothek München*, marked with Stöer's insignia of an "S" intertwined around an "L," and the inscription "*Lorentz Stöer Mahler* [painter] *in Augsburg 1567*," while all of the prints in *Geometria et Perspectiva* include the monogram set into the plinths at the solids' bases.[256] But more often, Stöer reserved his monogram for what he considered to be his finest geometrical work, very selectively populating his drawings with his attribution. Even rarer are the few instances when his experimentation appeared to yield a graphic result so exquisite and innovative that it required an imperial privilege, perhaps to protect the time and investment of skill in its invention and rendering (Witcombe 53) and thus substantiate that something truly new and special had been created [**Figure 5.27**]. At the base of a dazzling, nested polyhedral model towards the end of the Munich volume is the phrase

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[256] Universistätsbibliothek München, 2Cod. Ms 582a.

“*cum gratia et privilegio caesmaiest*”—“by the grace and privilege of his imperial majesty.” The imperial privilege fixed the graphic form in place, setting it beyond the endless transformability that otherwise could subsume it in a sea of variation. It seems that an end point, one of few, had been reached.

Chapter Six: The Ivory Towers

The polyhedral models in August, Elector of Saxony's (1526-1586) *Kunstkammer* in Dresden functioned as intermediaries between the technical knowledge gleaned from textbooks and the practical application of this knowledge in the production of new and experimental assemblages of geometrical forms. Polyhedra were scattered throughout the collection, where they were used as pedagogical aids for mixed-mathematical calculation and perspectival construction, as companion pieces to August's specialized research library, as ornate copies of these "working models" rendered in ivory on a lathe, and as components of larger turned-ivory assemblages destined for the masterpiece section of the collection. Both practical tool and artwork, polyhedra straddled the worlds of production and display, creating a perceptual continuity between the geometry necessary to understand contemporary mixed-mathematical literature, on topics ranging from surveying to ballistics and cosmology, and the performance of geometrical knowledge to astonished visitors in the *Kunstkammer.* In turn, geometrical models in ivory migrated from the *Kunstkammer* into the domain of diplomatic gifts, as symbols of sovereigns' dominance over matter and the technological capacities of their courts.

That polyhedra could be used as tools and produced as artworks themselves was implicitly acknowledged in the Belgian physician Samuel Quiccheberg's (1529-1567) *Inscriptiones; vel, tituli theatric amplissimi* (1565), an idealized organization system of a princely *Kunst-* and *Wunderkammer*, which called for "regular solids of various shapes, beautifully constructed of transparent rods" to be included in the same spatialized knowledge category (or inscription for Quiccheberg) as "mathematical instruments, such as astrolabes, spheres, cylinders, quadrants, clocks, geometric rods, and other objects to be used in measuring land and sea, in war and peace" (Quiccheberg 67).[257] Though Quiccheberg's *Inscriptiones* was not a direct reference for the Dresden *Kunstkammer*, he was a near contemporary and advisor to Albrecht V, Duke of Bavaria (1528-1579). His descriptions of a consortium of "Museums, Workshops, and Storerooms, Such as Are Meant for Furnishing Wisdom and Pleasing Arts, Which Are Sometimes Constructed Separately in Palaces and Sometimes Joined Together" (Quiccheberg 71) anticipates August's integrated research center, which connected mathematical instruments, a research library, a laboratory or working space, and a space for display of masterpieces in order to formulate mixed-mathematical research through turned ivory. For Quiccheberg, as for August, the ideal organization for making and learning would house "mathematical instruments" of all kinds, together with "instruments for workshops and laboratories used by the more skilled artisans, such as the tools of sculptures, turners, goldsmiths, foundry workers, woodworkers, and indeed of all artisans whom this world supports in our age" (Quiccheberg 68).[258]

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[257] Fourth Class, inscription 2. Trans. by M. Meadow and B. Robertson. The English title translates as *Inscriptions; or, Titles of the most ample theater.*

[258] Inscription 5, Fourth Class. This inscription belongs to the same class, and thus physical space, as the previously referred mathematical instruments. In reference to Prince Albrecht of Munich, whom he knew personally, Quiccheberg states that the turning workshop had been "enlarged by Albrecht himself for the most skillfully wrought items. All of these things can be comprehended here both collectively and individually" (Quiccheberg 76). In regards to the turning workshop, Quiccheberg states that "here, moreover, are diverse works on the lathe from wood, metal, ivory, alabaster, horn, bone, and perhaps still other materials, readily revealing their inner forms to their makers so that, beyond their ornamentation and elegance...a not trifling delight is experienced" (Quiccheberg 72).

Ivory lathe turning constituted a practice that necessitated a high degree of craft knowledge, intimate familiarity with geometrical calculations, a sophisticated understanding of the interrelationship between drawing geometry and modeling geometry on curved surfaces, and a commitment to precision measurement (Moran 1981, 259). To broach the evolving early modern categories of mixed-mathematical knowledge embodied by the ivory models is to view the complex spatial and geometrical issues raised by their forms as the end results of the matrix of tools and drawings used to produce their component parts, and the assembly of these component parts into one fantastical whole. It is also to address the importance of perception in turning—the inherent tactile pleasure of turning, the seemingly "magical" appearance of form on the lathe, the way the body needed to be attuned to the precise changes in pressure while turning, and how the variety of tools at the 16th century turner's disposal required precise imaginative leaps in order to anticipate the effect they were designed to have upon material. The focus on ivory polyhedra, and the significance of using ivory as a material for polyhedra, is used to relate the generation of these forms to the myriad phenomenologies of turning.

The ivory *corpora* that adorned the tops of the miniature decorative columns, or *Säulen*, for which the Dresden court became the most renowned site of production in Europe in the late 16th century [**Figure 6.1**], were perhaps the capstone expressions of virtuosity in the plethora of skills and techniques it would have taken to work with ivory. Yet unlike the numerous drawn models floating around Germany in this period, which remain as evidence of the open-ended experimentation with the Platonic Solids by artisans, their ivory counterparts were more strictly defined by the limits of the material and the practicalities of making. They tended to either be solid, symmetrical forms or fall into a singular category of nested geometries, in which one form, or several, were inserted inside another. While there was certainly a measure of artistic subjectivity in the act of deciding which polyhedra to embed within the other, the focus here shifted away from the conceptualization of new geometries towards the practicing of techniques required to successfully manufacture several mutually enclosed bodies.

Though ivory in both raw and processed form had been flowing into Europe since Antiquity, during the 15th and 16th centuries the numerous European excursions into Africa and India opened up new lucrative opportunities for the procurement of ivory, which in turn fueled the luxury markets for decorative art objects, as well as ivory sculptures, in the wealthy urban centers of Renaissance Europe.[259] The Munich *Kunstkammer* of Albrecht V of Bavaria (1528-1579) and the Hapsburg *Kunstkammern* included examples of carved African ivories (Mark 198). The *Wunderkammer* of Ferdinand II (1529-1595), Archduke of Tyrol in the Ambras castle at Innsbruck contained twelve items of African origin, all of which were made of ivory.[260] Carved ivory goods were a common enough import from Africa that they were mentioned in the *Casa de Guiné* (custom house), the book of accounts that related the receipts of payments

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[259] Titus Livius (Livy)(64 or 59 BC – AD 17) relates that Lucius Scipio's victories were celebrated in Rome with a procession that included, among the spoils of war, 1231 ivory tusks that had been confiscated from the battles in Asia Minor (Livy, Book 37). For a concise history of ivory trade in Late Antiquity and the Middle Ages, see Shalem 18-37, and also F. St. Anbyn's *Ivory: A History and Collectors Guide*, London: Thames and Hudson, 1987.

[260] See Bassani 3-8 for a list of the items in the Ambras Wunderkammer, based upon the 1596 inventory of the collection. They include one saltcellar, various pieces of cutlery and several oliphants. For further reference see W.B. Fagg, *Afro-Portuguese Ivories.* London: Bachworth Press, 1959?; and E. Bassani and W. B. Fagg, *Africa and the Renaissance: Art in Ivory.* New York and Munich: The Center for African Art and Presel-Verlag, 1988.

made by sailors and officials returning to Portugal.[261] And in his undated diary entry from after 1877, AW Franks (1826-1897), the British Museum's famous administrator, recorded and drew a "carved ostrich egg forming a box. Two negro heads carved on top. Open work ivory stand carved in four open compartments—in each a sitting figure" from the Grünes Gewölbe in Dresden.[262]

Under August's leadership, Dresden became the center of European turning innovation, exporting turning expertise and peerless ivory confections across Europe, attracting the world's most highly skilled turners, and equaling if not surpassing the skill of the Medici workshops in Florence. Substantial investment was necessary to establish the conditions required to manufacture these highly articulate and precious objects. To this effect, August assembled a world-class turning workshop and library of mixed-mathematical literature, stocked with the newest technologies and texts, and paid handsome sums to attract the most well known turners to Dresden. While he could not hope to rival the ivory stylings of his master court turners, August still actively participated in the process of transforming the raw African ivory, artifacts from a foreign world he would never get the chance to visit, into gleaming white exemplars of late Renaissance machine technology. For August, himself an enthusiastic lathe turner, ivory was a material worthy in value of receiving the sustainedly intense reworking and treatment required to

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[261] "And I received that same day (16 October 1504)…an ivory salt-cellar and three ivory spoons belonging to Diogo Lopes, Captain of the Mina, who paid for them all valued at 700 *reis*- 201 *reis*." Reference and quote in Ryder 1964, 363.

[262] 90 percent of ivory carvings amassed in the 16th century have disappeared from German collections (Mark 201), leaving approximately 150 in existence (Mark and da Horta 135-136). From the unpublished notebooks of A.W. Franks. Notebook 15, Department of Africa, Oceana, and the Americas at the British Museum, Ref. num. LS 15.

transform *exotica* into *artificialia.* The divine overtones of the spherical, lathe-produced forms would not have been displeasing to any sovereign, for lathe turning was the most suitable technical method to create a three-dimensional sphere, much as God, "the first lathe turner," had first created the sphere of the earth.[263] To create something as miraculous as the world was to create objects that exhibited the world's essential spherical continence, and had similarly come into being by means of the force of perpetual rotation. Unlike basic geometry or calculation, "basic" ivory turning—i.e. the creation of the type of elegant forms produced by August, as opposed to the hyper-articulate polyhedral towers manufactured by the master turners—resulted in beautiful objects which August could proudly display in his *Kunstkammer*.

The broader cultural agenda of Saxony's rulers was legible through the Dresden castle's spatial layout, which situated ivory turning facilities at the heart of power within a vertical series of hybrid spaces that included tools, books, ivory columns, and raw ivory from Africa. The impetus for the upper classes to work on mathematics through manual procedures and to collaborate with craftsmen in the production of mixed-mathematical objects or instruments was not uncommon during this period (Moran 1977, 211) [**Figure 6.2**].[264] Thus the practice of ivory turning, a pastime that Elector August evidently very much enjoyed, was used to produce decorative art objects whose very production was considered a serious course of mixed-mathematical study. The boundary between mixed-

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[263] See Friedrich Friese's *Der vornehmsten Künstler und Handwerker Ceremonial-Politica…* (1708) and Johann Martin Teuber's *Dreh-Kunst* (1756).

[264] Wilhelm IV of Hesse-Kassel (1532-1592) helped to provide the observations and calculations for the manufacture of an astronomical clock he financed in 1561, and frequently relayed ideas and suggestions to its designer, Eberhart Baldewein (Moran 1977, 222). See also Moran 1981, 254.

mathematical research, as touted by the late 16th and early 17th century authorities on the subject, and decorative art practice, as facilitated by the finest tools and equipment money could buy, made the Dresden court the site of a unique spatial and practical synthesis of textual and artisanal knowledge.

Although August may likely have been the most prolific royal turner, from the early 16th century and well into the 18th century ivory turning on a lathe was widely considered to be a suitable pastime for European nobility.[265] Emperor Maximilian I had a turning lathe installed in the Hofburg at Innsbruck in 1503 and obtained a second machine in 1505 from Tramin. Hans Wecker, the son of Georg Wecker, taught Emperor Rudolf II, as did the Nuremberg turner, Peter Zick the Elder. His son, Lorenz Zick, went on to become court turner for Emperor Ferdinand III (Maurice 2004, 21). The Danish daughters of King George II and Queen Caroline of Brandenberg-Nasbach, third cousin to Frederick William, Elector of Brandenberg and Duke of Prussia, were known to have turned ivory objects as well as engaged in gemstone working.[266] King Frederick III of Denmark (elected 1648), founder of the *Kunstkammer* in Copenhagen, was an avid turner and collector of decorative ivory pieces from southern Germany.[267] The Wittelsbach Dukes

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[265] See Maurice 1985 for turning as a princely pastime.

[266] There are twenty four amber knife handles in the Hague by Princess Ann dating from between 1730 and 1740 and a miniature turned ivory chandelier from 1740-1750 by Princess Luisa is in the Rosenborg Castle in Copenhagen.

[267] As reference see *Kongelige Kunstdrejere* (1998) by M. Bencard, *The Royal Danish Kunstkammer 1737* (1991) by B. Gundestrup, and J. Hein's *The Treasure Collection of the Rosenborg Castle I* (2009). The collection in Rosenborg is replete with evidence of ivory polyhedra as well as a sumptuous *pietre duré* cabinet that juxtaposes nature with abstract geometrical forms, as in the Castrucci table in Vienna and the *Kunstschrank* in Cologne. For a catalogue of Rosenborg's ivory collection, see Hein and Kristiansen 92-107.

were enamored with turning, and Duke Wilhem V of Bavaria sent the Florentine Duke Francesco de' Medici a hollow sphere by the master turner Giovanni Ambrogio Maggiore, who himself had travelled to Bavaria from Italy in 1574 on advice of the Milanese art dealer Prospero Visconti (Maurice 1985, 38).[268]

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The *Drechselkammer* (turning workshop), established by Elector August directly above the *Kunstkammer*, was a space alive with activity of producing objects.[269] August himself worked there, as did various court turners including Georg Wecker, who was employed on a permanent contract from 1578 (Syndram, Kappel, and Weinhold 54) and taught August's son and later successor Elector Christian I (1560-1591) to turn ivory, Egidius Lobenigk (d. 1595), appointed court turner in 1584 and the first to master the production of the counterfeit sphere in Dresden (Kappel 178), Jakob Zeller (1581-1620), court turner in Dresden from 1610, and Marcus Heiden (born 1597/98), who had previously worked for Duke Johann Casimir von Saxony-Coburg. [270] Though it is not known precisely where

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[268] Duke Francesco de' Medici was not the only Italian recipient to receive a turned object from Wilhelm. The 1585 Tribuna inventory at the Uffizi in Florence lists a present to Grand Duke Ferdinando I of a turned ebony ball enveloping a smaller ivory ball containing portraits of Duke Wilhelm, his wife, and son (Casazza 123). For further reading on ivory work in Italy see M. Mosco and O. Casazza's *The Museo degli Argenti – Collections and Collectors* (2004); and E. D. Schmidt and M. Sframeli's *Diafane Passioni – Avori barocchi dale corti europee* (2013). The first turner to produce a counterfeit sphere was the Italian Giovanni Ambrogio Maggiore (ca 1550 – d. after 1598).

[269] From the introduction to the 1684 inventory of the *Drechselkammer*. The fourth floor currently houses the plant and mechanical systems for the Grünes Gewölbe and is not accessible to the public, though I was allowed access to it due to my research.

[270] Scattered invoices in the *Finanzarchiv* attest to the payment provided to turners for their services and the terms of their contracts. The contract for Georg Wecker reveals that the court was exceedingly excited to have him in residence and sought to be unflaggingly generous, agreeing to pay him two hundred gulden Agünz for four yearly quarters. "In addition, as a sign of

the turning lathes within the *Drechselkammer* might have been situated, it is likely that they would have been set up facing west where the light is best and the view expansive onto the former garden that fanned out in front of the castle. Raw ivory was stored here as well and the floor would have been covered with shavings of the various material being turned, at least until the floor was swept. The smell of wood, used to test and practice the production of forms later to be rendered in ivory, would have permeated the space. From beneath the pitched wooden roof, resplendent with great beams which, even if the space was not very high still carried the air of majesty, ivory and wood particles in the air undoubtedly swirled like mist in the defuse light rays of the Saxon sun.

The *Kunstkammer* and the *Drechselkammer*, on the 3rd and 4th floors of the Dresden Palace respectively, formed the premier facilities for the research and production of ivory turning knowledge and practice in 16th century Europe. Though turning was popular throughout Europe in this period, with turners working for the Medici in Florence, Elector Maximilian I (1573-1651) in Munich, and King Friedrich II of Denmark (1534-1588) in Copenhagen, it was arguably championed and funded to a greater extent and with more of a single-minded focus by Elector August than by any of his contemporaries and consequently became very much of a speciality passed down through families of

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appreciation for his services, we have given and will pay every year 50 gulden groschen [German ten-pfenning piece] to his mother in Munich as long as she will live, so that he will be even more inclined to stay with us in this country." "*vnd zur erfözligkeit solcher seiner Dienste, wollen wir ime irhelich die zeit seiner Lebens, Vnd langen er vnnser besteltter Dreqler sein und bleiben wirdet, Zwei Hudert gulden Agünz, zur den Vier Quarteal zeitten…Vnd das er desto williger Im dieser Landen bei vns bleiben möge, seiner Mutter zur München, so lange sie am leben sein wirdet, alle jahr Funfzigt gulden groshen zu geben vnd folgen zulassen bewilligett*." Staatsarchiv, Finanzarchiv 10036, Rep. LII Gen 1925, Loc. 33342, fol. 506r-507v. For additional references to contracts and payment, see references and catalogue entries in Syndram and Scherner, 176-197; and "Elfenbeinkunst in der Dresdner Kunstkammer – Entwicklungslinien eines Sammlunsbestandes (1587-1741)" in Syndram and Minning.

artisans employed by the Saxon court. Long after August's death, Dresden persisted as a center of turning expertise, attested to by the fact that lathe-turned ivory remains one of the largest groups of art objects in the *kurfürstlich-sächsische Kunstkammer*, as well as by archival documents like the *Kunstkammer* inventories, which were compiled in 1587, 1619, 1640 and 1741; the fastidious detail of the 1622 and 1684 inventories of the *Drechselkammer*; and the inventories in the Dresden Staatsarchiv of the workshops of the two famous Dresden court turners, Wecker and Lobenigk.[271] It was in these interlinked courtly spaces, self-conscious incubators of innovation, that ivory was crucially collected in its raw form and through the interplay of sophisticated sets of tools and drawings, converted into the geometrical ivory columns that served both as exemplars of technical expertise and as sites for the mixed-mathematical faculty for mapping geometry onto curved surfaces.

When the 1587 inventory of the *kurfürstlich-sächsische Kunstkammer* was taken about eighteen months after the death of Elector August, the *Kunstkammer* consisted of six main rooms, including an entrance vestibule which connected to the northwest stairs leading to the *Großer Schlosshof* below and the workshop above [**Figure 6.3**].[272] The

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[271] The *Kunstkammer* inventories have been republished as four separate volumes. See bibliography. For the turners' inventories, see SachsHStA Loc. 9835/15, fol. 20 and fol. 23v-37r.

[272] A plan of the *Kunstkammer* was drawn in 1615 by its art chamberlain and sole keeper, David Uslaub. Given the *Kunstkammer*'s later expansion, the plan only corresponds to the state of the *Kunstkammer* at this particular period. Reconstructions of the *Kunstkammer*'s plan and collections of objects over the course of its history can be found in the republished *Kunstkammer* inventories. Because some inventories designated the *Kunstkammer*'s rooms with different numbers, the reconstructed plans are indispensable for determining which room was being referred to in the Kunstkammer. For further reading on the origins of the Kunstkammer see D. Syndram, "Princely Diversion and Courtly Display: The Kunstkammer and Dresden's Renaissance Collections," *Princely Splendor – The Dresden Court 1580-1620,* Dresden and Milan: Staatliche Kunstsammlungen Dresden and Mondadori Electa S.p.A. 2004, 54-69.

first of the *Kunstkammer*'s rooms described in the inventory contained all of August's mixed-mathematical instruments—his astrolabes, compasses, measuring tools, and tools for perspectival constructions.[273] The inventory also details the presence of a comprehensive set of polyhedra by Abraham Reise, son of the mathematician Adam Reise (1492-1559), under a subheading entitled "*Ahn regulirten corporibus, von holz und gepappten papir gemacht*"—"a regular body, made from wood and glued (stuck together) paper."[274] The next subheading refers to "*ahn perspectivischen instrument und derselben zugehörunge*," in other words, a category for perspectival instruments and all manner of drawing apparatuses. Furthermore, intricate geometrical models in wood also existed, such as the "four-sided cube with another eight-sided cube inside of it," to be used as preliminary tests for similar models to be later made out of ivory or perhaps as advanced

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[273] On visiting Dresden, the art dealer Philipp Hainhofer remarked on the collection of mathematical instruments found there, testament to the breadth of the instruments used for mixed-mathematical research: "astronomical, geometrical, geographical, scenographic, or perspectival and typographic, notably beautiful armillary spheres, celestial & terrestrial globes of wood, metal, and other material, astrolabes, quadrants, quadrata, declinatoria, nocturnalia, several solar clocks, cylinders, truncated [objects], various regular & irregular bodies" (Hainhofer 175); "*Astronomische, Geometrische, Geographische, Scenographische, oder perspectivische vnd typographische, insonderheit von schönen spaeris armillaribus vnd artificialibus, globis coelestibus & terrestribus, von metal, holz, vnd andern materien, Astrolabia, qvadranten, qvadrata, declinatoria, nocturnalia, mancherleÿ horaria solaria, cylindros, truncos, varia corpora regularia & irregularia*."

[274] The inventory list is as follows. It can be found on fol. 52r and fol. 52v in the *kurfürtliche-sächsische Kunstkammer* inventory of 1587: *1 Pyramis ex regularibus primum; 1 Cubus ex regularibus corporibus secundum; 1 Octohedrus ex regularibus tertium; 1 Icosahedrus ex regularibus quartum; 1 Dodecahedrus ex regularibus quantum; 1 Conus ambigonius; 1 Conus oxigonius; 1 Conus orthogonius; 1 Holtzern gevirter cubus mit einem andern achteckigten cubo darinnen; 1 Rundt corpus von mößenen drath, inwendigk mit den regulorten corporibus von guldenen fäden gezogen; 1 Holtzern birnbeumen cubus in einem kleinen khestlein. Hat Abraham Reise gemacht.* Abraham Reise was evidently intimately involved with the courtly practice and study of geometry at the Dresden court. He is also known to have given a geometrical treatise in 1589 entitled *Mathematicus* as a present to Elector Christian II, son of Elector August (Korey 42).

subjects for drawing.[275] Though there is no mention as to how the models might have been displayed, it is not inconceivable that they would have been hung on the wall, as in the Neufchâtel painting or the Faulhaber print, or suspended with string from the *Kunstkammer*'s ceiling as in Pacioli's *De divina proportione* or Carpaccio's *studiolo*.

In the 1640 inventory, Abraham Reise's contribution has been qualified as "*zur demonstration radices quadratae und cubicae dienlich*"—"useful for the demonstration of quadratic and cubic radiuses" (1640 Inventory, fol. 318r-318v). Additionally, six new nested polyhedra have been added, the first three of which had been produced by women —"*Diese drey sind von frauen eiß*." These three models were of an "octahedron, in which a hexahedron and in the same however a tetrahedron have been enclosed"—"*Octahedrum, darein ein hexaedrum und in selbigen hinwiederumb ein tetraedrum geschloßen ist*"—and two models of a "hexahedron, in which an octahedron has been enclosed"—"*Hexaedrum, darein ein octaedrum geschloßen ist*" (1640 Inventory, fol. 319r). The last three models were made "*von türckischen papir*" in rare acknowledgment of the specific type of heavyweight paper used to produce paper models. The three models are "1 small cube," "1 skeletal tetrahedron in which [there is] an octahedron," and a "prism, which one can dismantle into 5 pieces, made by Dr. Christof Pincker from

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[275] The polyhedral models are referred to for the first time in the 1640 inventory as "*mathematische und stereometrische corpora regularia und irregularia hin undt wieder in diesem zimmer zufinden*"—i.e. mathematical and sterometric regular and irregular bodies, which can be found here and there in the room." The descriptions of the *corpora* have also been fleshed out. For instance, the "*Conus orthogonius*" from 1587 is now given a further elaboration in 1640, as "*Conus orthogonius, darein ist section parabole geschniedten*"—Orthogonal cube, in which a parabolic section has been cut. The name of the "*Holtzern gevirter cubus mit einem andern achteckigten cubo darinnen*" from 1587 has been changed to reflect contemporary mid 17th century German terminology for nested polyhedra, "*Höltzern gevierter durchbrochener cubus mit einem octaedro*."

Leipzig"—"*1 Kleiner Cubus*," "*1 Durchbrochenen tetraedrum, inwendig ein octaedrum*," and "*Prisma, so man in 5 stuck zernehmen kan, welcher herr Doctor Christof Pincker zu Leipzigk gemacht*" (1640 Inventory, fol. 319r). One imagines the models, made from this heavyweight paper, both robust and integral enough to be included in the inventory, scattered around the room, surrounded by the drawing instruments that would have been used to translate them back into drawings.

The *Kunstkammer*'s second room, the room immediately to the right of the first room, included all of August's workshop tools, alongside more measuring tools and a specific research section of mixed-mathematical books that was separate from the rest of August's library collection.[276] The diaries of Philipp Hainhofer (1578-1647), written when the famed diplomat and art advisor to Duke Philipp II of Pomerania visited the Dresden *Kunstkammer* in 1629, describe August's research library as "a beautiful mathematical library for arithmetic, geometry, architecture, perspective, and other arts" right before mentioning "cones and pyramids, beauties of turned ivory, upon which can be seen several regular and irregular bodies" (Hainhofer 173).[277] Hainhofer notes that the third chamber of the Dresden *Kunstkammer* included "*ain schöner drehzug zum Agustain zu drehen"* ("a beautiful turning lathe used by August to turn") and the work of the court

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[276] The first and second room described in the 1587 inventory are referred to as the fifth and fourth room respectively in the 1619 inventory, though the physical location remains the same.

[277] "*Aine schöne Mathematische librej oder bibliotheca zu der Arithmetica, Geometria, Astronomia, Archietctur, perspective vnd andern künsten mehr. Schöne von helffenbain gedrehte coni vnd pyramides, darauf mancherleÿ corpora regularia & irregularia zu sehen*." Hainhofer is well known for his role in the design and procurement of art for Duke Philipp II's so-called *Pommerscher Kunstschrank* (1615-1617), which was destroyed during World War II. For a comprehensive study of the *Pommerscher Kunstschrank* see *Der Pommerscher Kunstschrank* (2009) by B. Mundt.

turner Georg Wecker. Hainhofer was particularly taken with the examples of nested polyhedral geometry found in the collection. He mentions "on a peculiarly turned goblet, an open-work ball, in which are located another 24 balls, all of which are moveable."[278] He also describes the contents of the interior of a "dense ball...in which one discovers through a hole, another beautiful star and small open-work spirals. Across it, similarly, is a box with two lids, in which are two contrafet [balls] carved by Elector Christian II..."(Hainhofer 171-172).[279]

The Elector, as well as all the court turners who worked upstairs on the 4th floor of the west wing, would all have had easy access to the state of the art literature in the mixed-mathematical library.[280] There are 288 books listed in the 1587 inventory, all of which

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[278] "*Aif ainem selzam gedreheten Becher aine durchbrochene kugel, in welcher andere 24 kugel alle beweglich zu befinden*."

[279] The entire quote reads "*Aine dichte kugel auswendig anzusehen, inwendig aber erkennt man durch ain loch aine andere mit schönen stern vnd schnecklen durchbrochen: veber das in selbiger ain bix mit zweÿen liden, in welcher bix zweÿ contrafet, Churfürsten Christiani secondi samt desselben gemahl geschnitten, die lider aber mit gulden bändern angeschlagen, alles aus ain stuck inwendig gemacht, vnd die kugel noch aus wendig di basso rilievo geschnitten ist*."

[280] I contend that a second Jamnitzer book mentioned in the inventory and no longer in Dresden, is in fact a book in possession of the National Art Library at the V&A in London, entitled *Ein gar Künstlicher und wolgetzierter Schreibtisch* and *Das Andertheil von Beschreibung der Künstlichen Silber und Vergulten Instrumenten* - MSL/1893/1600-1601. The connection has been previously mentioned, see Rosenberg 53, but without supporting evidence. I believe now that the provenance can be conclusively confirmed. The last reference to the book in Dresden is in the *Katalog des Mathematisch-Physikalischen Salon* 1874, Nr. 68; spätere Signaturen II B2 und L 90. The description of the book in the inventory reflects the content of the book—which is about the relative weights and masses of different metals and is accompanied by beautifully illuminated drawings. The 1587 inventory describes the book as "*Wenzel Gamitzers grundtlicher und eigentlicher unterricht und erklerunge des kunstlichen runden maß oder eichstabes aus die 7 metall, sambt den gar nutzlichen vierfußigen circkels und zweier kleinen masteblein, auch einem viesir mäßlein und seinem mastabe. Doran die goldische muntz gegen die marckh vorglichen wirdt*." For further description of the Jamnitzer volume see Watson 768-773. An additional book by Jamnitzer is mentioned in *Das Kunstkammerinventar Kaiser Rudolfs II*. 1607-1611. See Bauer and Haupt. The description of the book lists it as "*Wenzel Jamnitzers gemalt mit farben*

fall under the subheading "*astronomischen, astrologischen, geometrischen, perspectivischen, arithmetischen und anderen kunstbuchern*." Several books of relevance include (1) *Perspectivische kunststucken, so mein gnedigster churfurst und herr, herzogk Christian zu Sachen etc., selbsten gerißen*. This book is a collection of August's son, Duke Christian's, geometrical drawings, and may have been kept in the library because his father, August, was proud of it. (2) *Perspectiva corporum regularium Wenzel Gamitzers*; (3) multiple books by Albrecht Dürer, including *Underweisung der meßung*; (4) *Perspectiva Danielis Barbari in lingua italiana*; (5) *Hansen Lenckers von Nurnbergk perspectiva*, *illuminiert*; (6) *Augustin Hirschvogels eigentlich und grundtliche anweisung in die geometria…*;(7) *Erhardt Schön von Nurnbergk underweisung der proportion und stellunge der poßen*...The inventory also substantiates Hainhofer's account of a consortium of turned ivory objects ("*kunstlichen dröhwergk von helfenbein...welchs hogstgedachter hetzogk Augustus...selbsten gedröhet*"—"artful turned ivory work…which the well-regarded Elector August turned himself"), including ivory works by Lobenigk and Weckhardt (Wecker).[281] Although not the space where the actual turning took place, Room Two was clearly the more outward-gazing face of August's workshop. It was a carefully curated working environment geared towards contemplation and admiration, displaying the results of August's craft alongside the work of his court turners, and a place where the most relevant literature could be consulted and August's state-of-the-art tools admired by himself and select guests.

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perspectifbuch von vilen corporibus," (Kunstkammerinventar Kaiser Rudolfs II, Folio 382 no. 2712) proof that Jamnitzer made at least one other book of polyhedra in color.

[281] The section of turned ivory objects made by Elector August in the 1587 Inventory, fol. 131v/141v – fol. 136v/146v. For Lobenigk, see fol. 137r/147r – 137v/147v. For Wencker, see fol. 138v/148v – 139v/149v. These drawings will be discussed further in the chapter.

In addition to the two rooms which contained the bulk of mixed-mathematical artifacts, the *Kunstkammer* contained a vast array of artwork and objects, including paintings by Lucas Cranach the elder and Giuseppe Arcimboldo, magnificent prints by Dürer, lavish works in metal, *pietre duré* and elaborate carvings in wood, ivory, ebony, alongside maps, military busts, arms and armor, and collections of ceramics and glassware. The contents of the *Kunstkammer* also continued to expand in the years after August's death without sacrificing the preexisting collection, and an additional large room on the same floor at the end of the castle was annexed by the *Kunstkammer*, bringing the total number of rooms to eight. The eighth and far largest room became responsible for housing many of the collection's *Meisterstücke*, vivid assemblages in gold, silver, coral, and ivory. Turned ivory works by Wecker, such as a "large cup, turned three on top of the other, lid and base with little "see-through" [*durchsichtigen*] crowns…" and by August himself were brought together from their former locations in other rooms of the *Kunstkammer*, as befitted their status as courtly and princely works of art.[282]

The eighth room was also home to an expanded repertoire of polyhedral geometry which bore only superficial relation to those models contained in the collection of mixed mathematical instruments. "1 transparent ivory body, known as an octahedron. A

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[282] "*1 Großer becher, drey auf einander gedrehet, deckle und füßlein mit durchsichtigen crönlein… No. 18."* 1640 Inventory, fol. 439v. "*Helfenbeiner becher, so das letzte stück ist, welches weiland churfürst Augustus zu Sachßen hochlöblichster gedechtnus selbst mit eigener hand gedrehet hat. Ist von Georg Weckern, hofdrechslern, eingeantwortet worden de, 16. octobris anno 1567. No. 100*." 1640 Inventory, fol. 444v. The word "*durchsichtig*," which translates directly as "see-through" or "transparent" is more appropriately translated in this context as "skeletal." The word was often used in the 16th and 17th centuries to refer to those objects whose relief or edges were constructed from tracery, allowing one to "see through" the object. This fascination with transparency may well date back to the innovations of Leonardo, who first began working with "skeletal" geometrical bodies, and, following him, Dürer.

transparent pyramid on all sides above a transparent, circular base made from bone. It was fastened by my gracious sir, Anthonius Örtel, carpenter, on the day of Johannis, 1620; 2 angled, transparent regular bodies from ivory, with double tetraedra set one inside of the other, with a transparent base" [**Figure 6.4 + 6.5**].[283] These last descriptions refer to a set of exquisite geometrical masterpieces, characterized by facets filled with ornate, lacework-style ornament. As opposed to the previous "regular bodies" made from paper and wood, the degree of embellishment of their intricate ivory cousins had evidently elevated these objects from being commonplace or practical accouterments of the mixed-mathematical workshop, into paragons of technical achievement, in which the sides of the polyhedra were designated as opportunities to frame windows of fashionable tracery.

The most common genre for expressing geometrical virtuosity in the Dresden court were the ivory *Säulen* which the Dresden court turners excelled in producing, and which would all certainly have been considered *Meisterstücke*.[284] A number of these were topped by

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[283] *1 Durchbrochen corpus von helfenbein, so octaedrum genennt wird. U fallen flechen mit auch durchbrochenen pyramidibus erhaben uf einen beinern durchbrochenen schnirckelfuß. Hat meinem gnädigsten herrn Anthonius Örtel, tischler, darmit angebunden 1620, am tage Johannis.*
2 Eckigte durchbrochene helfenbeinerne corpora regularia, als duppelte teraedra in einem der gesetzt, mit durchbrochenen füßen. 1640 Inventory, fol. 455v.

[284] Though the ivory *Säulen* were plentiful in Dresden, they were present as well in all of the royal courts that employed turners. Notably, the Kunstkammerinventar Kaiser Rudolfs I (1607-1611) lists 160 items in the section entitled "*von Helffenbain, Ebenholtz und anderm gedrechselte Sachen"*—of ivory, ebony, and other turned things—including several small dishes (*geschirrlein*) by Duke Christian of Saxony given to the Kunstkammer in 1601 and works by Georg Wecker. There are numerous descriptions of *Säulen* that included geometrical shapes, some of which have been reproduced here. *Ein colonna a ovato schraubenweis von helffb: gedret uff einem tristaffelten ebnin fueß, dabey ein runde holgedrehte kugel, darin das corpus octoedron, auf die kugel gehört beyligendt eingeschraubter pyramus von helffenb*:—A spiral (screw-shaped) ovular column of turned ivory on a base of three staggered levels. On it is a round, hollow ball which has been turned, within which is an octahedron. On top of the ball has been added a screwed on ivory pyramid (Folio 95, no. 891); *Ein trifach ineinander geschlossen corpus dodecaedron von ebano,*

the types of open-worked balls described by Hainhofer, such as the two ivory columns [**Figure 6.6**] by Lobenigk from 1588 and 1591, and are characterized by continuous spiral incisions around the shaft and a capstone nested polyhedra (in this case an icosahedron made of twenty equilateral triangles and a pyramidal tetrahedron) at the summit.[285] Topping an ivory column with a self-conscious reference to experimental geometry, of which polyhedra were the premier emblems, was not an unusual aesthetic endeavor, as can be further seen in additional late 16th century examples by Georg Friedel and Georg Wecker in Dresden. These polyhedra were appended to lids of standard, if somewhat reinterpreted, ivory goblets, which in other similar compositions might be replaced by a carved figure of flowers, a royal crest, or a mythological scene. Clearly then, polyhedra were available as one option among many for ivory turners looking to distinguish their work. The imbedding of one form inside another had the dual rhetorical strategy of displaying the maker's virtuosity while requiring a viewer, or royal patron, to gaze even closer at the object.

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obenauf ein zierd von helffenb: und blümlein—A triple-enclosed, one inside the other, ebony dodecahedron, atop an ivory ornament: and small flowers (Folio 96, no. 905); *Inn einem langen gefierten mit leder überzogonen vergulten futral ein von helffenbain geschraufte colonna, obben darauf ein runde hol gelocherte cugel und mehr corpora darin, oben darauf ist ein triegget corpus, uff dessen spitz stehet ein von helffenb: geschnitzter Mercurius*—On a long gilded futral covered in leather and divided in quarters a spiral ivory column stands. Upon it is a round hollow ball with holes with more bodies inside. Above it is a three-cornered body, on whose point stands a carved *Mercurius* from ivory (Folio 97, no. 953); *Zwey durchbrochne corpora oder von helffenb: gedrehte kugeln, dern jedes fünffach ineinander geschlossen*—Two skeletal bodies or turned ivory balls, which have been enclosed five times inside of the other (Folio 99, no. 990); *Ein durchsichtig corpus dodicaedron, in wendig ein stern und ein kunststuckhle daran gedreht*—A skeletal dodecahedron with a star and a small surprise ornament turned inside (Folio 99, no. 1006).

[285] Inv. no. 7 and Inv. no. 99.

The seventh image in Jacques Besson's *Theatrvm Instrvmentorvm et machinarum* (1578) is one of three depictions of turning in which the turner operates an enormous lathe looming over him [**Figure 6.7**]. A sinuous wooden form is in partial production with a rope rigged up to a bow protruding from the wall providing the reciprocal motion required to cut into the material. In order to work this particular lathe, it appears that the turner would be required to use both feet and hands while simultaneously guiding the material, a tiring prospect that would not have befitted the image of the ruler of Saxony in so far as the hapless symbiosis of turner and machine did little to convey the mastery of learned mixed-mathematics to which Elector August clearly aspired.

The appearance of the 16th century lathe in royal collections is substantially due to the technological innovations that had brought the lathe out of the craft workshop and into the *Kunstkammern*, and later drawings rooms, of the upper classes.[286] Though many modifications and iterations of lathe would eventually be manufactured, the key 16th century innovation was the superseding of those lathes predicated on reciprocal motion for machines capable of continuous motion. Antique and pre-modern lathes had required turners or their assistants to manually rotate the turning material with one or both hands while using tools or bows to cut into and shape the material. Though the process could result in spectacular examples of craftsmanship, the dual imperatives to rotate the material directly as fast as possible, and balance this with the intricate manipulations of tools, made lathing a strenuous physical practice requiring years of apprenticeship. In

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[286] Perhaps the most extravagant machine at the Dresden court was the great wire turning machine of the Nuremberger Leonhard Danner, which was covered in intarsia and stretched 4.20 meters long. The wiring turning machine now resides in the *Musée de la Renaissance* in Ecouen. See M. Minning, "Les outils de la *Kunstkammer* de Dresde et leur histoire," *Le Banc d'orfèvre de l'électeur de Saxe*, Paris: Réunion des musées nationaux-Grand Palais, 2012.

Europe, improvements to the lathe sprang from the addition of new rigging mechanisms. Hartman Schopper's survey of trade professions, *Omnivm illiberalivm mechanicarvm…* (1568) [**Figure 6.8**], depicts a lathe which has one end of a string fixed to a pole while the other side was affixed to a treadle beneath it which enabled the lathe to be turned by pressing down on the treadle with one foot (Holtzapffel 16). This innovation left the hands free to work with tools, though cutting was still restricted to one half of the motion—that of the lathe rotation towards the tool.[287]

The introduction of the "flywheel" enabled the lathe to generate continuous movement as opposed to the limited, reciprocal movement of its earlier incarnations.[288] The central axis on a "center lathe" spun by means of its attachment to a wheel, itself powered by its connection to a treadle or independently powered by an assistant who would turn the wheel by hand himself (in which case the wheel would be called a "hand flywheel") [**Figure 6.9**]. A chord or string running from the wheel to the endpoints of the central axis to which the material to be turned would eventually be fixed, allowed the lathe to generate, in principle, a continuous revolution.[289] Combining the capacity for continuous

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[287] It was not necessarily the case that pole lathes were incapable of continuous motion. Holtzapffel describes a pole lathe belonging to the East India Company that was capable of continuous motion by using a pole flexible enough to overcome the friction caused by the cord, though even in this case, there were dead points at the ascent and descent of the treadle (Holtzapffel 27-28).

[288] Leonardo da Vinci may have been the first to design a flywheel capable of turning continuously in one direction, which he did in the *Codex Atlanticus* (Connors 218). His lathe has a drive mechanism made up of an offset shank, a crank, and a treadle, which permitted a turner to work without the assistance (Maurice 1985 133).

[289] The lathe was certainly one of the most advanced mechanical tools of its time, and is the subject of several in-depth technological histories. One such study can be found in the publications by Holtzapffel. For a history of the development of the center lathe in particular see Holtzapffel 29-57. See also Maurice 1985, Chapter 5. From the late 15th century onwards,

motion with precision-manufactured component parts from metal that could handle longitudinal and crosswise movements (of the cutting tools or the work-piece itself), while also rotating the central spindle, meant that the ivory could now rotate *ad infinitum* in three directions, while a cutting tool could remain solidly fixed to the lathe creating a reliably-exact cut. "Programming" tools called "mandrils" or "rosettes" were also integrated into the lathe (Maurice 1985, 134). Made from metal disks, mandrils guided cutting tools to make specific motions [**Figure 6.10**] that would not have been possible to execute by hand. Taken together, these advances conspired to make the late 16th century lathe more akin to a low tech computer or a Jacquard loom, in which various mandrils and tools could be "plugged" into the machine with the turner knowing full well before hand what the resulting forms would turn out to be. While master turners reached dizzying heights of complexity and fragility by combining mandrils, precision tools, and the newly continuous rotational capacities of the lathe to create the stackable component parts of their *Säulen*, the amateur noble turner, initially under the guidance of one of his court artists, would have been able to turn elegant shapes without undue exertion, leave them on the lathe when his patience flagged, and return again to finish them at his leisure.

In order to turn wood or ivory on a state-of-the-art 16th century lathe, the material was first sandwiched between two centers and fixed in place so that it could rotate at a high velocity. Likely the wood or ivory turned by August and his court turners would not have come into the *Drechselkammer* as a perfectly symmetrical column of material, though the more intact the ivory was when it reached Dresden and the greater its overall girth and

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regulating mechanisms known as rose-engines could also be mounted on the lathe spindle to engrave ornamental curvilinear lines upon the surface of the worked material.

circumference, the more creative options were potentially available to the litany of famous turners at the Dresden court.[290] Ivory was also a very good material for turning, given that its layers of dentine form an interwoven grain with structural strength in all directions, while oiliness between the grain helps to reduce brittleness.[291]

The first thing to do might very well have been to divide the ivory tusks into segments that roughly corresponded to the maximum size of the component parts anticipated as being required for a particular *Säule*. These segments could then be loaded into the lathe and the curvature of their profile made constant by applying pressure along its entire length, relative to the overall shape of the raw material itself, until the piece was reconstituted as a smooth cylinder.[292] It is possible to imagine that the thankless work of preparing the ivory for further articulation might have been undertaken by apprentices in the court workshop. They would have stopped the lathe periodically to check on the progress of their smoothing process, because too much pressure would require them to

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[290] The largest recorded tusks remain the Kilimanjaro tusks, believed to have been captured in 1896. Each tusk is more than ten feet long and two feet wide, weighing 237 and 225 pounds respectively, and belongs to the British Museum.

[291] From the introductory guide to the Ivory, Bone, Antler and Horn section of the Pitt Rivers Museum, Oxford (http://www.prm.ox.ac.uk/ivory.html). The dentine in ivory is formed by specialized cells called odontoblasts, which develop in a columnar pattern and are organized as one cone inside the other, radiating out from the center of the tusk to its edge. The coincidence between the columnar pattern of the cells and their concentric arrangement as cones is responsible for the strength and elasticity of the tusk (Shalem 14), and therefore makes it an ideal material to withstand the stress of lathe turning.

[292] In his *Mechanick Exercises* (1703), Joseph Moxon concisely described the basic principles behind evening out a work piece on a "center lathe" relative to a central axis. "As by placing one Foot of a pair of Compasses on a Plane, and moving about the other foot or point, describes on that Plane a Circle with the moving point; so any Substance, be it *Wood, Ivory, Brass*, &c. pitcht steddy upon two points (as on an *Axis*) and moved about on that *Axis*, also describes a Circle Concentrick to that *Axis*: And an Edge-Tool set steddy to that part of the outside of the aforesaid Substance that is nearest the *Axis*, cut off all the parts of the Substance that lies farther off the *Axis*, and make the outside of that Substance also Concentrick to the *Axis* (Moxon 167).

even out the rest of the material to the same width. As slight changes in pressure created difference, a trained sensitivity of touch was important. Thus it was easier to proceed with caution from the outside in—i.e. original shape of the material—towards the creation of a stable core and rely on feel and the sound of the tools against the material to confirm the distance from achieving a centered and constant base form.

Up until this point, the raw material had been irregular. In motion, this irregularity translates into a visual and aural fuzziness that serves also as a clue that the pieces have not yet been sufficiently prepared.[293] At the moment when the application of pressure causes the raw material to achieve a state of perfect roundness and even continuity, it suddenly snaps into focus even as it rotates, and the sound of the tool against the material dies away. The edges appear crisp and the object stable in motion. In this state of perfection, the rotating form exactly reflects the still (round) form of the column. It is as if the turned segment is at rest, even when it is rotating. The noise of the lathe dies away; the visual interference of the material's inconsistencies give way to the appearance of a stable object, which in the case of ivory, would also have gleamed a brilliant white as the wear and tear of the outer surface of dentine was ground down to its gleaming, virgin interior layers. Here, in front of August, once the numerous ivory shaving were swept away, the failures, fragments, the broken pieces, the very messy and physical process of preparation completed; after the journey from Africa to Lisbon, Antwerp, Hamburg and then down the Elbe had been erased; the weight of carrying tusks through the *Grosser Schloßhof* and up the stairs to the fourth floor of the castle forgotten; after all the sweat

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[293] These observations were recorded at the North Bennett Street School, Boston during a field visit organized by Jennifer L. Roberts and Ethan Lasser as part of the workshop Technologies of Turning: An Exploration of Matter and Meaning (June 2014).

and labor, the exchange of funds, the heat and poaching, the mosquitoes and the bribing, the bartering and the auctions, there would have lain a cylindrical segment, pristine in its simplicity, contextual-less, even a-material, an object so smooth and perfect that it could not have been created by human hands alone. Here lay an archetype of a new beginning, free from the aggression of imperfection, a blank slate whose surface was truly appropriate for enacting upon it the knowledge of geometry and form-making collected in his research library.

The first *Inventar Drechselkammer*, compiled by the Dresden court turner Johann Wecker on June 24th 1622, describes all the tools, accouterments, and machinery that were present in the workshop, including the number and contents of the workshops' many drawers and also several "*Drehe Bank*" and "*klein Drehebänklein*."[294] A fraction of the total number of tools were documented as drawings and grouped together by the types of incisions they would cut into turned ivory [**Figure 6.11**]. Each page of the inventory, then, preserves a typology of potential forms that were signified by the edges of the tool themselves. As with the mandrils, a turner would have been intimately familiar with the effects wrought by each of the tools. In other words, just by looking at the tools, the turner would have been able to imagine the form it would carve into ivory while the ivory was turning. Besides listing the contents of the workshop again, the later 1684 inventory includes a series of hardline elevation drawings of turned ivory vessels.[295] A subset of the drawings have been completed in a light-blue watercolor wash and dated from 1578.

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[294] Staatliche Kunstsammlungen Dresden, Inv. Nr. 68. The front page says "*Inventarium Uber den Drehewerck und Zeügt wekcher dem hoffdrechsler Johann Weckern in der Drehestube und Kammer uffn*(?) *Ehurfürstlichen Schlosse ober der Kunstkammer untergeben berschehenn.*"

[295] It is not possible to determine when these drawings were added to the 1684 inventory.

They depict a group of small turned vessels, with their profiles highlighted in pen [**Figure 6.12 + 6.13**]. In comparison with a drawing of tools from the 1591 inventory of Georg Wecker's workshop, it is possible to see how the profiles of tools could be matched with the profiles required by designs.[296] This meant that in order to create certain effects in ivory, a turner needed to own the tools necessary to create this effect. Thus, given the variety of turned ivory work in this period and the intense demand for prodigious displays of talent, it is highly probable that the *Eisen* and mandrils were to some extent thought of as templates for particular types of cuts and that they likely were traded, copied, circulated, and collected throughout 16th century and 17th century turning workshops.

The *Säulen* were a composition of effects, each exhibiting individual excellence and designed in advance of working on the lathe. Though few if any working drawings exist to support this assumption, it is probable that turners collaborated with artists who specialized in composing and mapping hardline designs for curved surfaces in ivory or, perhaps, were able to draw themselves [**Figure 6.14 + 6.15**].[297] Hardline elevation drawings done at 1:1 scale would have been an integral step in the process of manufacturing *Säule*, as they would have been used as a measurable visual reference which the turner would have matched to tools at his disposal capable of producing the

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[296] The 1591 inventory of Georg Wencker's workshop is in the SachsHStA Dresden. Drawings of some of his tools are on (23v-37r) represented in a light-blue wash similar to the drawings in the 1684 inventory. A listing of Egidius Lebenickt's tools is on (29v-30v). SachsHStA – Loc. 9835/15, fol. 20. There is one magisterial drawing by Wecker, possibly unrealized or imaginary and shaded in grey and blue, dated 1626.

[297] Several of the drawings evidence the presence of pin pricks, alluding to the use of precision instruments to make sure that the contours of the form would be absolutely symmetrical.

articulations of a *Säule*'s profile. Alternatively, elements of drawings could be recombined to suit a turner's or a client's taste. In the instance where a drawing proposed a new shape not available within the existing repertoire of available tools, a turner would have had to develop, or source, appropriate profile-cutting tools.

It is unlikely that finished *Säulen* would have been improvised backwards from the feel of the material on the lathe. Although in theory it was not impossible to experiment on the lathe in order to develop a repertoire of form-making**,** the high degree of perfection required would have made drawn lathe designs the easier option through which to experiment, infused as they certainly would have been with an intimate knowledge of the tools and techniques necessary to create the forms proposed by the design. Once the design was decided upon, one technique of translating it to ivory was to copy the profile of one side of the elevation to a flat piece of wood and hold or mount the wood parallel to the lathe's central axis of rotation, close enough away to enable the turner to read the lines on the profile but far enough away not to disturb the material when it was rotating. From this profile—essentially the translation of the edge of the design into an array of lines—the crucial points for the design would be marked lightly upon the ivory, giving the turner guidelines about where the cut. For comparing dimensions of a turned ivory piece with its original design drawing, a caliper set to the exact size proscribed in the drawing was used to measure the diameter of the ivory.

To create a column took an intense amount of planning, design work, and coordination. As techniques for turning nested polyhedra were not described in 16th century turning manuals and there are no *Lehrbücher* to guide the learning of the required techniques, these techniques must have been guarded as proprietary secrets of the turning trade.[298] The raw ivory surface had to be made regular; tools had to be matched to drawn elevations; the lengths of the various components had to be decided upon and cut, an assembly strategy devised, all while wasting as little of the precious material as possible. In order to make a nested geometrical showpiece, the turner would have needed to produce jigs and chucks—which were wooden parts used to hold the outermost geometrical form. If the outermost form was going to be a sphere, the chuck would have been hemispherical, whereas if a spiked star was part of the design, the chuck was square (Springett 24). Sphere-turning jigs could be used to turn spheres and templates were made to measure the accuracy of spheres, for instance, in order to ensure the regularity of their curvature. Once a sphere was turned, the turner would have removed it from the lathe and used a compass to mark out on its surface a series of twenty equally-spaced points (called constellation points) corresponding to the requirements of the design, six equidistant main points, as well as clearance points, which were equidistant from three adjacent main points.[299] At this step in the process, the ivory became, temporarily, a drawing surface of regular curvature upon whose face geometrical calculations were plotted. While in theory it was entirely possible to mark up the sphere with uneven intervals, within the conceit of turning a perfect sphere, it is possible to see how unevenness might have been perceived as a lack of rigor or accuracy, given also the

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[298] Consequently, I have reconstructed the process of turning an ivory polyhedron with reference to Springett and from discussion with turners at the North Bennett Street School.

[299] See Springett 44-47 on how to set out the surface of a sphere. Drilling or turning holes on a clearance point allows waste material that would not otherwise be easily accessible from the main points to be removed (Springett 47).

Euclidean definition of the Platonic Solids as being limited to those five polyhedra able to be inscribed into a sphere tangent to each face of the Solid at the center of its face.

The whole enterprise strove towards uniformity and centeredness. Moreover, uneven spheres would sit awkwardly within one another, making it exponentially more difficult to turn further interior shapes, which would have in essence multiplied the lack of precision and inexactitude. By sticking to geometrically-determined subdivisions, a turner gave himself the best chance of accurately anticipating where the material further within the sphere would be located when it came time to create further interior shapes.

After the sphere or cube was graphically prepared, it was reattached to the lathe and the turner would begin to work on the base form with a variety of tools, corresponding to his design intention.[300] To turn a series of spheres trapped one inside the other, a turner would use a specialized undercutting tool with a curved, bent edge made specifically to carve a sphere of a certain circumference. Each undercutting tool had a holder that kept the edge of the tool at a predetermined distance from the outermost surface of the outermost sphere, while each subsequent tool was inserted into the same opening hole in the outermost sphere, and a different tool deployed for each desired internal sphere, essentially grinding away smaller and smaller spheres that fit the curvature of the undercutting tool (Springett 147). To make a spiked star in a sphere, an essentially similar method was used. After drawing the opening holes on a prepared sphere, a curved undercutting tool and a specially made square-edged tool for forming the spikes were

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[300] For step by step instructions on how to turn a variety of shapes common in the 16th century, see Springett Chapter 12 – Spiked star in a cube; Chapter 13 – Spiked star in sphere; Chapter 16 – Chinese balls. Springett himself makes reference to Bergeron's *Manuel du Tourneur* (1816) for inspiration.

used to gently cut away the point of the star which protruded from each opening hole. The base of each spike terminated in a pentagonal base surface, with one surface corresponding to each spike, which was smoothed away using the square-edged undercutting tool.

Once these basic principles are grasped—that to turn nested geometries one needs specialized tools for each internal surface, that these tools need to directly correspond to the measurements drawn on the outermost sphere, that 1:1 scale drawings could be used to calculate the cutting angle, which would in turn define the angle of the undercutting tools, and that specialized jigs were used to hold tools at predetermined distances from the outermost sphere—it is perhaps easier to understand how this repertoire of techniques led to a prodigious if morphologically-related artistic output, even though the range of experimentation was somewhat limited by the utilized techniques. For instance, to create a spiked star whose points protruded through the outermost holes of a sphere, the above-described technique would have been used and then the outermost surface of the outer sphere carefully ground down towards its center point in order to give the impression that the larger star had somehow become trapped within a smaller sphere [**Figure 6.16**]. To introduce irregularity into the design of the column's capstone polyhedra would have been to fight the innate uniformity of motion of the lathe, which by design cuts towards stillness and regularity, the center of rotation. Still, providing the turner with machines capable of continuous motion freed his imagination to experiment within these primarily symmetrical limitations while the late Renaissance and early Baroque fashion for asymmetry, transformation, and expressive movement found itself displaced to the base of the columns, which could be assembled from individual parts and thus were not in theory as strictly bound by the logics of turning a singular object.

The result was a miraculous composition that unlike the figural flourishes that adorned domestic furniture in this period, strove to appear mysterious, impossible, and inhuman. These polyhedral components of the ivory *Säulen* were not freeform expressions of an artistic sensibility as much as they were formulaic, if recombinable, embodiments of precision calculation and manufacturing. As Joseph Moxon noted in his *Mechanik Exercises* (1703) in regards to turning slender ivory work, "some *Turners...*shew their Dexterity in *Turning*, and make others that know not the way how it is done admire their *Skill...*" (Moxon 214).[301] The seeming impossibility of the nested geometries was their essential appeal. Not merely *Saülen*, these columns were also *Siegesaülen*, commemorating the culmination of highly refined artistic skills combined with formidable geometrical understanding, both on the level of the material and in regards to the technology of their production. For makers and patrons these objects served as testimonies to the wealth and ambition of the court and its capacity to exert complete dominion over matter.

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It may be productive to ask why it was that the act of turning and the possession of finely wrought lathe-turned objects conveyed such pleasure to its noble practitioner/patrons.

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[301] Moxon also provides detailed descriptions of how to turn "several Globes or Balls of Ivory within one another, with a Solid Ball in the middle," as well as other permutations, such as how to turn "a Globe with several loose Spheres in it, and a Solid Cube, or Dy, in the middle of it." See Moxon 219-224.

With all the machines at their disposal, and all manner of other possible distractions, rulers returned again and again to the lathe. Yet if it had been too physically demanding to turn ivory, or if the practice had required many years of training, it is doubtful that the Elector, or for that matter many of the other members of Europe's ruling families, would have deigned to try their patience long enough to master the art. In truth, lathe turning is uniformly spellbinding and uncanny, regardless of whether the base material is as exotic and precious as the ivory used by Saxon nobility. Contours are coaxed into reality from a perpetually spinning form, not the hacking into and subtle refining of a stable object with hammer and chisel. As such, successful or prodigious lathe work aspired to aesthetic objectives that differed from the figurality that dominated contemporary European sculpture. Rather for turning, the key conceptual operation was in redirecting the rotational energy provided by the lathe through applying pressure with a variety of tools. The forms were generated by moments of pressure whose effect was carried around the entire circumference of an object and thus spoke, first and foremost, about the seductive and hypnotic power of rotation. The fragments, failures, broken pieces, ivory and wood byproducts and the consortium of specialized tools, jigs, mandrils, machines—indeed the messy very physical process of creation—remained masked behind the flawlessness of the *Säulen*. The fascination has nothing to do with the shifting cultural associations bound up with "work"; it is a plainly pleasurable, ultimately learnable activity that, in concert with state of the art equipment, produces results that far outstrip the invested physical effort.

The nested polyhedra that topped the summits of ivory *Säulen* were as much expressions of court turners' mastery of mixed-mathematics as they were forms that developed backwards from the innate capabilities of the lathe. The smoothness of the *Saülen*'s surfaces functions as a new kind of technological ornamentation that, in its aim for a kind of machined grace, sought the abstract perfection embodied by the Platonic Solids. Technology here was not merely a tool to execute a preconceived idea. It was precisely because of the lathe's capabilities that polyhedra found their way into ivory. Though August was in possession of Jamnitzer's *Perspectiva corporum regularium* and other graphic investigations of geometry, the repertoire of form that developed on paper was substantially broader than its ivory counterparts. Rather, the polyhedra found on *Säulen* were invested less in formal experimentation than they were in increasing degrees of imbedded-ness; the elegance of wrapping of one body inside the other, the surprise of concealing seemingly irreconcilable Platonic Solids, and the extent of the thinness to which ivory could be pushed. As Johannes Kepler would theorize in his *Mysterium Cosmographicum* (1596), most certainly after having seen *Säulen* in the collection of his first patron, Friedrich I, Duke of Württenberg (1557-1608), the act of nesting Platonic Solids inside of themselves could even be used to understand the distance of the known planets in the solar system. In the context of the workshop, the imbedding of Solids was a feat of technique and technology; an ontology of the impossible, manifested in polyhedra which self-consciously strove to defy explanation.

In as much as the turned Dresden columns were paradigms of cutting-edge turning technology and had been conceptualized and constructed according to the material and epistemological logics of turning as a method of generating form, they also embodied a calculated rejection of the very materiality from which the models had been made.[302] Imprints of soverigns' taste for the complex, the columns were not odes to the properties of ivory as a material, in that they did not strive to highlight their ivory-ness; ivory was used because it was uniquely capable of "disappearing" beneath the models' geometrical intricacy. Matter here becomes abstract. Though the stacked geometries which characterize the Dresden columns could have been made in wood, it was ivory, polished to a brilliant white, that seems to have most closely captured in three dimensions the materialness-ness pursued by the mixed mathematical research of artists like Wenzel Jamnitzer, whose printed work would surely have been known to the court turners and may even have been a source of inspiration. There is something about the whiteness of ivory that must have been perceived as innately intangible, both in terms of its exotic provenance and in contrast to the richly textured, wood paneled surfaces that covered the domestic interiors of central European Renaissance aristocracy. In its whiteness, ivory maintained as close a connection as possible to the drawn models and design drawings rendered on the whiteness of the pages that would have served as the references for the turners. To use ivory was to preserve a perceptual continuity between the prominent "paper projects" of the 16th century geometers and their further expression and elaboration as priceless physical models; through skill, precision, and technology to broach likenesses of what had heretofore been an unbuildable graphic reality, in seeming transcendence, through formalism, of nature.

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[302] For recent work on the phenomenon of cylindrical motion, see H. Müller-Sievers, *The Cylinder: Kinematics of the Nineteenth Century*, Berkeley: University of California Press, 2012.

Epilogue: *Mysterium Cosmographicum*

The gaining of technical expertise and professional prominence through the practice of mixed-mathematics was accompanied by a rise in the visibility of the regular bodies and their innumerable irregular variations. As covetable emblems of new geometrical knowledge, polyhedra journeyed from the southern German workshops that had perfected their representation into courtly *Kunstkammer* and onto the surfaces of luxurious domestic interiors across Europe. The transformation of the Platonic Solids comprises the century-long circular journey of polyhedral geometry out from cloistered establishments of academic learning and back again in a new and expanded role. The range of polyhedral interventions and effects was inestimably wide, spanning from their wireframe manifestations as images illustrating Euclid, available to only a specialized mathematical audience, to their broad and widely-recognizable status as emblems of geometrical knowledge, such as in the portrait of the scholar and humanist Oswald von Eck (before 1539 – 1573) by Hanns Lautensack (ca. 1520-1564/66) [**Figure E.1**]. For it was in the hands of goldsmiths, surveyors, architects, cabinet-makers, and turners that the representation of the Platonic Solids evolved into an intense area of experimentation that sought to explore the limitless possibilities of geometrical invention, in the process yielding new facilities with the manipulation of geometrical forms.

The famous Tabula III image, crowning visualization of Johannes Kepler's (1571-1630) *Mysterium Cosmographicum* (1596) [**Figure E.2**], depicts a model of the universe wholly dependent upon the nesting of polyhedral geometry to form the intervals between the six known planets.[303] If one were to circumscribe the *Mysterium* model within Kepler's broader oeuvre and contributions to astronomy, it would be difficult not to regard it as an eccentricity, albeit one to which Kepler would return throughout his life. But as a late outcropping of German experimentation with polyhedral geometry, the model opens up provocative new avenues for exploring the intersection of the work of 16th century artisans and the development of courtly science. The epilogue to the dissertation revisits Kepler's failed design for a physical model to accompany the *Mysterium Cosmographicum*.[304] Incidentally, the issues underlying the construction of the "*Credentzbecher*" remain the most complete documentation of anyone's attempt to think through and three-dimensionally realize a polyhedral model in the 16th century. Correspondence between Kepler, Michael Mästlin—Kepler's dissertation advisor and a prominent professor at the University of Tübingen—the intended addressee of the model, Kepler's patron Duke Friedrich I (1557-1608), and various functionaries in his court, as well as a collection of collaborators, reveal in remarkable vividness not only the problematics of constructing a polyhedral "*Meisterstuck*," but also the extent to which polyhedra had completely suffused Kepler's experience with mathematics and, consequently, his engagement with astronomy. The network of letters documenting the saga of the *Mysterium* model reveals Kepler to be less naïve than he describes himself.

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[303] Tabula III was printed by Georg Gruppenbach, a printer in Tübingen who maintained close relations with the university and Michael Mästlin. The artist of the drawing upon which the print is based remains unknown, although it is not attributed to Kepler. Dr. Friedrich Seck in Tübingen speculates that the artist is one Anton Ramsler, court artist of the University of Tübingen in the late 16th century, due to the fact that Mästlin, in reference to the *Mysterium Cosmographicum*, refers to a "copperplate sketched and etched by the university painter" (Seck 623). "*Das Kupferstick had der vniversitet Mahler gerißen und geetzt*" (GSW 13, nr. 63, p 109).

[304] See also E. Aiton, "Johannes Kepler and the 'Mysterium Cosmographicum',"*Sudhoffs Archiv*, 1977, 173-194; A. van der Schoot, "Kepler's search for form and proportion," *Renaissance Studies*, Vol. 15, No. 1, March 2001, 59-78.

He emerges as savvy and ambitious, intimately aware of trends in the decorative arts, and bent on seeking further employment by capitalizing upon the tastes of 16th century nobility. Kepler's trials, tribulations, conceptual compromises, and ultimate inability to produce a model to communicate his cosmological theory demonstrate the gap between real-world and theoretical knowledge practices, incidently exposing his fundamental misconception of their natural continuity.

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In October 1587, Johannes Kepler formally commenced his graduate studies in astronomy at the University of Tübingen on a scholarship underwritten by Duke Friedrich I of Württenberg. Shortly after completing his *Magister* under the tutelage of Professor Michael Mästlin, Kepler pursued further schooling in Theology with the intention of becoming a clergyman. However, having no independent means of financial support, in his final year of study Kepler accepted a post teaching mathematics at a Lutheran school in Graz, Austria, a position intended only as a first step towards other more prominent and lucrative professional opportunities.[305] Still, Kepler feared that his relocation to the Austrian provinces would threaten his options for future employment by removing him from the milieu of those patrons capable of financing his research. In a form of self-promotion that he would maintain from 1595 onwards, Kepler proffered his astrological skills to a select group of influential figures, a common practice for Renaissance court astronomers but a precocious one for an unknown twenty-four year-old teacher of mathematics, and cast unsolicited prognostications for the following year which he

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[305] See Johannes Kepler, *Complete Dictionary of Scientific Biography*.

distributed to authorities in Styria and the high counsel of war (*Hofkriegsräte*), as well as to his former professors in Tübingen. It was to a copy sent to Duke Friedrich himself that Kepler attached his first proposition for a model based on the cosmological theory from his soon to be published *Mysterium Cosmographicum* (1596), the book he hoped would establish his reputation as an astronomer.[306]

If we were to take Kepler at his word, as described in his preface to the *Mysterium*, while drawing out the conjunctions of Saturn and Jupiter for his students on July 9/19, 1595, Kepler was struck by an epiphany concerning a problem that he had been contemplating that past summer, namely, the geometrical relationship between the orbits of the six known planets. The central discovery which the entire book is devoted to explicating and justifying, hinges on the moment when Kepler apparently jumped from working with two dimensional polygons to considering the possibility that the intervals between the planetary orbits might best be captured by nesting the three dimensional Platonic Solids one inside the other in a particular order and orientation. And though the theory was still to be proven, and at the time was merely premised in "a clumsy conjecture drawn from the known distances of the planets," Kepler declared himself pleased at having successfully recognized the nature of these geometrical relationships to the extent "that there was nothing which I could later change in them when I was working with the ratios captured in detail" (Kepler 1596 [1981], 69).

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[306] For references to some of Kepler's earlier correspondence relating to his prognostications, see *Johannes Keplers Gesammelte Werke* (heretofore abbreviated GSW) 13, nr. 10, 11, 12, 13, 15, 27, 28. All translations from the GSW are my own.

Contemporaneous to Kepler's discovery, the Stuttgart court, newly under the rule of Duke Friedrich as of 1593, had steadily begun to amass luxury items for a new *Kunstkammer.* Invoices from the *Rentkammer* and *Finanzverwaltung* cover payments for copious amounts of watches, shields, decorative metalwork, exotic materials such as ivory, and scientific instruments commissioned and purchased by Friedrich during his tenure in power.[307] They incidentally reveal longstanding relationships between the Stuttgart court and networks of artisans, several of whom would eventually become embroiled in Kepler's attempt to realize a physical model to accompany the publication of the *Mysterium Cosmographicum*.[308] In addition to the stockpiling of precious objects, Duke Friedrich, like his counterparts in other German courts, was in the process of developing a comprehensive section of mixed-mathematical literature, which would subsequently be termed *Libri Architectonici* in the 1624 catalogue of the private library of his son, Duke Johann Friedrich von Württemberg (1582-1628). Included alongside the works of Vitruvius, Palladio, and the *Opera mathematica* of Vreedeman de Vries were copies of Euclid's *Elements* as well as Hans Lencker's *Perspectiva* (1571) and Jamnitzer's *Perspectiva corporum regularium* (1568) as representatives of the southern German polyhedral literature.[309]

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[307] These bills can be found in the *Rentkammer, Güter und Finanzverwaltung,* A256 – *Landschreiberei*, Hauptstaatsarchiv Stuttgart. The inventory is spread across many volumes in the *Inventare, zugangs- und Abgabeverzeichnisse 17. Jahrhundert*, Hauptstaatsarchiv Stuttgart. For further information on Duke Friedrich's *Kunstkammer*, see Fleischhauer 1-12.

[308] The inventory was compiled by the librarian and archivist Johann Jakob Gabelkover (1578-1635). See Cod. Hist 2 1068, Stuttgart Landesbibliothek.

[309] It is likely that the Jamnitzer belonged to the elder Friedrich, given that it had been published over fifty years prior to the library inventory. Of note also is a copy of Jacques Besson's mixed-mathematical opus, *Theatrum instrumentorum et machinarum*, translated from the original Latin

In a letter to Mästlin, dating October 3rd 1595, Kepler describes how the only possible geometries appropriate to structuring the intervals between the planets are the five regular solids. "Remove the irregular bodies, namely the foundation that creation is of the most well-ordered nature."[310] Kepler uses the phrase "*irregularia corpora,"* evidencing that he was aware of the existence, and by now prevalent use, of the concept of irregular bodies in Germany, to the extent that he felt compelled to invalidate them and their untamable multiplicity as inapplicable, or even threatening, to his theory of celestial order.[311] By 1595, Kepler may well have had ample opportunity to see irregular bodies up close. The library of the University of Tübingen possessed copies of Augustin Hirschvogel's *Geometria* (1543), Lorentz Stoer's *Geometria et Perspectiva*, and Heinrich Lautensack's *Des Circkels und Rechtseyts* (1564)—a significant acknowledgement of the importance subsequently granted to the artisanal explorations of polyhedral geometry. The ivory turner Sebald Burrer from Nuremberg was engaged under Duke Friedrich from 1595-1600, as was his son Georg from 1597/98-1627, during which time numerous *Säulen* with turned ivory polyhedra were manufactured in the style popularized by the Dresden

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of 1578 into a German edition in 1595 and dedicated to Duke Friedrich, meaning that Friedrich had had the *Theatrum* translated just one year before Kepler published his *Mysterium Cosmographicum*.

[310] "*Removenda verò irregularia corpora, nempe in conditu ordinatissimae creaturae"* (GSW 23, p 35).

[311] In the extended addendum to the letter, Kepler gives his comparison, in table form, between the numerical values of the maximum and minimum distances of the eccentricities of the planets derived from Copernicus and from the proportions derived from the Platonic Solids in order to demonstrate that the idealized values derived from the Solids are relatively close to the values determined by Copernicus. This rhetorical maneuver reconstitutes the Copernican planetary intervals as approximates of a geometrically-derived set of intervals, at once both seeking to benefit from the status of Copernicus' mathematics while also improving upon it. Had the irregular solids been deemed to be acceptable options under consideration for the planetary intervals, Kepler would not have been able to substantiate his own table of intervals as superior to Copernicus'.

and Medici courts. And the base of the Tabula III image is suspiciously close to the bases for the graphic polyhedral models depicted in Jamnitzer's *Perspectiva.*[312]

Kepler's awareness of Fredrich's keen interest in mixed-mathematical geometry and princely objects of scientific value spurred the initiative to materialize his cosmological theory in gold with the aim of being included in the Duke's new *Kunstkammer.* Thus by February 17th, 1596, Kepler was already lobbying Duke Friedrich with a proposal for a physical model of the planetary intervals, ostensibly to accompany the publication of the forthcoming *Mysterium Cosmographicum.* "I would like to know already, Your Grace, which goldsmiths have been allotted for me to this commission. Each goldsmith would work on part of the project (nothing else is peculiar here) and without being familiar with the entire project. Then these pieces will easily be able to fit into each another."[313] While duly anticipating the complexity of his design, Kepler appears to underestimate the difficulties required to potentially coordinate several workshops, and the resultant incompatibilities in material tolerances and dimensions that outsourcing the labor to multiple craftsmen in separate locations would generate. A simple "woodcut or copper-engraved outline in plan of a paper body [*papüren corpus*]," possibly the schematic drawing in planometric view of the proposed model attached to the letter [**Figure E.3**], was intended to convey an initial design that had very little relation to the Tabula III

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[312] Kepler was certainly aware of the work of Daniele Barbaro and Augustin Hirschvogel, even referencing Barbaro's edition of Vitruvius' *De architectura* (GSW 20.1, 82. 31f. 569) and Hirschvogel's maps of Austria (GSW 17, 175 and 478). References to Euclid's *Elements* and Plato's *Timaeus* are copious in his published and unpublished work.

[313] "*Woltte ich schon wissen, denen Goldschmiden, so E. F. G. zu mir verordnen woltte, solliche ordnung zu geben, das jeder ein stuckh (die sonsten nichts seltzams) arbeittete, und doch kheiner wuste, warzue enete: dan selbige leichtlich in einander gesetzt warden mögen*" (GSW 13, nr 28, p 51).

model and more to do with Kepler's predilections for developing mechanistic waterworks.[314] "I would like the planets to be cut from gems; Saturn from a diamond, Jupiter as a Hyacinth, Mars as a ruby or spinel, the Earth as a Turkish [stone] or magnet, Venus as a 'yellow-colored eyestone' ["*ein Augstein gelber farb*"] or something similar, Mercury in crystal, the Sun in a red garnet, and the Moon in pearl. Furthermore…I would like there to be delectable drinks inside. Therefore, to bring this about there would be seven spigots in the outer rim of the chalice covered with images of the seven planets, out of which seven different drinks will be able to be drawn, and would cause an unexpected surprise" (GSW 13, nr 28, p 52).[315]

In consideration of Kepler's proposal, the Duke still had his reservations and conveyed a set of instructions to his subordinates in his own hand: "The proposal should first be produced in copper and if we would view it after and decide whether it is possible to execute in silver, we would stop being adversarial [to the idea]."[316] But perhaps sensing that Duke Friedrich had been too easily swayed by Kepler's wishlist, and knowing that a *Mysterium* model was at the moment nothing more than a fanciful and alarmingly

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[314] See F. D. Prager's "Kepler als Pneumatiker und Erfinder der Zahnrad-Pumpe," *Blätter für Technikgeschichte*, 1966, on Kepler's later designs for water-powered mechanical objects.

[315] "*…möchten die Planeten stern auß Edelgsteinen geschnitten werden, als Saturnus auß einem Adamant, Jupiter ein Hyacinth, Mars ein Rubin order Balagius, die erd ein Türkis oder Magnet, Venus ein Augstein gelber farb oder dergleichen, Mercurius ein Crystall, Sol ein Carfunckel, der Monde in Perlstein… möcht darinnen eine ergeßlitheitt im Trinken gesuecht und gar wol also zugericht werden das im eüsseristen rand siben zapffen, mitt der siben Planeten bildnussen verdecket würden daraussen sibenerley underschidliche getränck gesogen, vnd einem vnwissenden ein schimpff zugericht würde...*"

[316] "*Die prob sol zuvor auß Kupfer gemacht warden und wann wir darnach die prob ersehen und befinden daß solches werdt in silber zu fassen, sol es hernacher khein Not haben*" (GSW 13, nr 28, p 51).

expensive idea by an untested if ambitious young astronomer, the director of the chancery, Balthasar Eisengrin, required Kepler to first present a complete model in paper to the court before any further work was to be commissioned.[317]

Twelve days later, on February 29, 1596, Kepler sent Duke Friedrich the required model accompanied by another letter in which he claimed to have "with my unseasoned hand, diligently attempted to apply myself for the past eight days. I present a contemporary paper model whose content is as follows." Kepler goes on to justify the "unequal width of separation" ["*ungleicher weitte*"] between the planets in the model on the basis that they reflect the proportions of the five Platonic Solids to each other before returning to the question of the model's material. "Your Grace, though I have, in deep gratitude, hurriedly produced the current model, I promise that future copies of the creation…will much better represent the importance of the [scientific] material than the low quality displayed by this model."[318] Given the curvilinearity of the model design, and the inability to model a liquid-containing system in paper in eight days, Kepler most likely would have

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[317] Beneath Friedrich's handwriting, Eisengrin writes that "Our Grace and Master, who has recorded within the margins the 18th resolution which will be shown to me to sign, would like a proposal made from paper and offered by you to His Grace, which he thereupon will examine." "*Unsers G. F. und Herns Intus ad marginem veraichnete Resolution ist jme den 18ten hujus durch mich underschriebenen angezeigt worden. Darauff er sich vernehmen lassen, er wölle ein prob auß papir machen, und iren F. G. offerieren*" (GSW 13, nr 28, p 52).

[318] "*Nämlich mit meiner zuvor vndersuchten hand, doch anwendung müglichstes fleiß, dise acht tag vber ein Muster in Papeyr vnd gegenwürtige form gebracht, dessen Inhalt ist wie folgt…E.F.G. zu schuldiger Danckbarkeit in gegenwertigen Muster ehegemeltes Ebenbild der Erschaffung, zum ersten vnder allen Menschen, in massen ich versprochen, in vnderthönigkait zu praesentirn, deren vnderthönigen bitt, E.F.G. wöllen mrin Gehorsam genaigt Gemuth, auch die würdigkait der Materj vil mehr, als des Musters vnbeschaffenhait ansehen*" (GSW 13, nr 30, p 66).

attempted to use paper solids to convey the five planetary intervals to the Duke.[319] These solids may have been some of the phantom paper models (*"von papir geschnitten*") mentioned in the *Kunstkammer* inventory.

The letter bears a note from the court indicating that they have received Kepler's astronomical work and also documenting their own request that Kepler's former professor from Tübingen, Michael Mästlin, "should send on his judgment [*iudicium*] and thoughts to Our Grace." Promptly on March 12th, having apparently received Kepler's proposal from the Court, Mästlin replied with a glowing letter of recommendation to the Duke that sought to bolster Kepler's methodology of revisualizing the existing observational data around the Platonic Solids.[320] Though expressing a modicum of doubt about the polyhedral harmony of the planetary distances, Mästlin contextualizes the *Mysterium* as providing much needed order in a discipline swamped by inaccuracy, conceding that the existing data on planetary motion included many errors and that for this reason Kepler's innovative approach might end up being a very useful way to

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[319] We know from Kepler's voluminous literary remains that he was familiar with the *Underweysung*, given that he had criticized it for a false definition of a heptagonal figure (GSW 6,55). Kepler also refers to the Dürer's construction of ovals in the *Underweysung der Messung.* See GSW 3, 295 and 15, 249 and 528.

[320] "Because the celestial spheres have been studied and written about, he [Kepler] came up with his own reasoning from the astronomical observations, that is, *à posteriori*. Namely that while astronomy would like to have several *à priori* rationales, from natural, geometrical, and proper proportions, and also from his own observations, [Kepler] had the hypothesis to examine and regulate the quantities and magnitudes of the celestial spheres, as has been presented through this invention." "*Dann was noch bis hieher von der höhe, vnd größe aller sphaerarum coelestium ist disputiert vnd geschriben worden, hatt seinen grund allein ex observationibus Astronomicis, hoc est, à posterior. Das aber Astronomia möchte einigen Behälf à priori haben: vnd das oder wie au seiner naturlichen, Geometrischen, richtigen proportion auch die observationes selbs, vnd dann die hypotheses, quantitates et magnitudines Orbium coelestium zu regulieren vnd zu examinieren weren, wie aus disem invento zu verhoffen, hatt sich keiner niemals vnderstanden*" (GSW 13, nr. 31, p 68).

accurately revise the data [**Figure E.4**].[321] "Having deeply ruminated and recalculated the effect of this invention, and also having spoken with Kepler himself, I find it to be completely appropriate for princely instruction and an ingenious invention by a learned man, also with use for astronomy."[322] Apparently satisfied by Mästlin's endorsement, Duke Friedrich approved the model's production: "Because it is [just] such a work, we are content for it to be realized through workmanship."[323]

On March 18th all seems well, and the chancellor updates the Duke in another private communiqué, relaying to him that a local goldsmith, Carl Seckhler the younger "who is known from all the goldsmiths around for producing the best and most artful work" had

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[321] "Indeed I find that the proportion of the regular bodies do not fully apply to the spheres of the world [the elliptical trajectories of the planets N.A.], (many of which have already been known as Copernicus demonstrated). However, after diligent observation of the facts, I see that many astronomical imperfections have been produced on one side of the tables. Many observational errors regarding the motion are still not sufficiently accounted for. Hence, I have also proceeded to reform the observations for several years thus far, as well as the motion, through the grace of God, and hence to recalculate certain Ephemerides and the like." "*Ich befind zwar das die proportio corporum regularium mit den Sphaeris mundi (so vil die alberait bewußt, vnd von Copernico demonstriert seind) nit gar punctlich zutrifft: Jedoch nach fleissiger erwegung aller umbstend, sehe ich das solchs vil mehr imperfectioni Astronomiae zuzuschreiben ist, seitenmal die tabulae vil mal ab observationibus fählen, vnd die motus noch nit gnug bekandt seind. Derwegen auch ich mich bisher ettlich Jar lang auf die observationes begeben, in willens die motus, per gratiam Dei, zu reformieren vnd daraus gewissere Ephemerides (darzu E.F.G. mit gar gnädigen Bevelch newlicher Zeit, mich gnädig gemanet hatt) vnd was dergleichen ist, zu rechnen*" (GSW 13, nr 31 p 69).

[322] "*Nun aber, da auf E.F.G. gnädigen bevelch, ich disem invento, seidther, fleißig nachgesinnet vnd nachgerechnet: Auch mi time M. Kepplern selbs vnderredt, befinde ich, das es gar wol für ein herrlichs eruditum, vnd kunstreichs hominis eruditi inuentum, auch in Astronomia nützlich, zu halten seye*" (GSW 13, nr. 31, p 68).

[323] "*weil es ein solch werkh seind wir zufriden das solches ins werckh gericht werdt.*" In an unpublished letter from the court chancellor to Friedrich, a letter to which Kepler would not have been privy, the chancellor double checks whether Friedrich wants the model to be designed in the shape of a *Credentzbecher* (he does), whether Friedriech ultimately would like the model in silver or another material (he chooses silver), and whether Friedriech would like to use a specific goldsmith or to let Kepler choose someone himself (Friedrich replies that they should use the best goldsmith). Nr 5 Cod math fol 28, Stuttgart Landesbibliothek.

been chosen for the project.[324] But by May 28th, the updates from Kepler to the Duke on "the earlier manufacture of the astronomical feat" are ridden with anxiety. Kepler admits to having given "all kinds of poor guidance to the goldsmith that Your Grace agreed to commission," and that although he had already spent several months on the project, the goldsmith was still requesting more time. Given then the expectation that the project would take still more time than anticipated, Kepler requests a salary to cover his lost wages for this period and also, in recognition of the difficulty of manufacturing a prototype, advocates simplifying the material and building parts of the model not from metal but from more easily workable material like wood or paper. "The 1022 stars and the 48 images have not even begun and still have to be distributed (for which a great effort is required to affix them all). But whereas this whole time I have been futilely waiting for the budget from Your Grace…the increasing costs of the astronomical work are not accruing alone for Your Grace, but rather also for me, as I have been neglecting my official duties and also my other private affairs for too long …I propose to construct the remaining [parts] from paper, as they would be in the actual sphere, and a part in wood."[325]

Yet a month later on June 27th, the design is not close to being completed, and in a letter to the Duke, Kepler is left railing against the goldsmith, "the same phony who after a few

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[324] See Nr 6 Cod. Math.fol.28, Stuttgart Landesbibliothek.

[325] "*…vnd dannoch die 1022 sterne, vnd 48 bildeer sampt der Außteilung (wölliches eine grosse mühe zustechen) noch im geringisten nicht angefangen: Ich aber hingegen dise gantze Zeitt yber in E.F.G. cost aller müessig gewesen vnd gewartet…es würde nicht allein E.F.Gn. gar zu grosser costen auff ein Astronomicum opus aufflauffen, sondern auch mir zu vil langer versaumnuß meines officij, hinderstellung meiner besoldung, vnd dan auch zu anderer meiner privat sachen vnwiderbringlichem schaden vnd verwirrung gedeyen…Ich aber das yberig, so in die kugel kompt, auff Papir auffreissen, vnd theils in ein holtzformirn, solle*" (GSW 13, nr. 42, p 83).

days without my continual presence dared to stop working."[326] Without waiting for a reply, Kepler dashed off a second plea on July 3rd in regards to his debts accrued during "the first two months from the time since I made the paper model until Your Grace assigned me a table in the court with provisions, a wooden sphere, and heaps of pasteboards."[327] On receipt of this letter, the Duke relents and grants an order for 30 gulden to be paid to Kepler.

After a year and a half of increasingly expensive work and yet with nothing to show for his labor, in a January 6th 1598 letter to Mästlin, Kepler pivots to a new version of the model more closely resembling a mechanical armillary sphere. No longer holding liquid or concealing tubes, this version was intended to move in a proportionate rhythm to the movement of the planets, and included the Platonic Solids as physical elements in its center. Kepler states that he intends "the solids to be therefore in a fixed order, so that if they are lifted out [of the model] they will not be able to be pulled apart or destroyed. I have also thought that there will be no order if one cannot detach all the orbs [signifying the planetary orbits] and solids from each another but yet at the same time still be able to disassemble them. If it becomes too difficult, then one should just place all the orbs and solids one inside the other (from the smallest onwards) and rivet them together so that they will not be able to disassembled." Furthermore, he continues, "I would like the

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[326] "*...desselben Angebers, der Goldschmid nach wenigen tagen, ohne meine weittere gegenwart fort zuarbeitten getrawett*" (GSW 13, nr. 49, 91).

[327] "*So befinde ich aber hingegen, das ich die zweyn erste Monat, von der Zeitt an, da ich das Papyrene Muster gemacht, biß E.F.Gn. mir den tisch zu hoff assignirt, mitt Zehrung, hültzene kugel dräen, Papendeckelthauffen…"* (GSW 13, nr. 50, p 92). This invoice for the 30 gulden is reprinted in GSW 19, 7.20 (p 324).

entirety of the internal part of the model to be transparent and around four posts in the middle will be wrought [zodiac signs]."[328]

Kepler's fallback plan to construct the Solids as physical objects in the model would have, in a sense, brought the design closer in principle to the Tabula III image from the *Mysterium Cosmographicum*, although these printed Solids, in the same vein as Jamnitzer's unbuilt, graphic models, were conceived as conceptual spacers never intended to be physically produced. Nevertheless, the new design was also dismissed by Friedrich's goldsmiths for its impracticality and expense. Moreover, Friedrich himself seems to have received unsettling reports from Mästlin, namely that the existing model "has some imperfections around its edge that need to be corrected. It is possible to distinguish a noticeable flaw also on the great star [the sun], and everything has not yet been improved, but would rather be better to begin again anew."[329]

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[328] "*Mitt den corporibus aber hab ich wöllen kunst brauchen, vnd also anordnen, das jnen im abheben khein beyn…zerbrochen oder zerteiltt werde. Auch hab ich vermeindt, es werde khein kunst sein wan man nicht alle orbes vnd corpora auß einander lösen vnd gleich als ein vhr zerlegen khönde. Wan es aber je zu schwer woltt werden: So thue man eins, man mach alle corpora vnd orbes (vom kleinesten anzufahen) in einander, vnd lätt oder niette sie zusamen, das sie nicht zerlegt werden khönden...Zum andern, hab ich zu behauptung des names (Theoria primi et secundorum mobilium) ve ordnen müessen, das das gantze opus internum, so da durchsichtig ist, auff vier pfosten in medio* [zodiac signs] *angeleinet herumb getriben werde*" (GSW 13, nr. 85, 163).

[329] "*Der* [Mästlin] *hatt Vnns nuhn diser tagen bericht, daß sollich werckh an der runde zimbliche vnnolkhommenhait habe, auch an der Sternnen größe vnnd vnderschaydt ein mercklichs mangle, vnnd solliches alles nit mehr zuverbeßern: Sondern von Newen wider zu machen seye*" (GSW 13, nr. 90, p 188).

Writing to Mästlin on June 11, Kepler is flabbergasted by the developments, and places the blame for the model's poor execution squarely on the goldsmith.[330]

> There is no way to understand [why there are problems], since in the first communication I provided [a model] in paper and placed it before the eyes of the goldsmith, and finally drew onto a different sphere what needs to be changed on the paper model in a sensibly light and artful manner very well executed by me, as well as provided a drafted report…I have observed, firstly, that His Grace wants to have such a work in his *Kunstkammer*. 2. That its considerable cost will rise… so it must follow that the spectators and surveyors will be able to see nothing else of the work's interior other then the 5 geometrical bodies, and so they will believe…that the cost has been well accounted for…Hereto I would give the reason for Kretzmayer's last conversation in which he said, after I had left Stuttgart, that the interior of the work would be too difficult for a goldsmith, and thus he wanted to run a rigid support through its middle, upon which all the elements could be better and more conveniently installed.[331]

The substantive part of Kepler's letters addressing his dissatisfaction with the production of the *Mysterium* model ends here, though Kepler does include scattered references to it throughout the rest of his life and never seems to have stopped using polyhedra as a medium for thought, whether in regards to packing problems, snowflakes, so-called

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[330] Kepler also describes how the idea of a moving celestial globe has been around since ancient times, and lists examples of relevant works around Europe that are capable of representing planetary motion. Kepler's list is a form of proof of how prevalent these machines are, and therefore the goldsmith's insistence that the model is impossible is unfounded. He continues that he wants his model to have motion because he wants to represent nature itself as much as possible, "*sovil müglich die natur selbsten repraesentiren*" (GSW 13, nr. 99, p 223)—and also wants to reproduce the exact speed of the planetary orbits, which he describes in great detail.

[331] "*Dan es soll kheins Wegs den verstand haben als hett ich an der ersten angebung verzagt oder geirretm sintemahl ichs zuvor in papir gebracht, vnd dem goldschmid für augen gestalt, vnd was an dem papirenen Muster zuändern gewest, auff underschidliche kugel gerissen, jn einen bericht verfasset…hab ich…betrachtet, erstlich, das Ire. Fürl. Gnaden sollich Werckh in dero Kunstkamer haben wollen. 2. Das ein zimlicher costen darauff lauffen werde…so mueß folgen, das die Spectatores vnd besichtiger das jnnere Werckh für nichts anders, als für die 5 Geometrische Corpora ansehen, vnd vermeinen werden, es sey vnvonnötten gewest, das man den costen drauff wende…Hatt mir auch hierzue Ursach gegeben des Kretzmayers letze red, als ich von Stuetgart gescheiden, der gesagt, wan das jnnere Werckh dem goldschmid zu schwär sein wölle, so wöll er einen stefft mitten hindurch gehen lassen, darauff alle stuck desto füeglicher könden eingesetzt werden*" (GSW 13, nr. 99, p 219-220).

Archimedean Solids, or even his revised edition of the *Mysterium Cosmographicum* published alongside his commentary in 1621.[332] In aiming to take advantage of the polyhedral turn that had peaked in the late 16th century, Kepler revised his design so as to exhibit and justify its costliness through geometrical complexity, a strategy with a lineage directly emanating from those makers who produced the highly ornate objects desired by princely *Kunstkammern*. Nevertheless, the material ambiguities inherent in the history of the *Credentzbecher* confesses to the difficulties in conceiving and constructing irregular and regular bodies, let alone communicating these bodies to craftsmen that did not specialize in their production. The nesting of skeletal polyhedra in precious metal was always going to fail, given that the technological constraints of their manufacture limited them to being produced on a lathe, hence the goldsmith's suggestion that the Solids be mounted (and constructed) separately onto a central supportive column. In attempting to realize a cluster of polyhedra firmly grounded in the mixed-mathematics and decorative arts of late 16th century Germany, Kepler, it seems, may have mistaken their ubiquity for an innate, ontological drive towards being and discounted the contributions, and rigidities, of makers and materials. Without the skills to represent his ideas or to manage the project or project personalities, the *Credentzbecher* was destined to remain stranded in the interstitial spaces between court and workshop. And yet its failure speaks to the metaphysical durability of the Solids, which despite the violence of Dürer's unfolding and the subsequent proliferation of geometrical irregularity, kept haunting the

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[332] On Kepler's polyhedra see Field's "Kepler's Star Polyhedra," *Vistas in Astronomy*, Vol. 23, 1979, 109-141; and "Kepler's Mathematization of Cosmology," *Mysterium Cosmographicum 1596-1996*, National Technical Museum of Prague, 1996, 27-48.

imagination of science as Classical ghosts that offered the promise of order and mathematical definition in an unbridled world.

Appendix

Euclid, *Elements*, *Book 13, Proposition 13: To construct a dodecahedron and comprehend it in a sphere* [excerpt]

Let *ABCD*, *CBEF*, two planes of the aforesaid cube at right angles to one another, be set out, let the sides *AB*, *BC*, *CD*, *DA*, *EF*, *EB*, *FC* be bisected at *G*, *H*, *K*, *L*, *M*, *N*, *O* respectively, let *GK*, *HL*, *MH*, *NO* be joined, let the straight lines *NP*, *PO*, *HQ* be cut in extreme and mean ratio at the points *R*, *S*, *T* respectively, and let *RP*, *PS*, *TQ* be their greater segments; from the points *R*, *S*, *T* let *RU*, *SV*, *TW* be set up at right angles to the planes of the cube towards the outside of the cube, let them be made equal to *RP*, *PS*, *TQ*, and let *UB*, *BW*, *WC*, *CV*, *VU* be joined.

I say that the pentagon *UBWCV* is equilateral, and in one plane, and is further equiangular.

For let *RB*, *SB*, *VB* be joined.

Then, since the straight line *NP* has been cut in extreme and mean ratio at *R*, and *RP* is the greater segment, therefore the squares on *PN*, *NR* are triple of the square on *RP*.

But *PN* is equal to *NB*, and *PR* to *RU*; therefore the squares on *BN*, *NR* are triple of the square on *RU*.

But the square on *BR* is equal to the squares on *BN*, *NR*; therefore the square on *BR* is triple of the square on *RU*; hence the squares on *BR*, *RU* are quadruple of the square on *RU*.

But the square on *BU* is equal to the squares on *BR*, *RU*; therefore the square on *BU* is quadruple of the square on *RU*; therefore *BU* is double of *RU*.

But *VU* is also double of *UR*, inasmuch as *SR* is also double of *PR*, that is, of *RU*; therefore *BU* is equal to *UV*.

Similarly it can be proved that each of the straight lines *BW*, *WC*, *CV* is also equal to each of the straight lines *BU*, *UV*.

Therefore the pentagon *BUVCW* is equilateral.

I say next that it is also in one plane.

For let *PX* be drawn from *P* parallel to each of the straight lines *RU*, *SV* and towards the outside of the cube, and let *XH*, *HW* be joined; I say that *XHW* is a straight line.

For, since *HQ* has been cut in extreme and mean ratio at *T*, and *QT* is its greater segment, therefore, as *HQ* is to *QT*, so is *QT* to *TH*.

But *HQ* is equal to *HP*, and *QT* to each of the straight lines *TW*, *PX*; therefore, as *HP* is to *PX*, so is *WT* to *TH*.

And *HP* is parallel to *TW*, for each of them is at right angles to the plane *BD*; and *TH* is parallel to *PX*, for each of them is at right angles to the plane *BF*. [*id.*]

But if two triangles, as *XPH*, *HTW*, which have two sides proportional to two sides be placed together at one angle so that their corresponding sides are also parallel, the remaining straight lines will be in a straight line; therefore *XH* is in a straight line with *HW*.

But every straight line is in one plane; therefore the pentagon *UBWCV* is in one plane.

I say next that it is also equiangular.

For, since the straight line *NP* has been cut in extreme and mean ratio at *R*, and *PR* is the greater segment, while *PR* is equal to *PS*, therefore *NS* has also been cut in extreme and mean ratio at *P*, and *NP* is the greater segment; therefore the squares on *NS*, *SP*are triple of the square on *NP*.

But *NP* is equal to *NB*, and *PS* to *SV*; therefore the squares on *NS*, *SV* are triple of the square on *NB*; hence the squares on *VS*, *SN*, *NB* are quadruple of the square on *NB*. But the square on *SB* is equal to the squares on *SN*, *NB*; therefore the squares on *BS*, *SV*, that is, the square on *BV* —for the angle *VSB* is right—is quadruple of the square on *NB*; therefore *VB* is double of *BN*.

But *BC* is also double of *BN*; therefore *BV* is equal to *BC*.

And, since the two sides *BU*, *UV* are equal to the two sides *BW*, *WC*, and the base *BV* is equal to the base *BC*, therefore the angle *BUV* is equal to the angle *BWC*.

Similarly we can prove that the angle *UVC* is also equal to the angle *BWC*; therefore the three angles *BWC*, *BUV*, *UVC* are equal to one another.

But if in an equilateral pentagon three angles are equal to one another, the pentagon will be equiangular, therefore the pentagon *BUVCW* is equiangular.

And it was also proved equilateral; therefore the pentagon *BUVCW* is equilateral and equiangular, and it is on one side *BC* of the cube.

Therefore, if we make the same construction in the case of each of the twelve sides of the cube, a solid figure will have been constructed which is contained by twelve equilateral and equiangular pentagons, and which is called a dodecahedron.

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Figure I: Archimedes (?) (ca. 1518-1520) by Ugo da Carpi (ca. 1480 - 1520/1532).
Albertina, Vienna. Inv. Nr. DG2002/524.

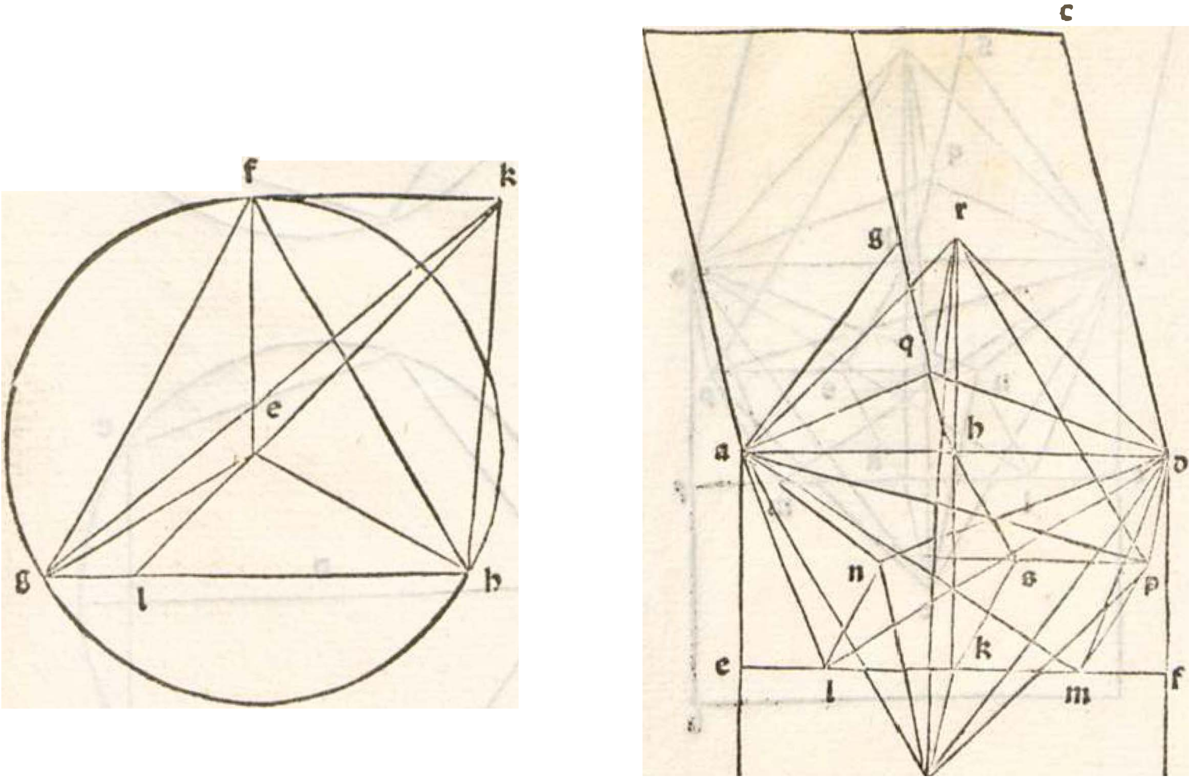


Figure II: (Left) Diagram of a pyramid inscribed in a circle, Book 13, Proposition 13; and (Right) Diagram of a dodecahedron from Book 13, Proposition 17. Both images are from the first printed edition of Euclid's Elements from 1482.

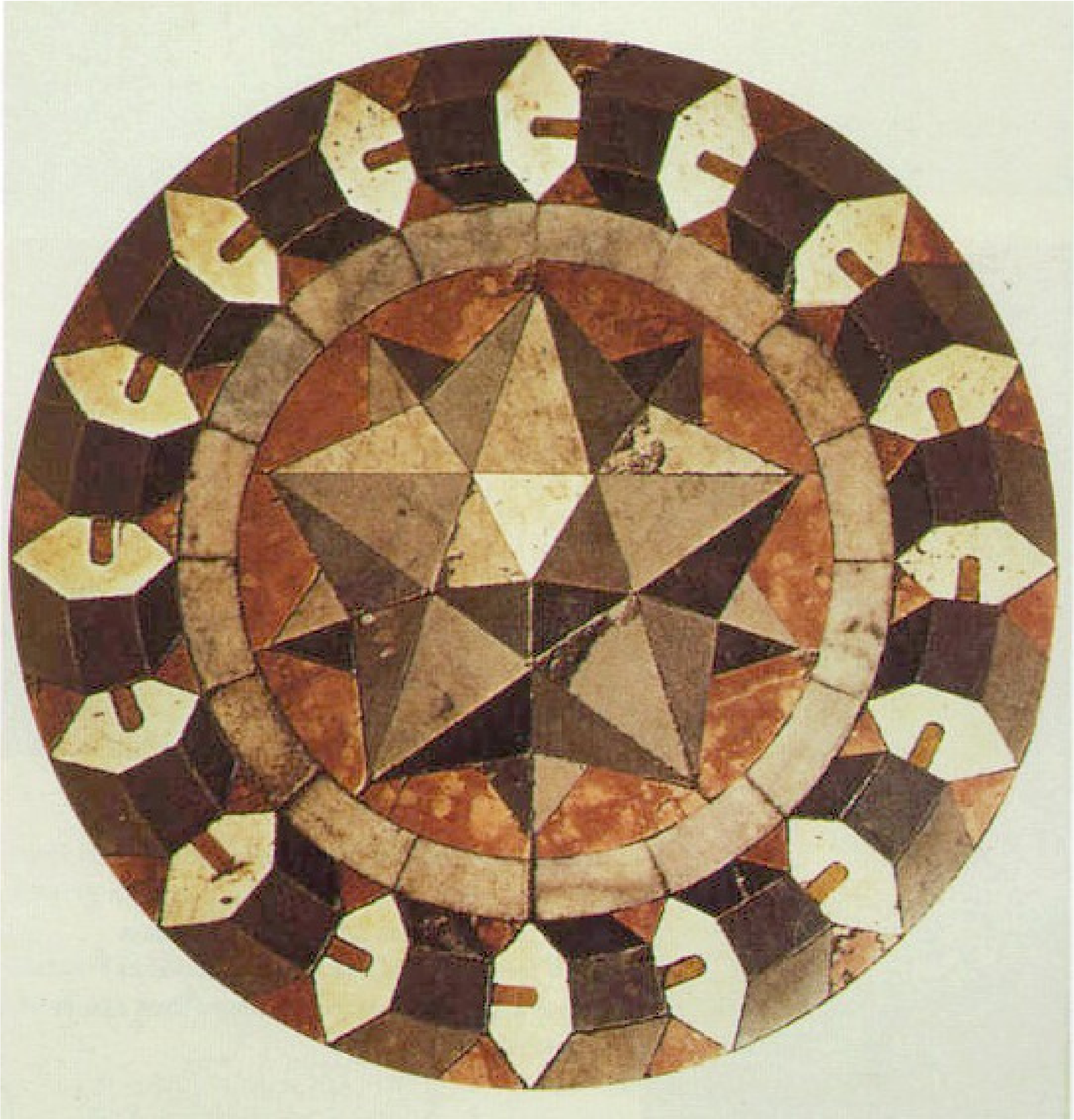


Figure III: Paolo Uccello’s (1397-1475) marble stellated dodecahedron (15th century) from the floor of the *Basilica di San Marco* in Venice.

Figure IV: Studies of polyhedra by Parmigianino (1503-1540). Museo di Capodimonte, Naples. Inv. 1366 verso.

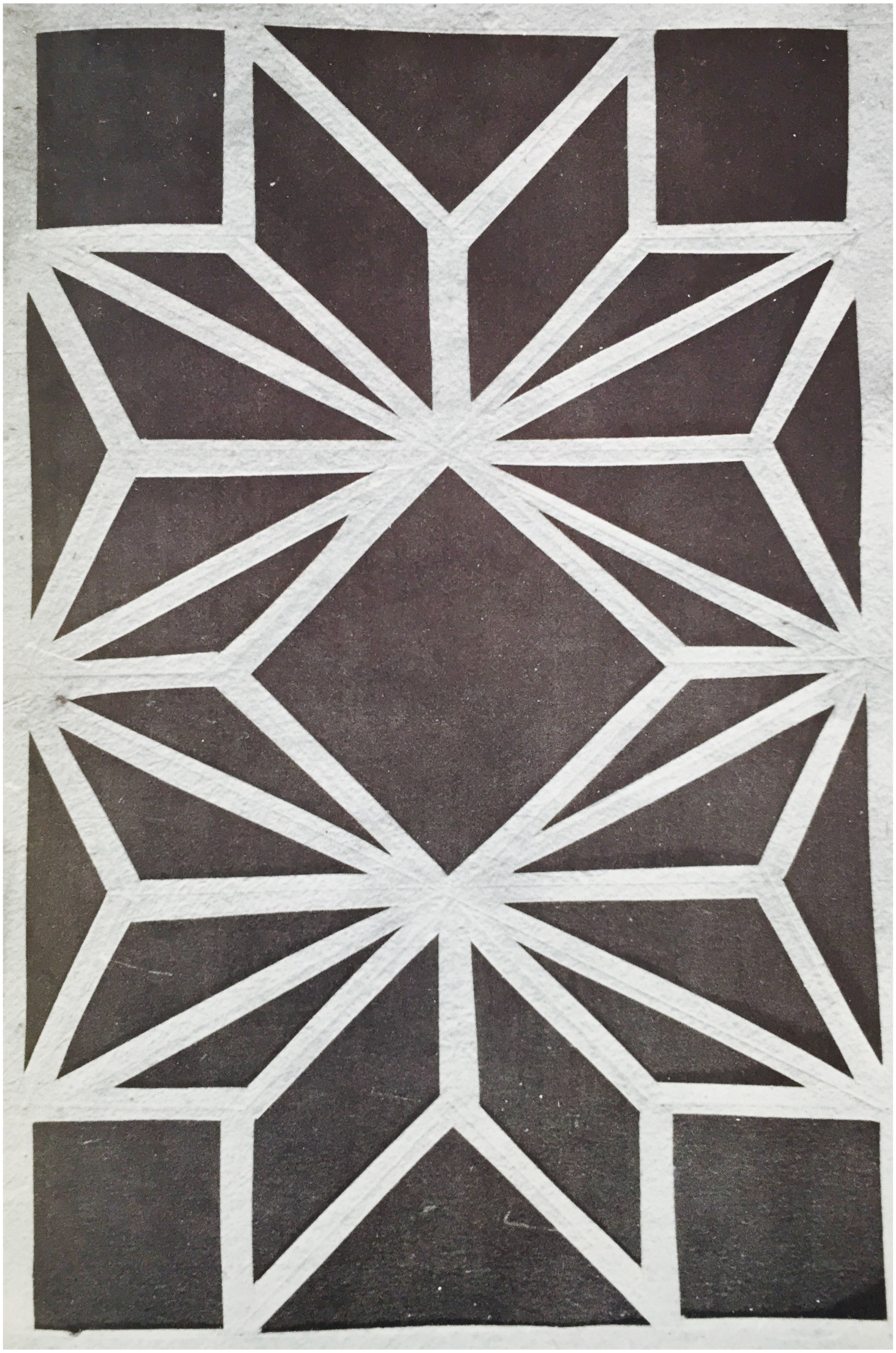


Figure V: Net vault from Sketchbook and Pattern Book. Master WG (active 1560-1572). Sädelsches Kunstinstitut, Frankfurt-am-Main. Inv. Nr. 8-494, WG 138.

Figure VI: Still Life with a Book, a Geometric Sphere, Dividers and Spectacles. Anonymous, Italian. 15th Century. Princeton University Art Museum, Princeton, NJ. Inv. Nr. x1945-54.

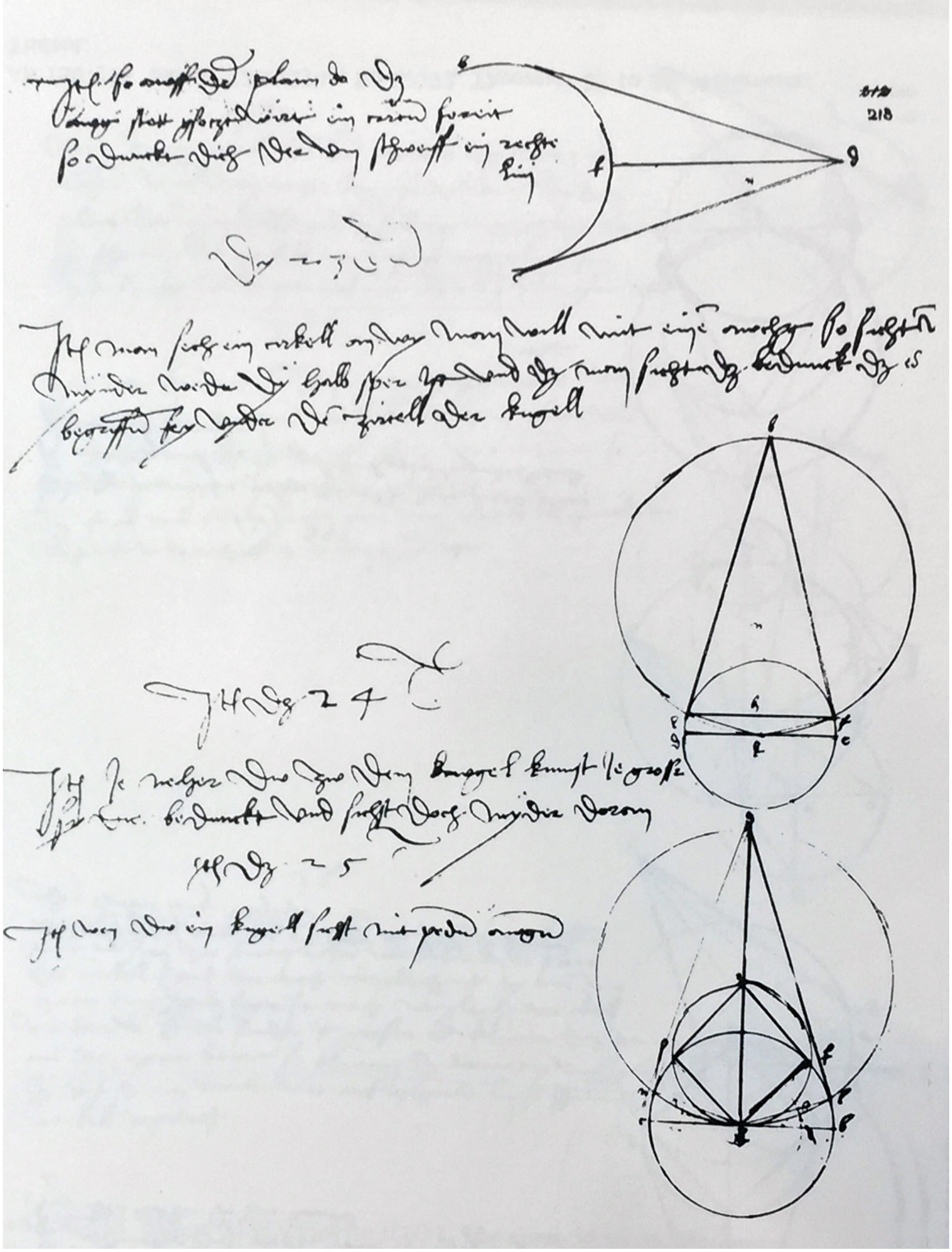
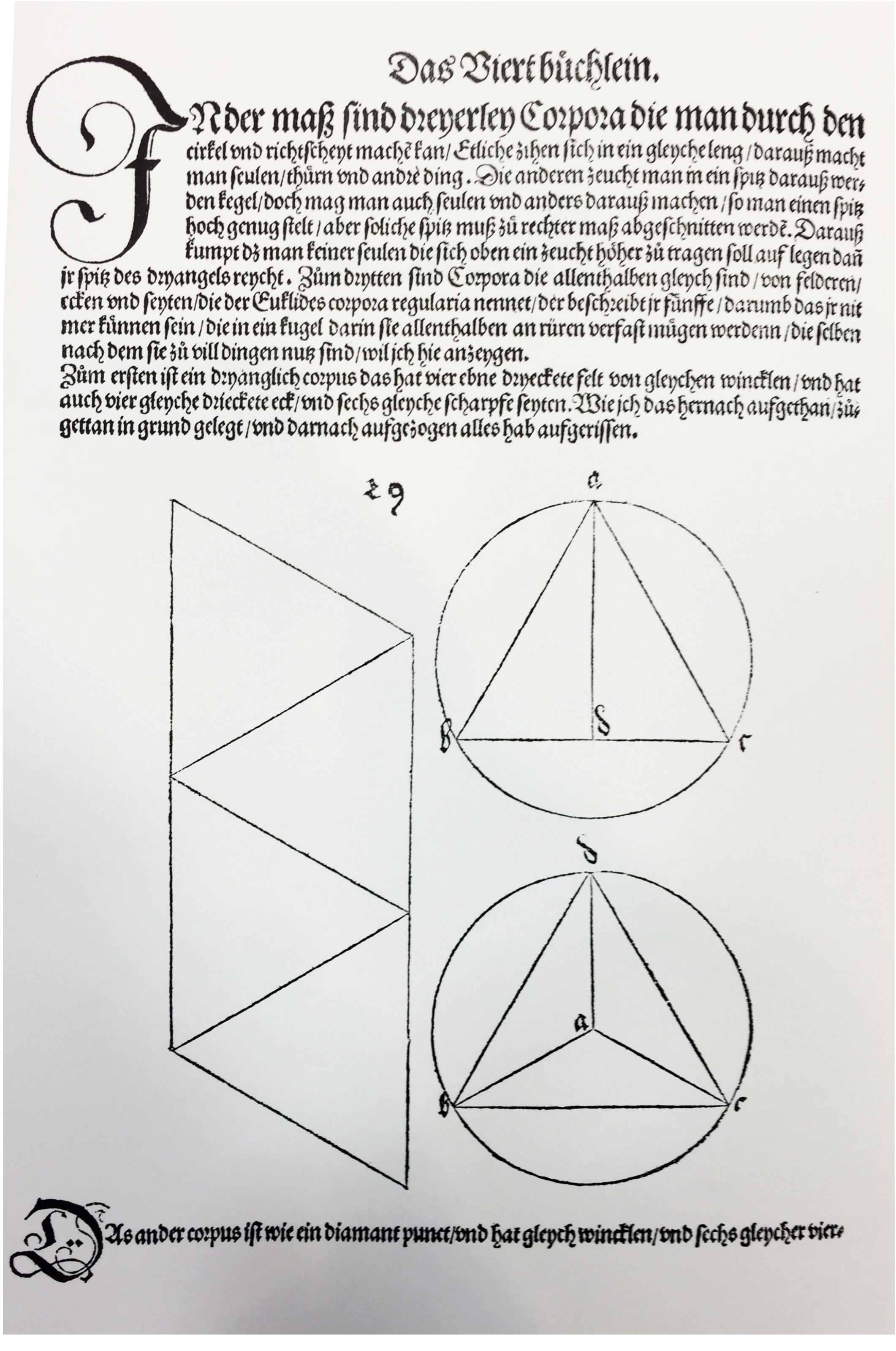


Figure 1.1: Translations and Diagrams from Euclid's *Elements* by Albrecht Dürer (1471-1528). Sloane Collection, British Museum, London. Sloane 5228/213r.



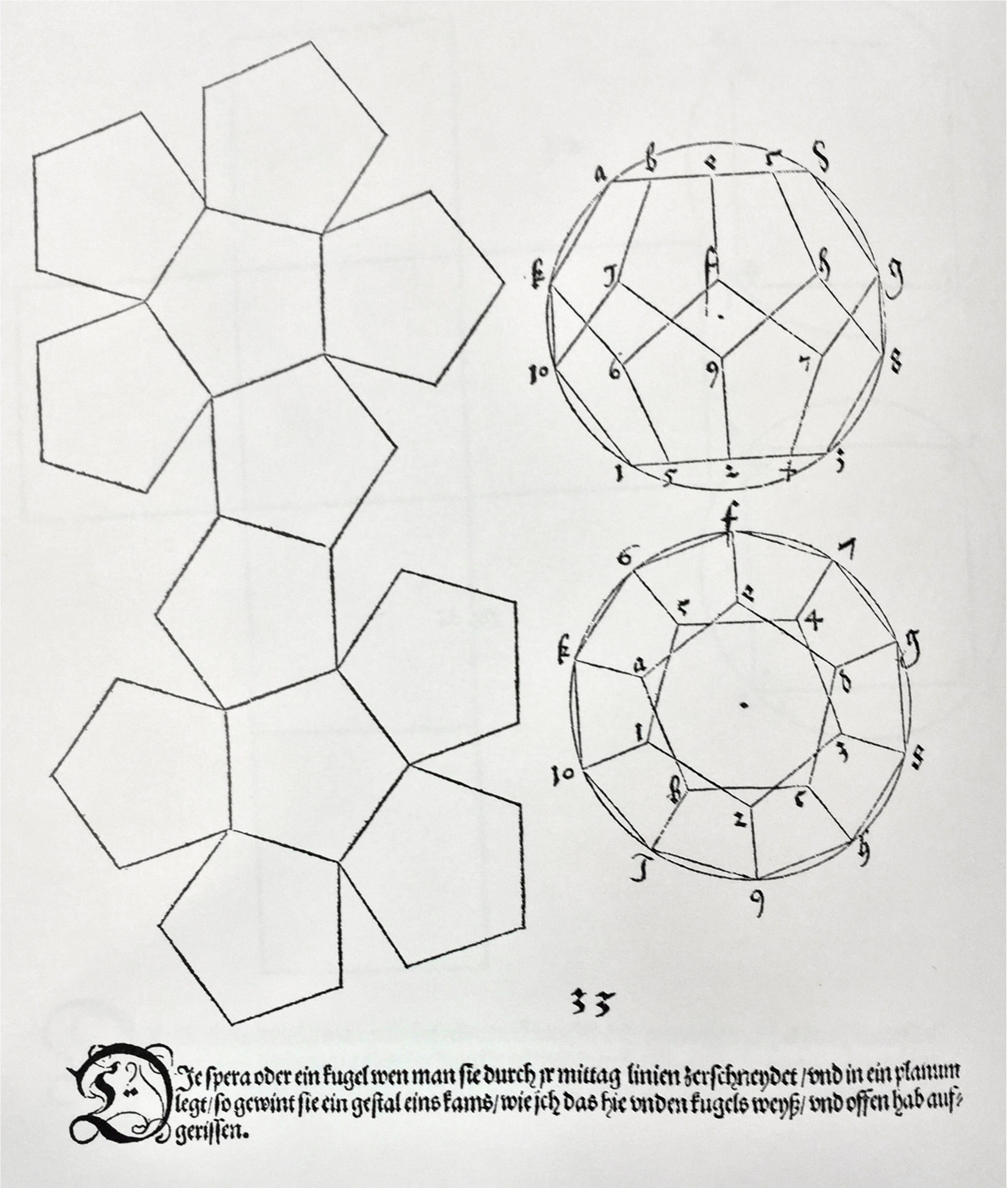
Das Viert bůchlein.

IN der maß ſind dreyerley Corpora die man durch den cirkel vnd richtſcheyt machē kan/ Etliche zihen ſich in ein gleyche leng / darauß macht man ſeulen/ thürn vnd andrē ding. Die anderen zeucht man in ein ſpitz darauß werden kegel/ doch mag man auch ſeulen vnd anders darauß machen / ſo man einen ſpitz hoch genug ſtelt / aber ſoliche ſpitz muß zů rechter maß abgeſchnitten werdē. Darauß kumpt dz man keiner ſeulen die ſich oben ein zeucht höher zů tragen ſoll auf legen dañ jr ſpitz des dryangels reycht. Zům drytten ſind Corpora die allenthalben gleych ſind / von felderen/ ecken vnd ſeyten/ die der Euklides corpora regularia nennet/ der beſchreibt jr fünffe / darumb das jr nit mer künnen ſein / die in ein kugel darin ſie allenthalben an rüren verfaſt mügen werdenn / die ſelben nach dem ſie zů vill dingen nutz ſind/ wil jch hie anzeygen.

Zům erſten iſt ein dryanglich corpus das hat vier ebne dryeckete felt von gleychen wincklen / vnd hat auch vier gleyche drieckete eck/ vnd ſechs gleyche ſcharpfe ſeyten. Wie jch das hernach aufgethan/ zůgettan in grund gelegt / vnd darnach aufgezogen alles hab aufgeriſſen.

DAs ander corpus iſt wie ein diamant punct/ vnd hat gleych wincklen/ vnd ſechs gleycher vier-

Figure 1.2: Page with net of a pyramid from *Underweysung der Messung* (1525) by Albrecht Dürer.



33

Die ſpera oder ein kugel wen man ſie durch jr mittag linien zerſchneydet/vnd in ein planum legt/ſo gewint ſie ein geſtal eins kams/wie jch das hie vnden kugels weyß/ vnd offen hab aufgeriſſen.

Figure 1.3: Page with net of a dodecahedron from *Underweysung der Messung* (1525) by Albrecht Dürer.

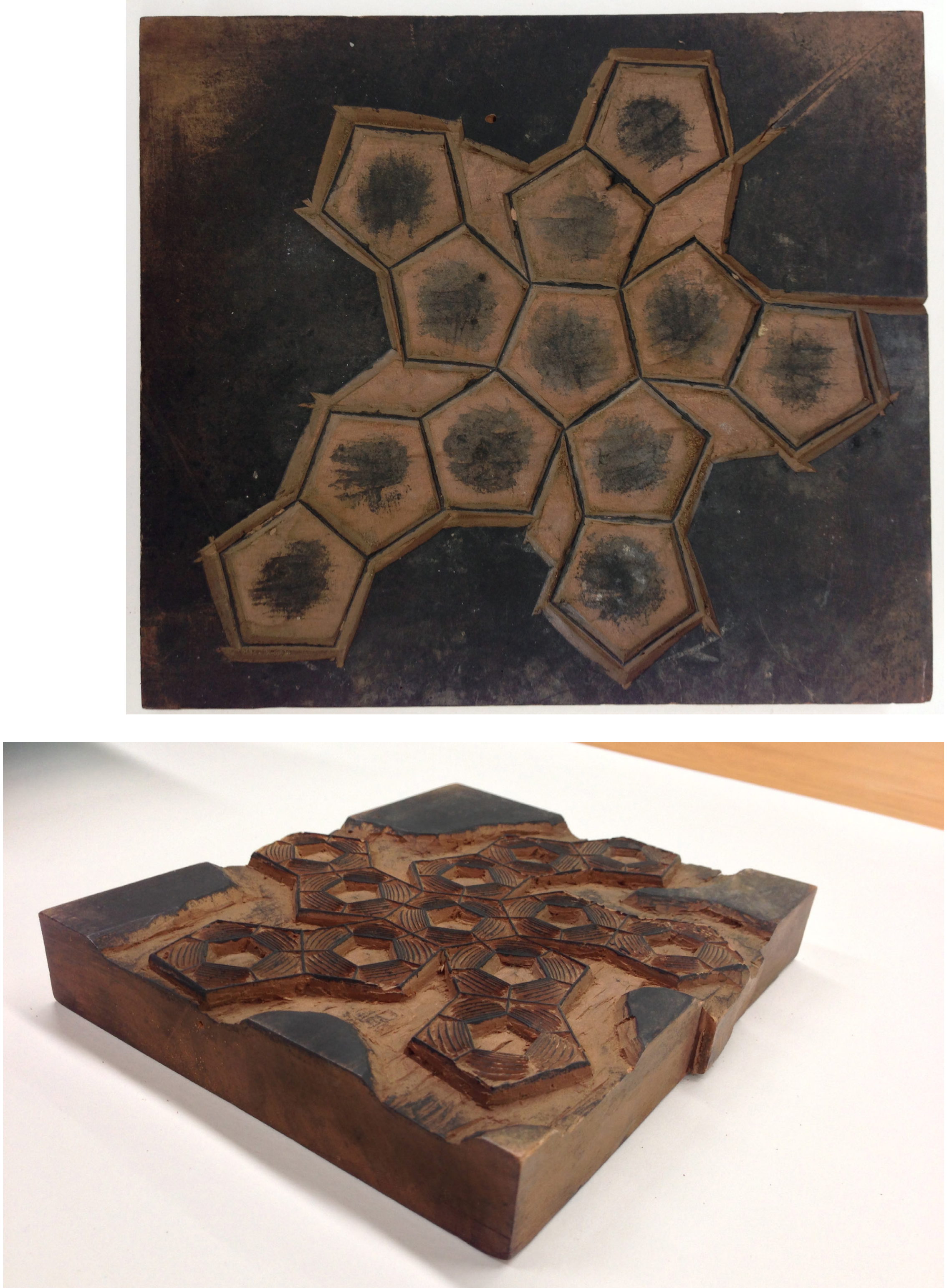
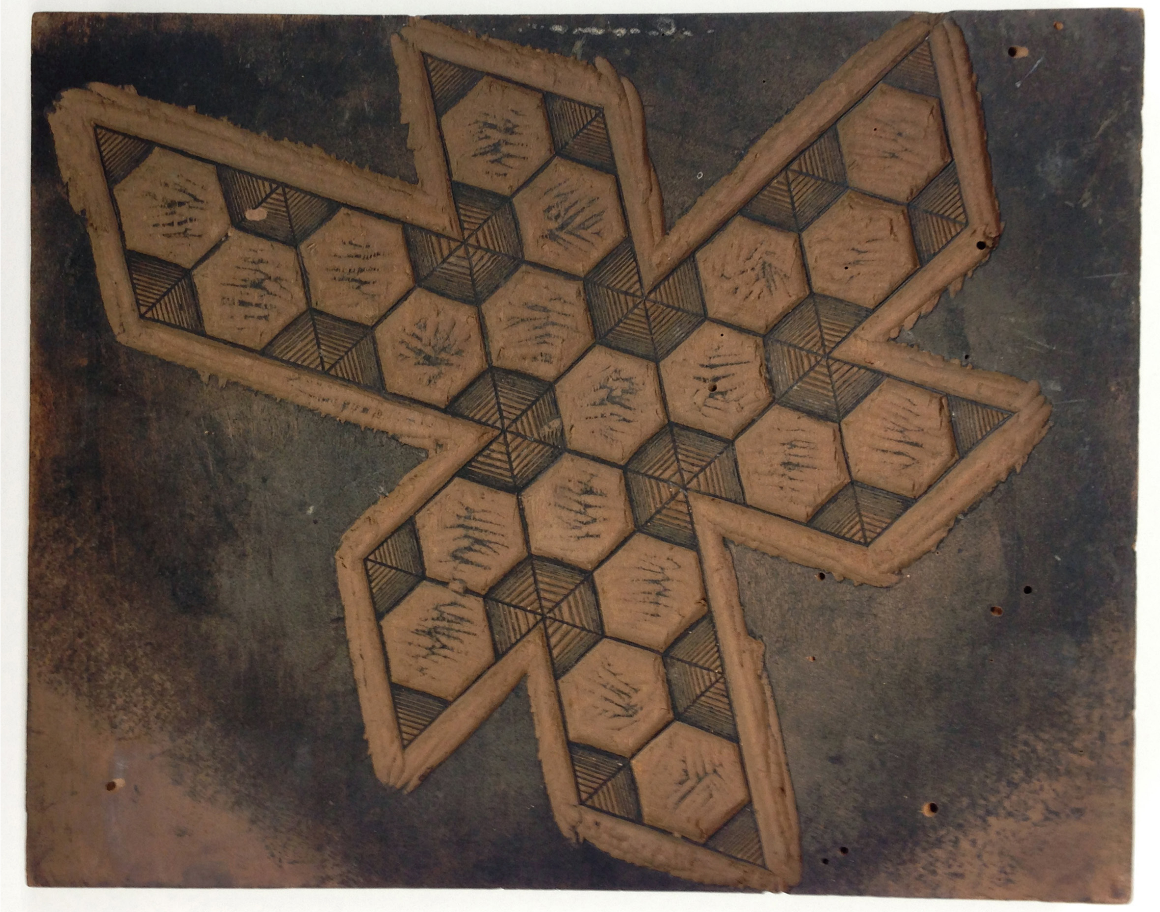


Figure 1.4 (top): Net of an unfolded dodecahedron (after 1538) - laid out differently than in Dürer's *Underweysung* (Diagram 33). Inventarnummer HO 2006/693. (bottom): View of the net of a dodecahedron (after 1538). Unknown artist. Woodblocks by Hieronymous Andreae (died 1556). Albertina, Vienna.



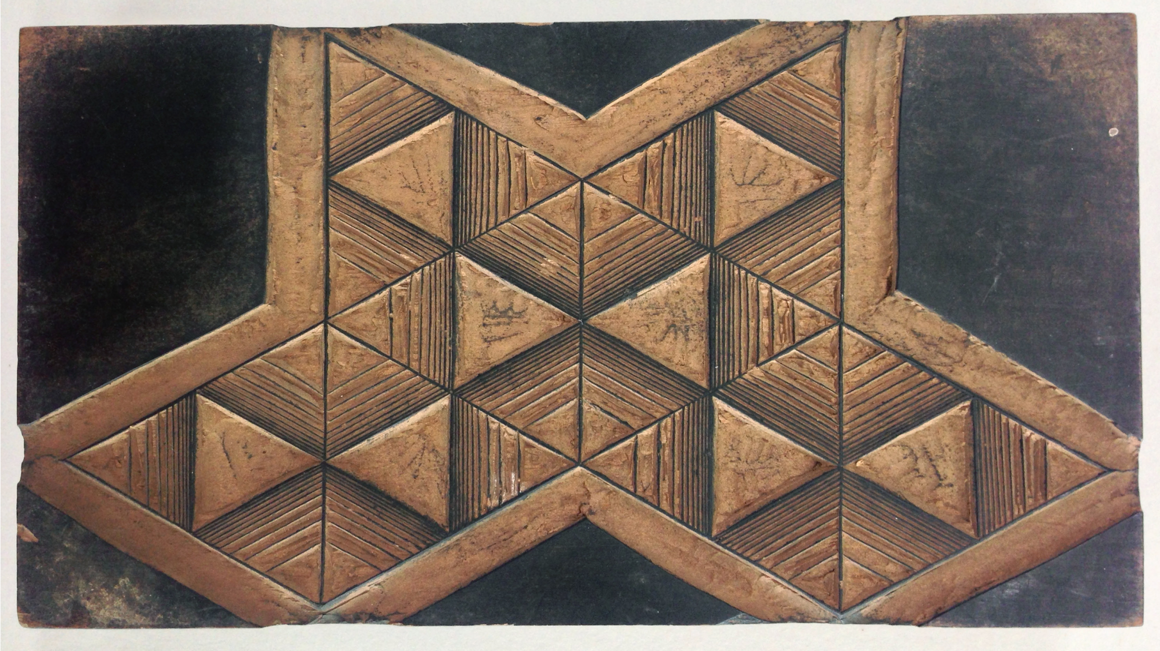
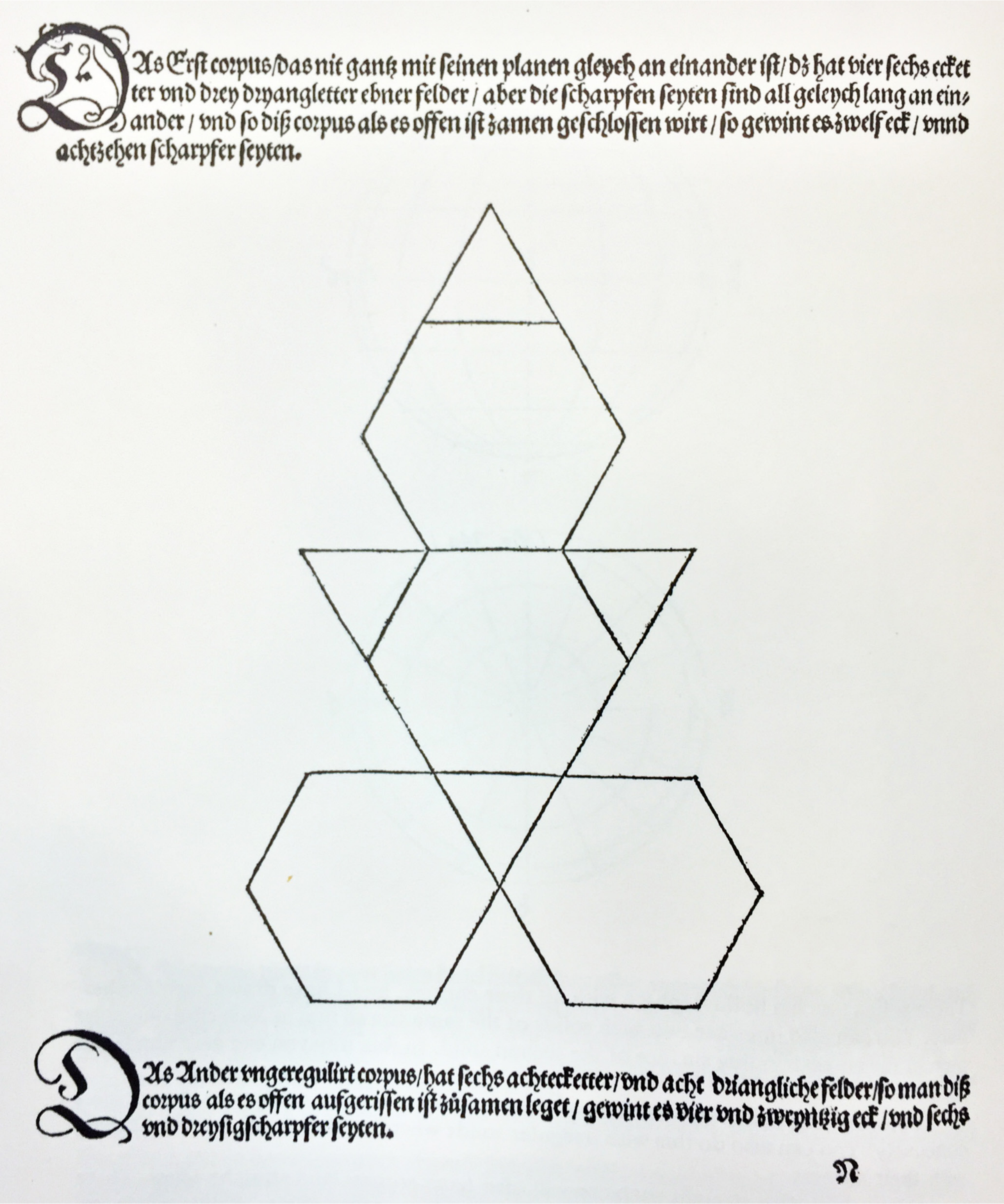


Figure 1.5 (top): Net of an icosahedron (after 1538) - Inventarnummer: H02006/685
(bottom): Net of an octahedron (after 1538) - Inventarnummer H02006/706. Albertina, Vienna.



DAs Erſt corpus/das nit gantz mit ſeinen planen gleych an einander iſt/ dz hat vier ſechs ecketter vnd drey dryangletter ebner felder / aber die ſcharpfen ſeyten ſind all geleych lang an einander / vnd ſo diß corpus als es offen iſt zamen geſchloſſen wirt / ſo gewint es zwelf eck / vnnd achtzehen ſcharpfer ſeyten.

DAs Ander vngeregulirt corpus/hat ſechs achteckketter/vnd acht driangliche felder/ſo man diß corpus als es offen aufgeriſſen iſt zůſamen leget / gewint es vier vnd zweyntzig eck / vnd ſechs vnd dreyſigſcharpfer ſeyten.

N

Figure 1.6: Page with net of a truncated tetrahedron from *Underweysung der Messung* (1525) by Albrecht Dürer.

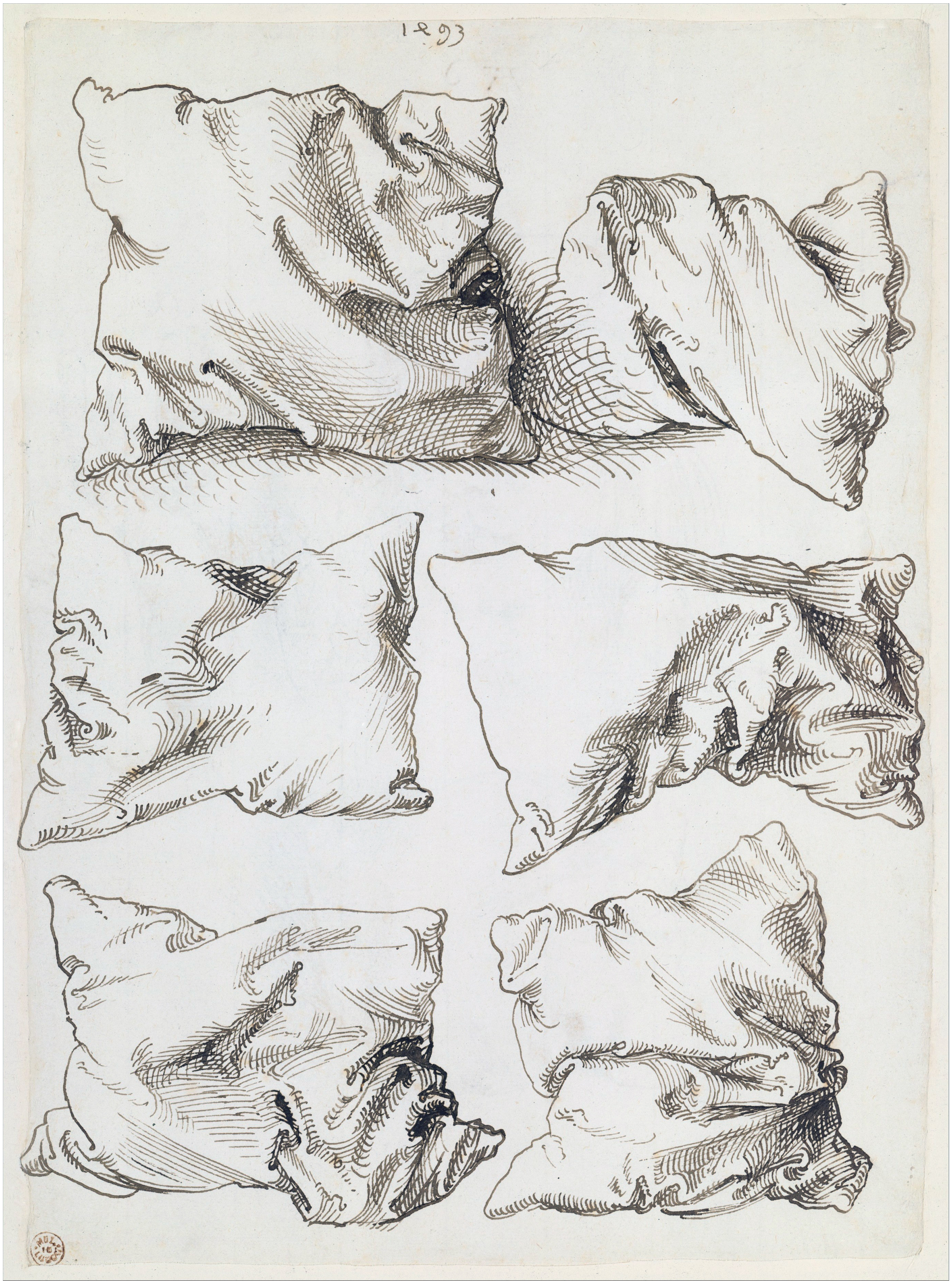


Figure 1.7: Six Studies of Pillows (1493) by Albrecht Dürer. Metropolitan Museum of Art, New York. Accession number 1975.1.862.

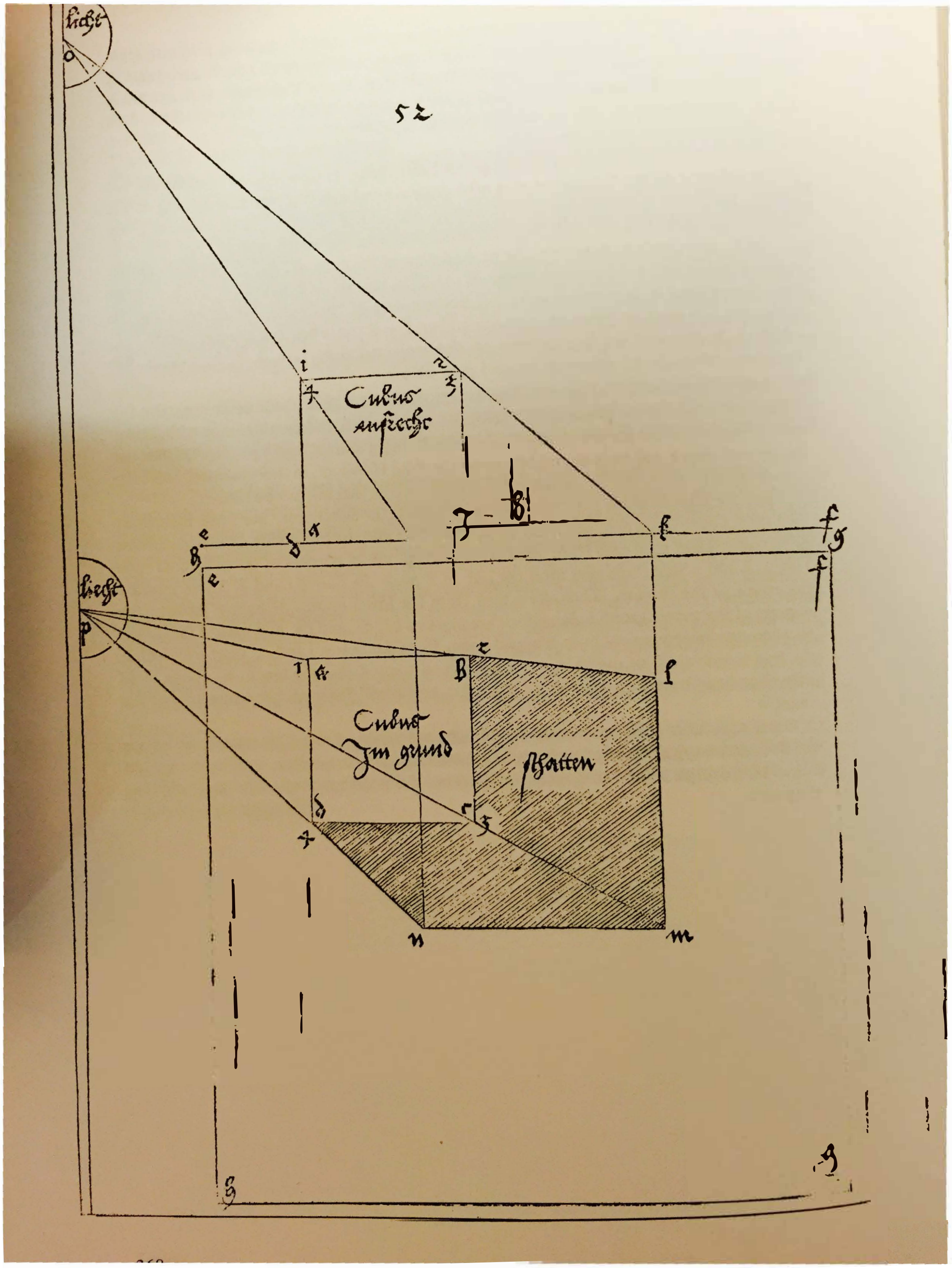
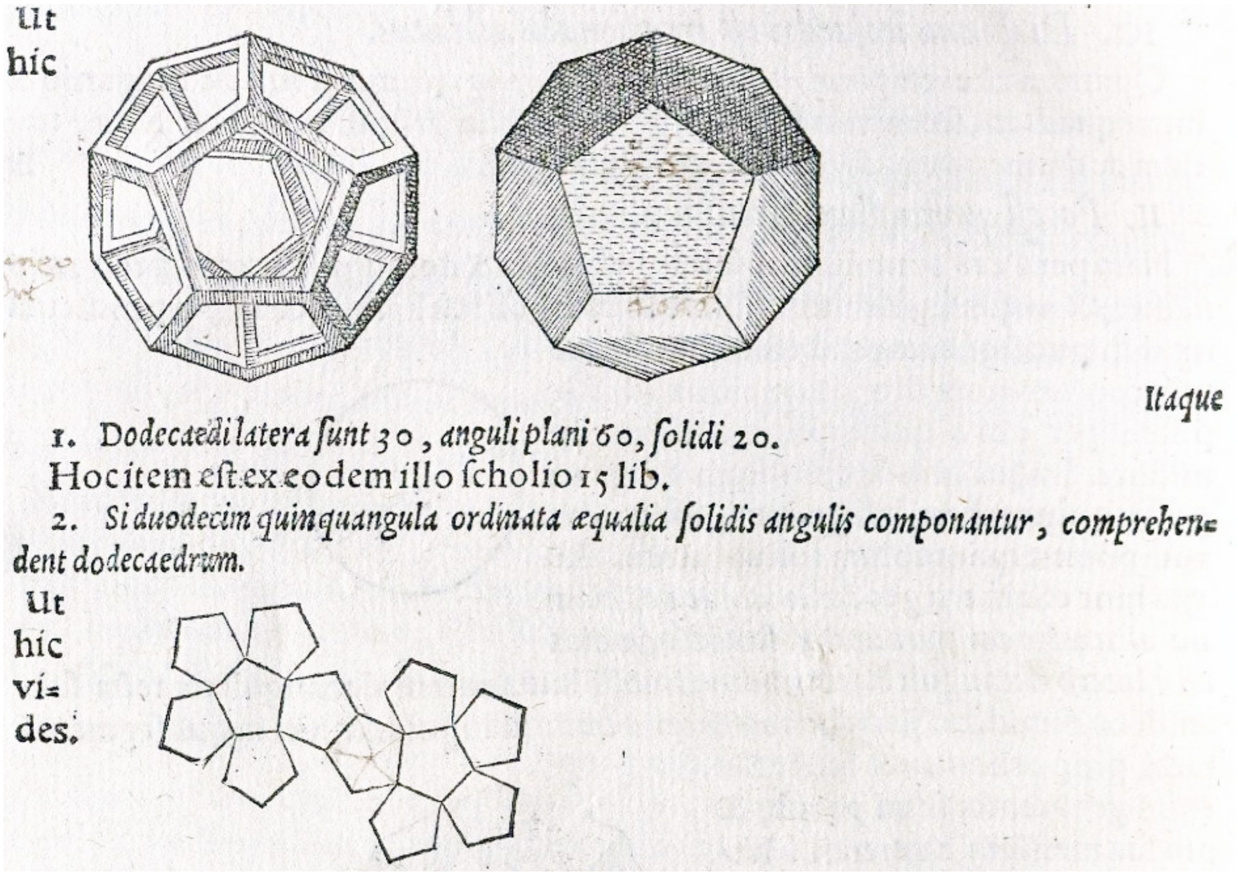


Figure 1.8: Construction of a cube in perspective from Dürer's *Underweysung*, diagram 52.



Ut hic

Itaque

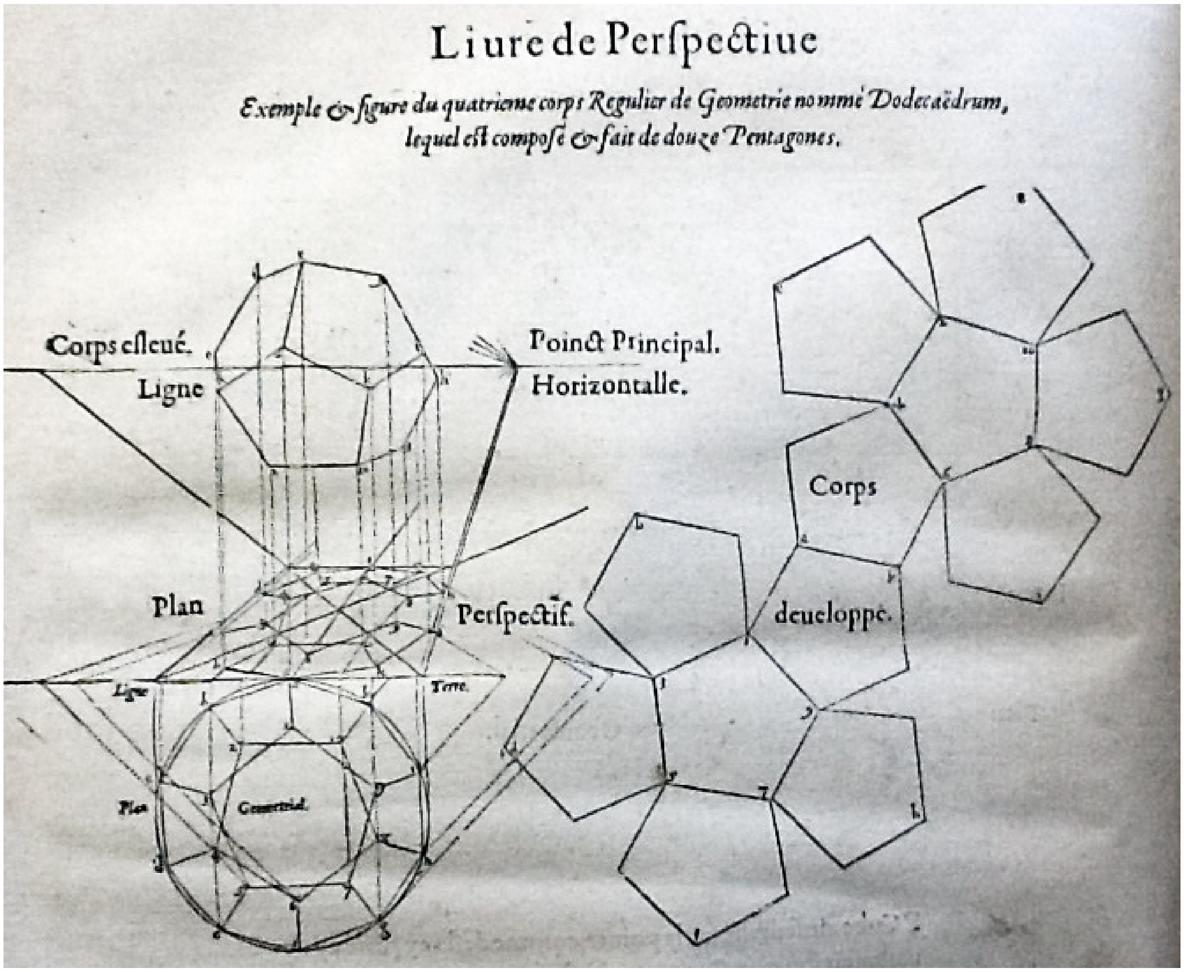
1. Dodecaedi latera ſunt 30, anguli plani 60, ſolidi 20.

Hoc item eſt ex eodem illo ſcholio 15 lib.

2. Si duodecim quinquangula ordinata æqualia ſolidis angulis componantur, comprehendent dodecaedrum.

Ut hic vides.

Figure 1.9: Page from *Arithmeticae libri duo: Geometriae septem et viginti* (1569) by Petrus Ramus, showing three ways of visualizing a dodecahedron: “1. Dodecahedra have 30 sides, 60 angled planes, and 20 solids. 2. If twelve equal pentagons with solid angles are orderly combined, they will create (comprehend) a dodecahedron. As you see here.”
"*Dodecaedi latera sunt 30, anguli plani 60, solidi 20. 2. Si duodecim quinquangula ordinate aequalia solidis angulis componantur, comprehendent dodecaedrum. Ut hic vides*"
(Ramus 1569, 178).



Liure de Perſpectiue

Exemple & figure du quatrieme corps Regulier de Geometrie nommé Dodecaëdrum, lequel eſt compoſé & fait de douze Pentagones.

Figure 1.10: Page from Jean Cousin's *Livre de perspective de Iehan Cousin senonois, maistre painctre a Paris* (1560). The text says "Example and figure of the four Regular body of Geometry, called the Dodecahedron, which is composed and made from twelve pentagons." "*Exemple and figure du quatrieme corps Regulier de Geometrie nommé Dodecaedrum, lequel est compose & fait du douze Pentagones.*"

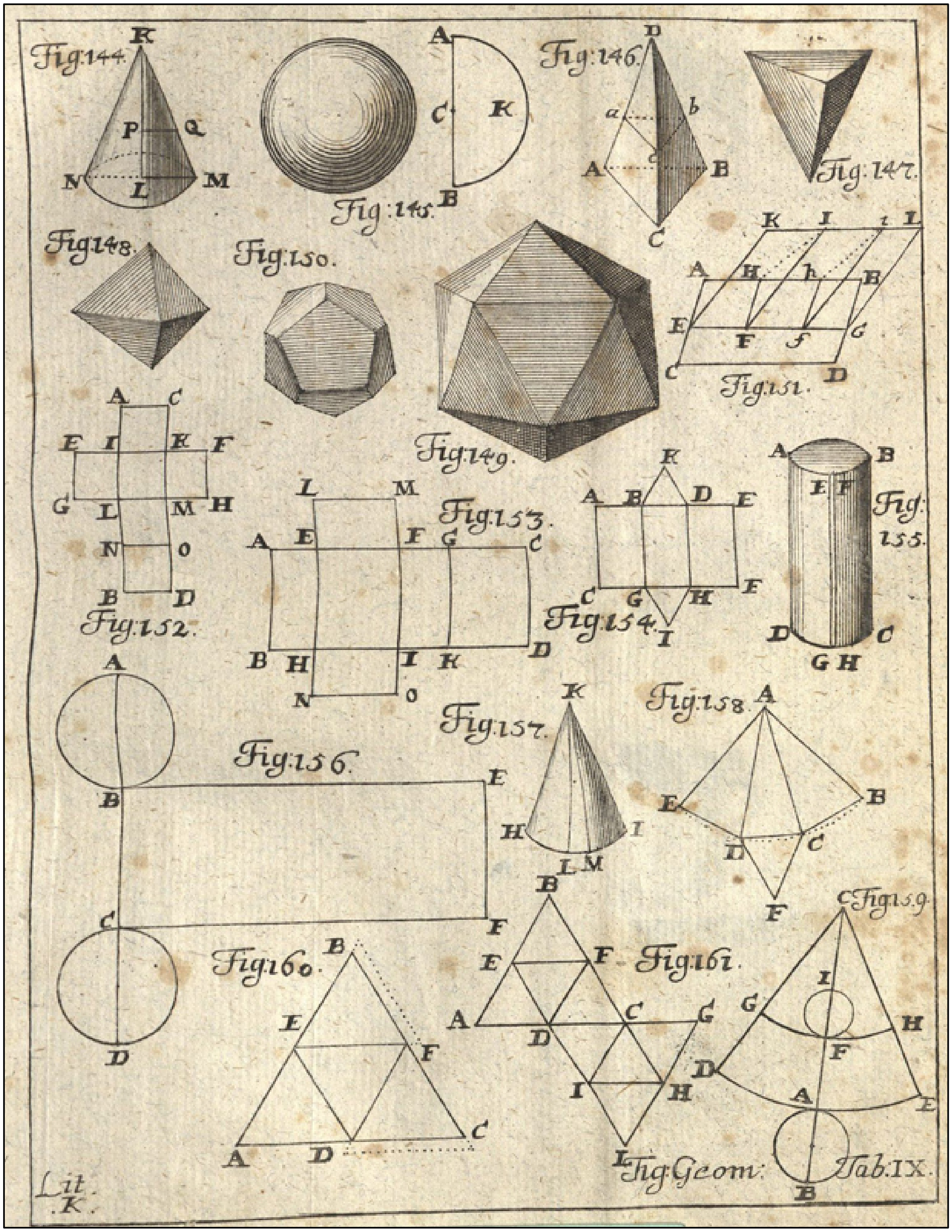


Figure 1.11: Page from the section illustrating Platonic Solids in Christian Wolff's *Elementa Matheseos Universae* (1730-1741).

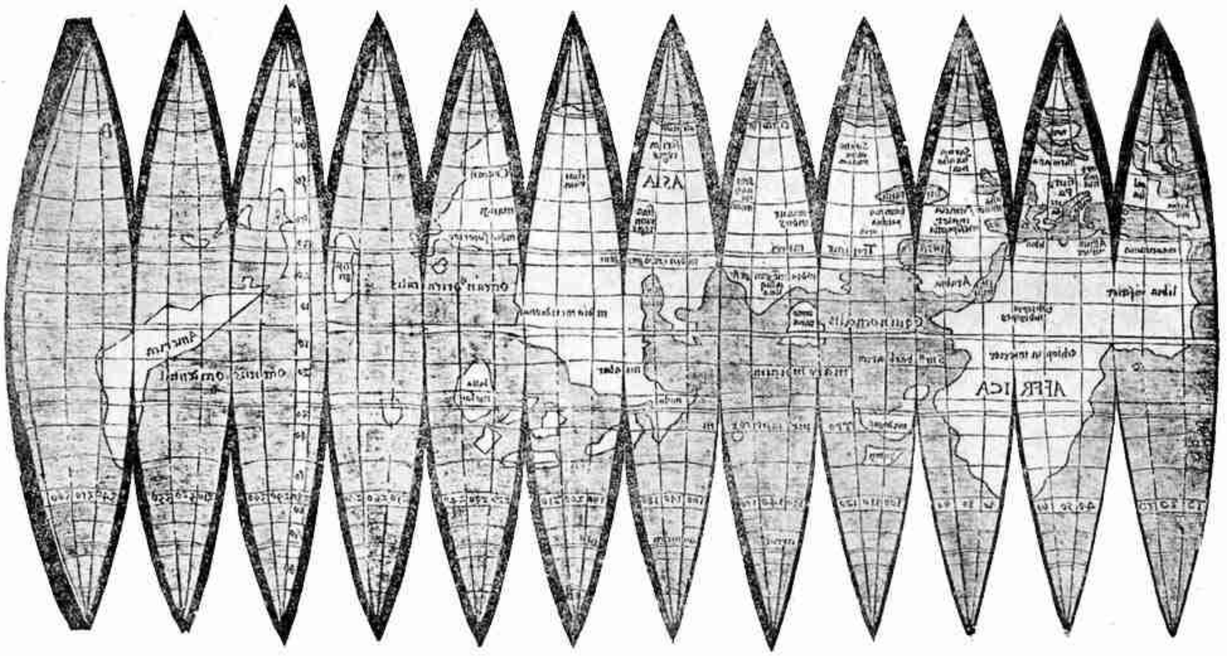


Figure 1.12: Globe Gores attributed to Martin Waldseemüller (1509).

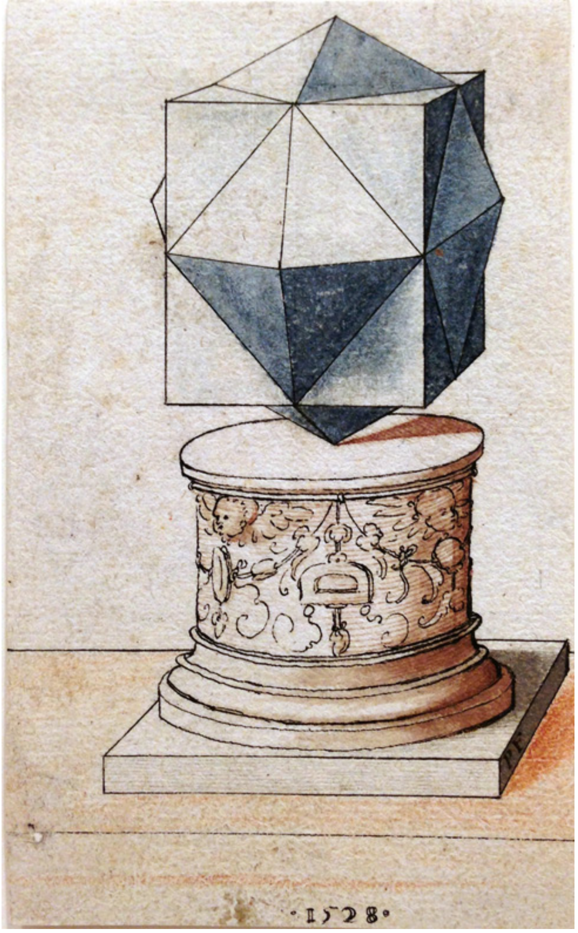


Figure 2.1: Perspectival drawing (1528), Peter Flötner, Metropolitan Museum of Art, Accession number 2007.223.5.

Figure 2.2: "A nude man measuring a block of stone" (1530) Erhard Schön, British Museum, inventory number 1978,1216.40 .

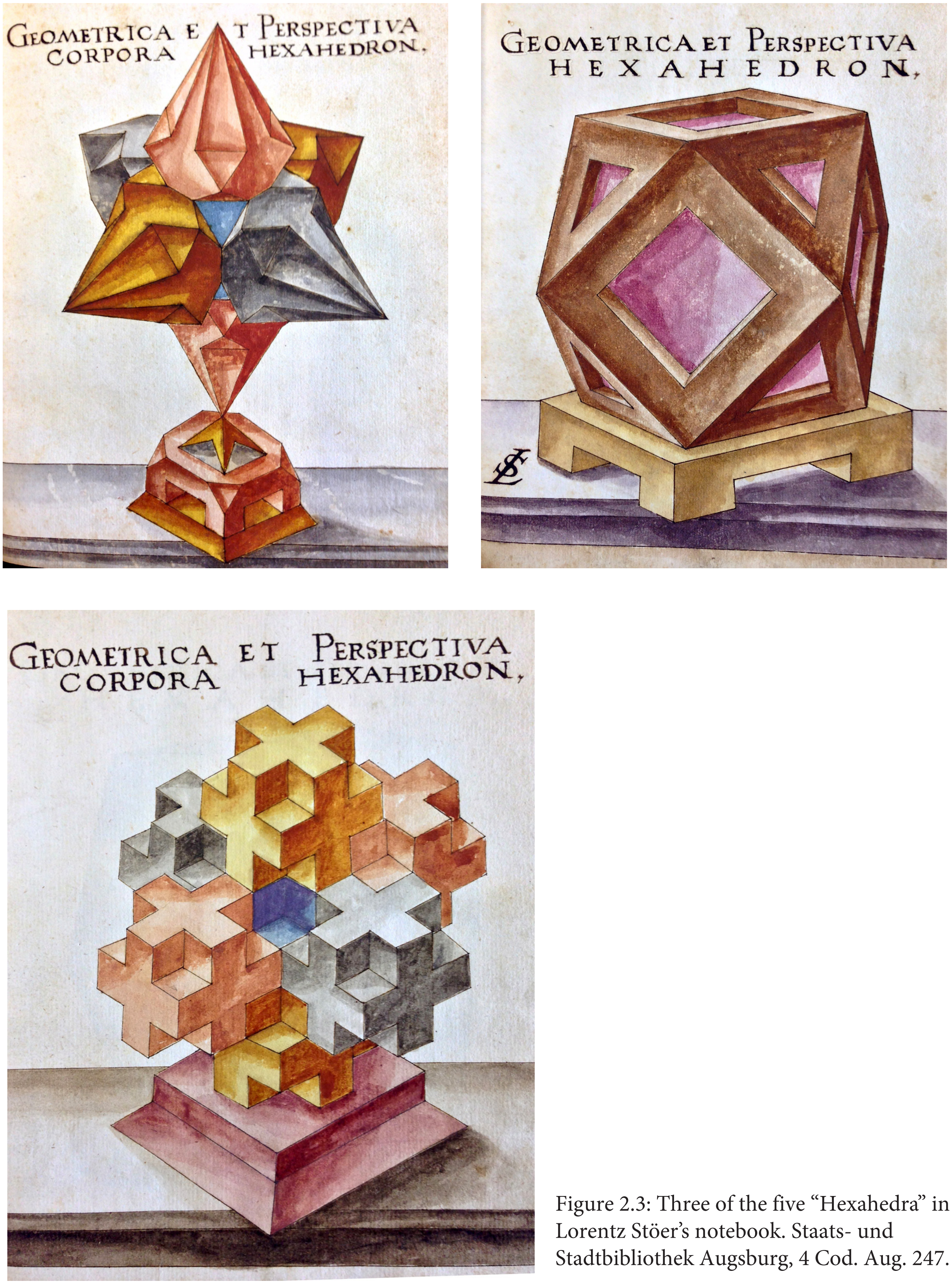
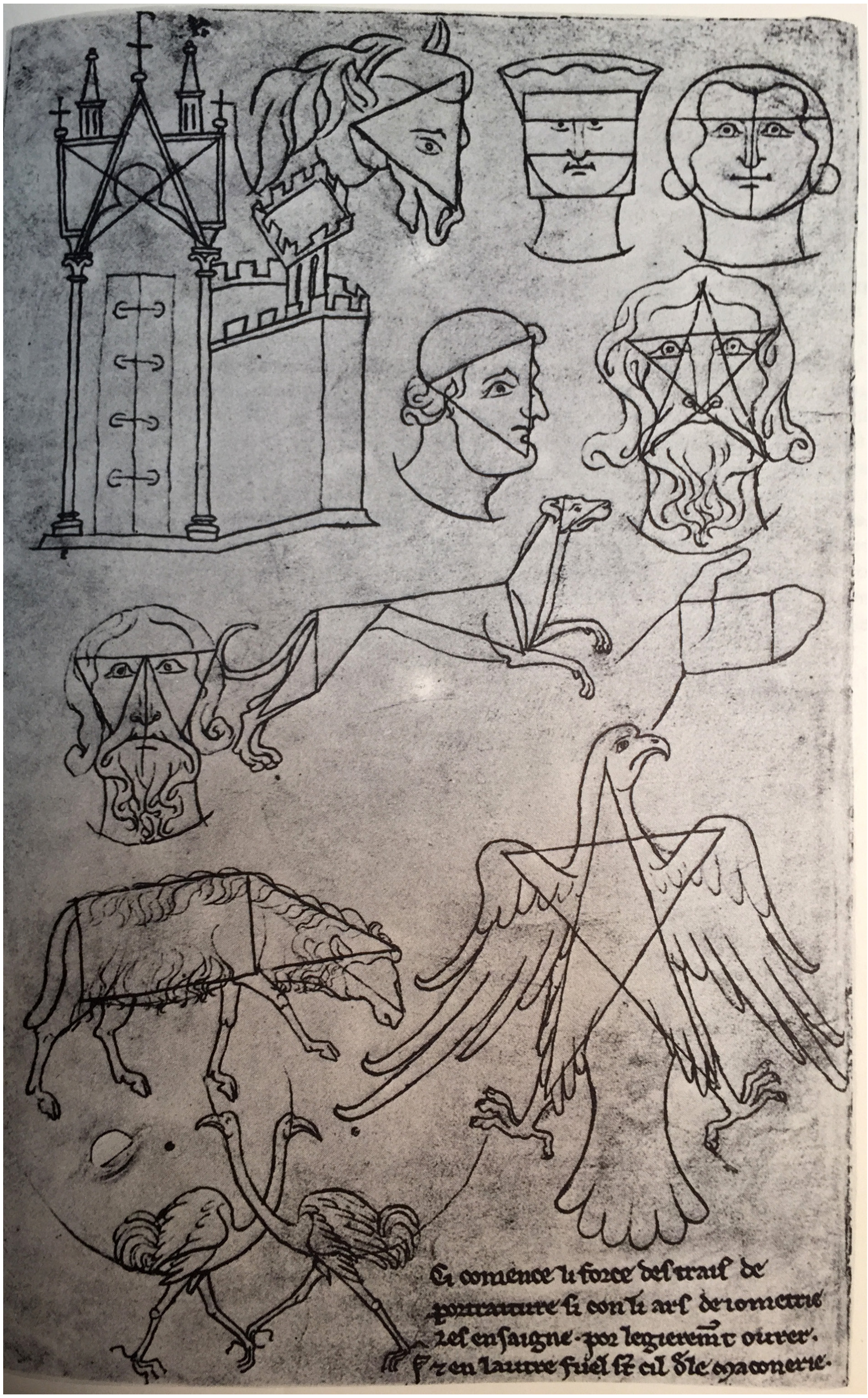


Figure 2.3: Three of the five “Hexahedra” in Lorentz Stöer’s notebook. Staats- und Stadtbibliothek Augsburg, 4 Cod. Aug. 247.



Ci comence li force deſ traiſ de
portraiture ſi con li arſ de iometrie
leſ enſaigne · por legierement ouvrer.
Et en lautre fuel ſont cil de le maconerie.

Figure 2.4: Page V36 from the Lodge Book (*Bauhüttenbuch*) of Villard de Honnecourt (ca. 1175-1240). Bibliothéque Nationale, Paris. Ms fr. 19093.

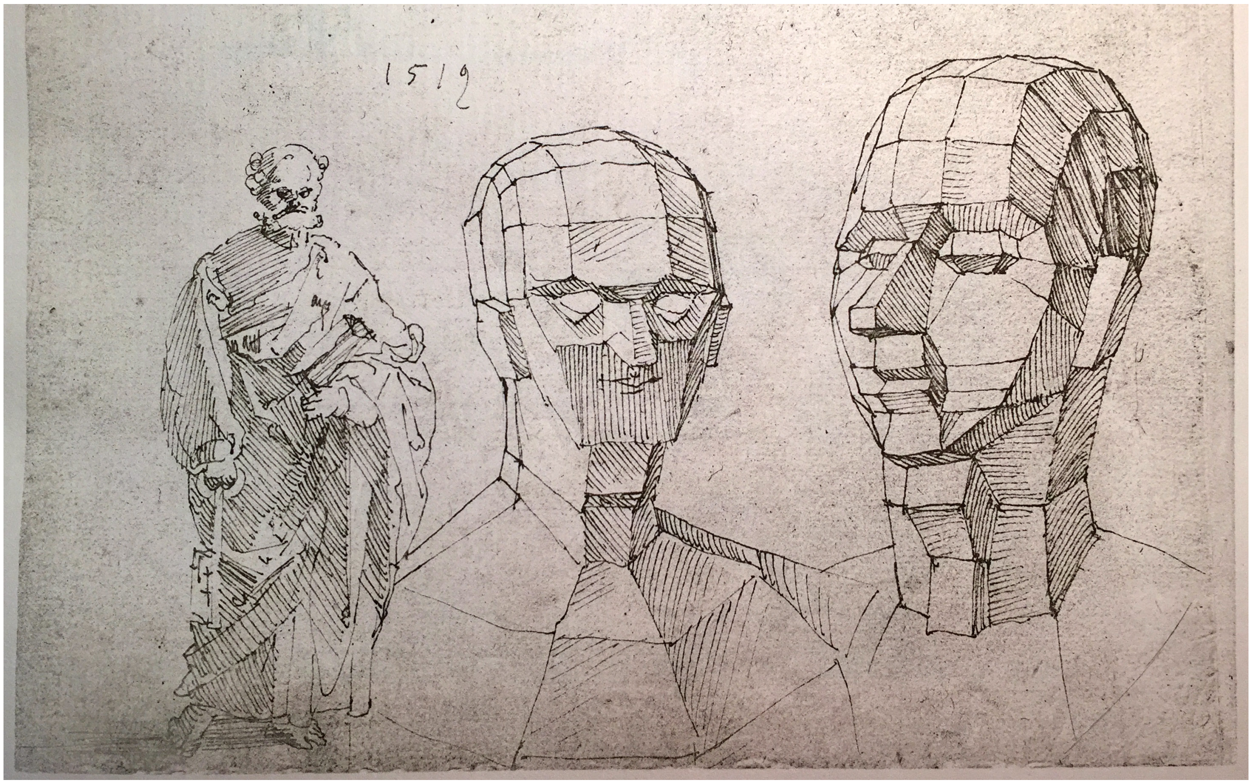


Figure 2.5: Two stereometric heads, Verso of No. 115 by Albrecht Dürer. Dresden Skizzenbuch, Sächsische Landesbibliothek, Dresden.

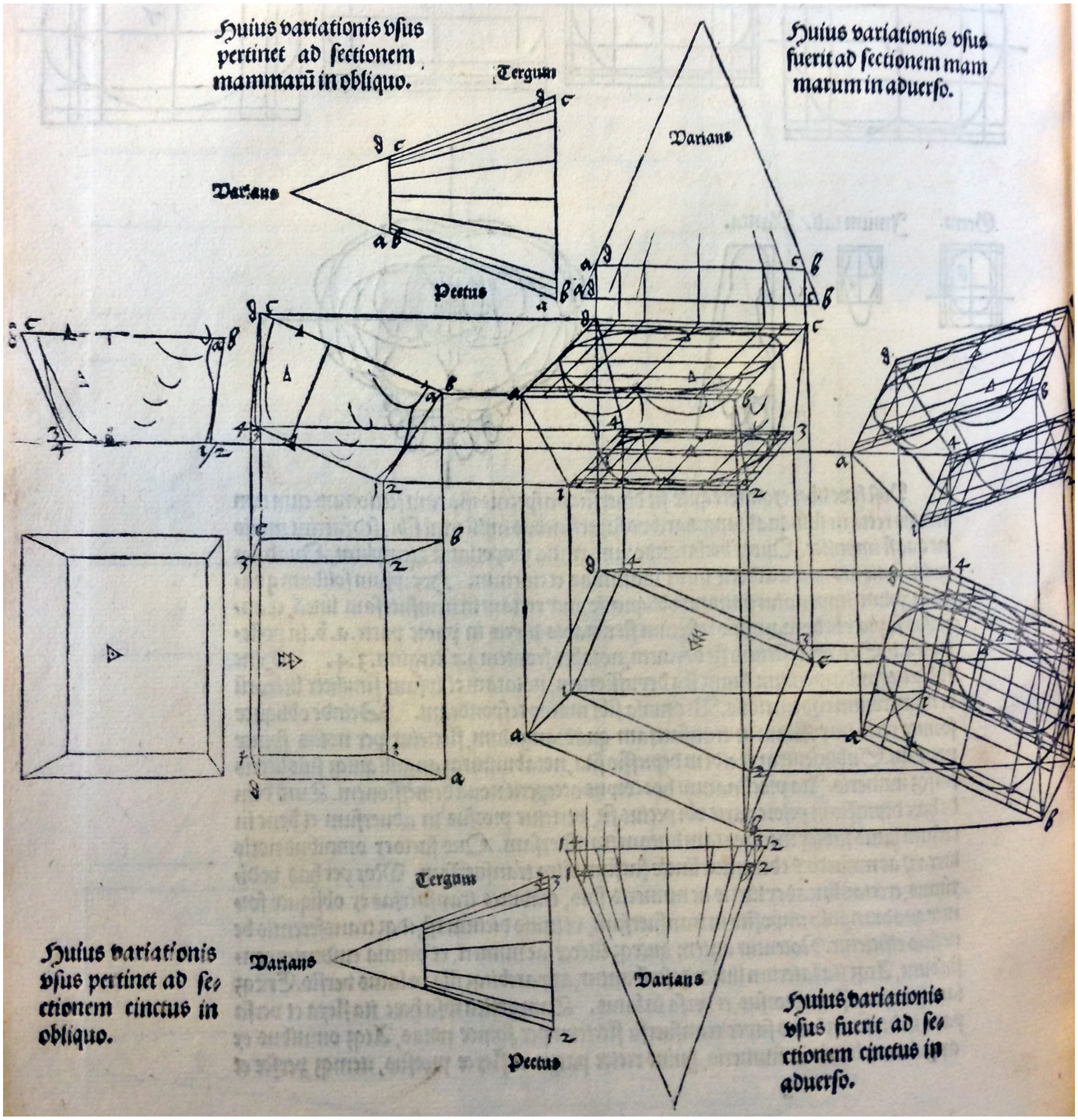


Figure 2.6: Graphic techniques for rotating a man's chest from *Vier Bücher von menschlicher Proportion* (1528) by Albrecht Dürer.

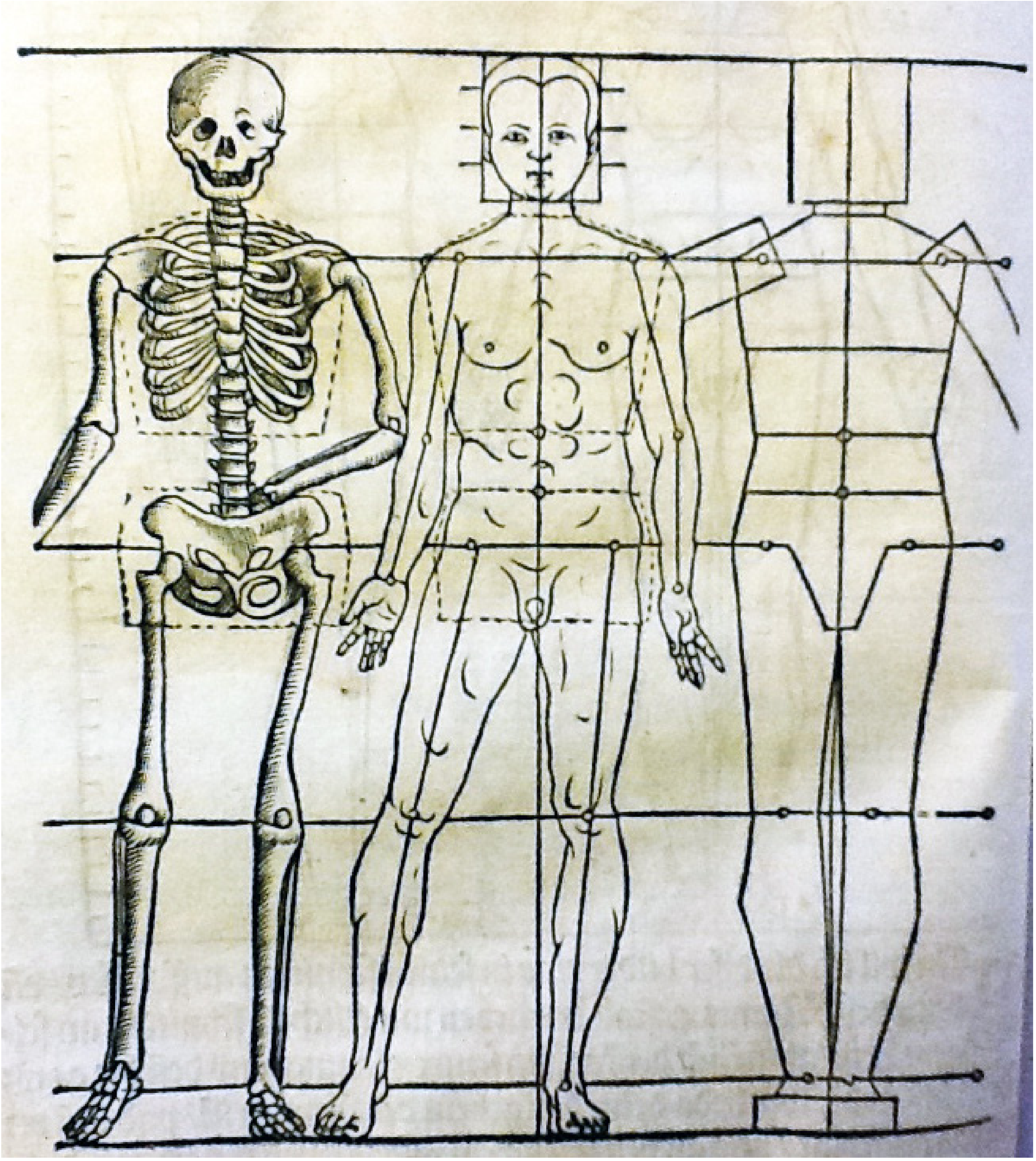


Figure 2.7: Depictions of the human figure flanked by two linear structures - the skeleton beneath the skin and the skeletal (invisible) abstract geometrical lines in which his form can be graphically constructed. *Des Circkels unnd Richtscheyts* (1564) by Heinrich Lautensack (1522-1568).

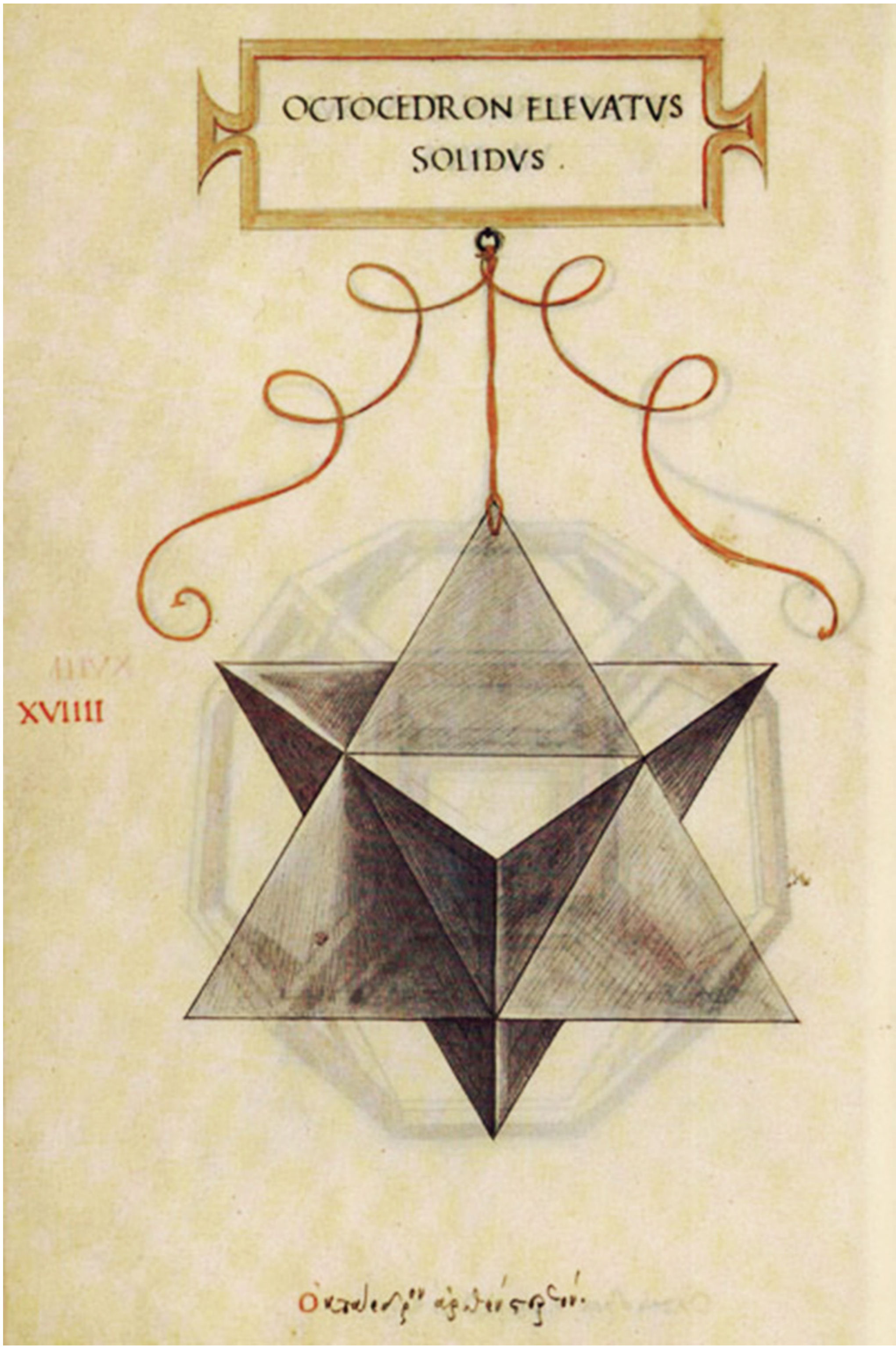


Figure 2.8: Stellated Octohedron from Luca Pacioli's (1447-1517) *De divina proportione* (manuscript completed in Milan 1496-1498). Biblioteca Ambrosiana, Milan.

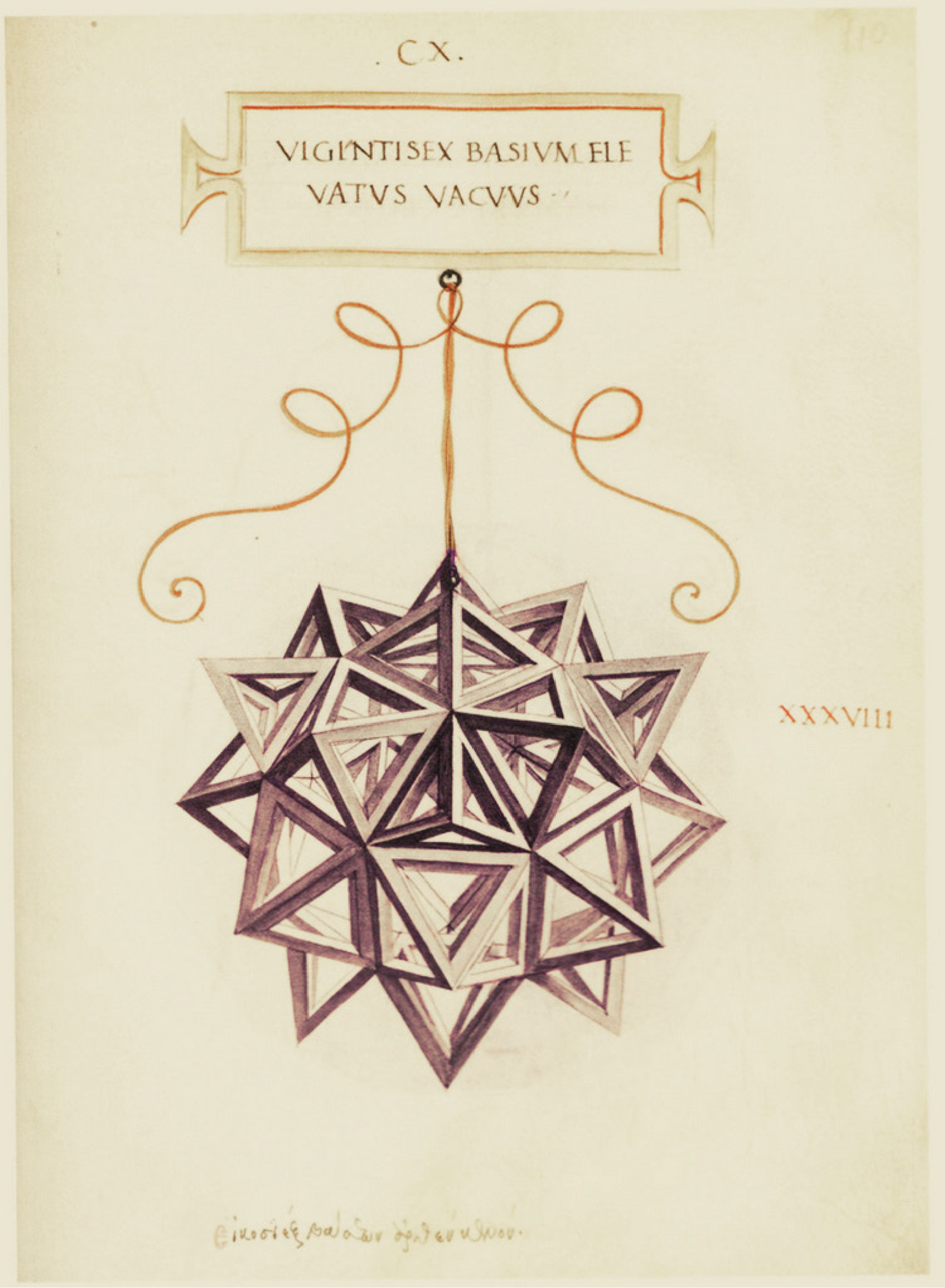


Figure 2.9: Skeletal Elevated Icosidodecahedron from Luca Pacioli's (1447-1517) *De divina proportione* (manuscript completed in Milan 1496-1498). Biblioteca Ambrosiana, Milan.

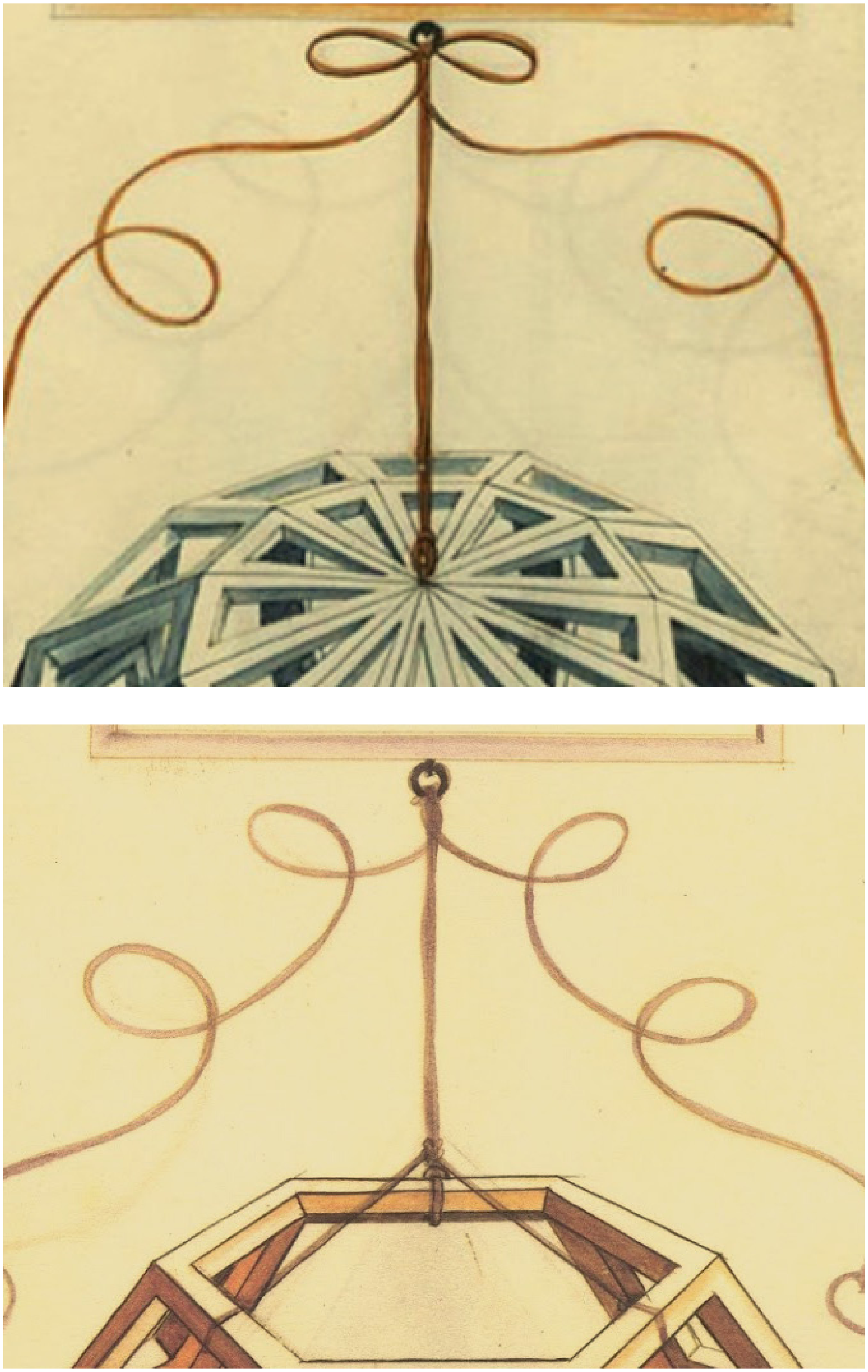
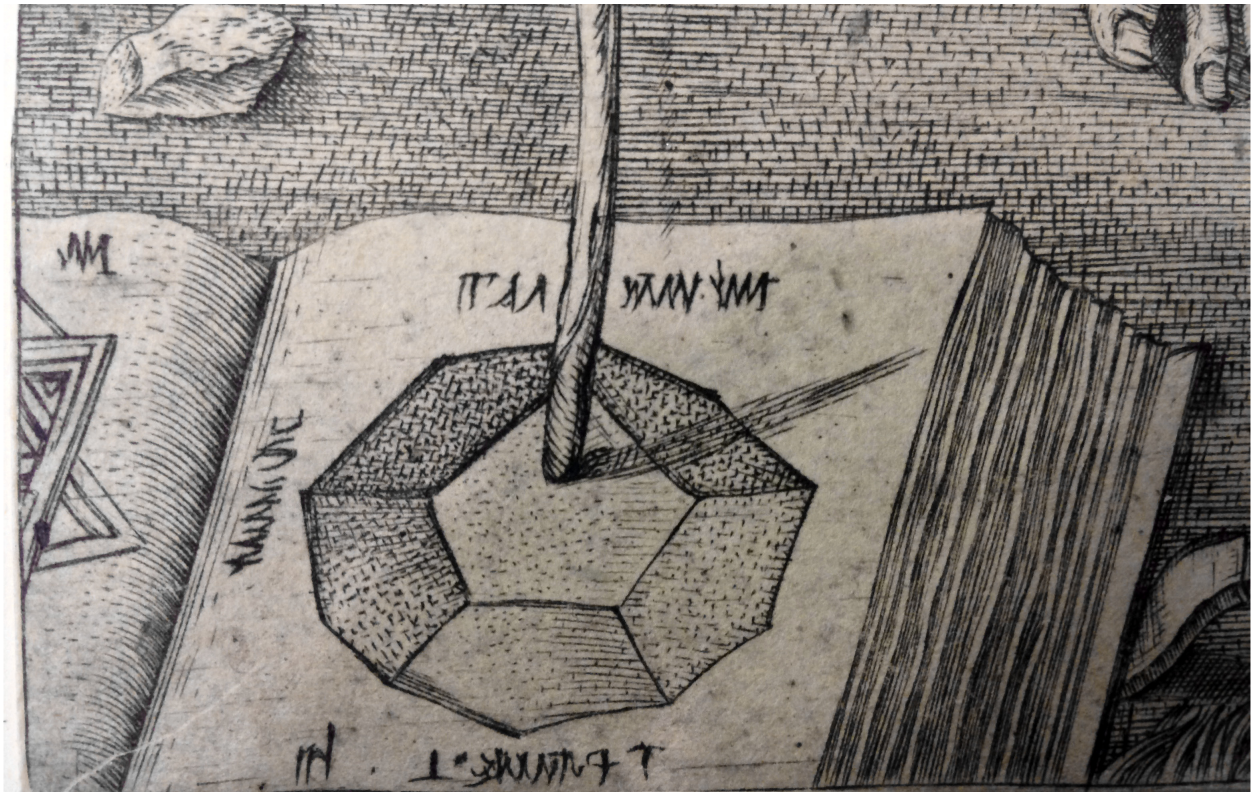


Figure 2.10: Details of hanging supports for the polyhedral models in Luca Pacioli's (1447-1517) *De divina proportione* (manuscript completed in Milan 1496-1498). Biblioteca Ambrosiana, Milan.

Figure 2.11: *Portrait of Luca Pacioli* (ca. 1500) by Jacopo de'Barbari (ca. 1460/1470 - 1516). Capodimonte Museum, Naples, Italy.

Figure 2.12: *Diogenes* (ca. 1524-1527) by Giovanni Jacopo Caraglio (ca. 1500/1505 - 1565) after Parmigianino. Metropolitan Museum of Art, New York. Accession number 17.3.3416.



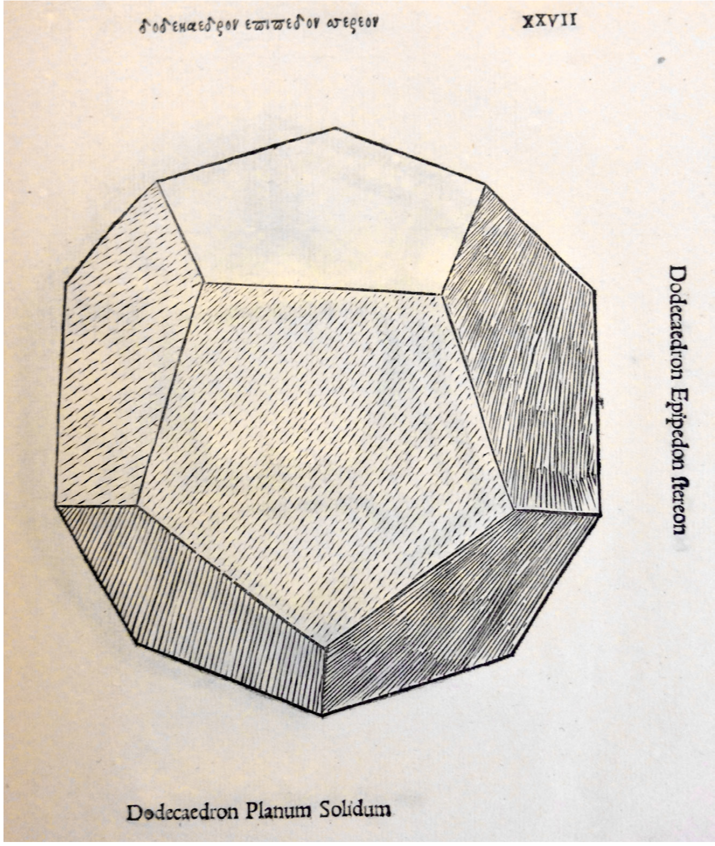


Figure 2.12 continued: Detail from *Diogenes.*

Figure 2.13: Dodecahedron from Pacioli's *De divina proportione* (1509).

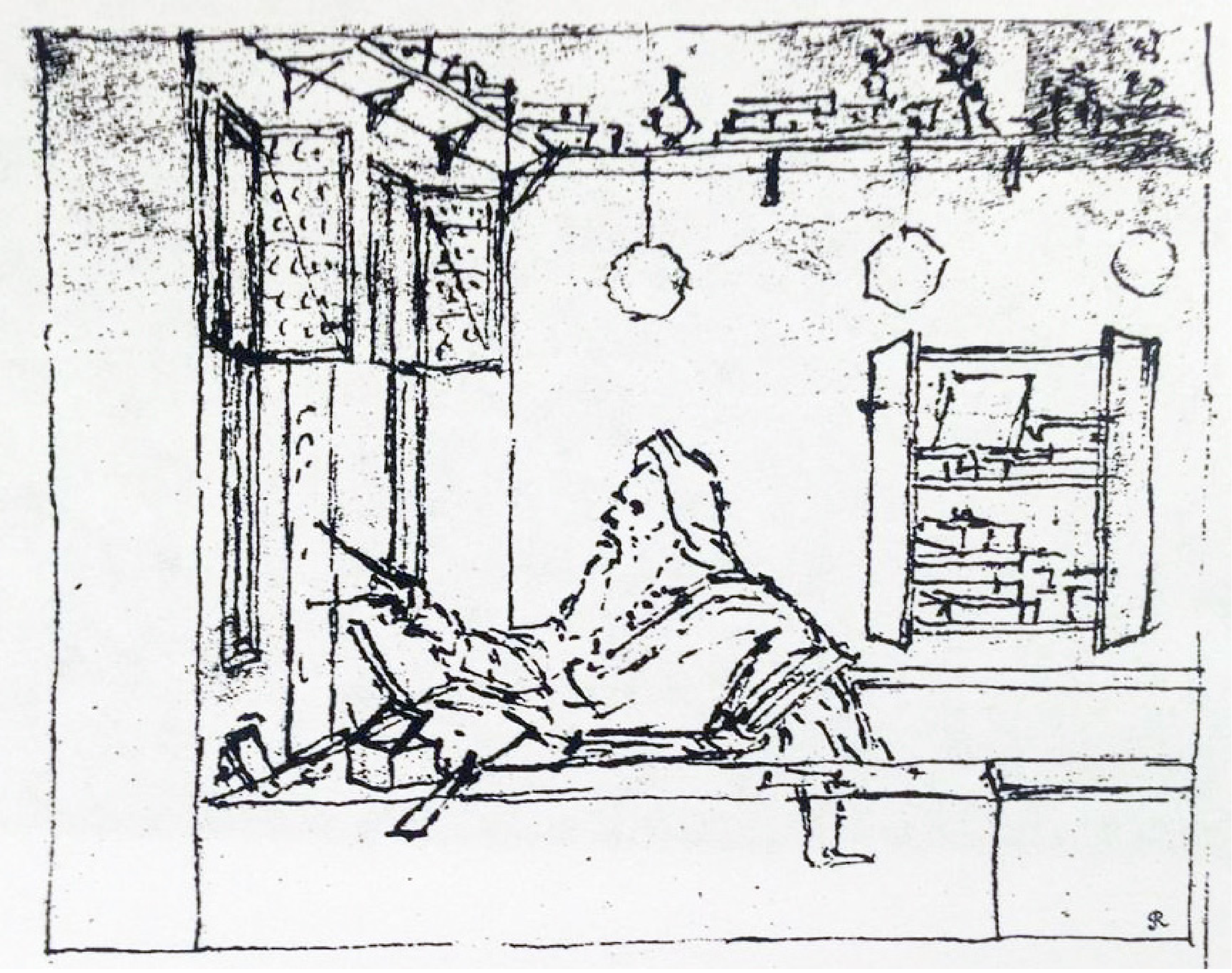


Figure 2.14: Scholar with polyhedra (ca. 1500) by Vittore Carpaccio (1460-1520). Pushkin Museum, Moscow.

Figure 2.15: Image of a mathematician in his workshop (with hanging polyhedra) from Johann Faulhaber's (1580-1635) *Neue Geometrische und Perspectivische Inventiones* (1610).

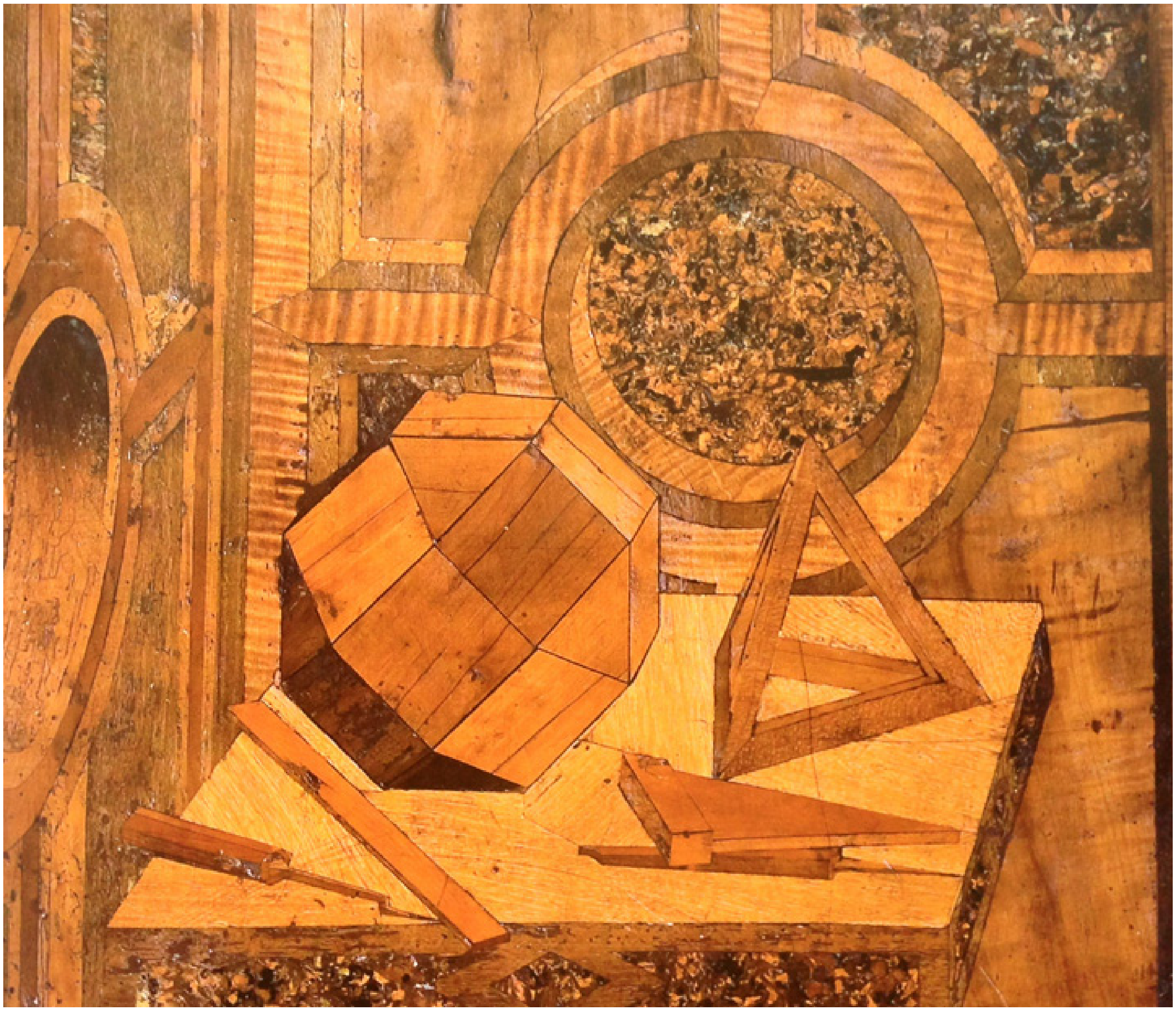


Figure 2.16: Detail from *Tools of Intarsia* (1538) from the Choir door of the Basillica di San Domenico in Bologna, Fra Damiano da Bergamo aka. Damiano di Antoniolo de Zambelli (ca. 1480-1549).

Figure. 2.17: Der *Nürnberger Schreibmeister Johann Neudörffer mit einem Schüler* (1561) by Nicolas Neufchâtel (1527-1590). Germanisches Nationalmuseum, Nuremberg.

Figure 2.17 continued: Detail of a nail supporting the hanging cube in Der *Nürnberger Schreibmeister Johann Neudörffer mit einem Schüler* (1561).

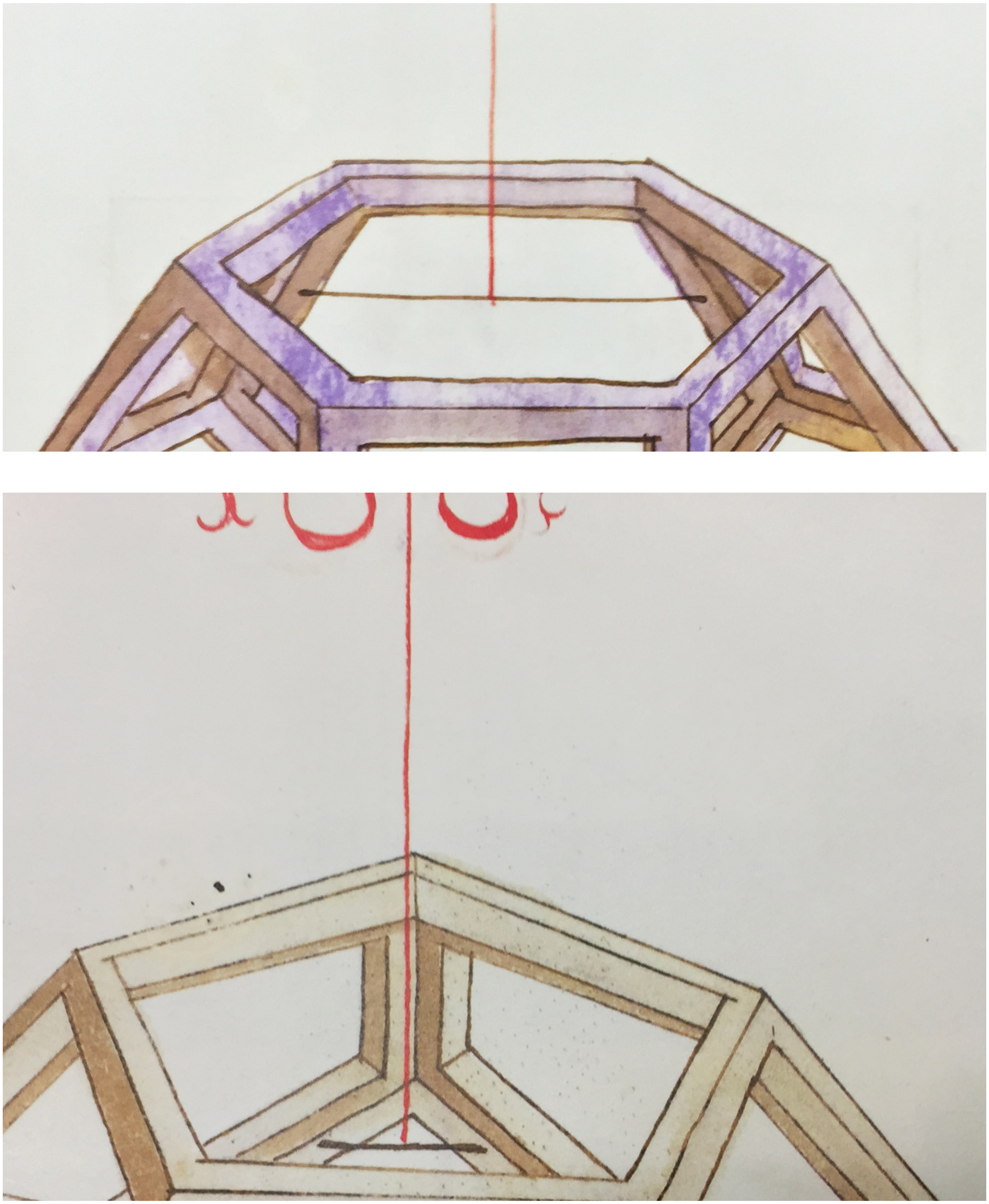


Figure 2.18: Notional hanging supports, details from a second copy of Luca Pacioli's (1447-1517) *De divina proportione*, manuscript in the Bibliothèque de Genève, Geneva.

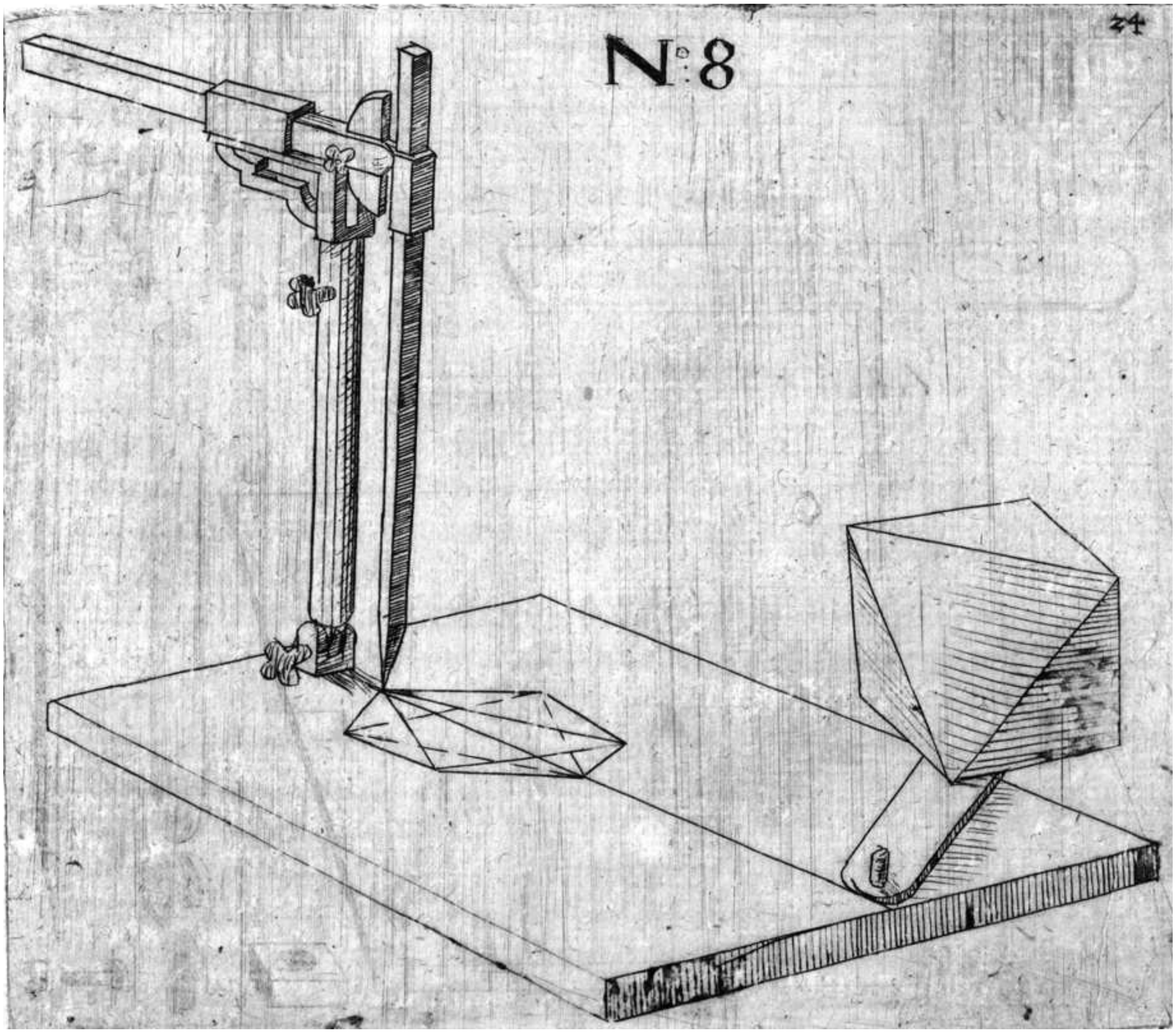


Figure 2.19: Image of a drawing device, Image No. 8, *Perspectivische Reiss Kunst* (1625), Peter Halt.

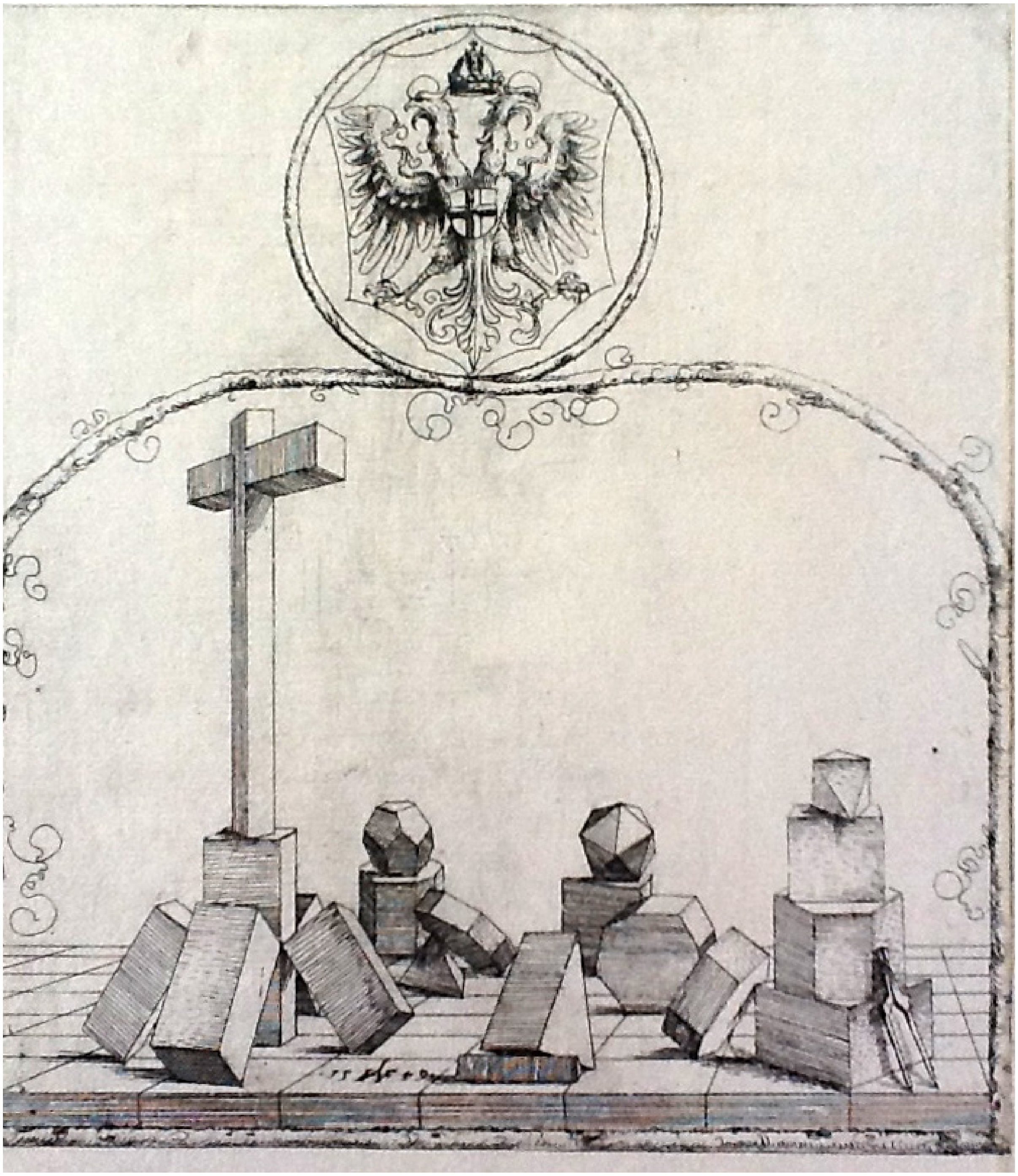


Figure 2.20: Etching of geometrical forms, including the imperial eagle of the Habsburg Emperor, by Augustin Hirschvogel from *Perspectiva* (1543).

Figure 2.21: Wenzel Jamnitzer (16th century), Jost Amman, Metropolitan Museum of Art. Accession Number 56.510.2.

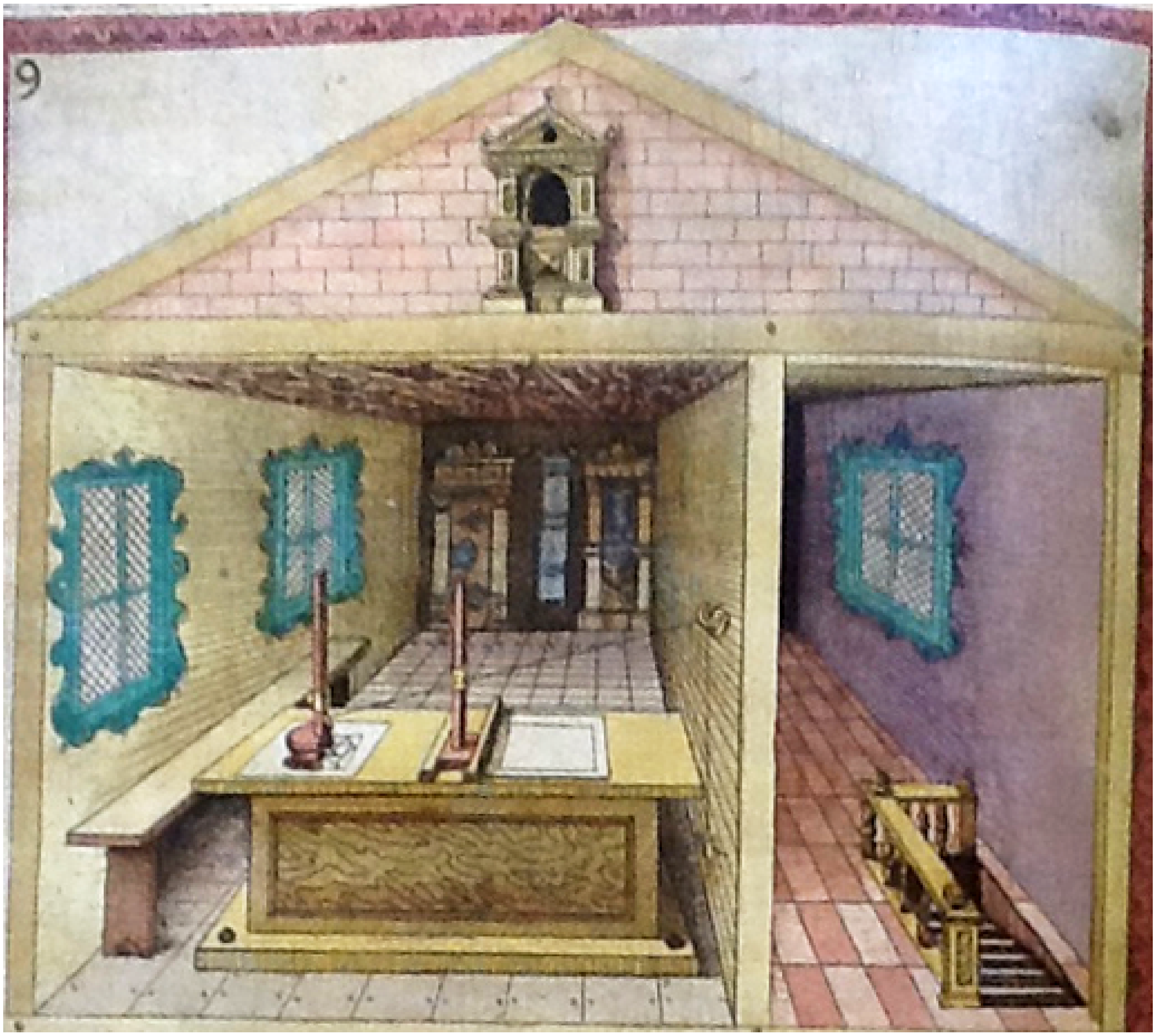


Figure 2.22: A colored representation of Wenzel Jamnitzer's workspace for making perspectival drawings by Paul Pfinzing, from *Ein Schöner Kurzer Extract der Geometriae und Perspectiva* (1599).

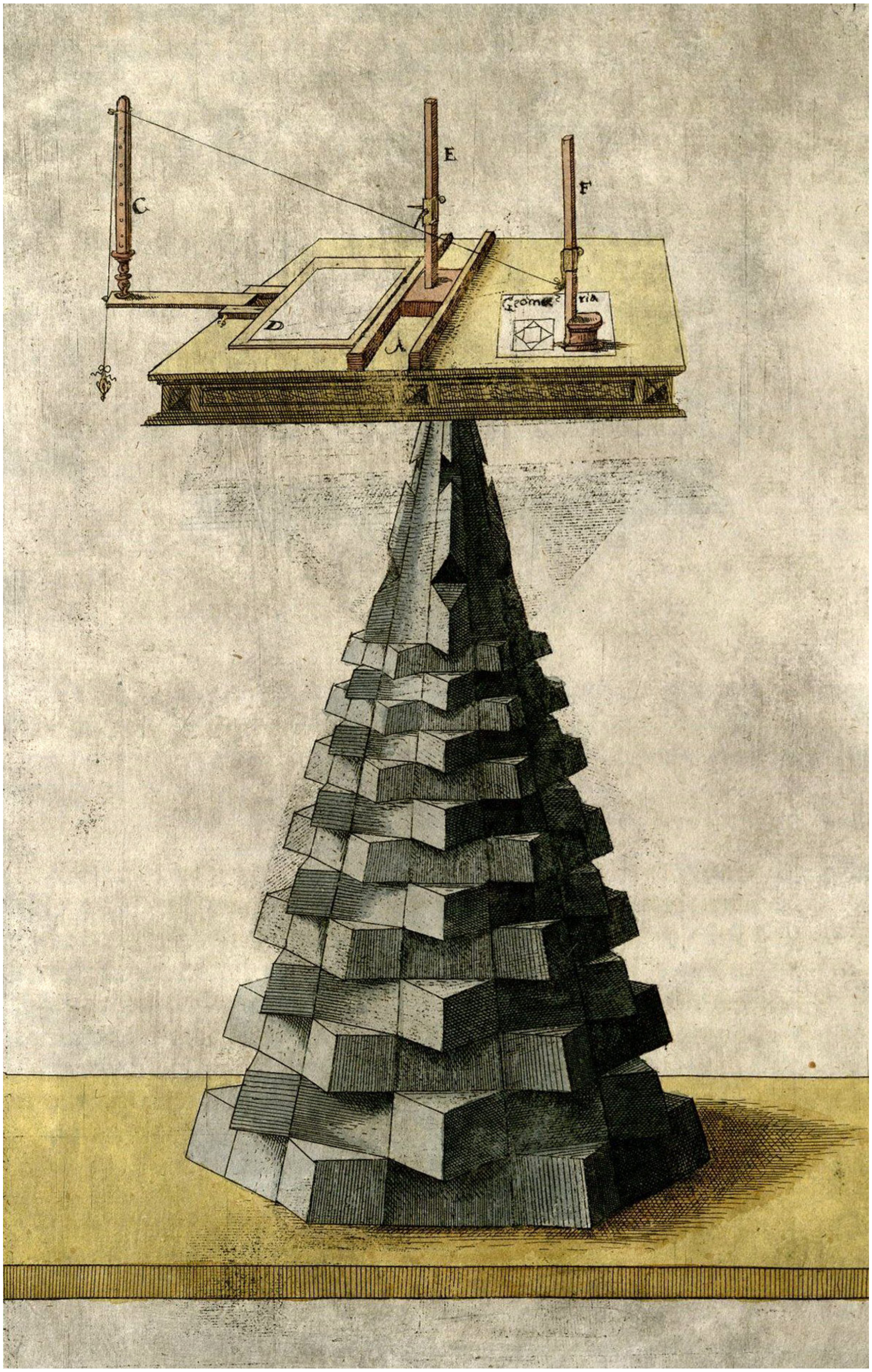
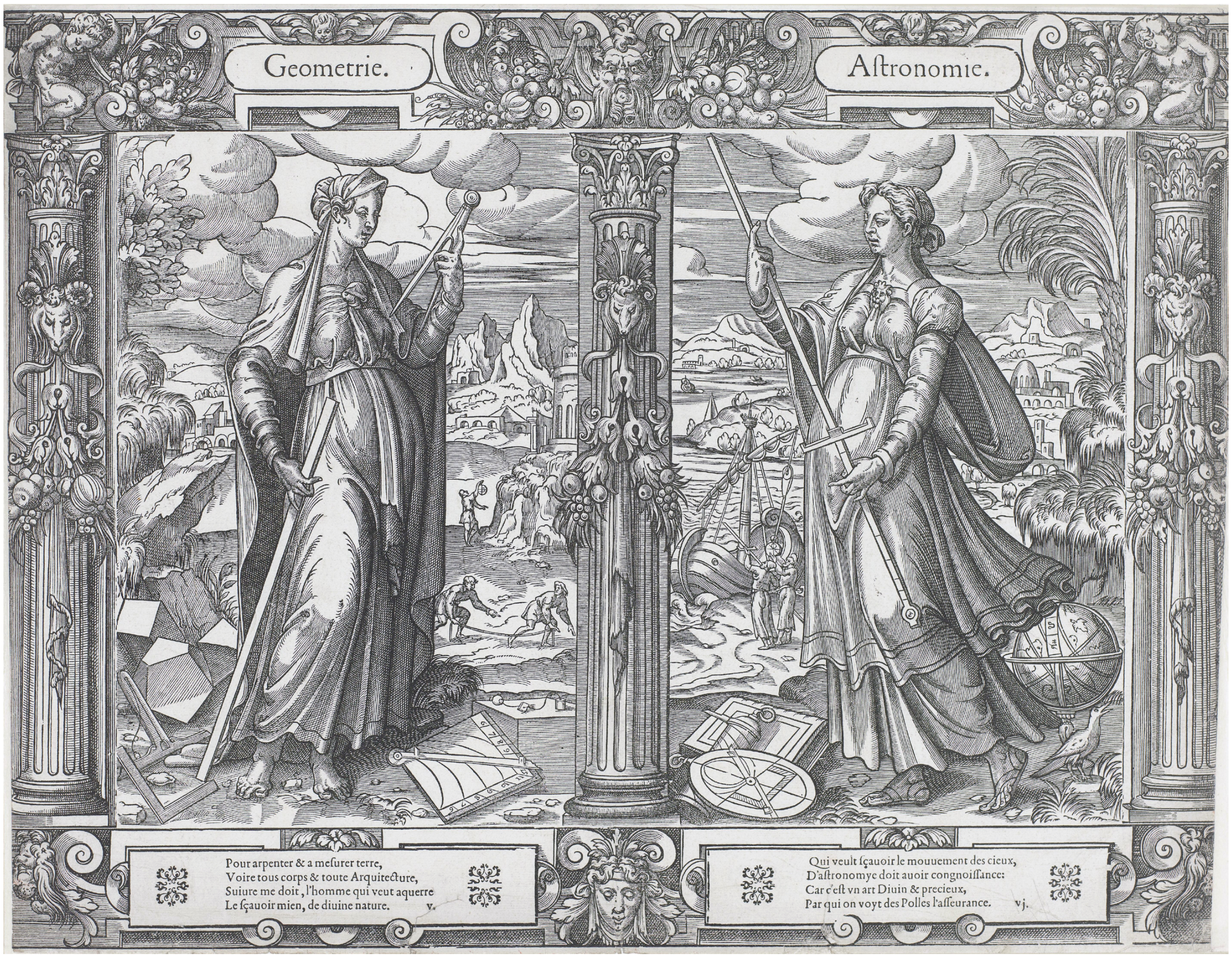


Figure 2.23: Image from Jamnitzer's Perspectiva, with drawing apparatus balanced on top, Paul Pfinzing, *Ein Schöner kurzer Extract der Geometriae und Perspectivae* (1599).

Figure 2.24: Melencolia I (1514) by Albrecht Dürer. Metropolitan Museum of Art. Accession Number 43.106.1.

Figure 2.25: *Geometrie* (late 16th century) by Johann Sadeler (I) (1560-1600). Rijksmuseum, Amsterdam. Object Number RP-P-OB-7493.



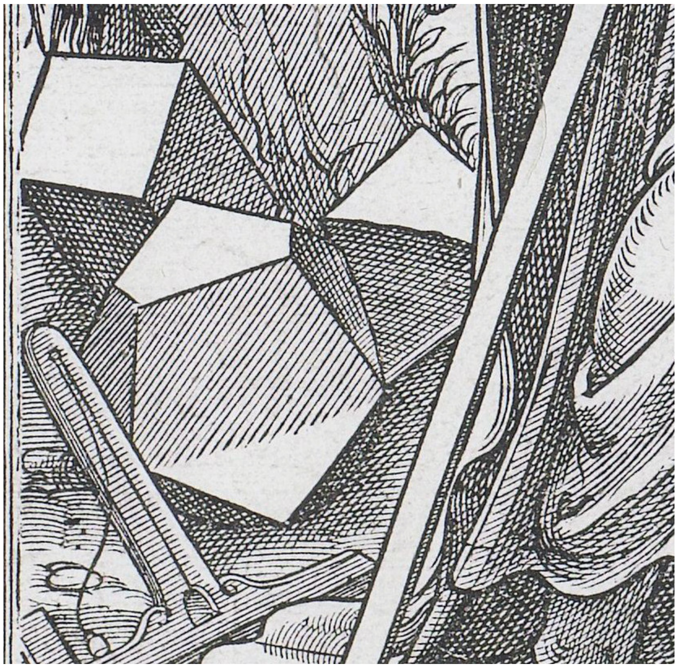
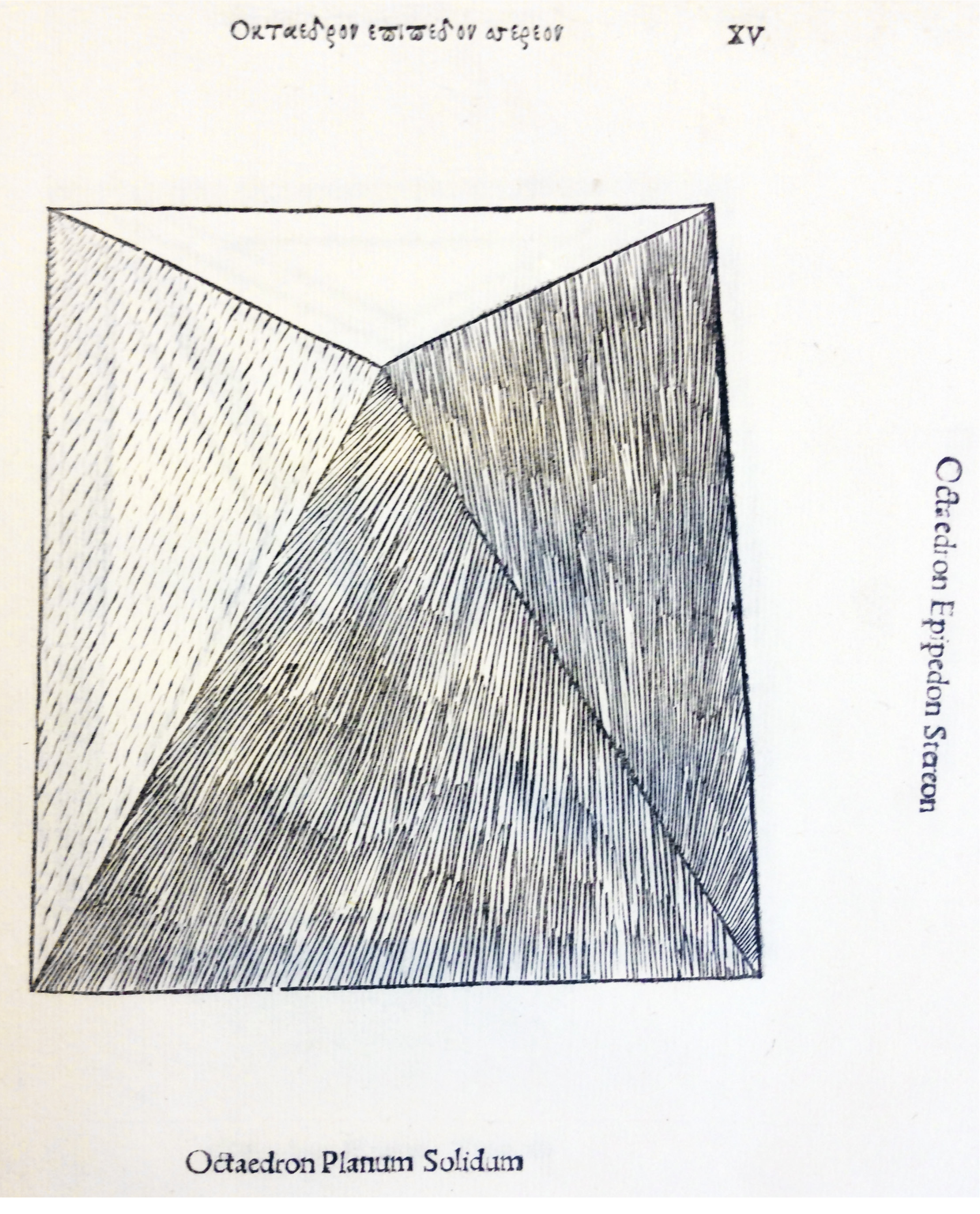


Figure 2.26 (top): *Geometry and Astronomy* (16th century) by Marin Bonnemer (died 1584). Rijksmuseum, Amsterdam.
Object Number RP-P-OB-33.909.
(bottom): Detail of polyhedra

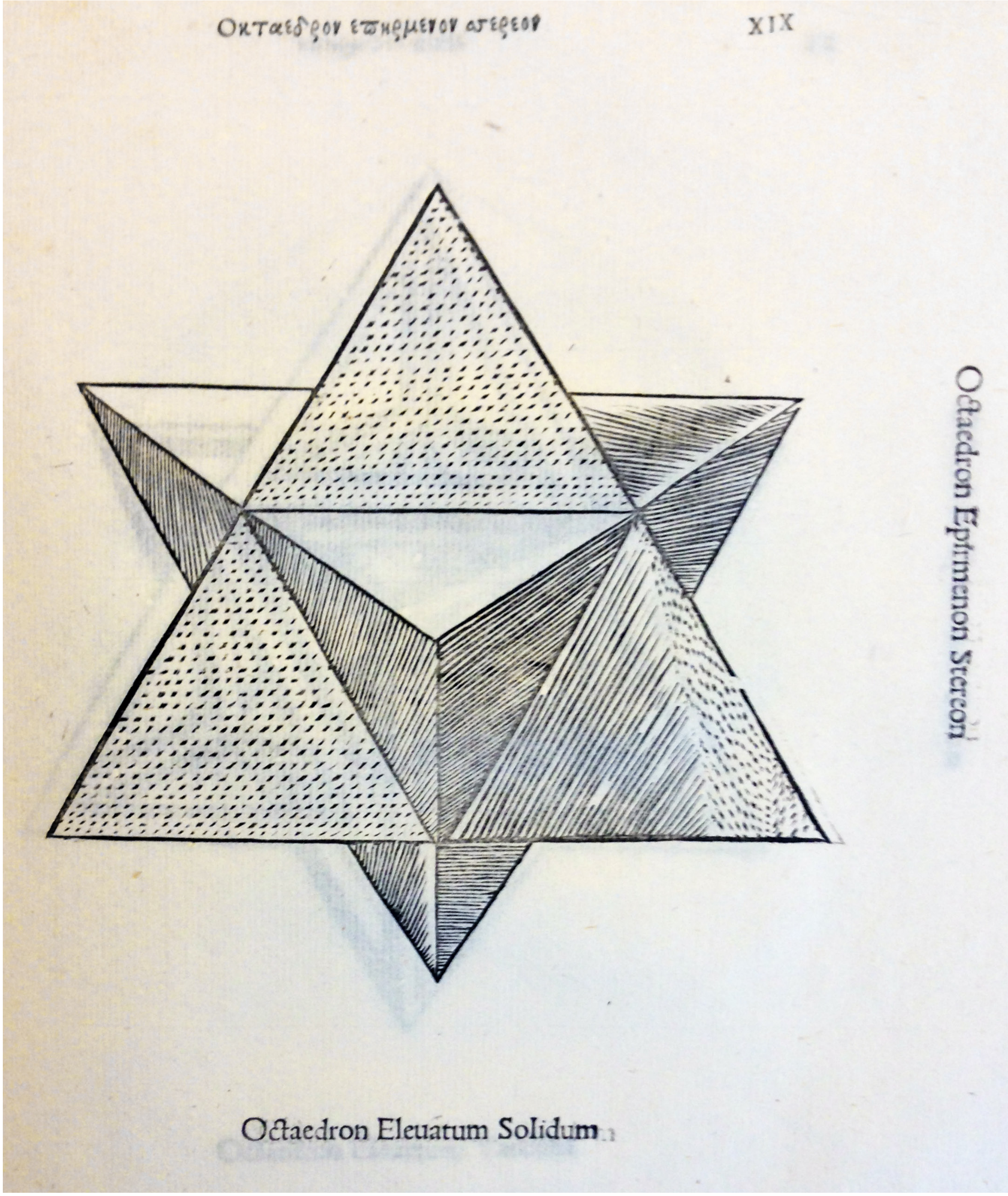


Οκταεδρον επιπεδον στερεον XV

Octaedron Epipedon Stereon

Octaedron Planum Solidum

Figure 2.27: Octohedron from the printed edition of Luca Pacioli's (1447-1517) *De divina proportione* (1509).



Οκταεδρον επηρμενον στερεον XIX

Octaedron Eptmenon Stereon

Octaedron Eleuatum Solidum

Figure 2.28: Stellated Octohedron from the printed edition of Luca Pacioli's (1447-1517) *De divina proportione* (1509).

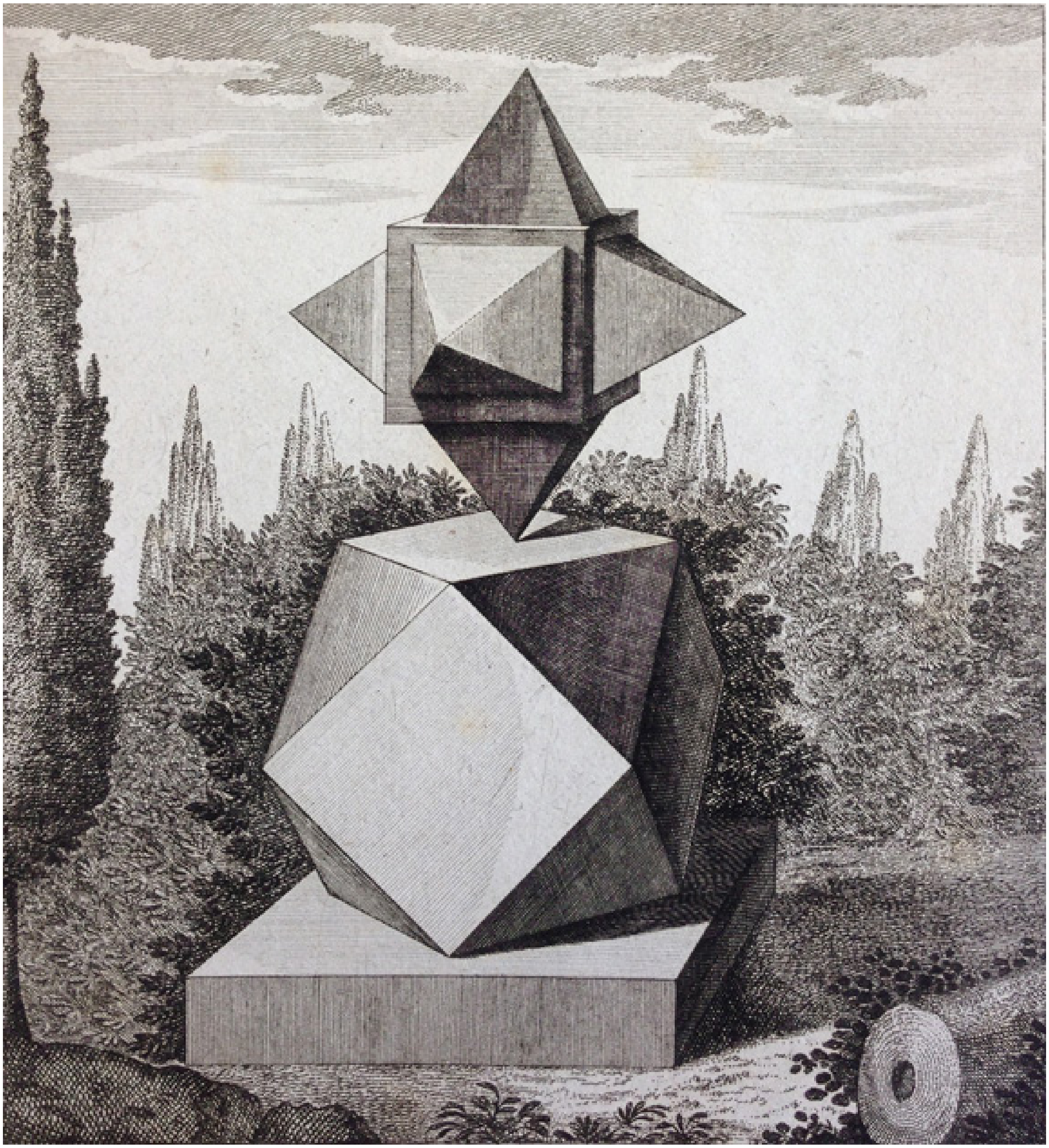
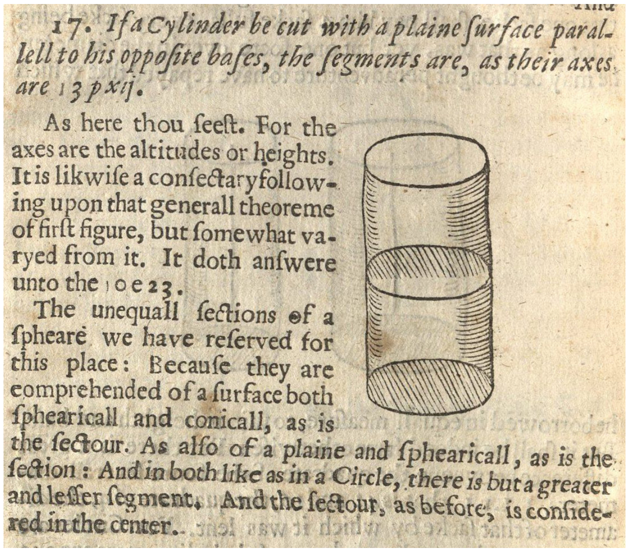


Figure 2.29: Image of stacked polyhedral geometry, *Anleitung zur Linearperspective* (1812), Christoph Andreas Nilson.



17. *If a Cylinder be cut with a plaine ſurface parallell to his oppoſite baſes, the ſegments are, as their axes are 13 p xij.*

As here thou ſeeſt. For the axes are the altitudes or heights. It is likwiſe a conſectary following upon that generall theoreme of firſt figure, but ſomewhat varyed from it. It doth anſwere unto the 10 e 23.

The unequall ſections of a ſpheare we have reſerved for this place: Becauſe they are comprehended of a ſurface both ſphearicall and conicall, as is the ſectour. As alſo of a plaine and ſphearicall, as is the ſection: And in both like as in a Circle, there is but a greater and leſſer ſegment. And the ſectour, as before, is conſidered in the center.

Figure 2.30: Instructions for measuring a cylinder bisected by a plane and a real world scenario of a well being surveyed. *Via regio ad geometriam* (1636), Petrus Ramus.

Figure 2.31: Melencolia (1539) by Hans Sebald Beham (1500-1550). Metropolitan Museum of Art. Accession Number 66.529.45.

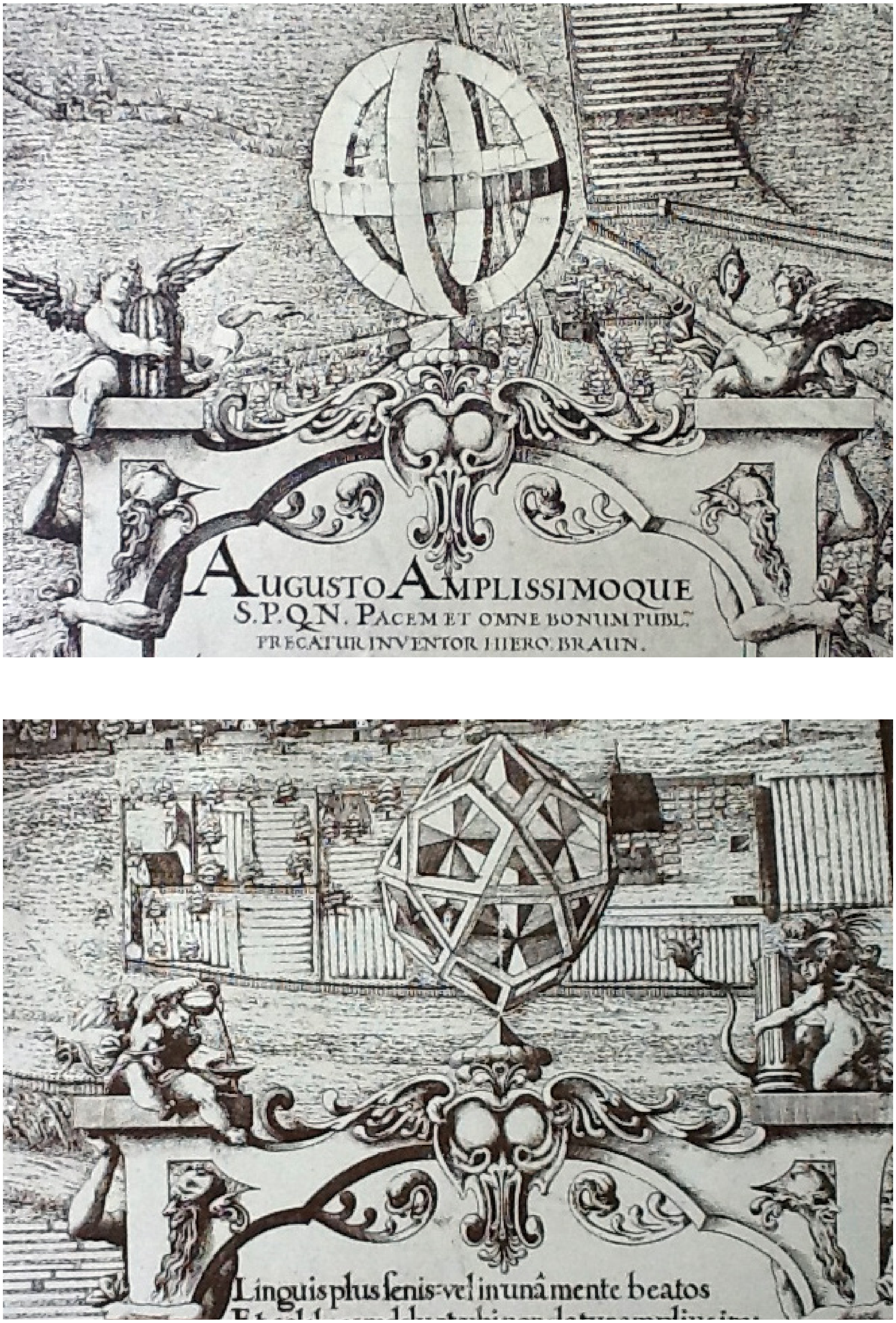


Figure 3.1: Details from *Prospekt der Reichsstadt Nürnberg* (1608), Hieronymous Braun.

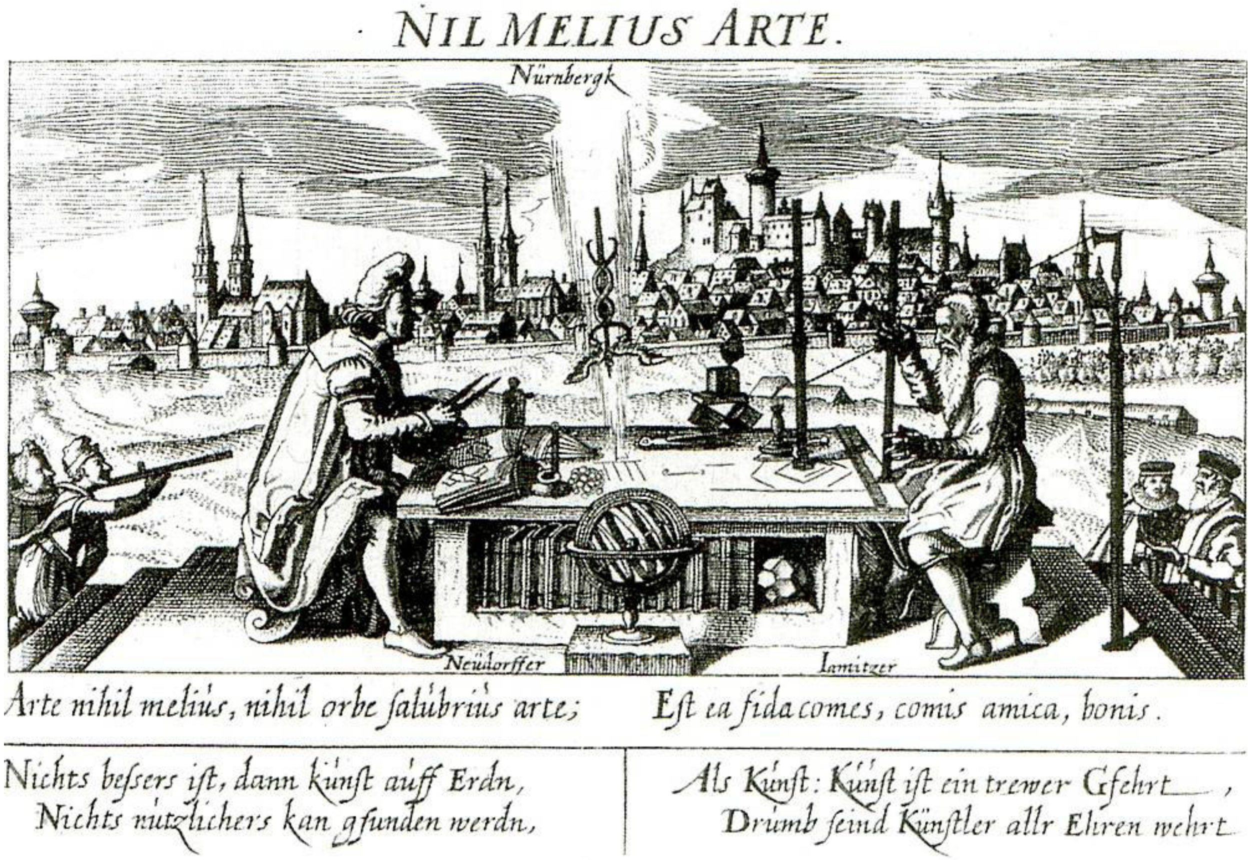


Figure 3.2: Image of Wenzel Jamnitzer and Johann Neudörffer from Daniel Meisner's *Politisches Schatzkästlein* (1625-1626).

Figure 3.3: Allegory of Knowledge in *Margarita Philosophica* (1503), Gregor Reisch.

GEOMETRIA.

DAS BVCH GEOMETRIA IST MEIN NAMEN
ALL FREYE KVNST AVS MIR ZVM ERSTEN KAMEN
ICH BRING ARCHITECTVRA VND PERSPECTIVA ZVSAMEN.

15 43

MIT RO. KO. MA. ALLER GENEDIGISTEN PRIVILEGIA, MIR NOCH MEINEN EHLICHEN LEYBS ERBEN NIT NACH ZV DRVCKEN FERFAST.

Figure 3.4: Title page of *Geometria* by Augustin Hirschvogel (1543).

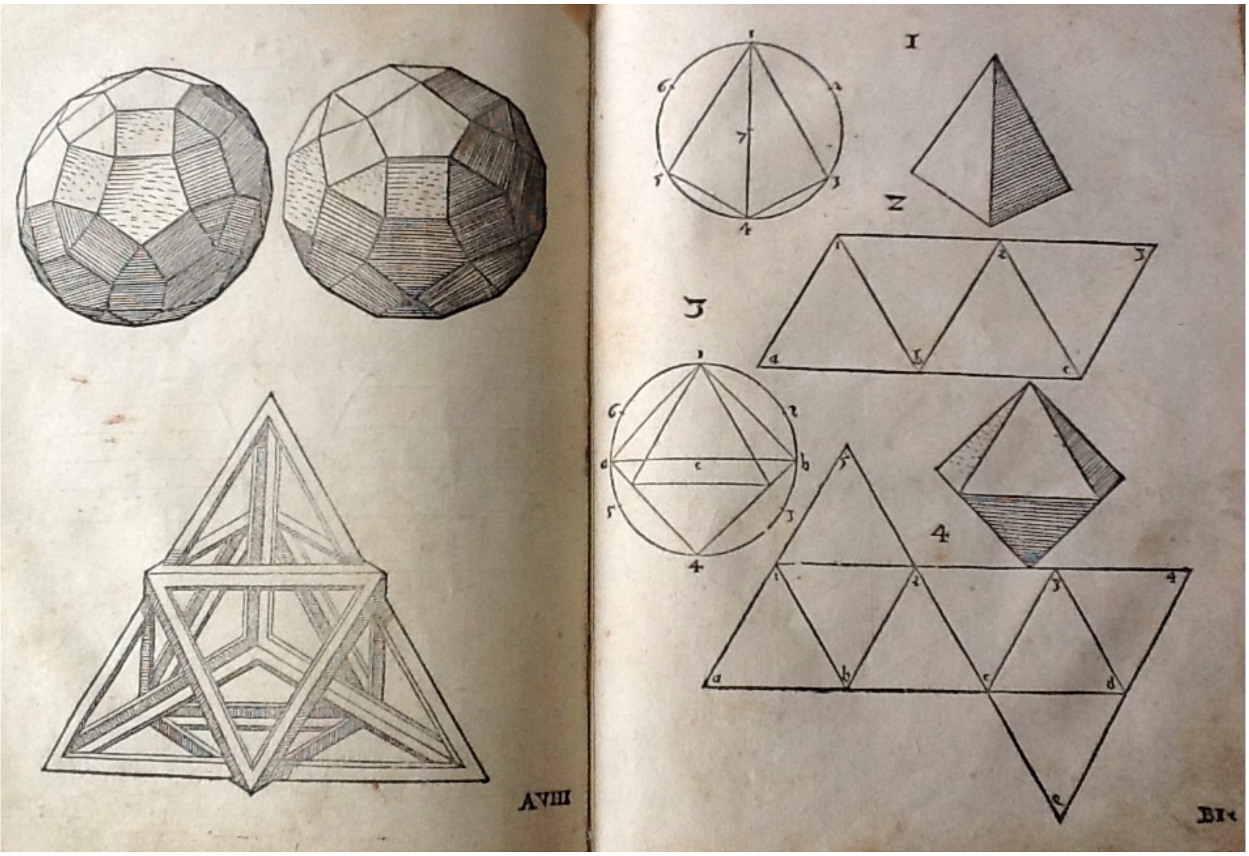


Figure 3.5: Pages from Hirschvogel's *Geometria* (1543).

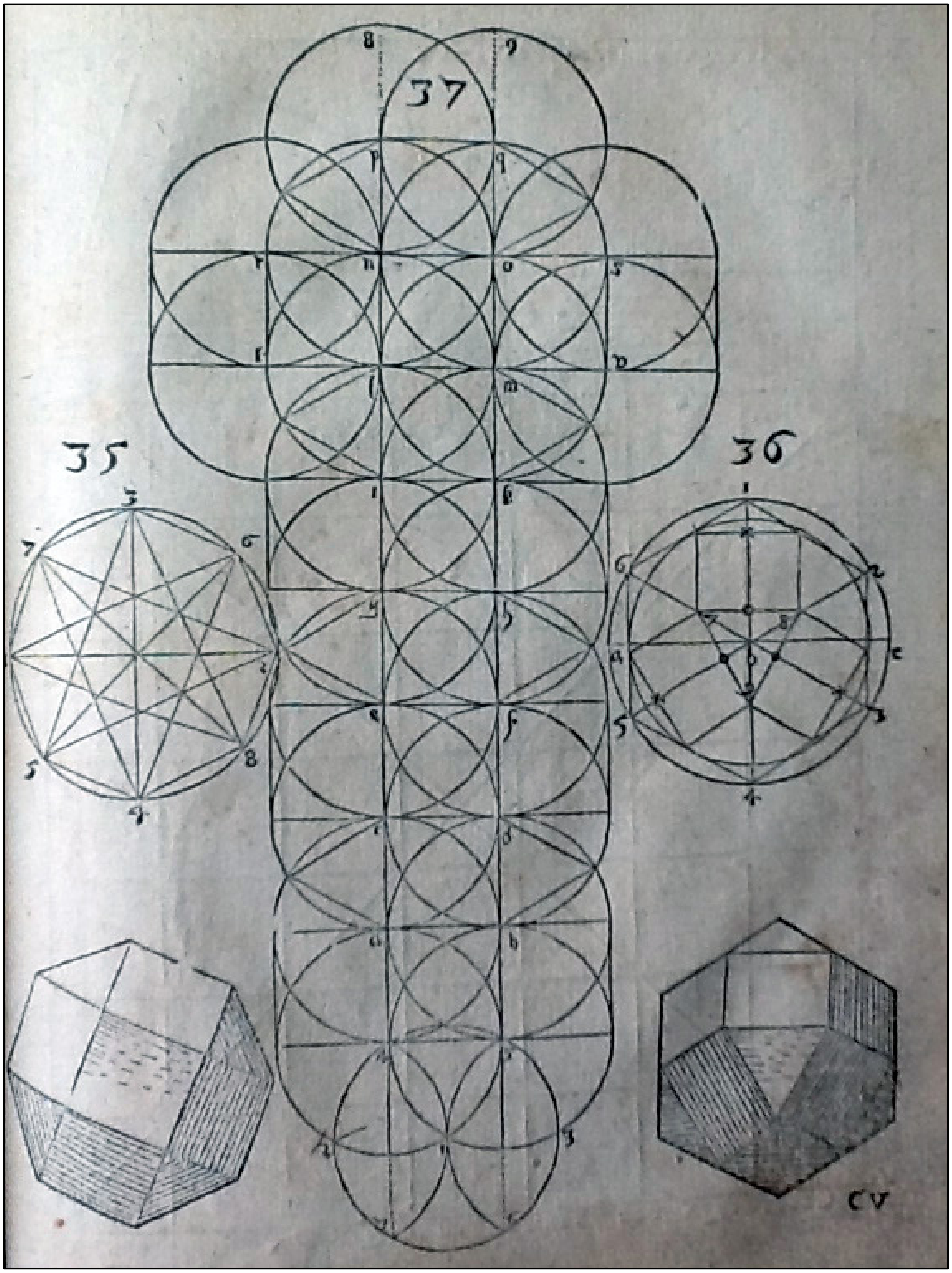
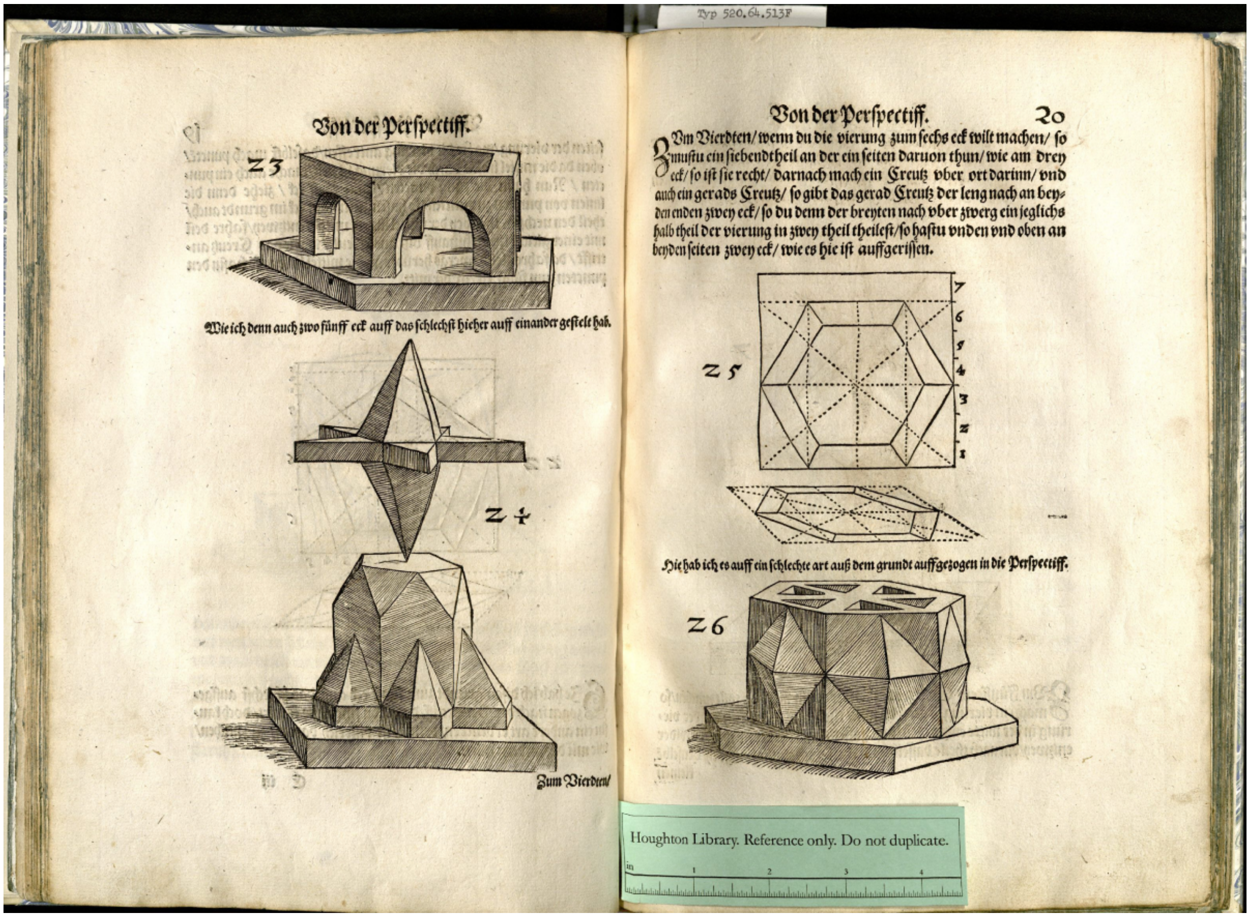


Figure 3.6: An unfolded truncated dodecahedron from Hirschvogel's *Geometria* (1543).



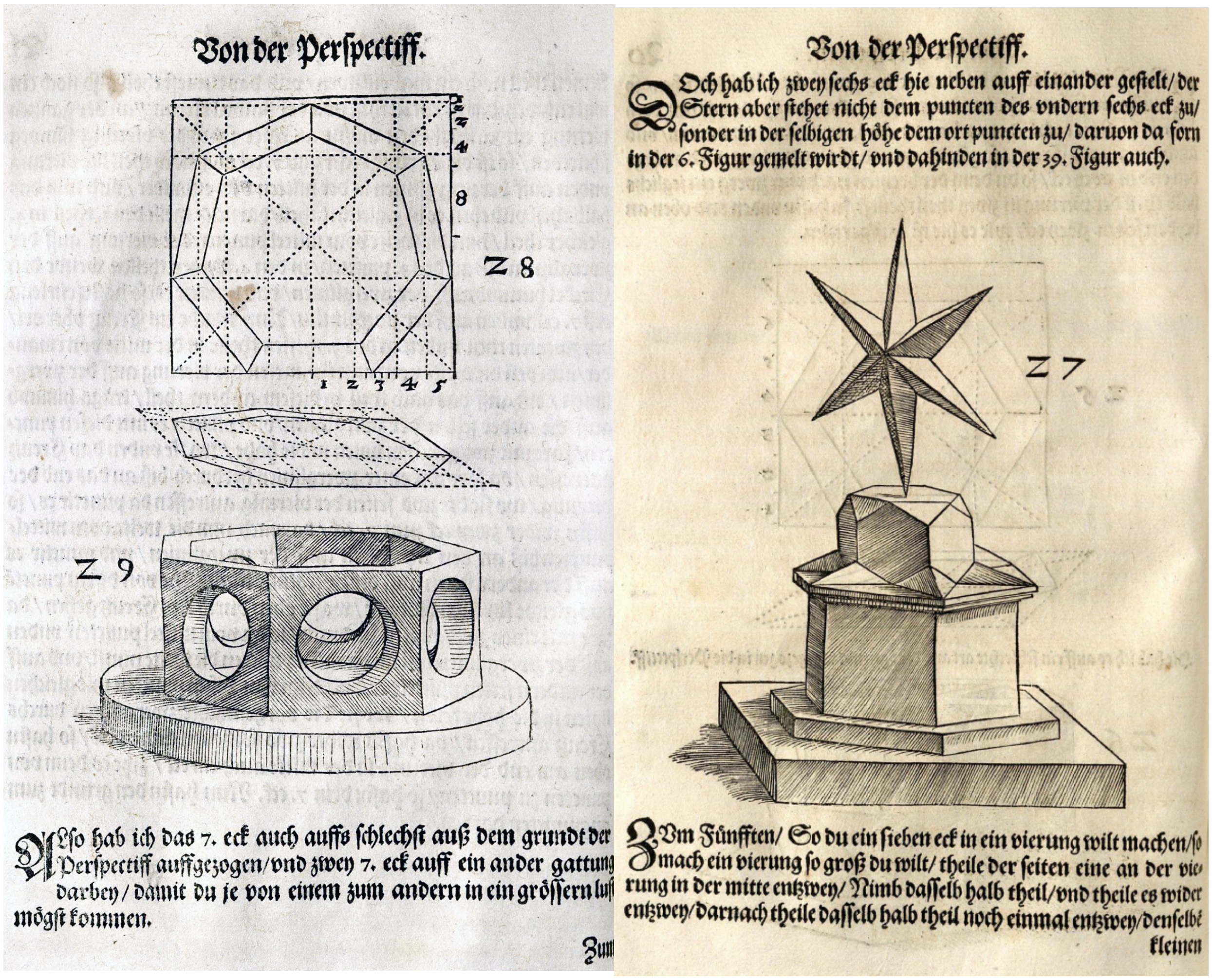
Wie ich denn auch zwo fünff eck auff das ſchlechſt hieher auff einander geſtelt hab.

Zum Vierdten/

Zum Vierdten/ wenn du die vierung zum ſechs eck wilt machen/ ſo muſtu ein ſiebendtheil an der ein ſeiten daruon thun/ wie am drey eck/ ſo iſt ſie recht/ darnach mach ein Creutz vber ort darinn/ vnd auch ein gerads Creutz/ ſo gibt das gerad Creutz der leng nach an beyden enden zwey eck/ ſo du denn der breyten nach vber zwerg ein ieglichs halb theil der vierung in zwey theil theileſt/ ſo haſtu vnden vnd oben an beyden ſeiten zwey eck/ wie es hie iſt auffgeriſſen.

Hie hab ich es auff ein ſchlechte art auß dem grundt auffgezogen in die Perſpectiff.

Figure 3.7: Pages depicting perspectival construction from Heinrich Lautensack's *Des Circkels vnnd Richtscheyts auch der Perspectiua vnd Proportion der Menschen vnd Rosse kurtze doch gründliche Vnderweisung* (1564).



Von der Perſpectiff.

ALſo hab ich das 7. eck auch auffs ſchlechſt auß dem grundt der Perſpectiff auffgezogen/vnd zwey 7. eck auff ein ander gattung darbey/ damit du je von einem zum andern in ein gröſſern luſ mögſt kommen.

Zum

Von der Perſpectiff.

DOch hab ich zwey ſechs eck hie neben auff einander geſtelt/ der Stern aber ſtehet nicht dem puncten des vndern ſechs eck zu/ ſonder in der ſelbigen höhe dem ort puncten zu/ daruon da forn in der 6. Figur gemelt wirdt/ vnd dahinden in der 39. Figur auch.

ZVm Fünfften/ So du ein ſieben eck in ein vierung wilt machen/ ſo mach ein vierung ſo groß du wilt/ theile der ſeiten eine an der vierung in der mitte entzwey/ Nimb daſſelb halb theil/vnd theile es wider entzwey/darnach theile daſſelb halb theil noch einmal entzwey/denſelbē

kleinen

Figure 3.8: Pages from Lautensack's *Des Circkels vnnd Richtscheyts…* (1564).

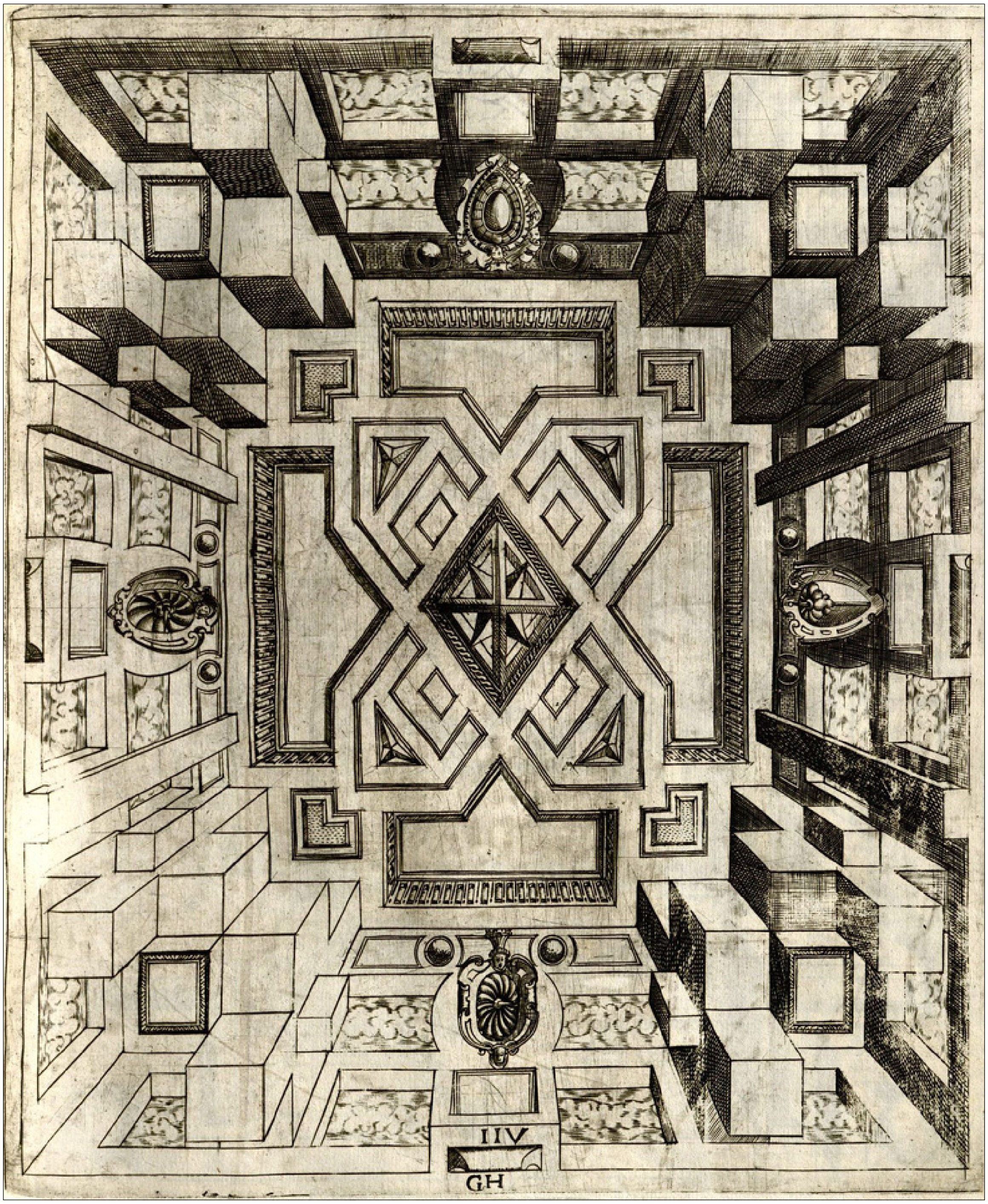


Figure 3.9: Geometrical ceiling design by Georg Hass, from *Künstlicher, und zierlicher newer vor nie gesehener, funfftzig perspectifischer, stück oder Boden...*(1583).

Figure 3.10: Portrait of Oswald von Eck with polyhedra (1553), Hanns Lautensack (ca. 1520-1564/66), Acc. Num. 41.1.143, Metropolitan Museum of Art.

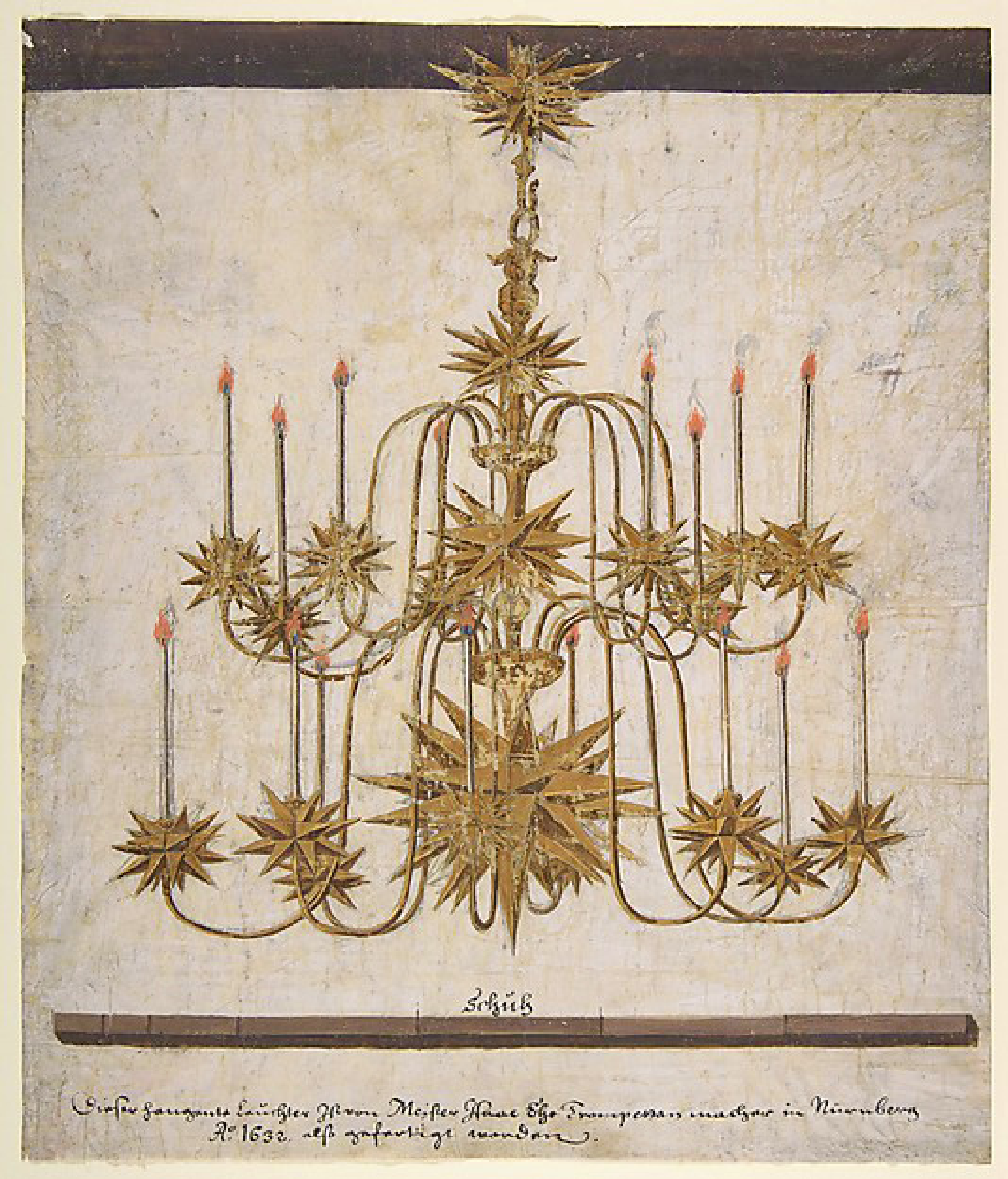


Figure 4.1: *Design for a Chandelier with Sixteen Candles* (1632), Johann Isaak Ehe, Acc. Num. 53.600.32, Metropolitan Museum of Art.

Figure 4.2: Wainscotting on columns in the Hirschvogelsaal (1534), Nuremberg, Peter Flötner (1490-1546).

Figure 4.3: Steel strongbox, Nuremberg, (late 16th or early 17th century). Its elaborate locking mechanism consists of nine bolts and various leaf-shaped shields. Metropolitan Museum of Art. Accession number 90.13.1.

Figure 4.4: Thomas Erngast, Buckle-maker (1535), *Das Hausbuch der Mendelschen Zwölfbrüderstiftung zu Nürnberg,* Amb. 279.2° Folio 24 recto (Landauer I).

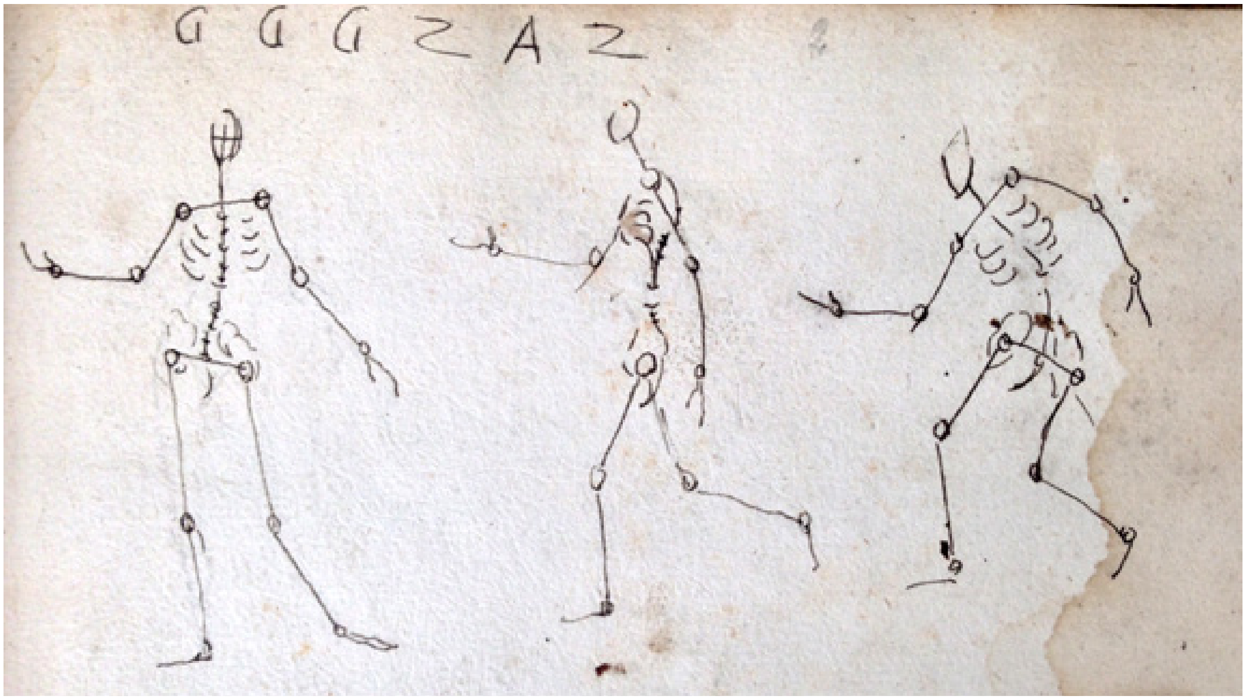


Figure 4.5: Drawing of three skeletal figures from Wenzel Jamnitzer's *Skizzenbuch*, Kunstbibliothek, Berlin.

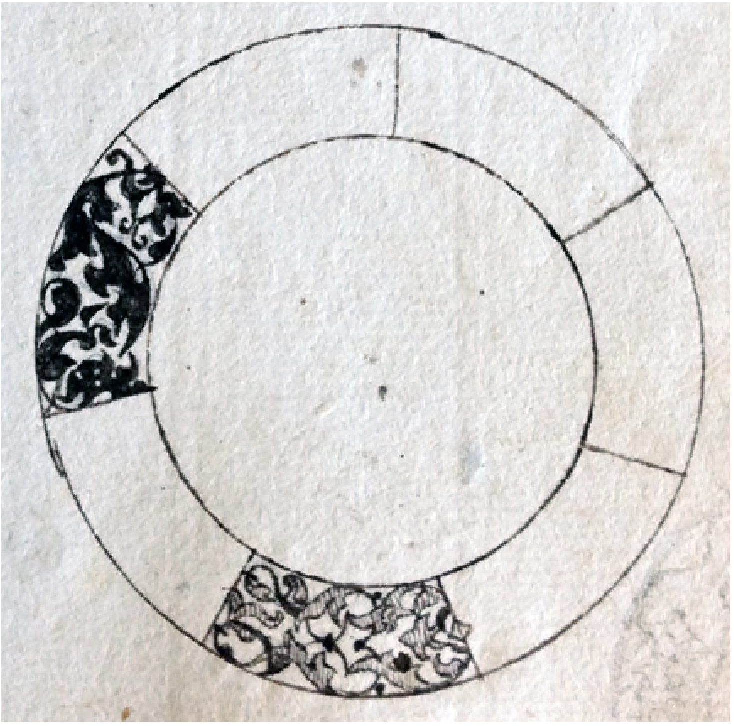


Figure 4.6: Ornament drawing from the Jeweller's Pocket Book (ca. 1550). British Museum. Inventory Number 1978,1216.2.

Figure 4.7: Ornament drawing from the Jeweller's Pocket Book (ca. 1550). Inventory Number 1978,1216.8. British Museum.

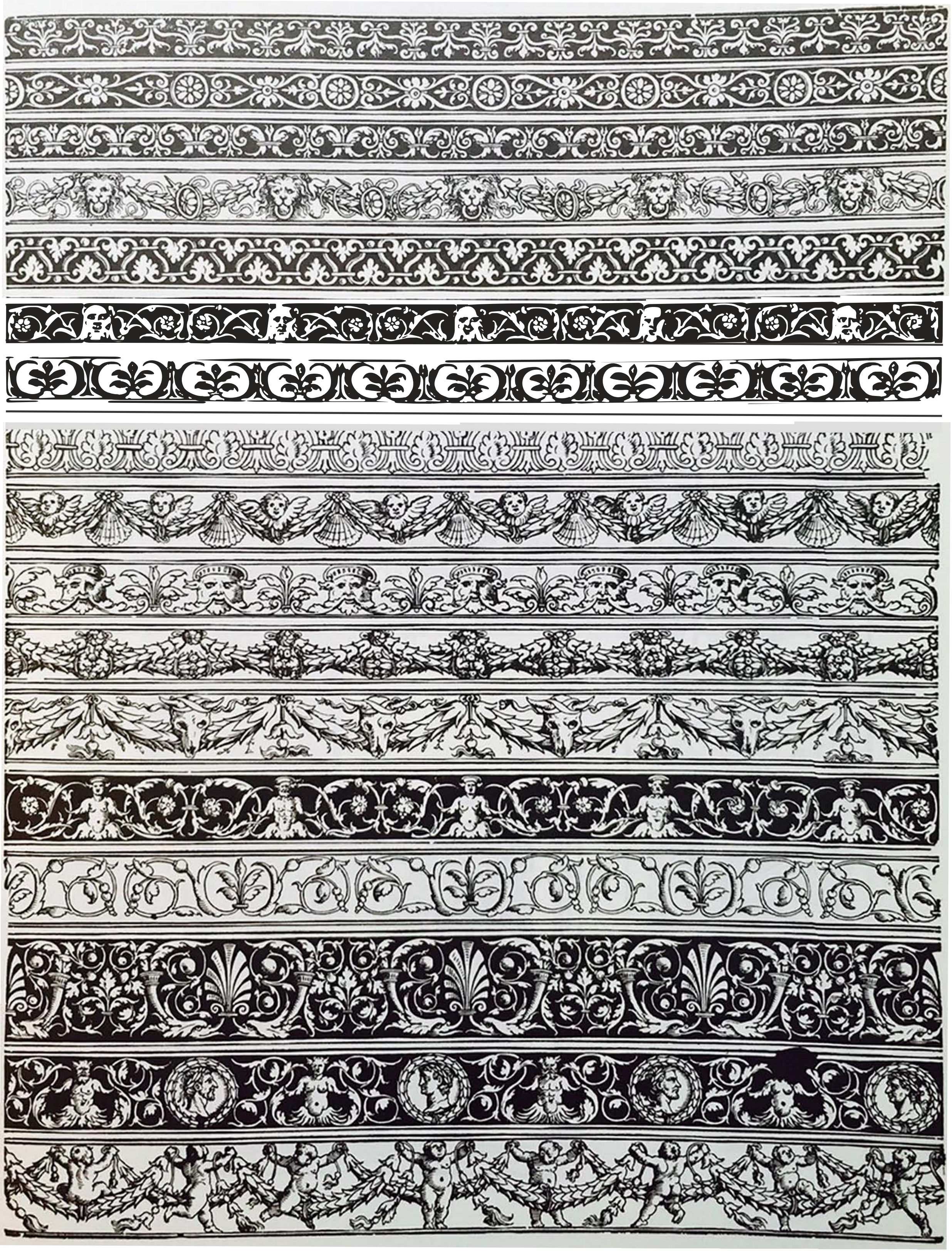


Figure 4.8: Patterns from the workshop of Peter Flötner, Nuremberg (1546).

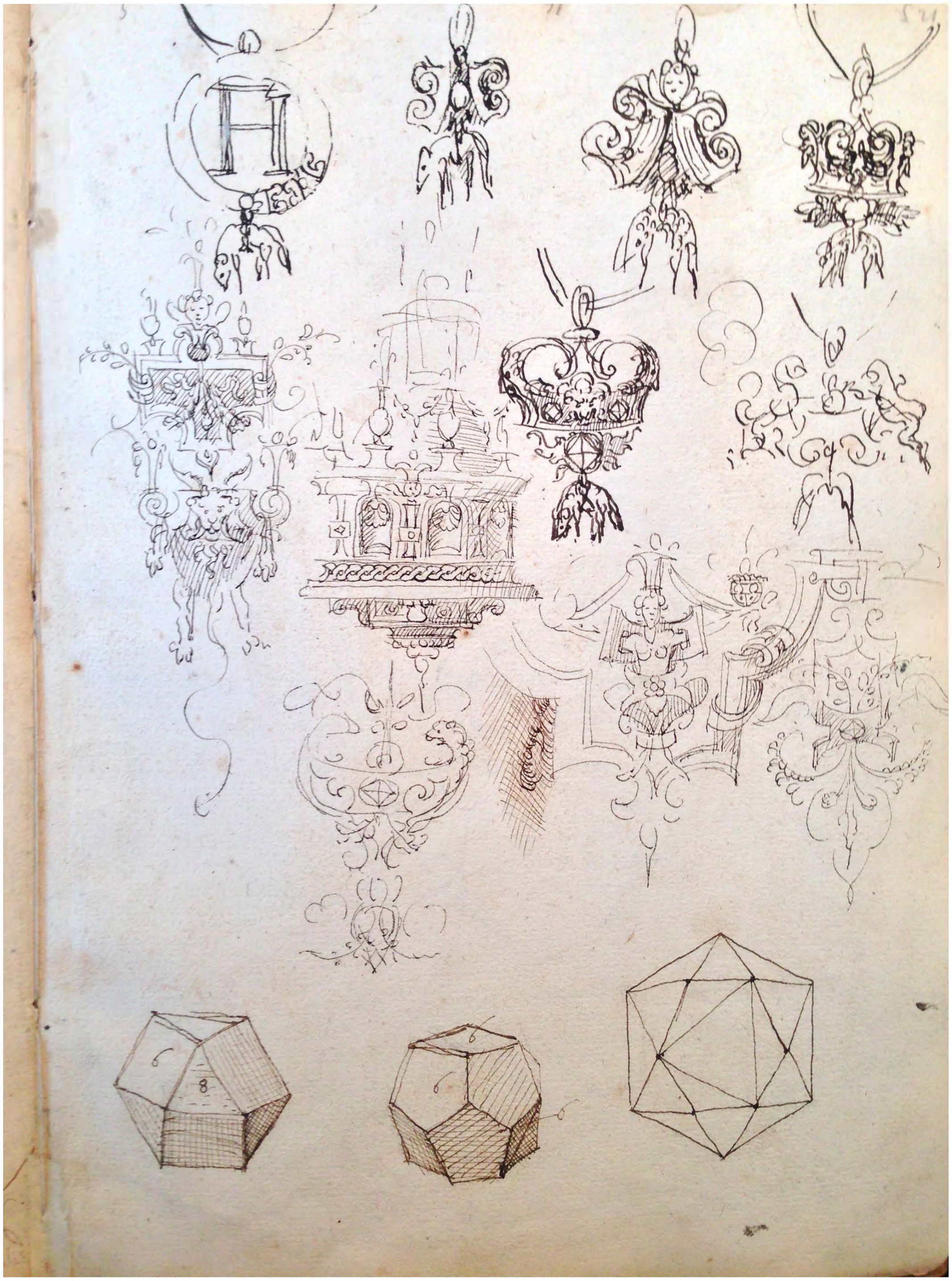


Figure 4.9: Sketchbook page, Wenzel Jamnitzer, Kunstbibliothek, Berlin.

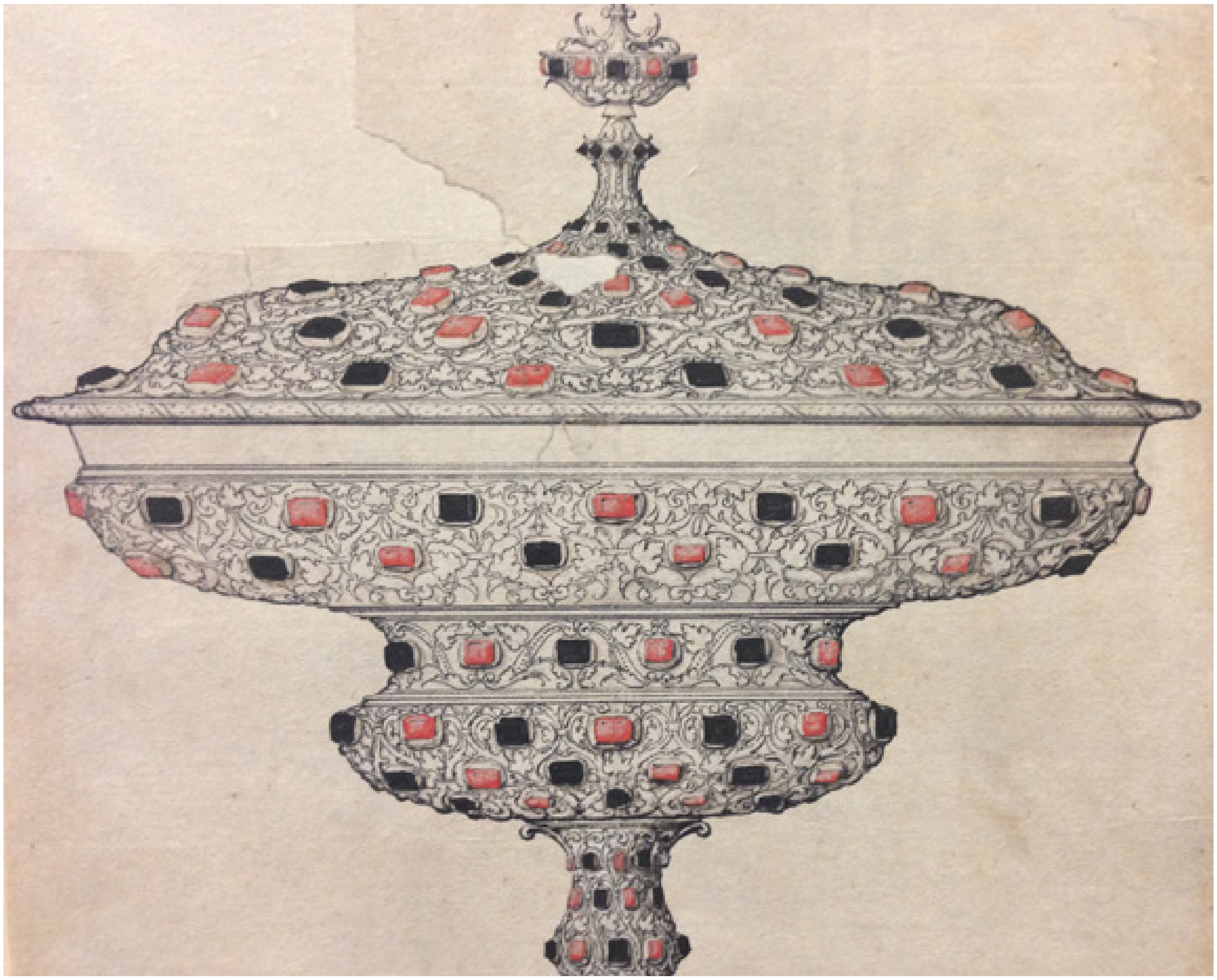


Figure 4.10: Design for a covered goblet, Hans Holbein the Younger, Kunstmuseum Basel.

Figure 4.11: Design for a goblet, Basler Goldschmiedriss, Inventory Number U.XII.65. Kunstmuseum Basel.

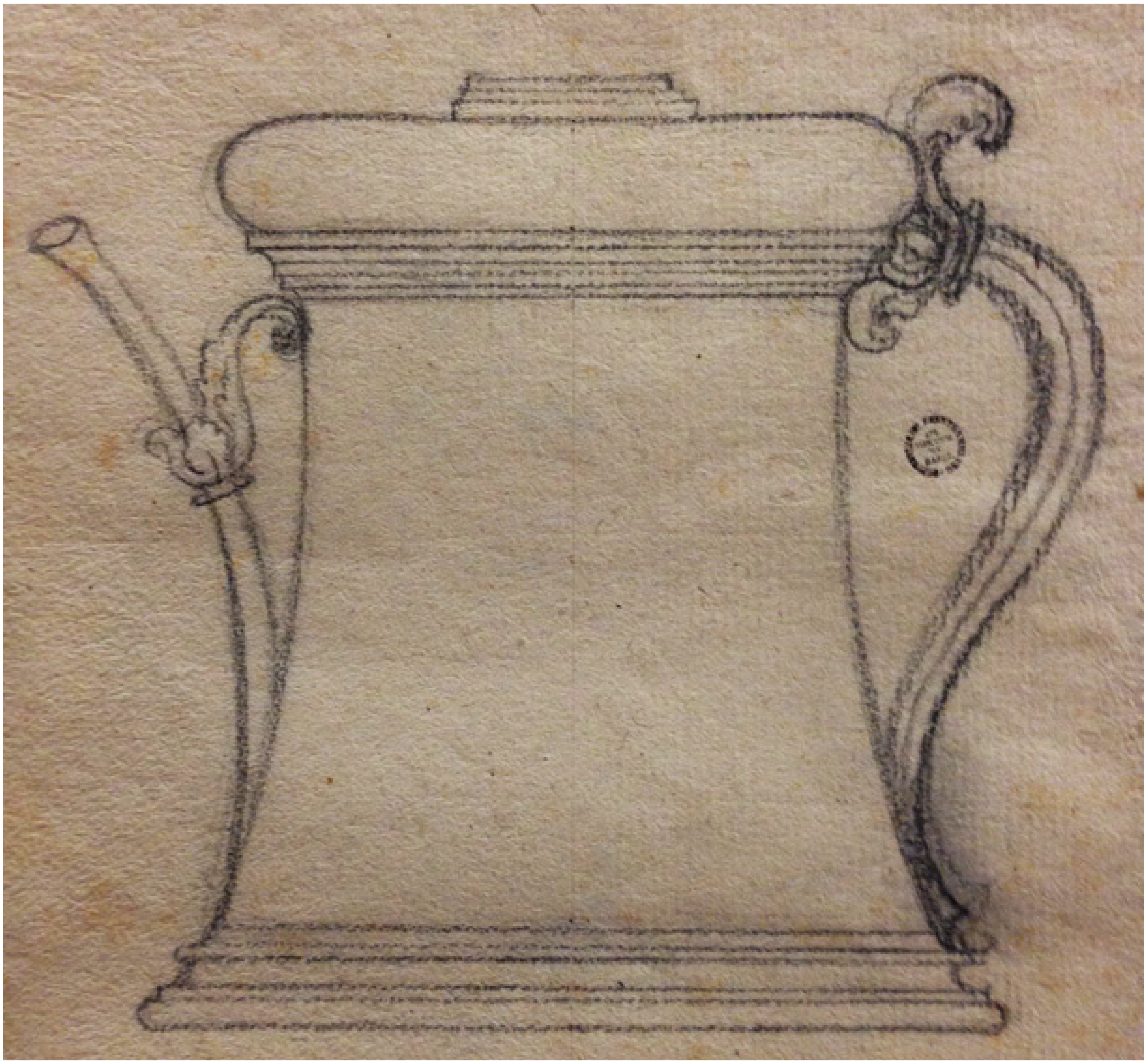


Figure 4.12: Drawing of a pitcher, Basler Goldschmiedriss. Inventory Number U.XII.3, Kunstmuseum Basel.

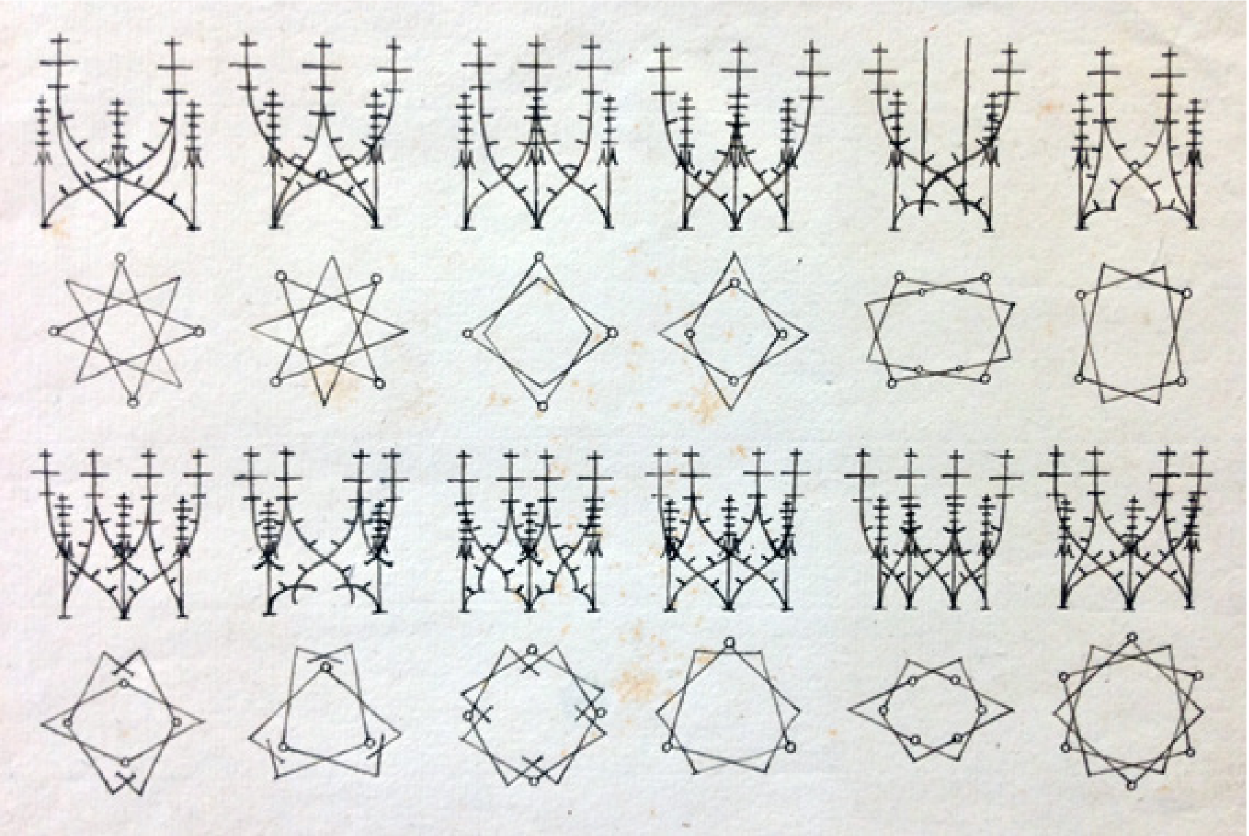


Figure 4.13: Design for baldachins and wimpergs, Basler Goldschmiedriss, Inventory Number U.XI.12, Kunstmuseum Basel.

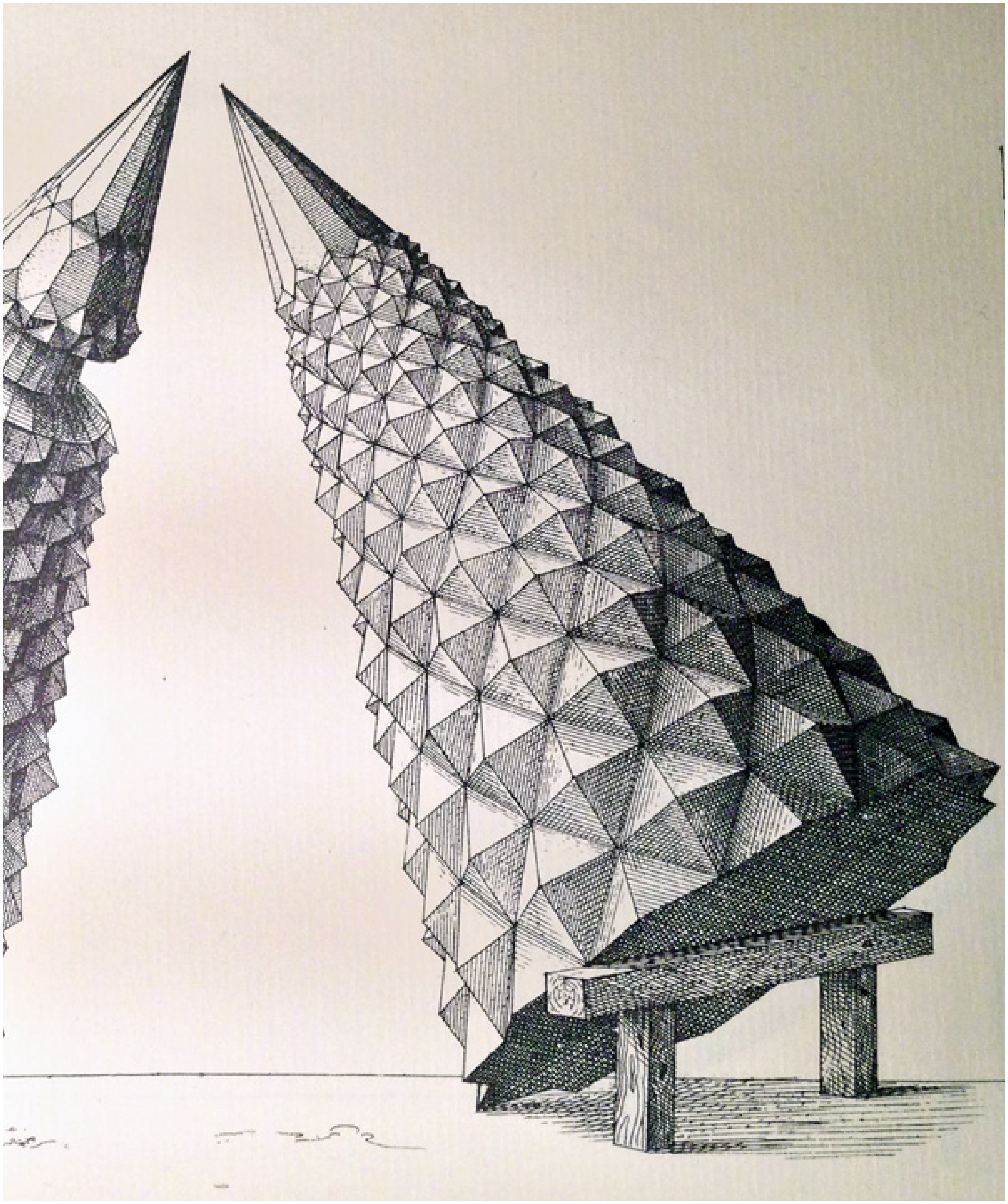


Figure 4.14: Page HIII, Geometrical model, Wenzel Jamnitzer, *Perspectiva corporum regularium* (1568).

Figure 4.15: Wenzel Jamnitzer's grave, St. John's cemetery, Nuremberg.

A. i.

IGNIS. Das Fewer.

TETRAEDRON.

Siue Pyramis trilaterata. Ein trianglichter Kegel.

Der Erſt vnter den Fünf regulirten Cörpern Iſt ein Corpus gemacht/von vier gleichſeitigen triangeln/Flechen oder Pöden gerader Linien/ darauff es geſtelt werden mag/hat ſechs ſeiten oder gerader Linien Zwölff flache Winckel vnnd vier Cörperlicher Eck. Auß diſem drianglichten Corpore oder Kegel/ ſind verner drey vnd Zwaintzig ander Corpora geurſacht/ vnd vf mancherley vnterſchidliche Art zu werckh gezogen. Wie hernach geſehen wirt.

A.I.

Figure 4.16: Title page to the Tetraedron section of *Perspectiva corporum regularium* (1568).

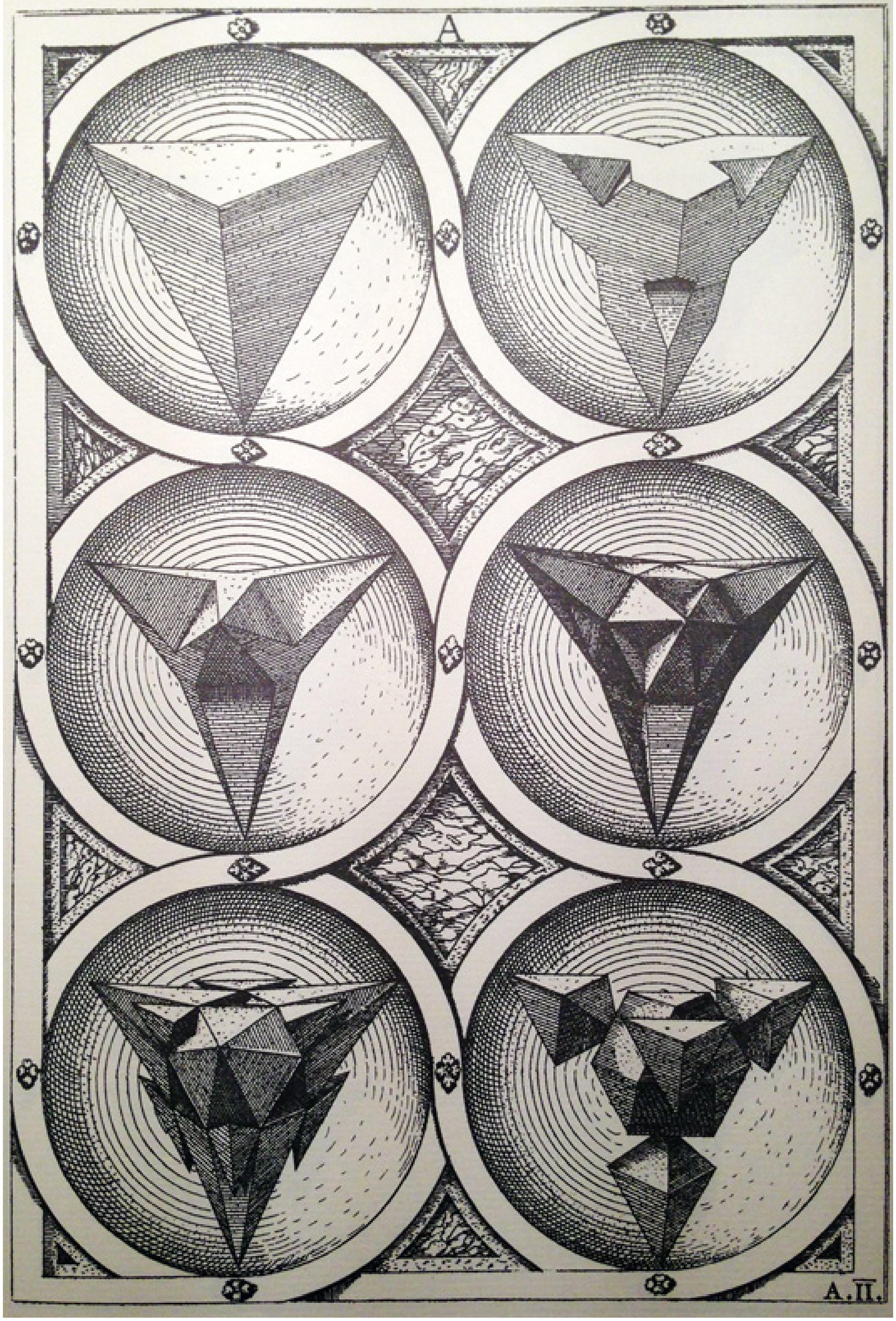


Figure 4.17: Page AII, Wenzel Jamnitzer, *Perspectiva corporum regularium* (1568).

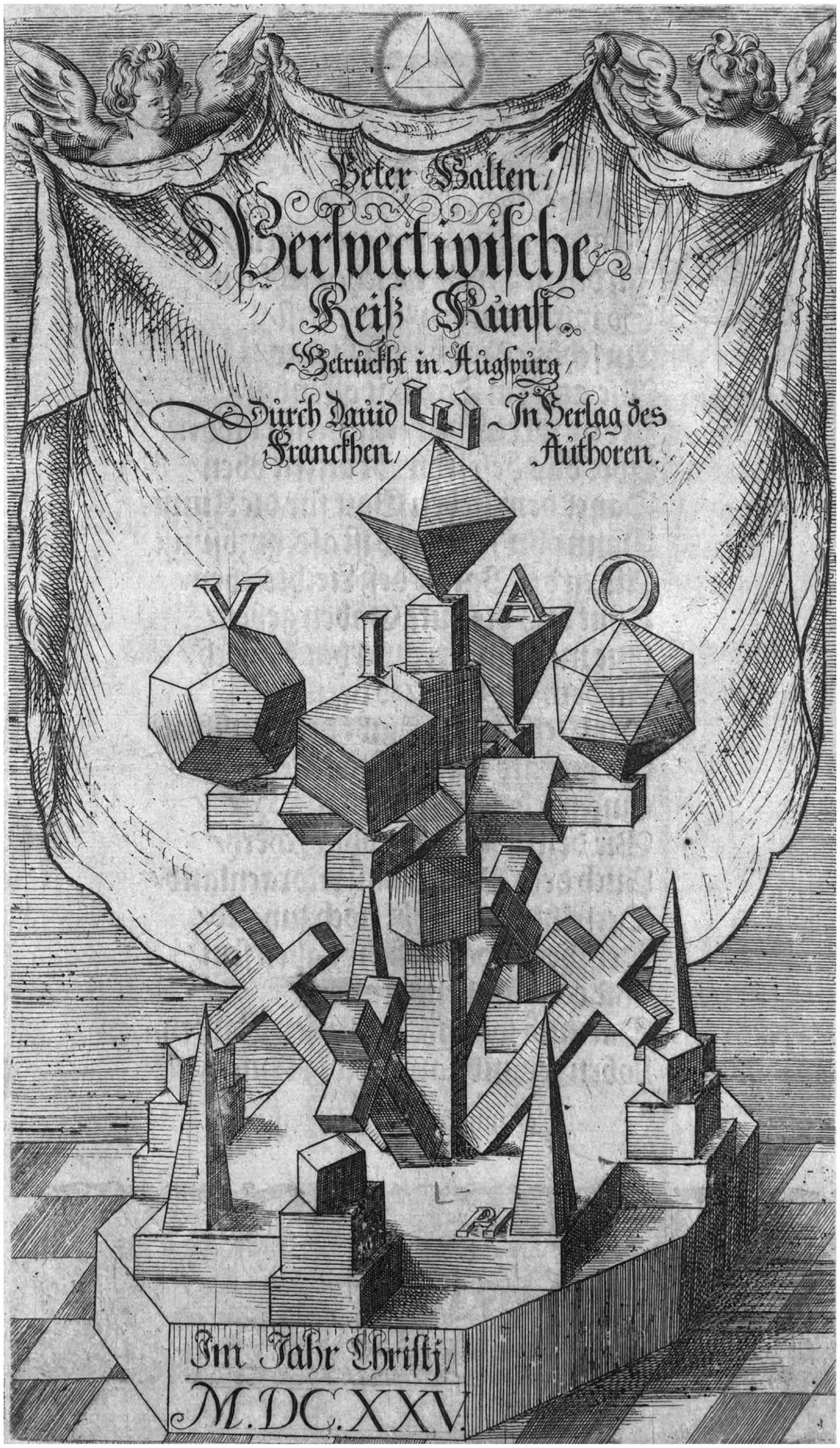

Peter Halten /
Perſpectiviſche
Reiß Kunſt.
Getruckht in Augſpurg /
Durch Dauid Franckhen /
In Verlag des Authoren.
Im Jahr Chriſtj
M.DC.XXV

Figure 4.18: Title page to *Perspectivische Reiss Kunst* (1625) by Peter Halt.

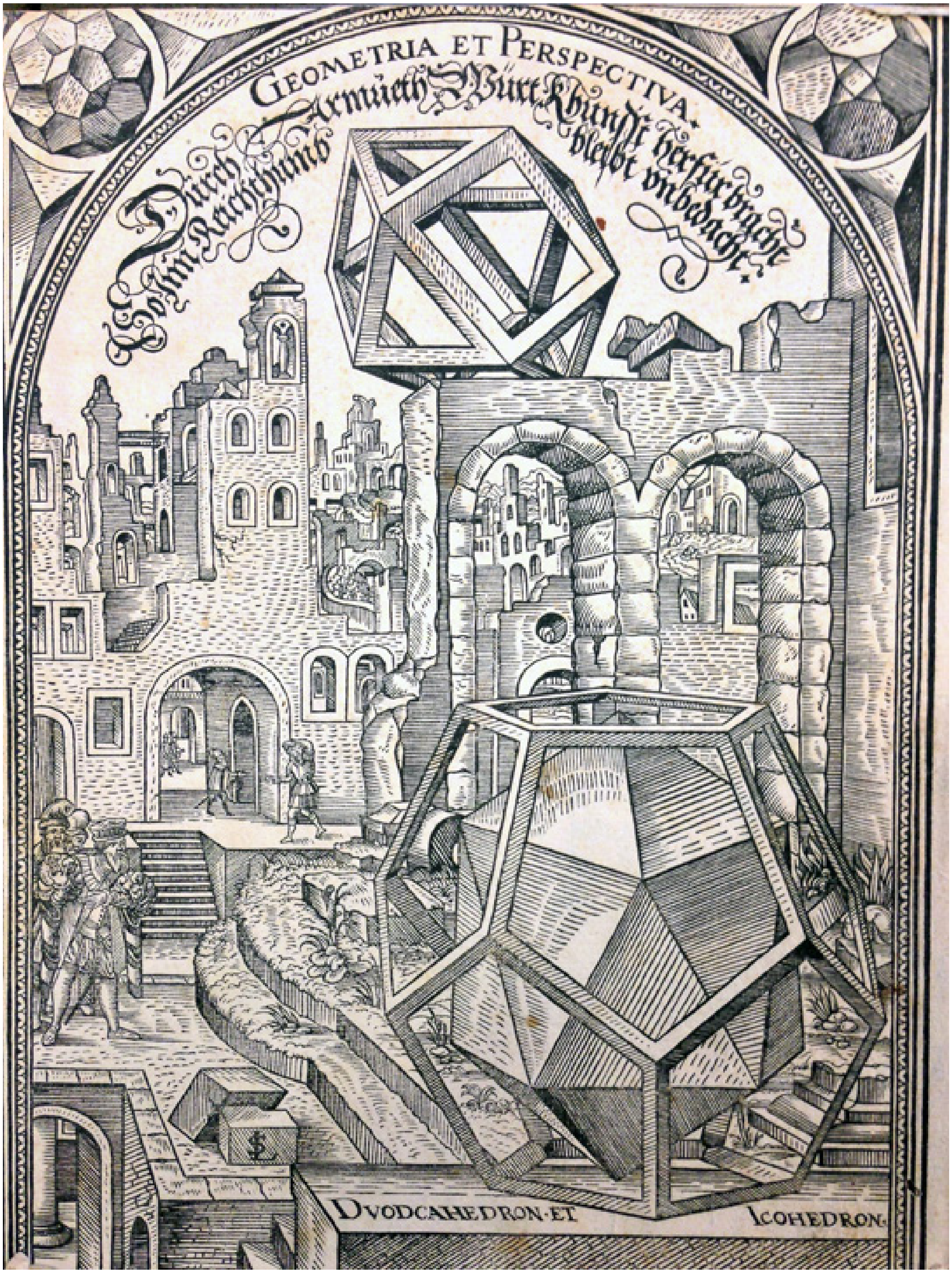


Figure 5.1: “Geometric Forms among Ruins” (1567). Alternative frontispiece for Lorenz Stöer’s *Geometria et Perspectiva*. Metropolitan Museum of Art. Accession Number 984.1085.5.

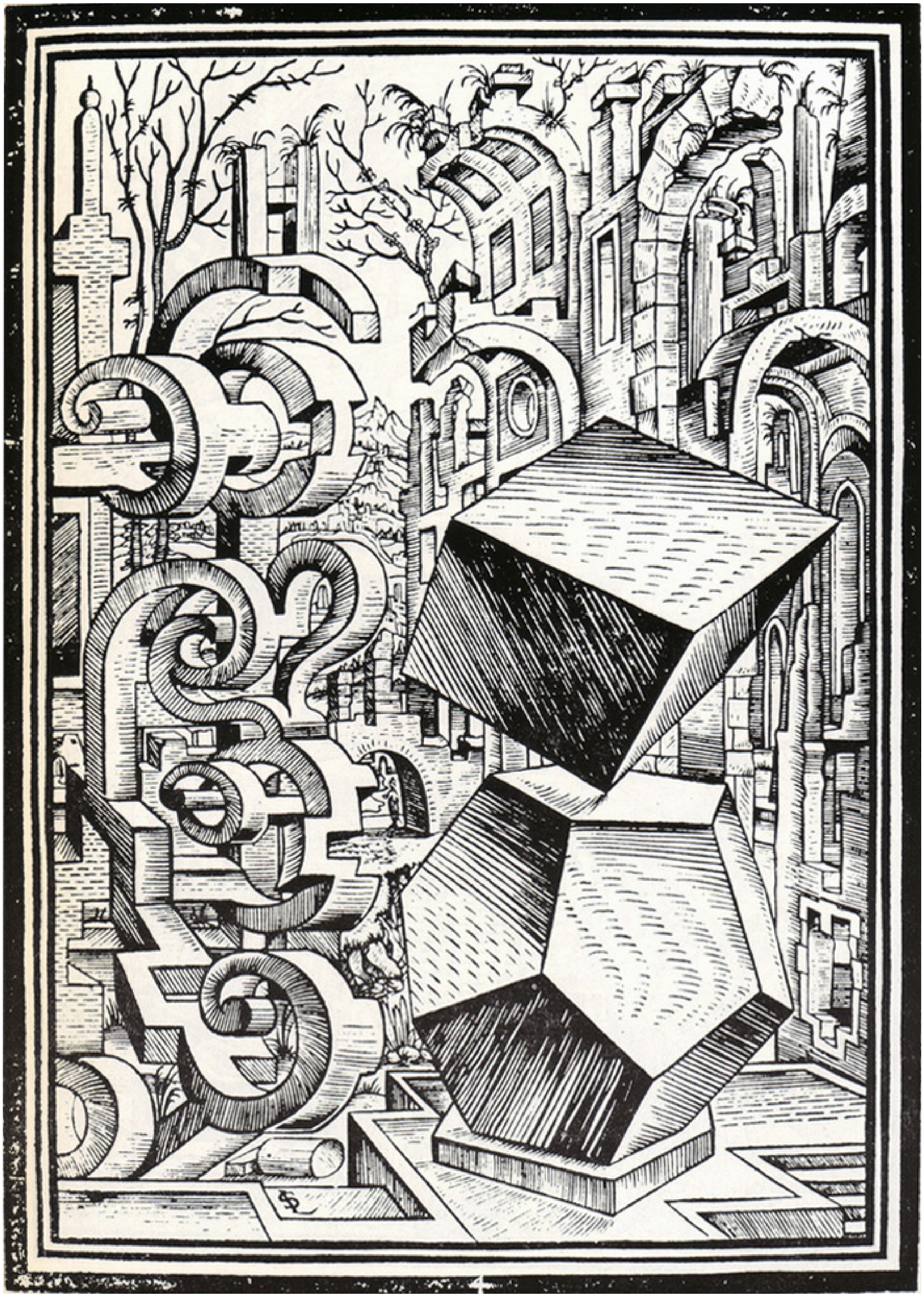


Figure 5.2: Plate Four from *Geometria et Perspectiva* (1567), Lorentz Stöer.

Figure 5.3a: Elevation of 16th century Augsburg Cabinet (with geometry), Georg Laue Gallery, Munich.

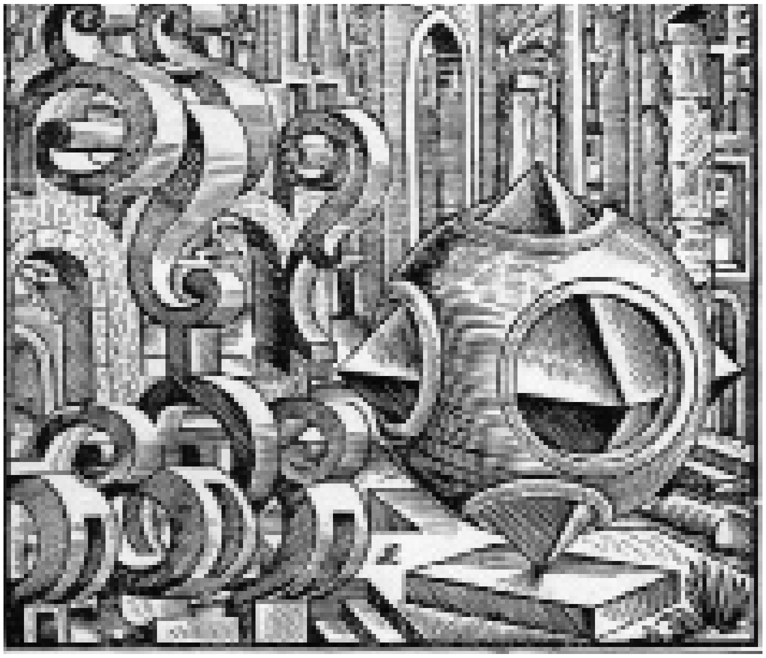


Figure 5.3b: Detail from Plate 8 of Lorentz Stoer's *Geometria et Perspectiva.*

Figure 5.4: Wrangelschrank (detail). Anonymous, 16th century, South German.
Westfälisches Landesmuseum, Münster.

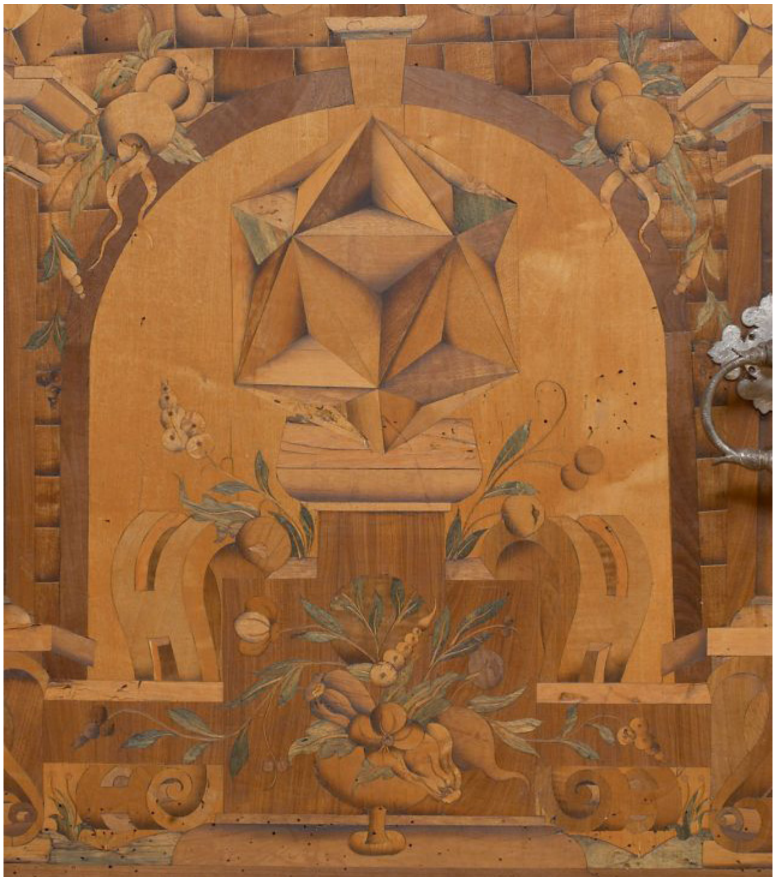


Figure 5.5: Two panels from a Coin Chest in the Ambras Castle, Conrad Gottlieb, ca. 1580. Inventory Number PA 21.

Figure 5.6: Detail of nested polyhedra from tabletop belonging to Prince Karl I of Liechtenstein, Castrucci Cosimo di Giovanni workshop, ca. 1620, The Princely Collection of the Liechtenstein Museum, Vienna. Inventory Number SK 1401.

Figure 5.7: Pair of miniature cabinets, 16th Century Germany. Georg Laue Gallery, Munich (top) and Metropolitan Museum of Art (bottom).

Figure 5.8: Cabinet with inlaid polyhedra. Around 1600. Museum für Angewandte Kunst, Köln. Inv. Nr. A 1451.

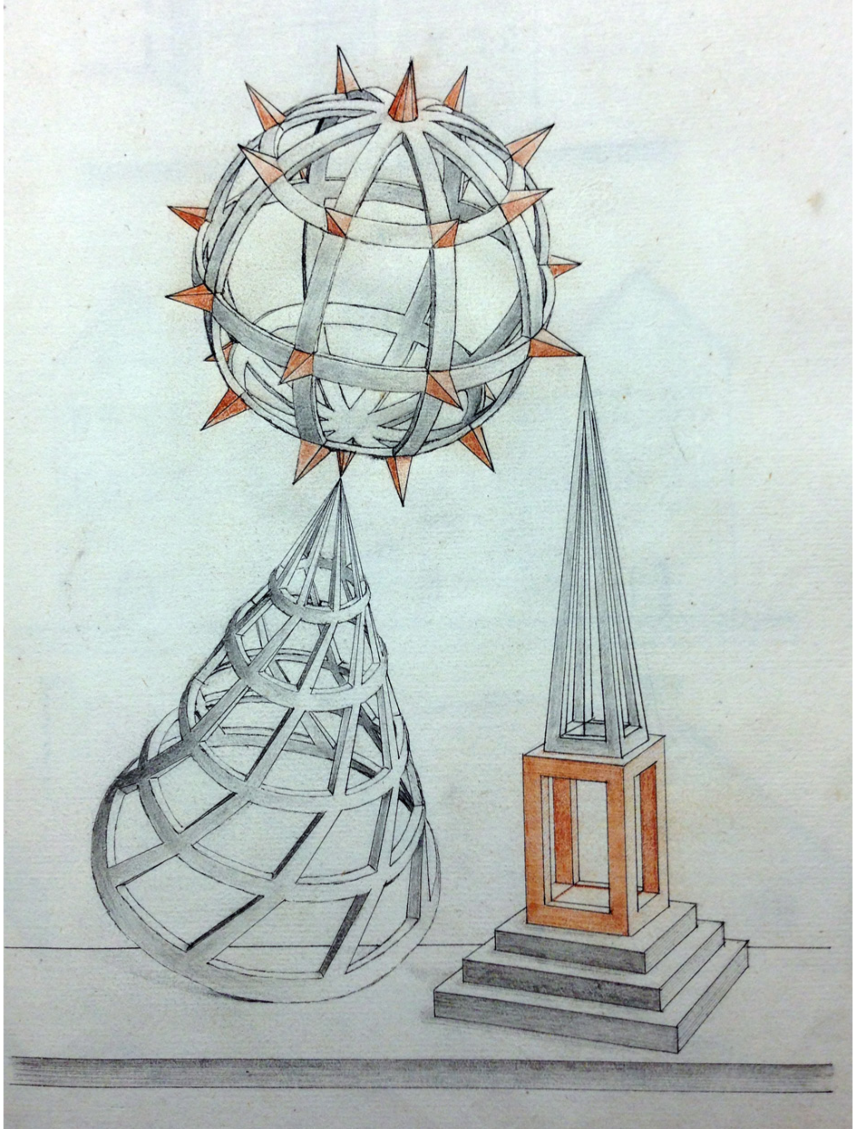


Figure 5.9: Recto and Verso side of page 30 of Duke Christian's *Perspectief Buch*. The spherical form on the verso shows evidence of copying - pinpricks and the raised lines indicative of scoring with a sharp instrument.

Figure 5.10: Recto from Lorentz Stoer's *Geometria et Perspectiva* (1567). Universistätsbibliothek Erlangen.

Figure 5.11: Verso from back of pagae depicted in Figure 5.10, showing selective copying techniques. Lorentz Stoer's *Geometria et Perspectiva* (1567). Universistätsbibliothek Erlangen.

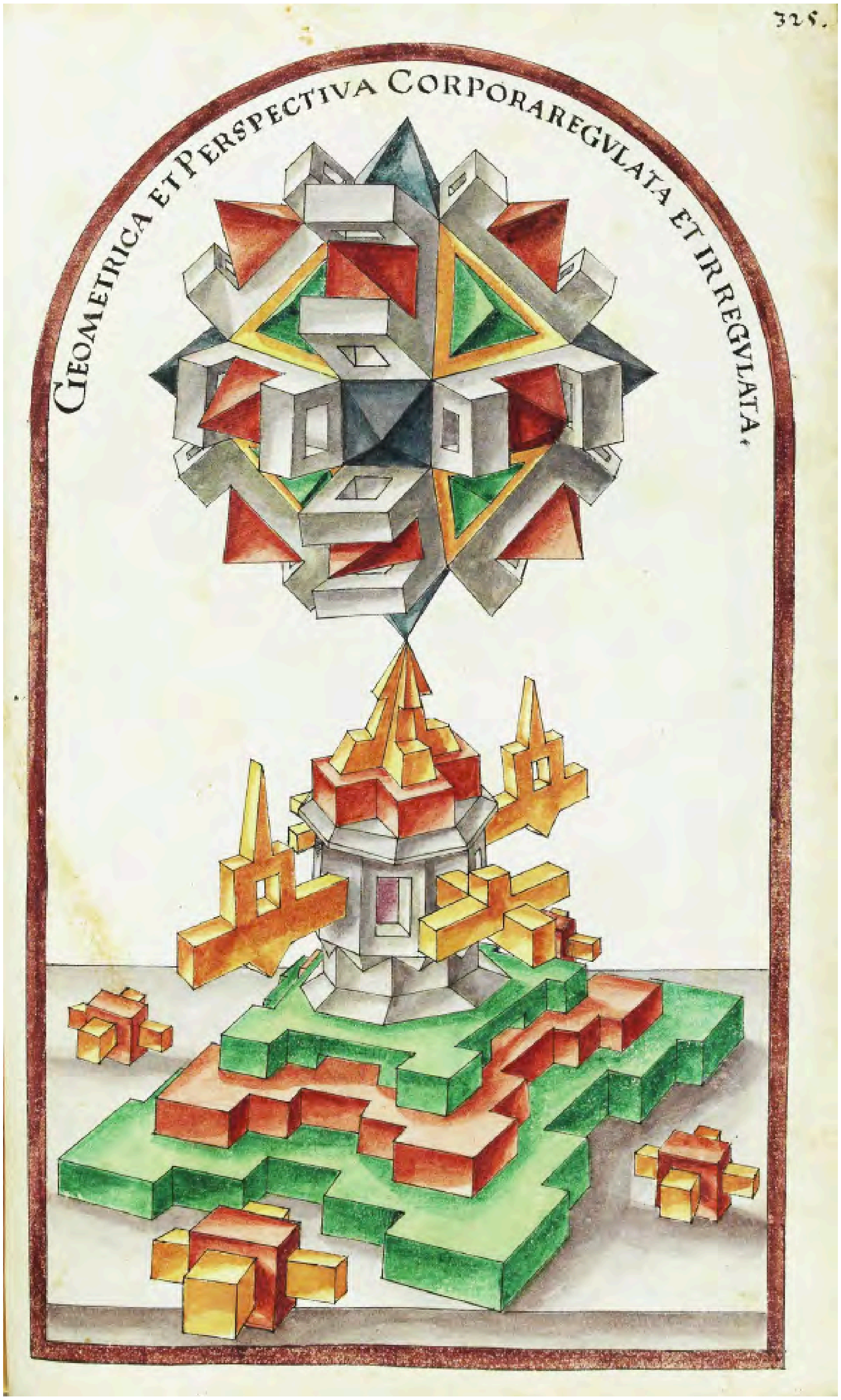


Figure 5.12: Drawing of an “irregular solid” from Lorentz Stöer’s *Geometria et Perspectiva Corpora Regulata et Irregulata* (late 16th century), Universitätsbibliothek der LMU, München.

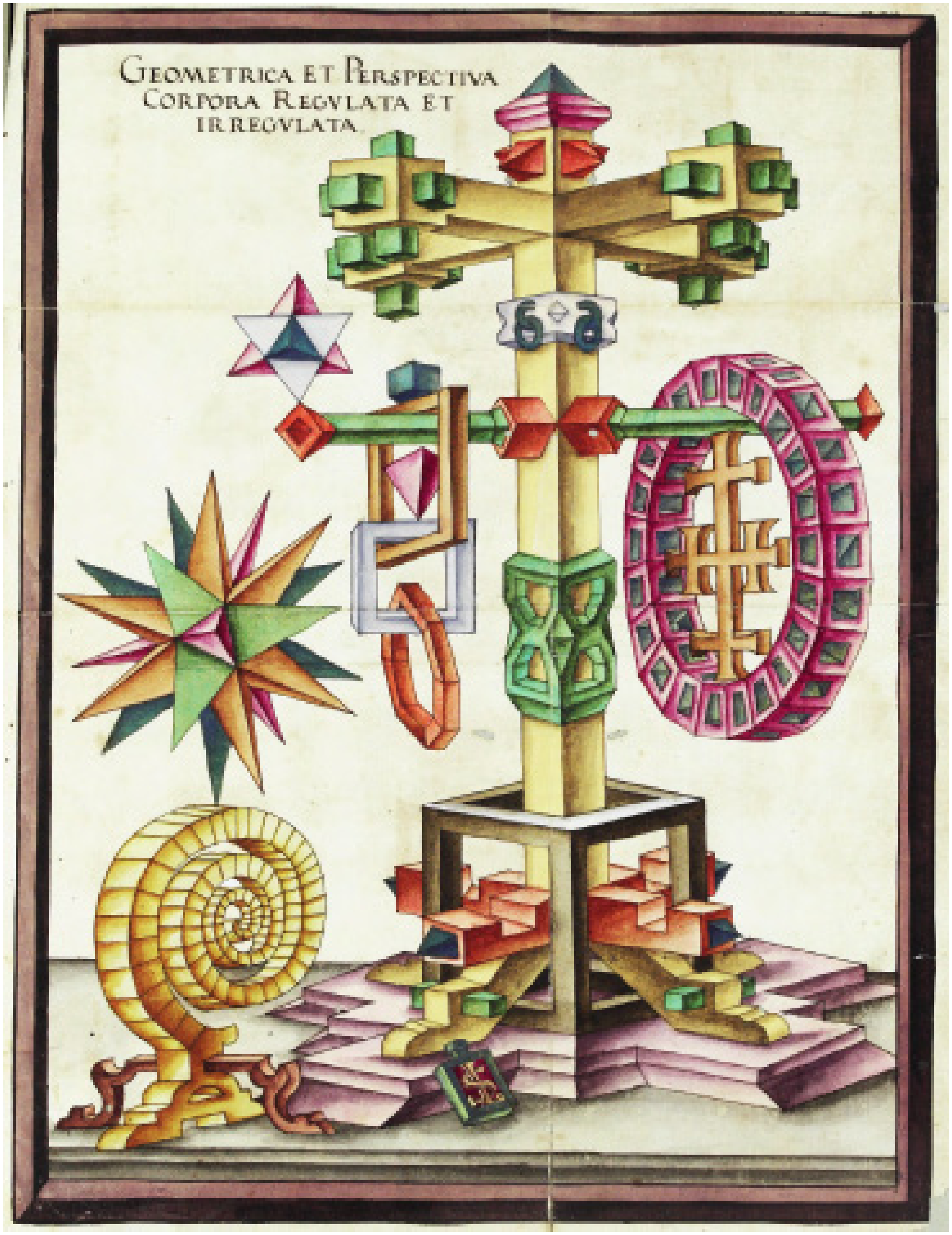


Figure 5.13: Composite geometrical model of irregular solids from the Munich volume of Stoer's drawings. Universitätsbibliothek München. Cim 103.

Figure 5.14: Cover page from the Munich Compendium of geometrical models. Lorentz Stöer. Universitätsbibliothek München. Cim 103.

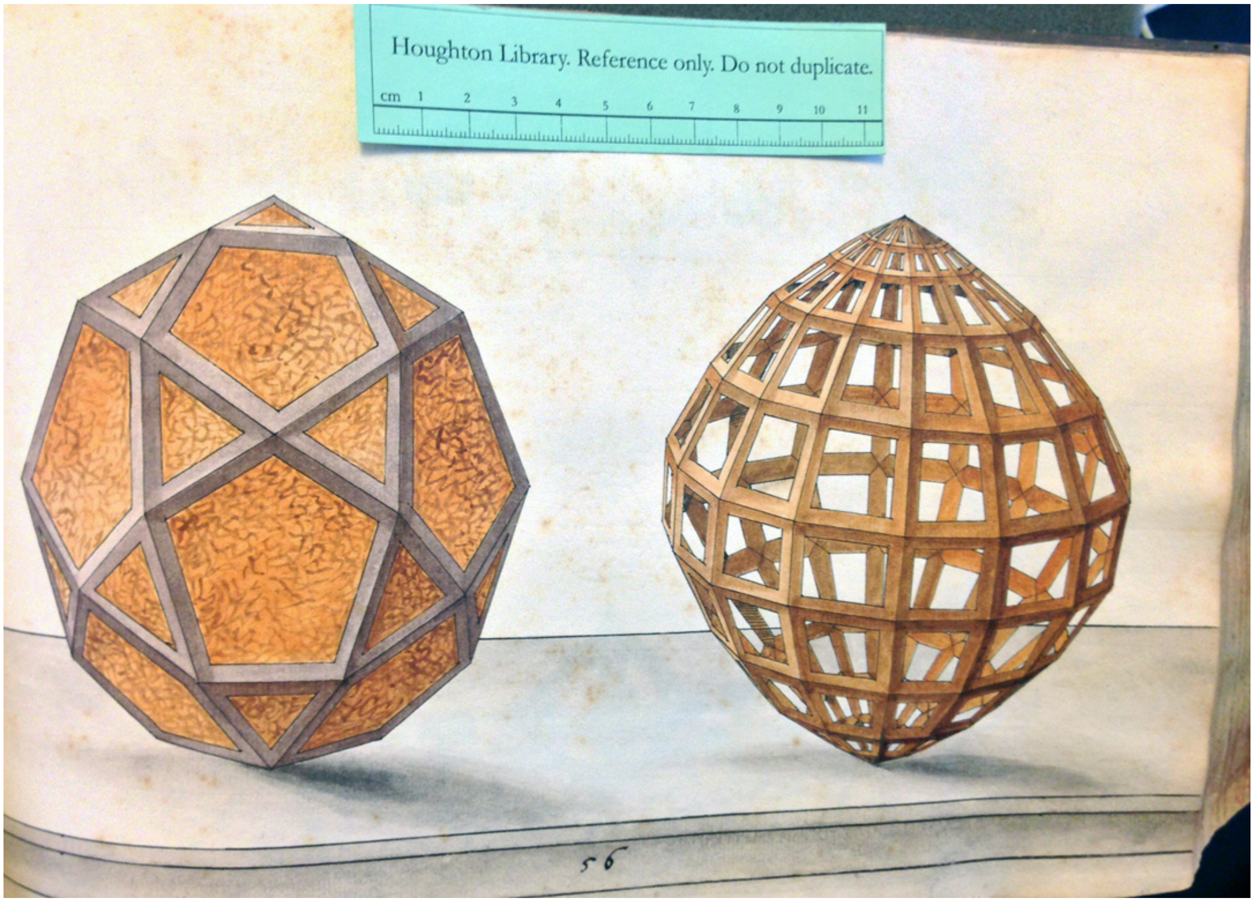


Figure 5.15: No. 56 from the Harvard Compendium of Stoer drawings. Showing two models made from wood. Houghton Library, Typ 520.67.810.

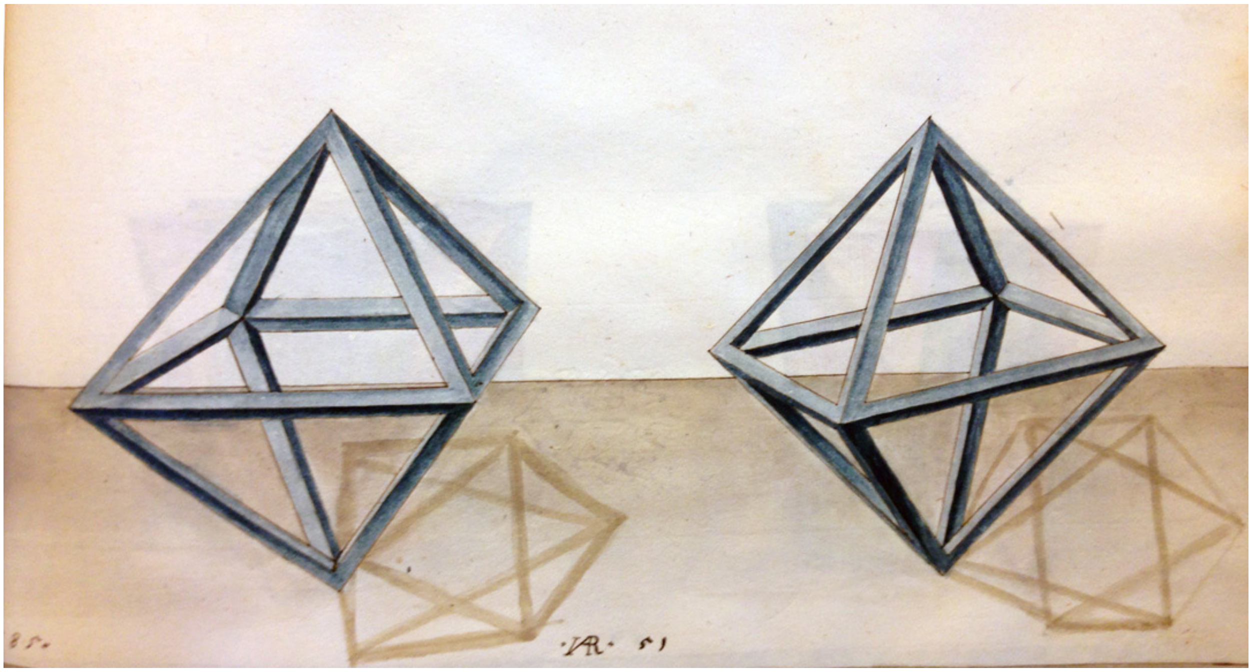


Figure 5.16: Drawing No. 51 (1585) of two skeletal diamonds from the second, “less proficient” section of the Harvard compendium of Stoer drawings. The monogram may refer to Rogel. The shadows are notional and have not been accurately constructed. Hougton Library, Typ 520.67.810.

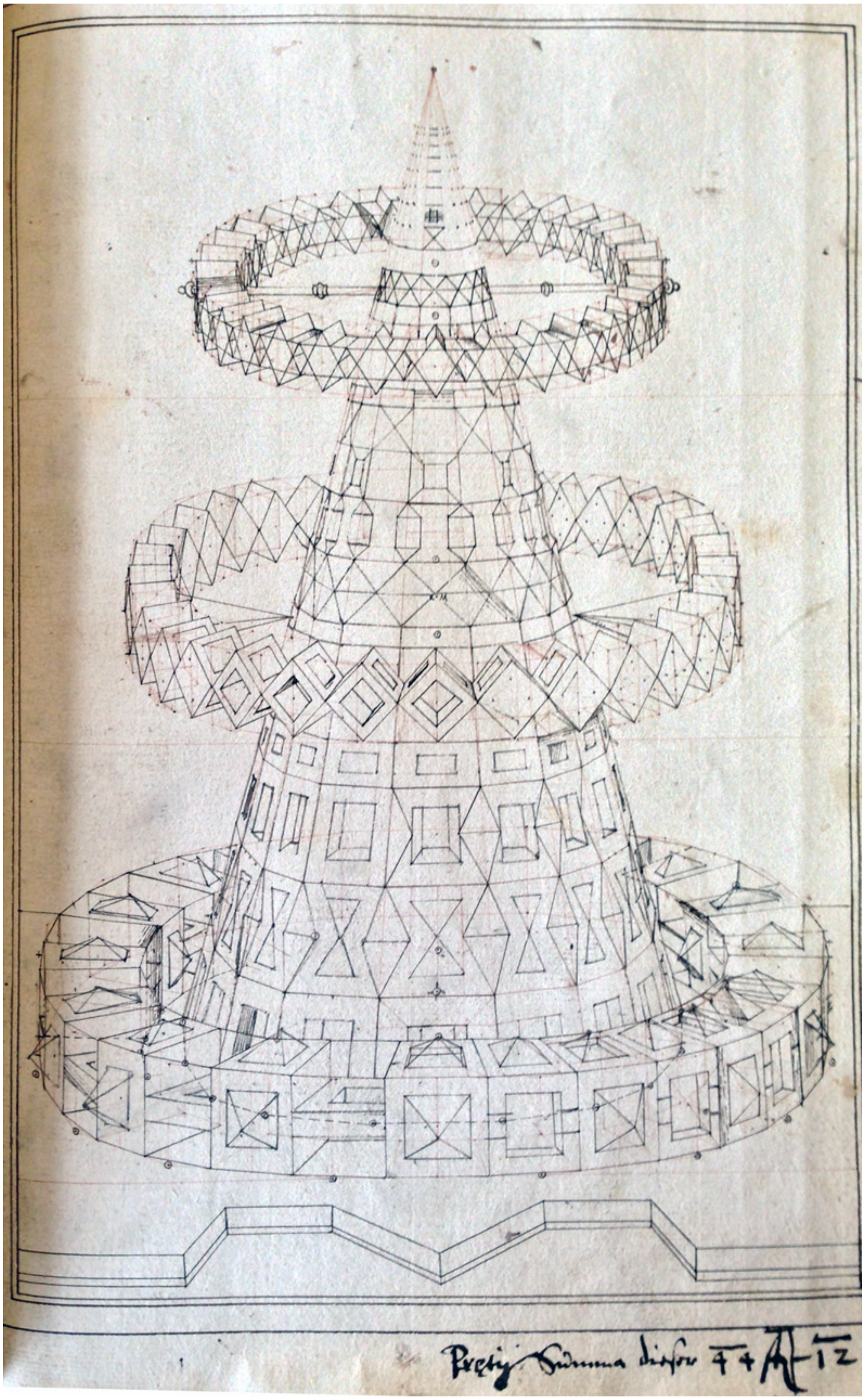


Figure 5.17: Construction drawing for a geometrical model from Lorentz Stöer's *Optica*. Universistätsbibliothek Erlangen.

Figure 5.18: Geometrical model from Munich's Stoer volume. Universistätsbibliothek München.

Figure 5.19: Verso side of a construction drawing for a geometrical model from Lorentz Stoer's *Optica*. Only half of the spherical form has been scored, due to its symmetricality. Universitätsbibliothek Erlangen.

Figure 5.20: Detail of verso side from the latter, less-proficient section of Harvard's Stoer volume, showing pin pricks at the vertices of a mathematical model.

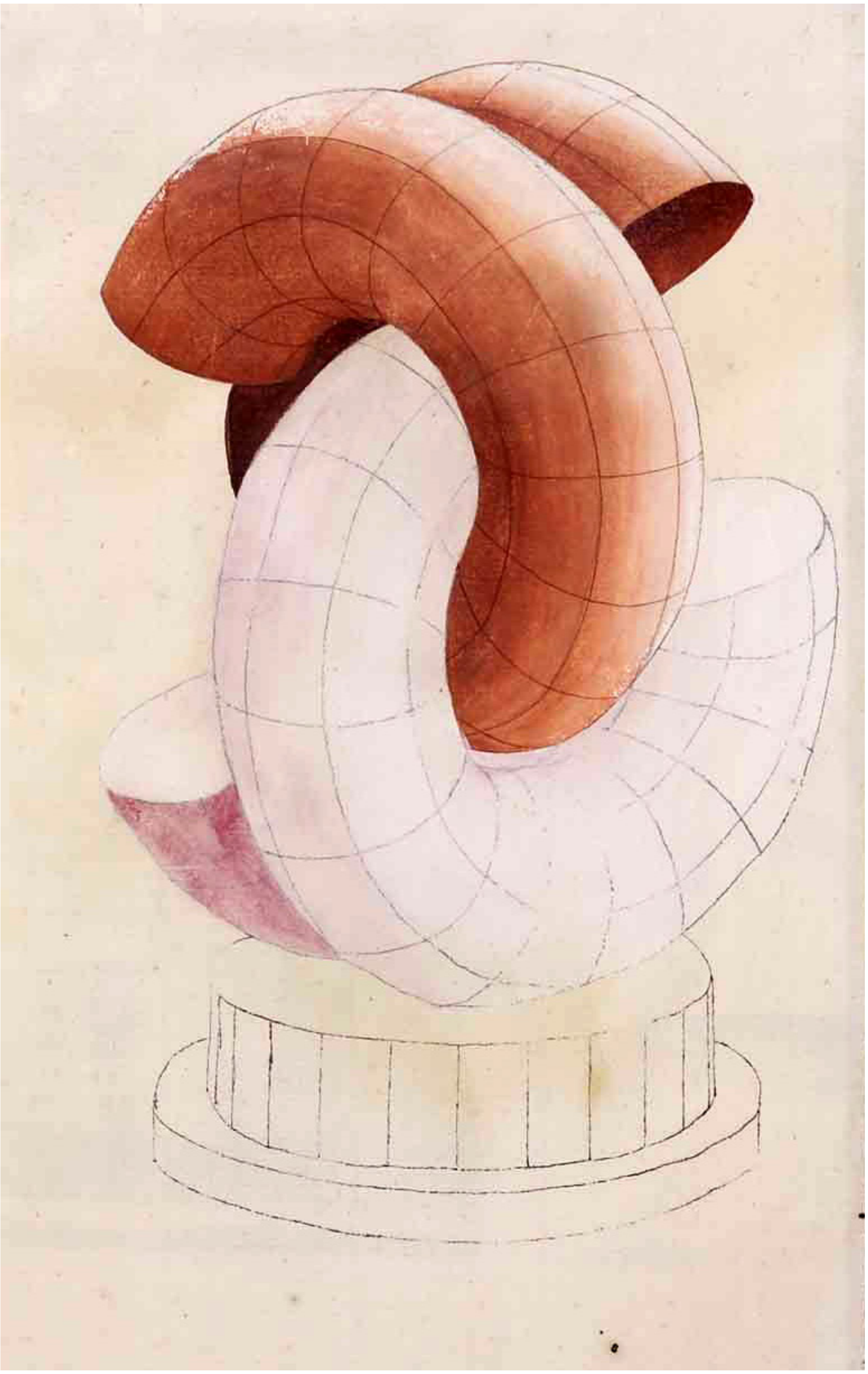
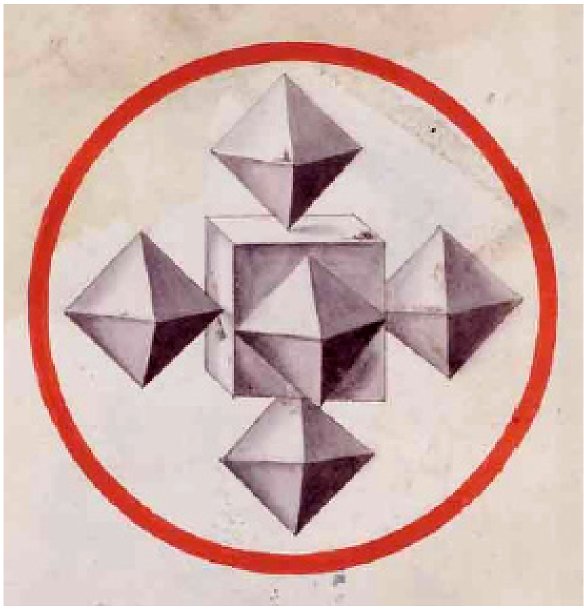
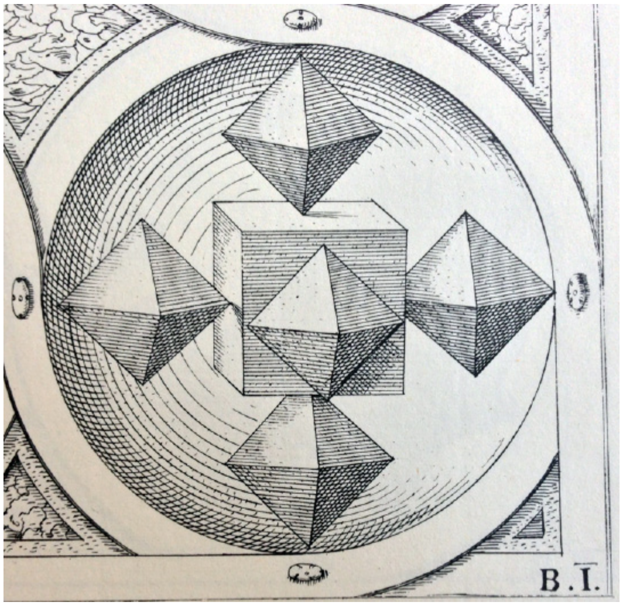


Figure 5.21: Geometrical model from Cod Guelf. 74.1 Aug. 2° in Wolfenbüttel. It is a copy of a mathematical model from Hans Lencker's *Perspectiva Literaria* (1557).





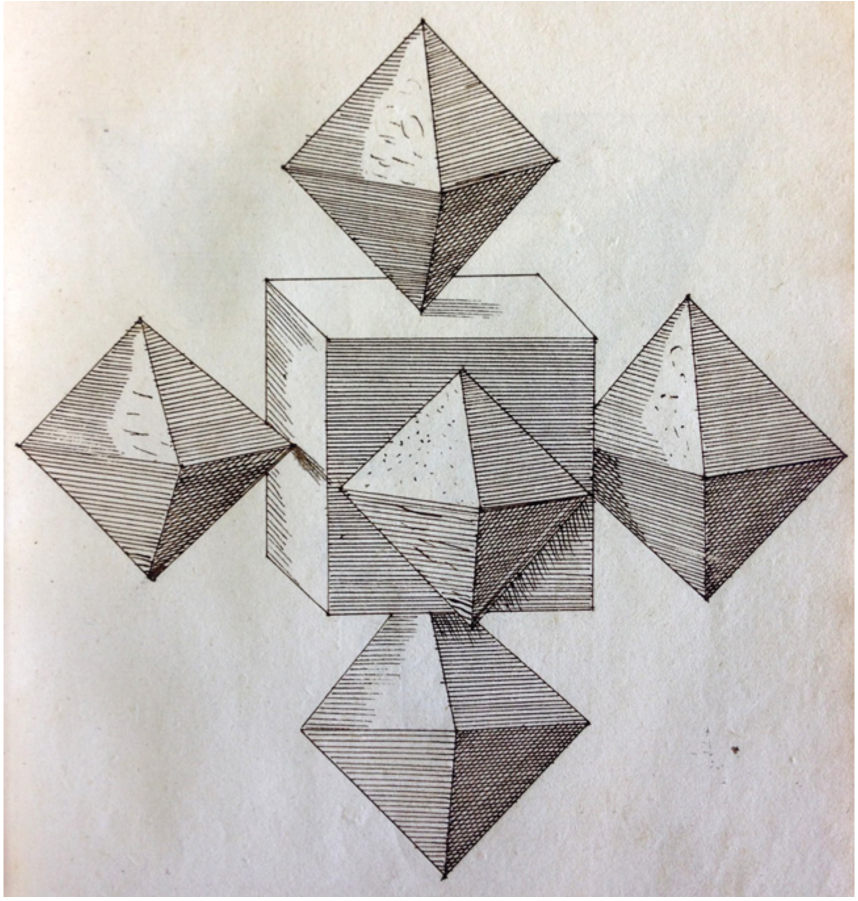


Figure 5.22: [top left] Octohedron from Cod Guelf. 74.1 Aug. 2° in Wolfenbüttel. [top right] The orginal BI Octohebdron etching from Wenzel Jamnitzer's *Perspectiva corporum regularium* (1568). Copy of Jamnitzer's octohedron in Stoer's notebook (in pen) at the Staats- und Stadtbibliothek Augsburg. 4 Cod. Aug. 247.

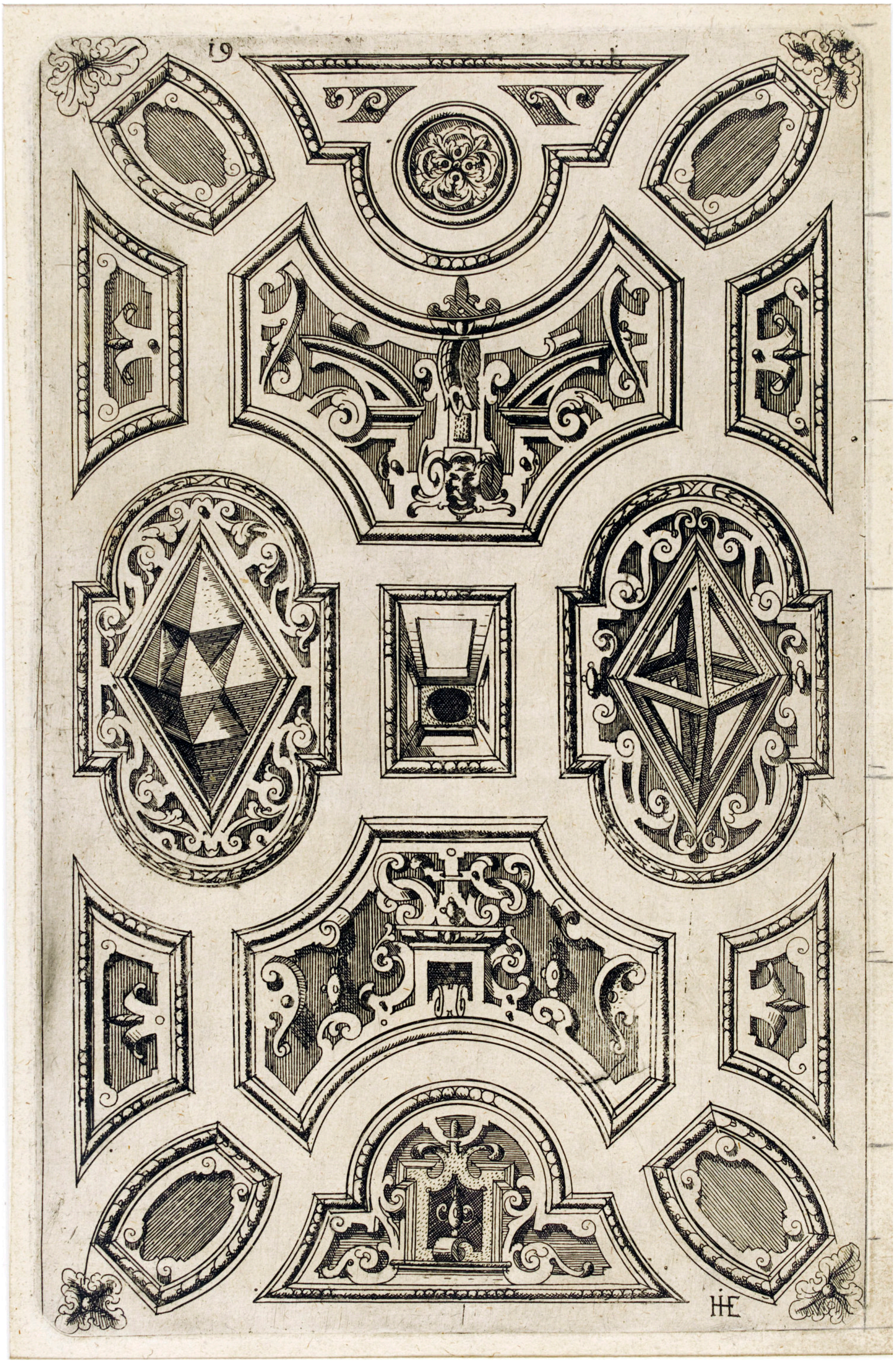


Figure 5.23: Ceiling design from Hans Jakob Ebelmann's *Gebäude Bekrönungen Decken* (1609).

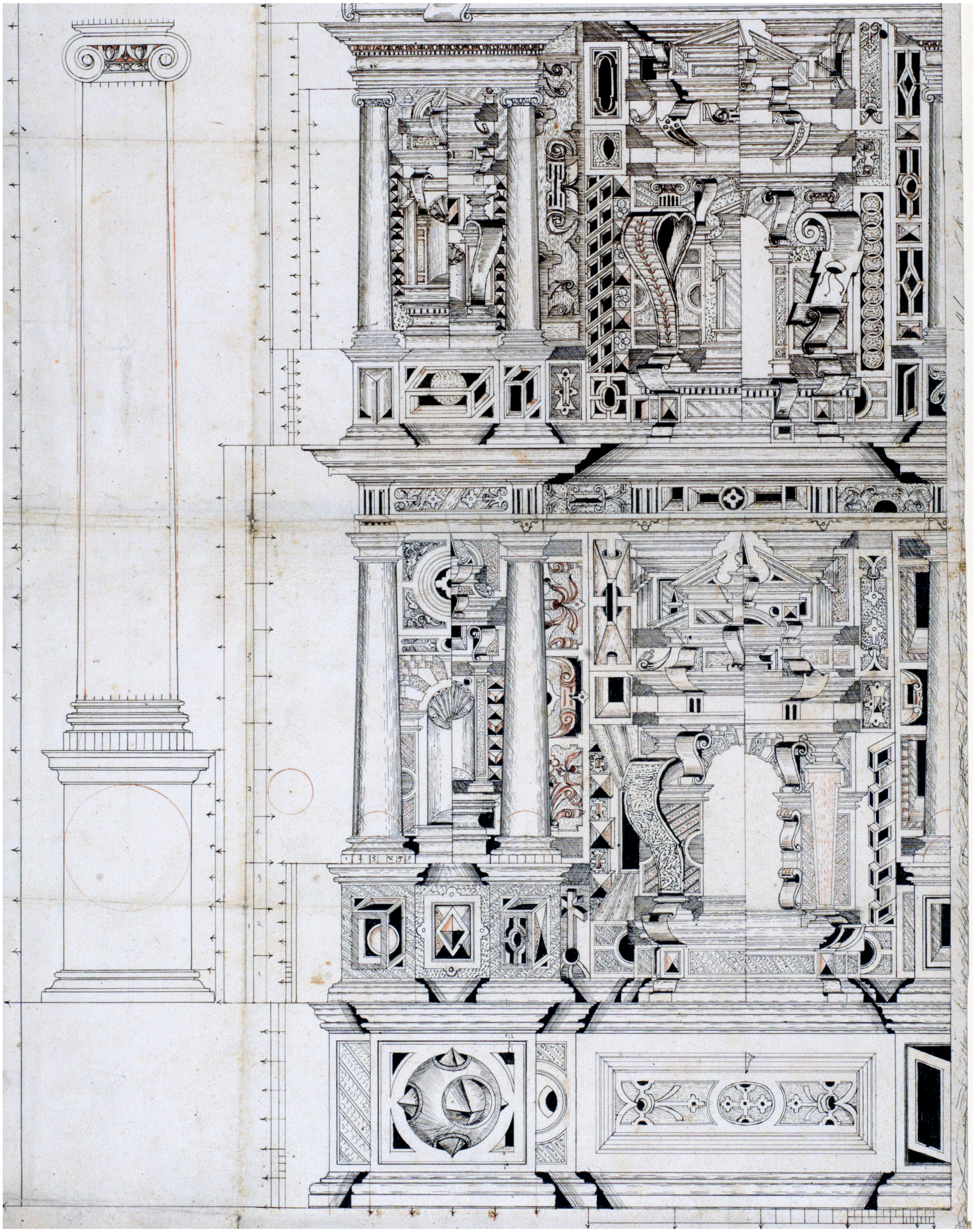


Figure 5.24: Drawing of building facade options, detail, Lorenz Stöer (late 16th century). Germanisches National Museum, Nuremberg. Hz 6979.

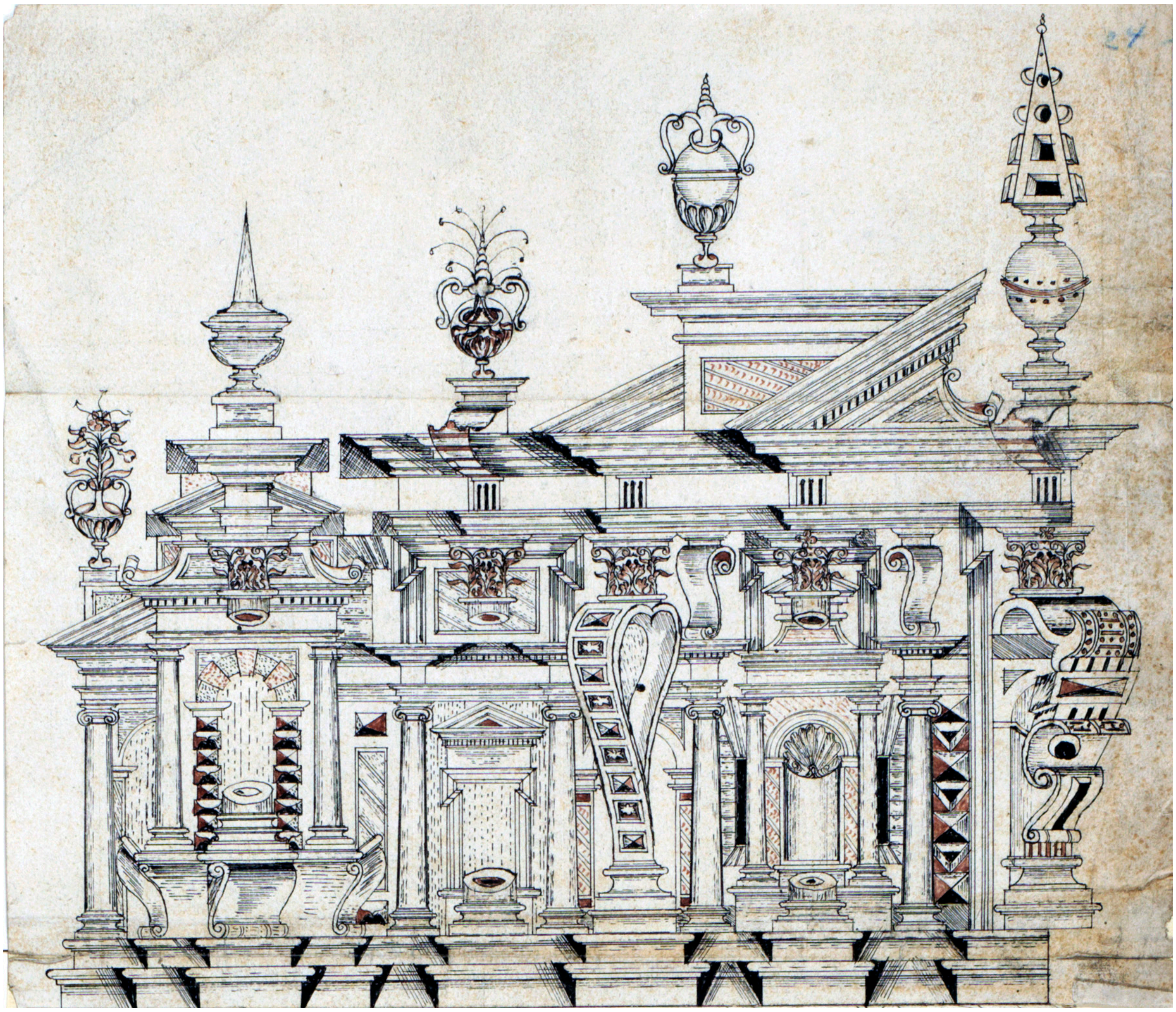


Figure 5.24 continued: Drawing of building facade options, detail, Lorenz Stöer (late 16th century). Germanisches National Museum, Nuremberg. Hz 6979.

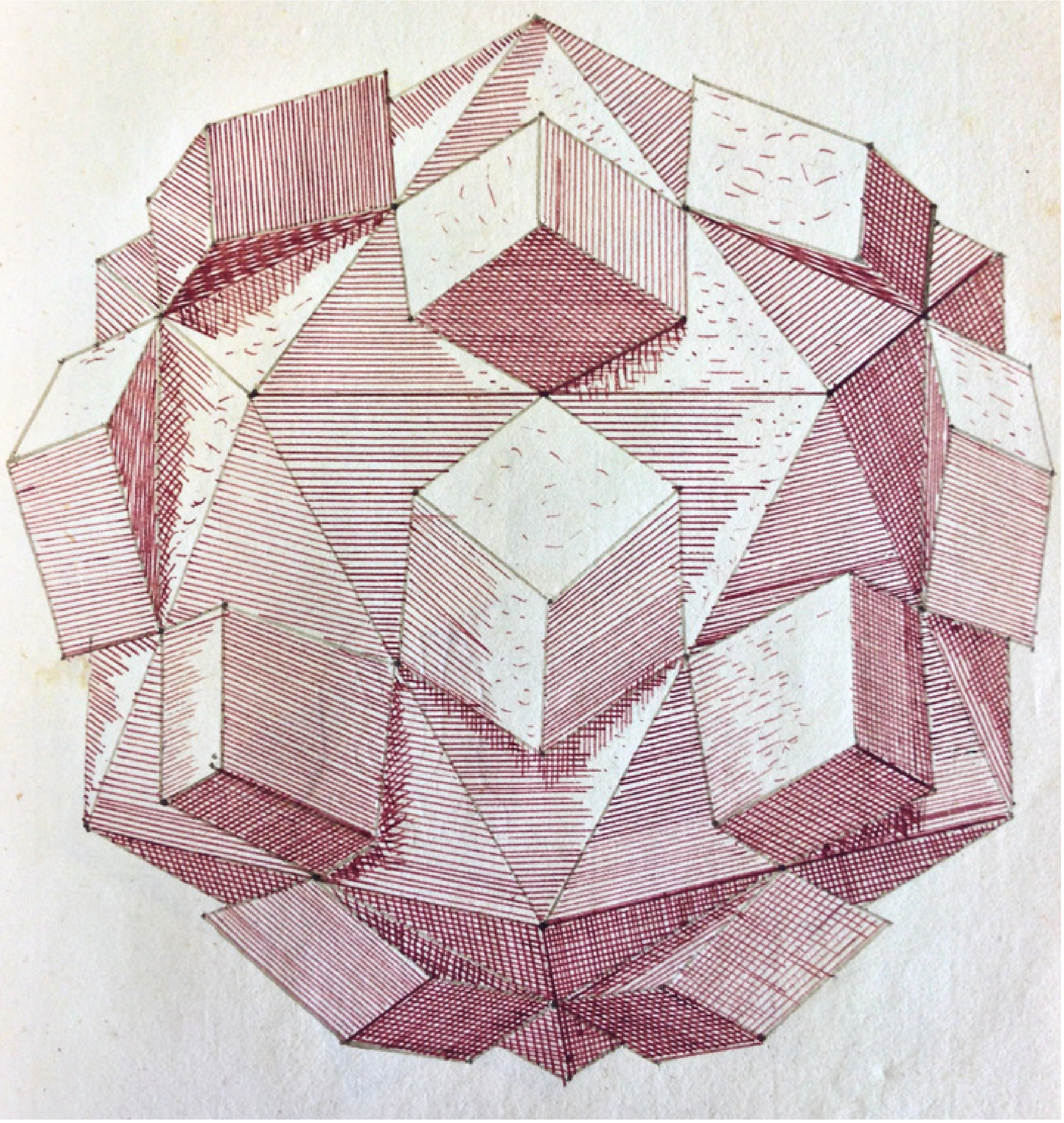


Figure 5.25: Icosahedron variant (p 32) from Stoer's notebook at the Staats- und Stadtbibliothek Augsburg. 4 Cod. Aug. 247.

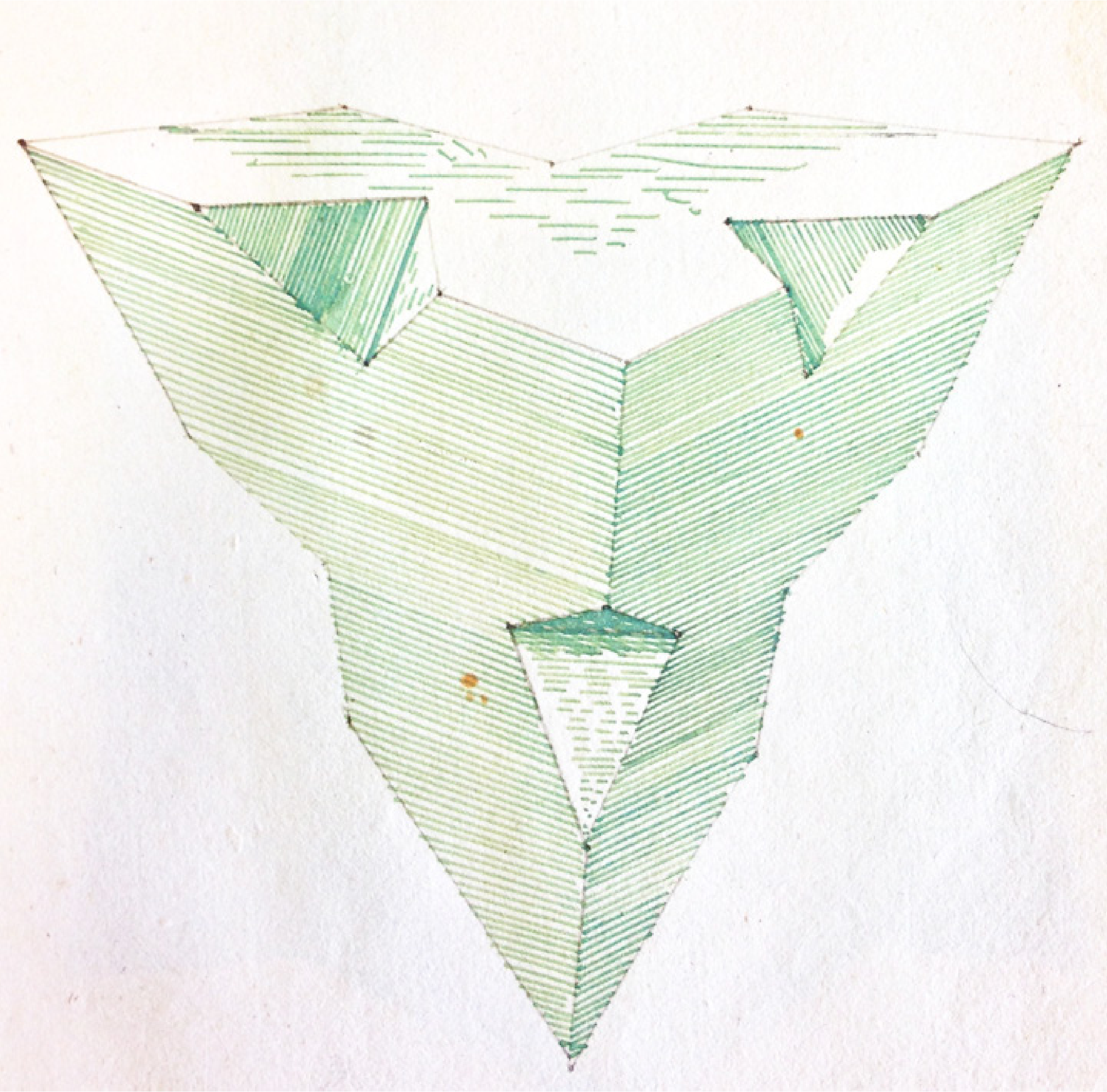
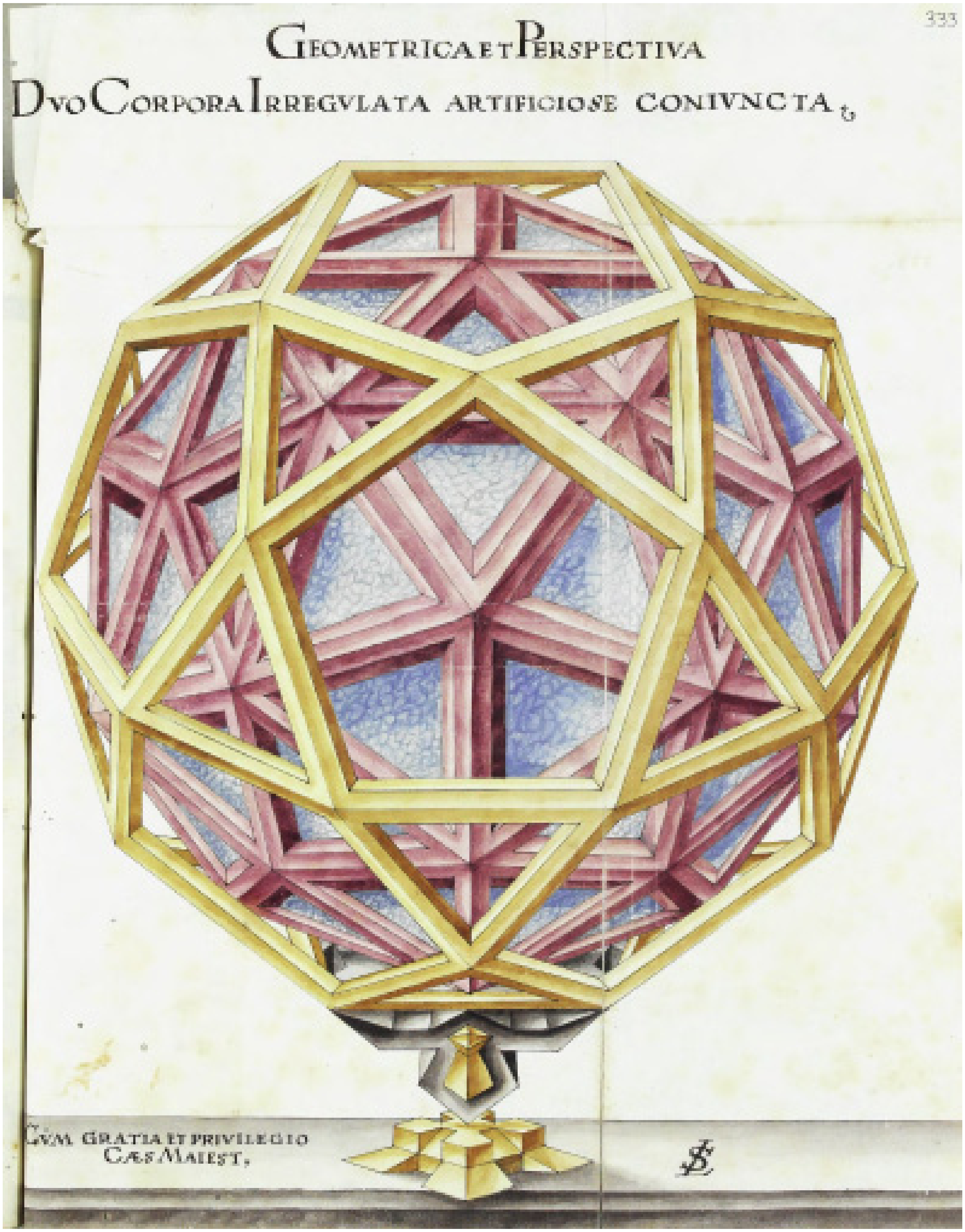


Figure 5.26: Tetrahedron variant (p 33) from Stoer's notebook at the Staats- und Stadtbibliothek Augsburg. 4 Cod. Aug. 247.



333

GEOMETRICA ET PERSPECTIVA

DVO CORPORA IRREGVLATA ARTIFICIOSE CONIVNCTA

CVM GRATIA ET PRIVILEGIO
CÆS MAIEST.

LS

Figure 5.27: Page 333 from the Stoer volume showing a royal priviledge. Universitätsbibliothek München. HS Cim 103.

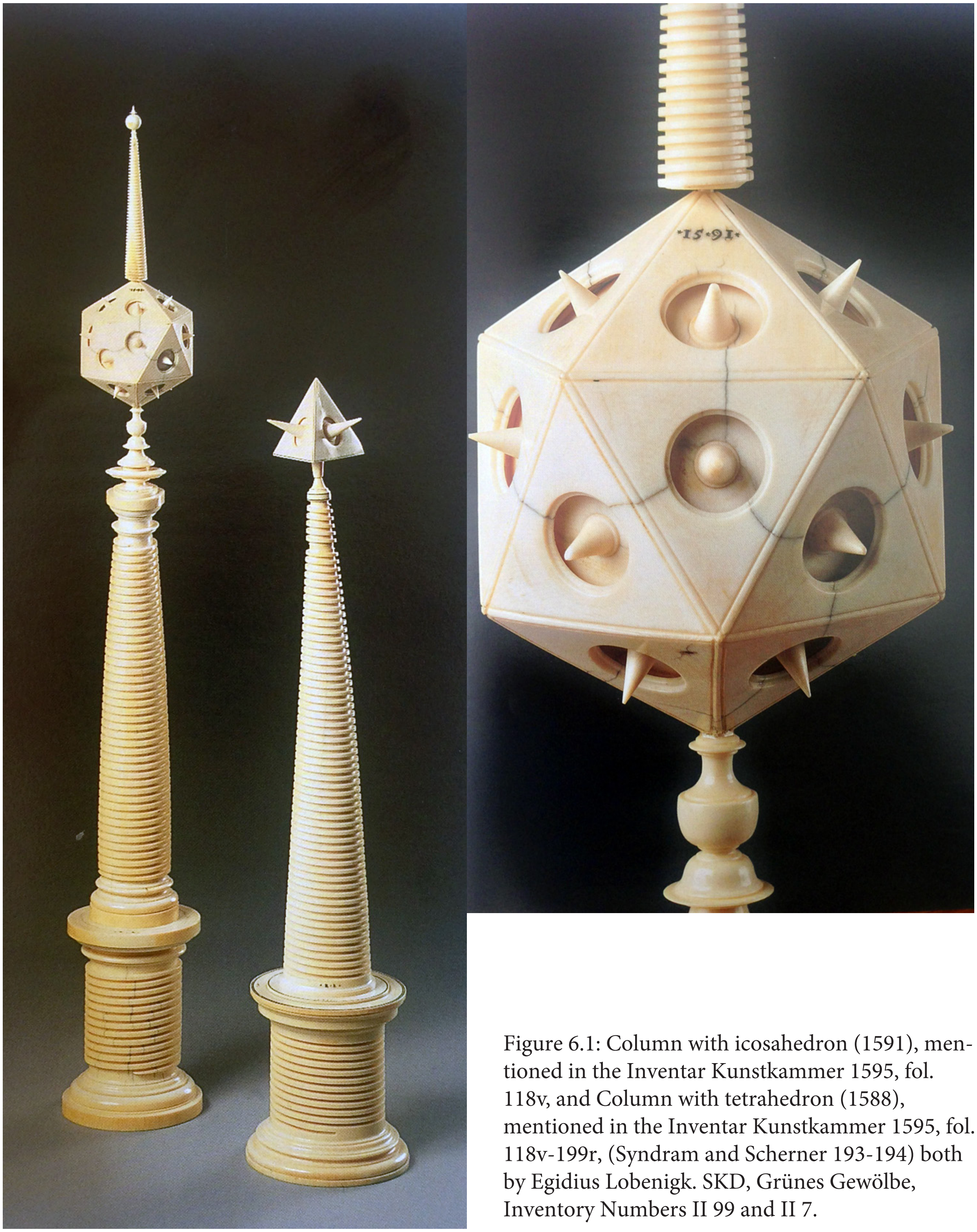


Figure 6.1: Column with icosahedron (1591), mentioned in the Inventar Kunstkammer 1595, fol. 118v, and Column with tetrahedron (1588), mentioned in the Inventar Kunstkammer 1595, fol. 118v-199r, (Syndram and Scherner 193-194) both by Egidius Lobenigk. SKD, Grünes Gewölbe, Inventory Numbers II 99 and II 7.

Figure 6.2: Portrait of the collector Manfredo Settala with a hollow sphere, Daniele Crespi attrb., ca. 1640, Milan, Pinacoteca Ambrosiana.

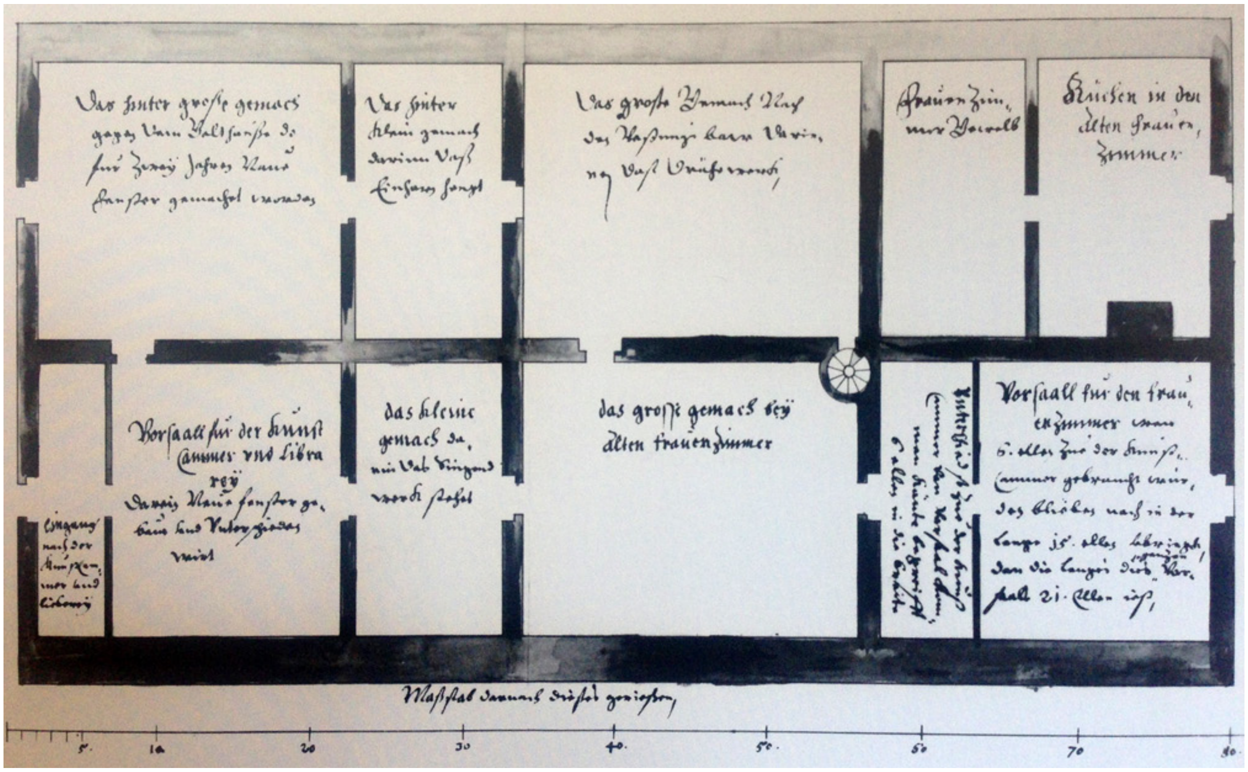


Figure 6.3: 1615 plan of the Royal Kunstkammer by David Uslaub. StA-D, 100024 Gehimer Rat (Geheimes Archiv), Loc. 7324/1, fol. 258r.

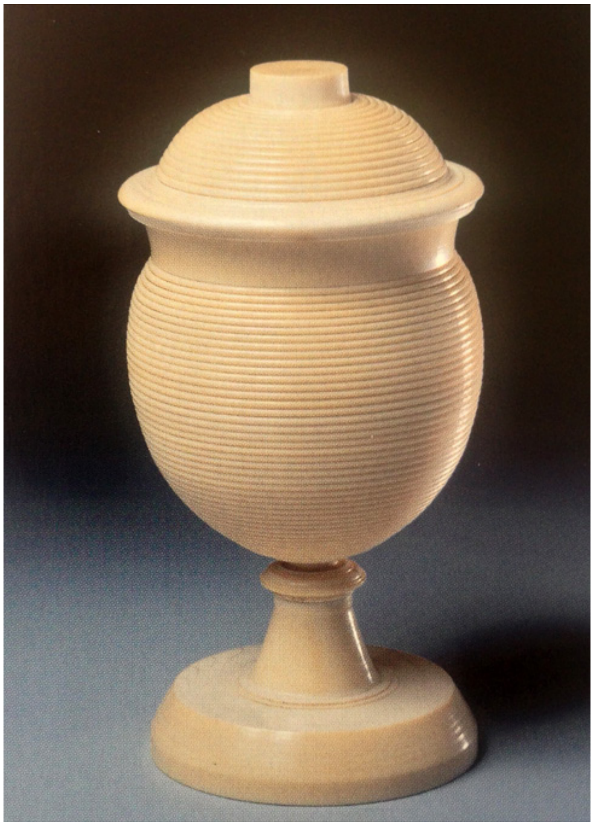


Figure 6.4: Goblet (1586) by Elector August of Saxony from Inventar der Kunstkammer 1640, fol. 44v. No. 100. Grünes Gewölbe, SKD. Inventory Number II 65.

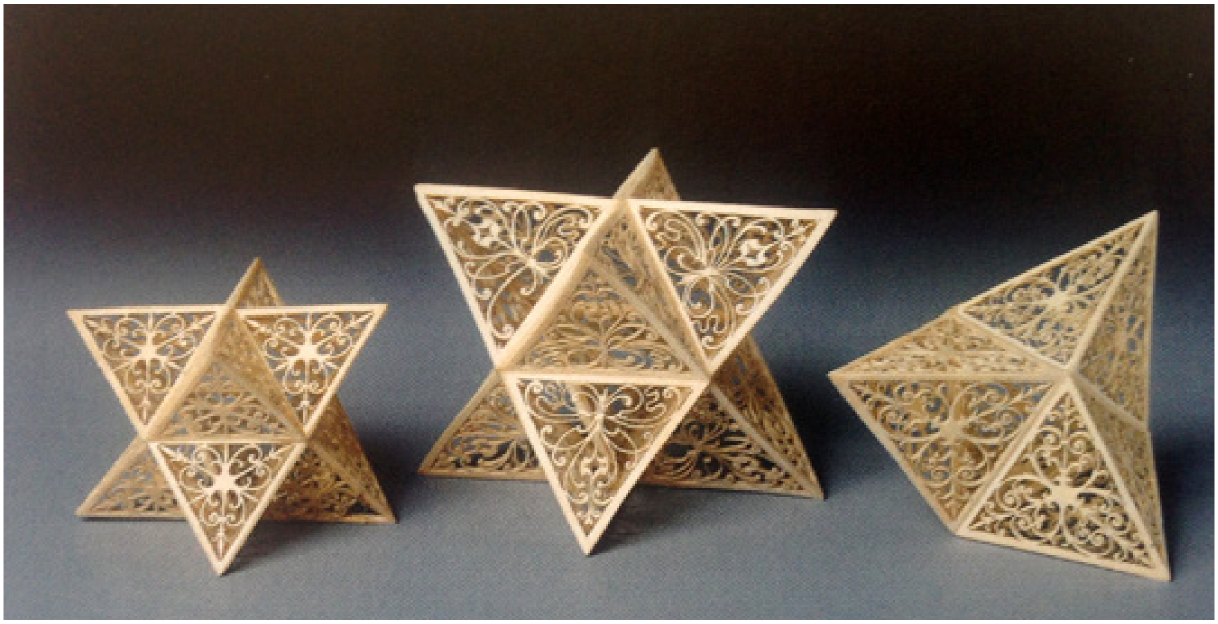


Figure 6.5: Two ornamental stars and a polyhedron in ivory (ca. 1620), Anton Örtel, Inventar der Kunstkammer 1640, fol. 455v. Grünes Gewölbe, SKD. Inventory Number II 269 uu/2, II 269 uu/1 and II 269 uu/4.

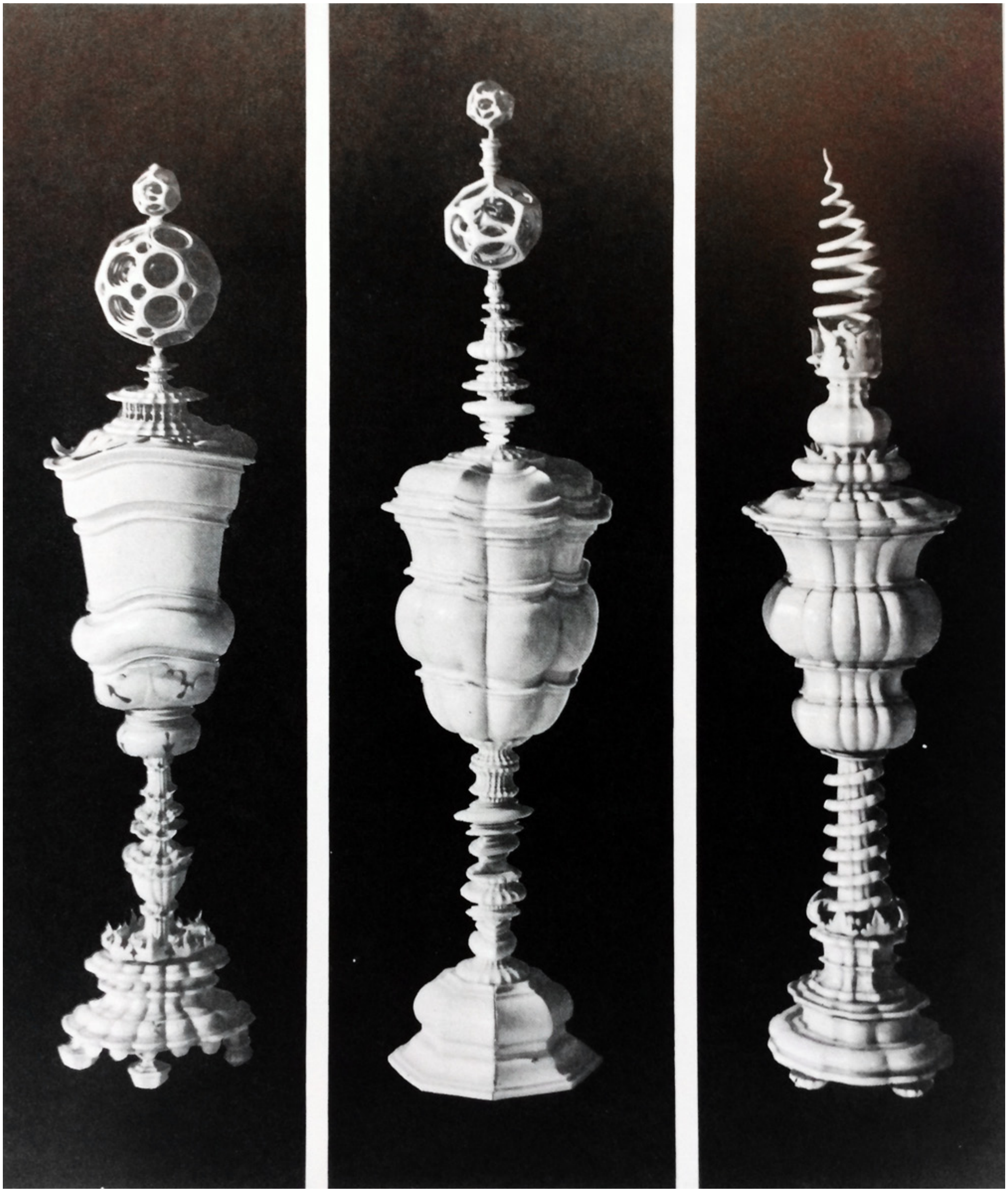


Figure 6.6: Ivory Goblet, Georg Friedel, Dresden, late 16th century, Grünes Gewölbe; Ivory Goblet, Georg Wecker, Dresden, 1581, Grünes Gewölbe; Ivory Goblet, Jakob Zeller, Dresden, late 16th century, Grünes Gewölbe.

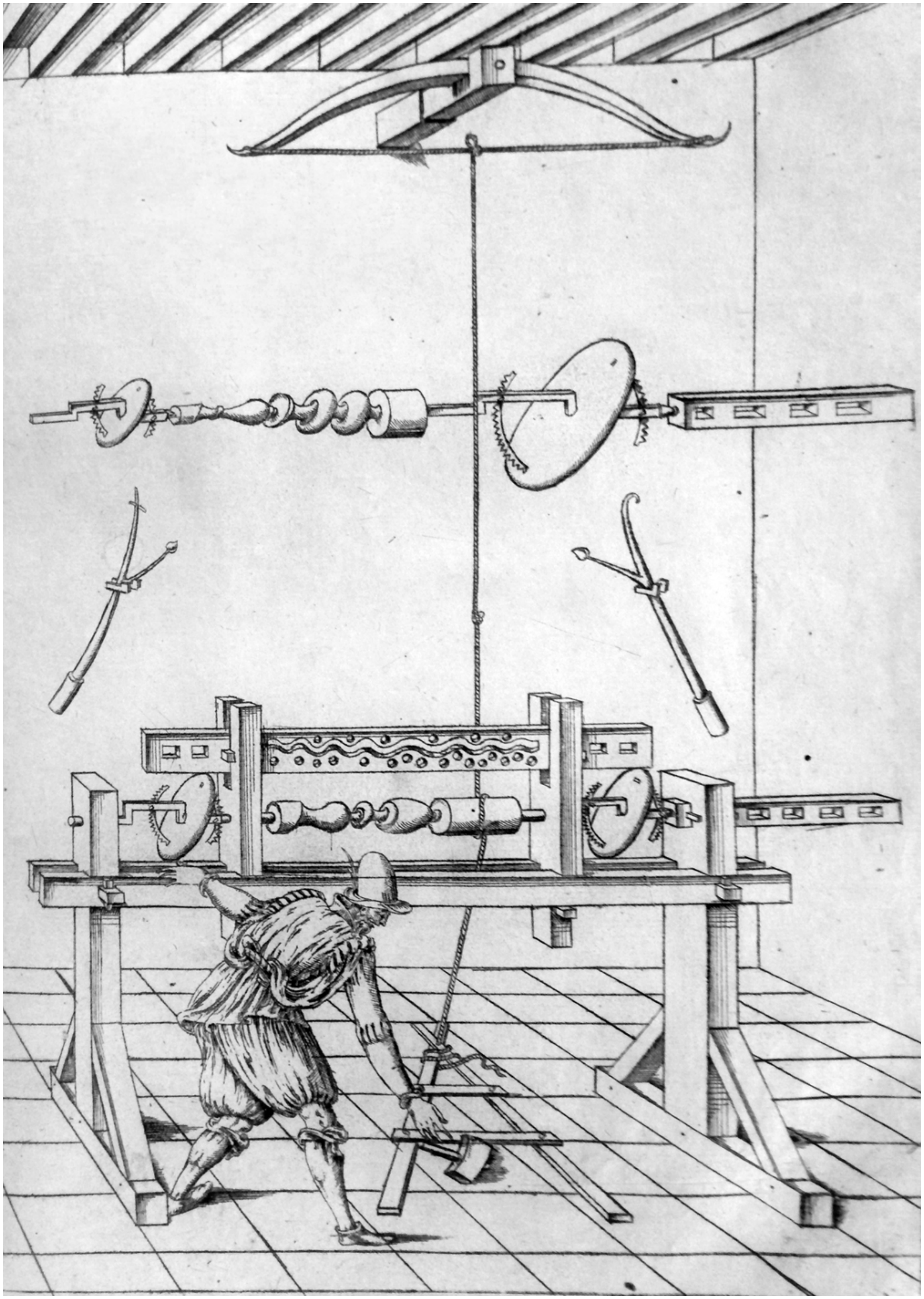
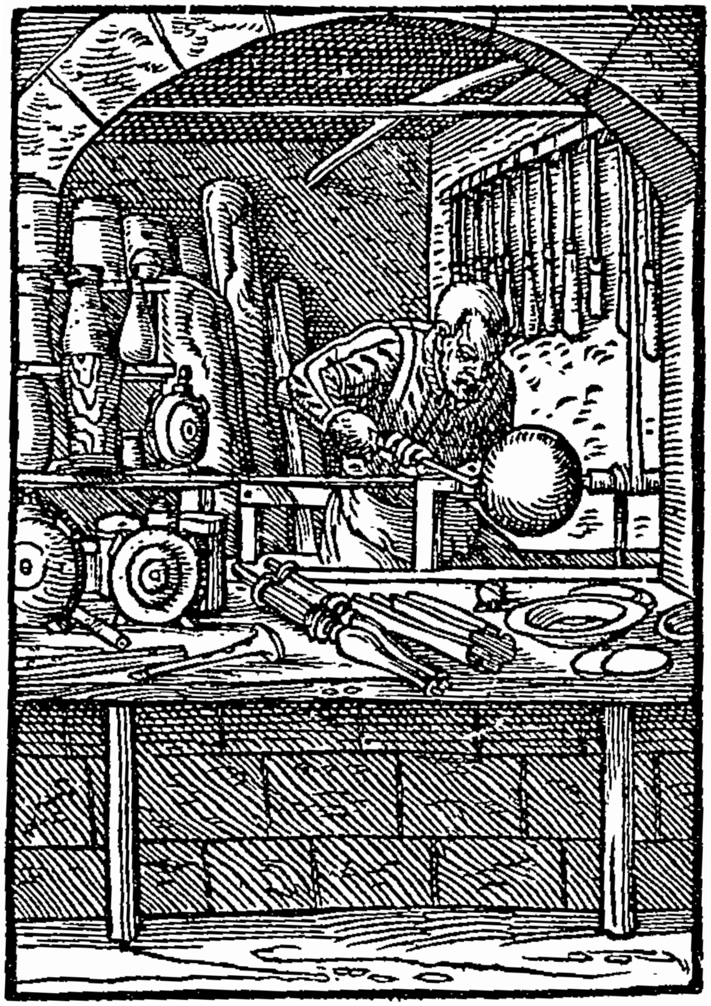


Figure 6.7: A turner at work from Jacques Besson's *Theatrum instrumentorum et machinarum* (1578).

Tornarius, Der Holtzdreher.

SEdulus è flaua tornarius omnia buxo,
Torno meo torno, quicquid habere voles.
Pyxidas innumeros hominum formamus in vſus,
Immenſa quæ non vtilitate carent.



In quibus abſcondens rerum tibi mille colores,
Clam penitus ſerues, nobile quicquid amas.
Hîc pila conficitur, miraq́ volubilis arte,
Huc illuc baculis fortibus icta ſalit.
Hîc nec abeſt pueris, qui concitat acrius iram,
Verbere quem verſes per ſola plana, trochus.

L 5 Bali-

Figure 6.8: Page depicting a lathe worker from Hartman Schopper's *Omnivm illiberalivm mechanicarvm* (1568).

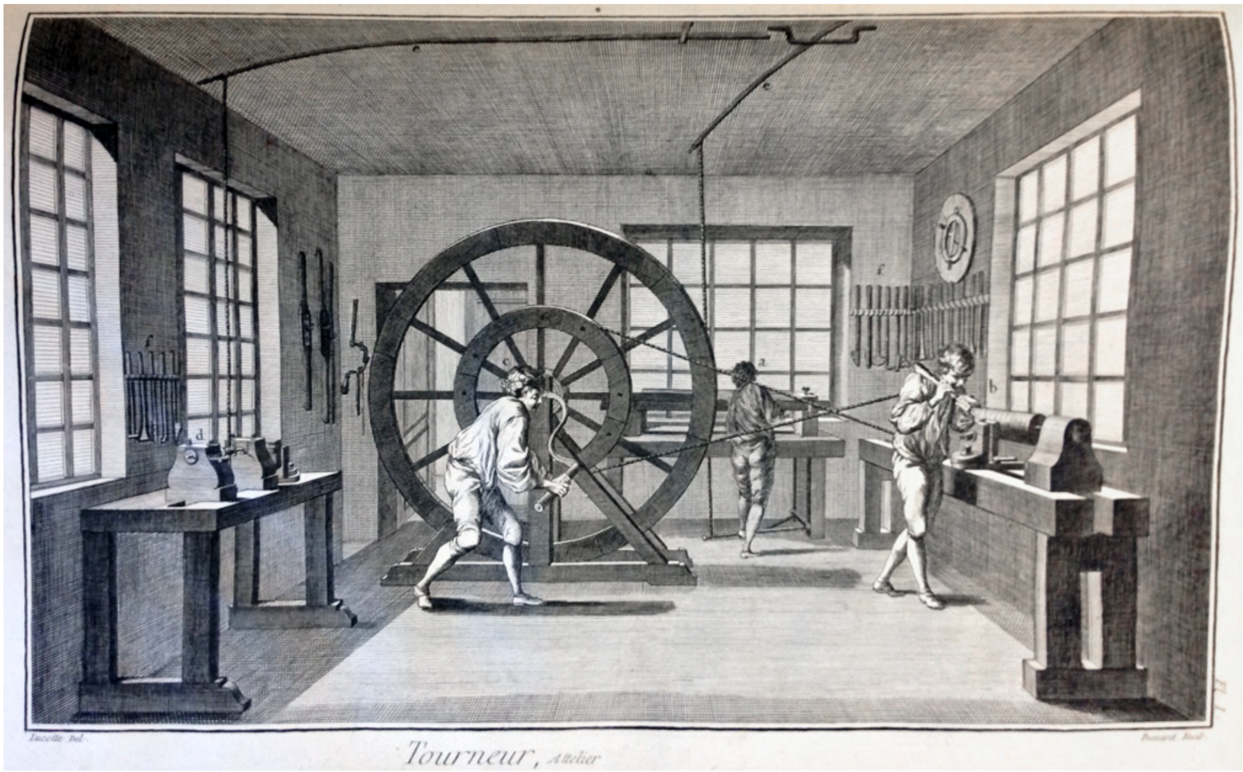


Figure 6.9: Atelier of a turner with a "hand fly wheel" from Diderot’s *Encyclopédie* (1751-1765).

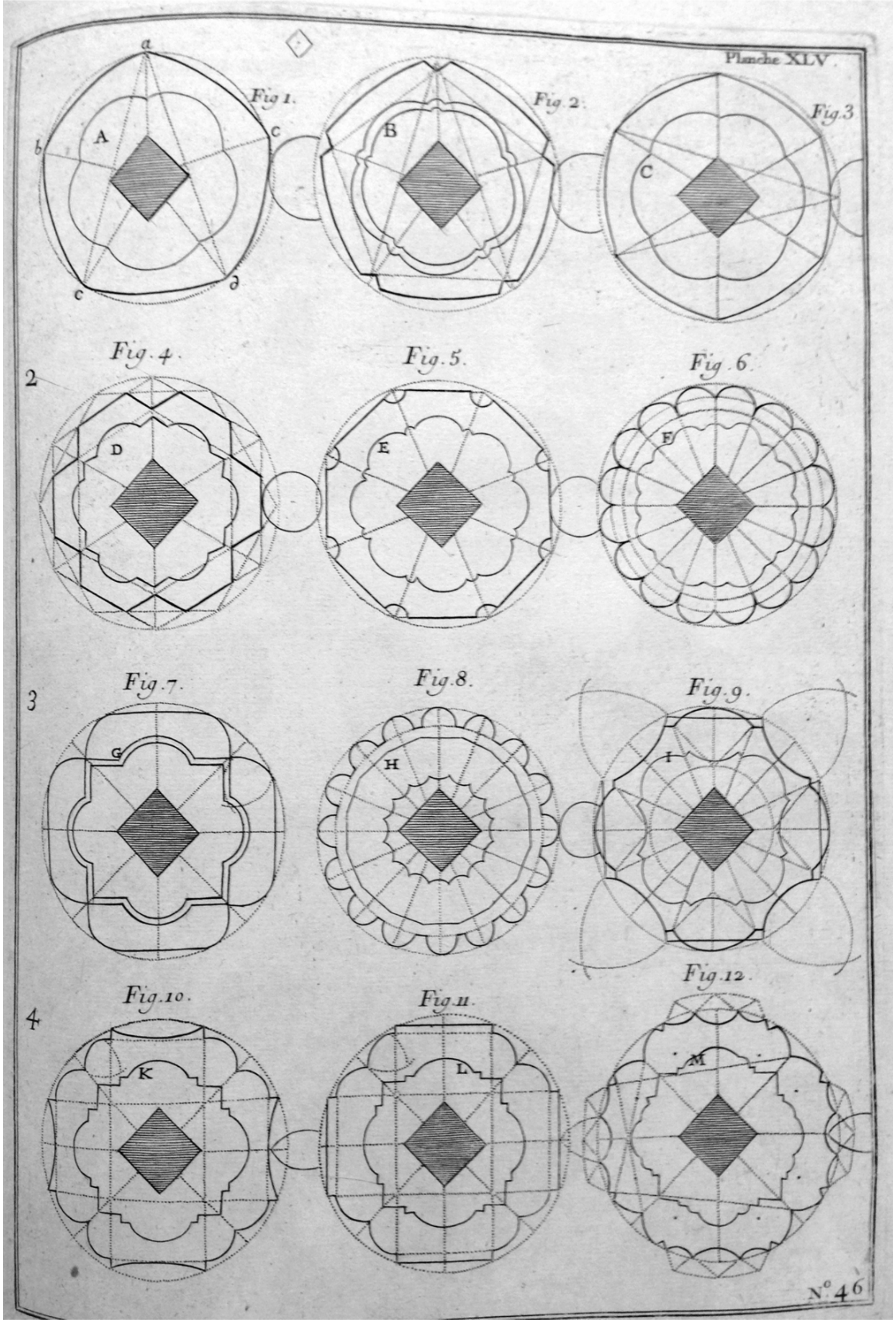


Figure 6.10: Mandrils (Plate XLV) from Charles Plumier's *L'art de tourner...* (1749).

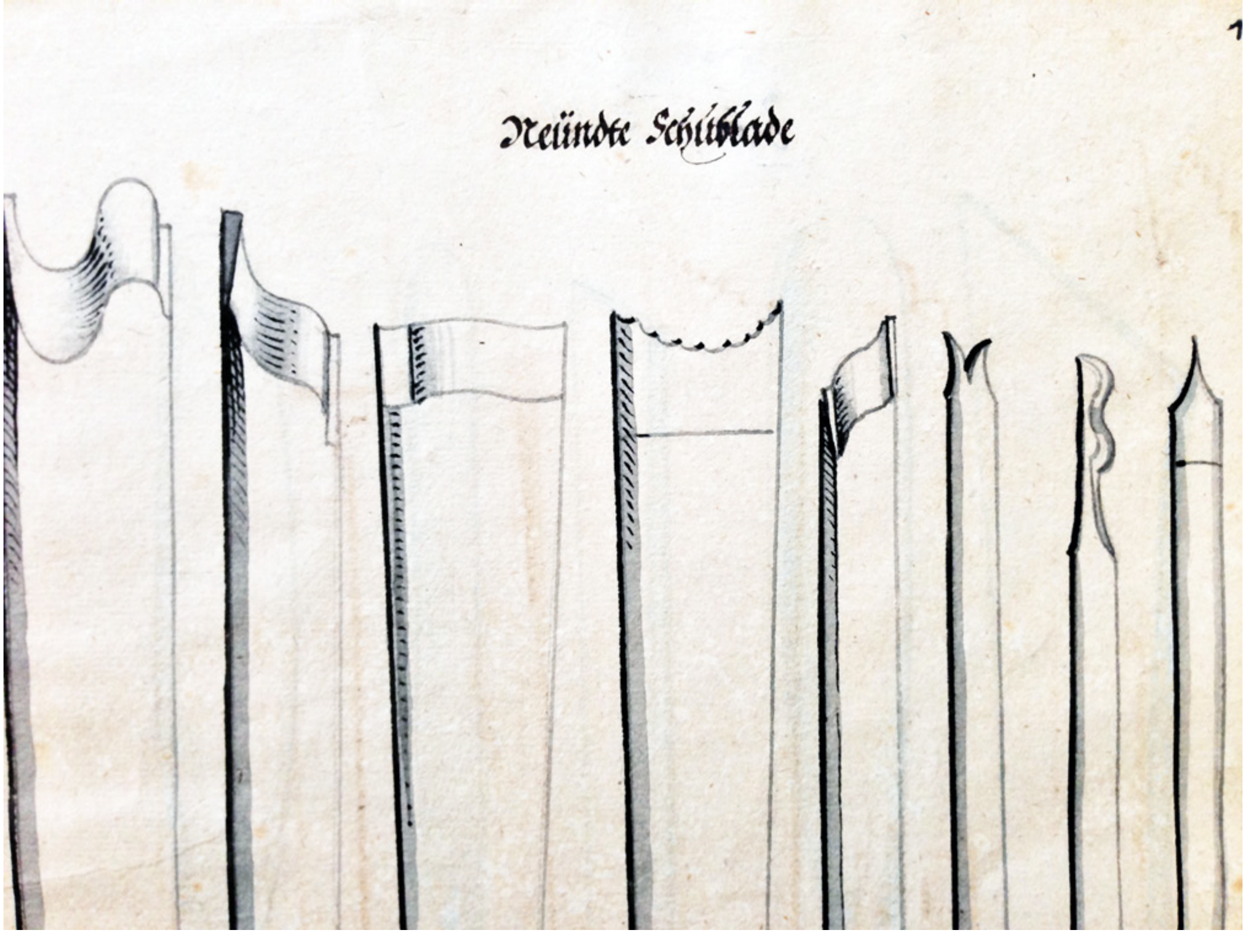


Figure 6.11: Page from the Inventar Drechselkammer 1622 depicting lathe-turning tools in the "ninth drawer." The tools' profiles gesture towards the incisions they would make in ivory. SKD, Inventory Number 68.

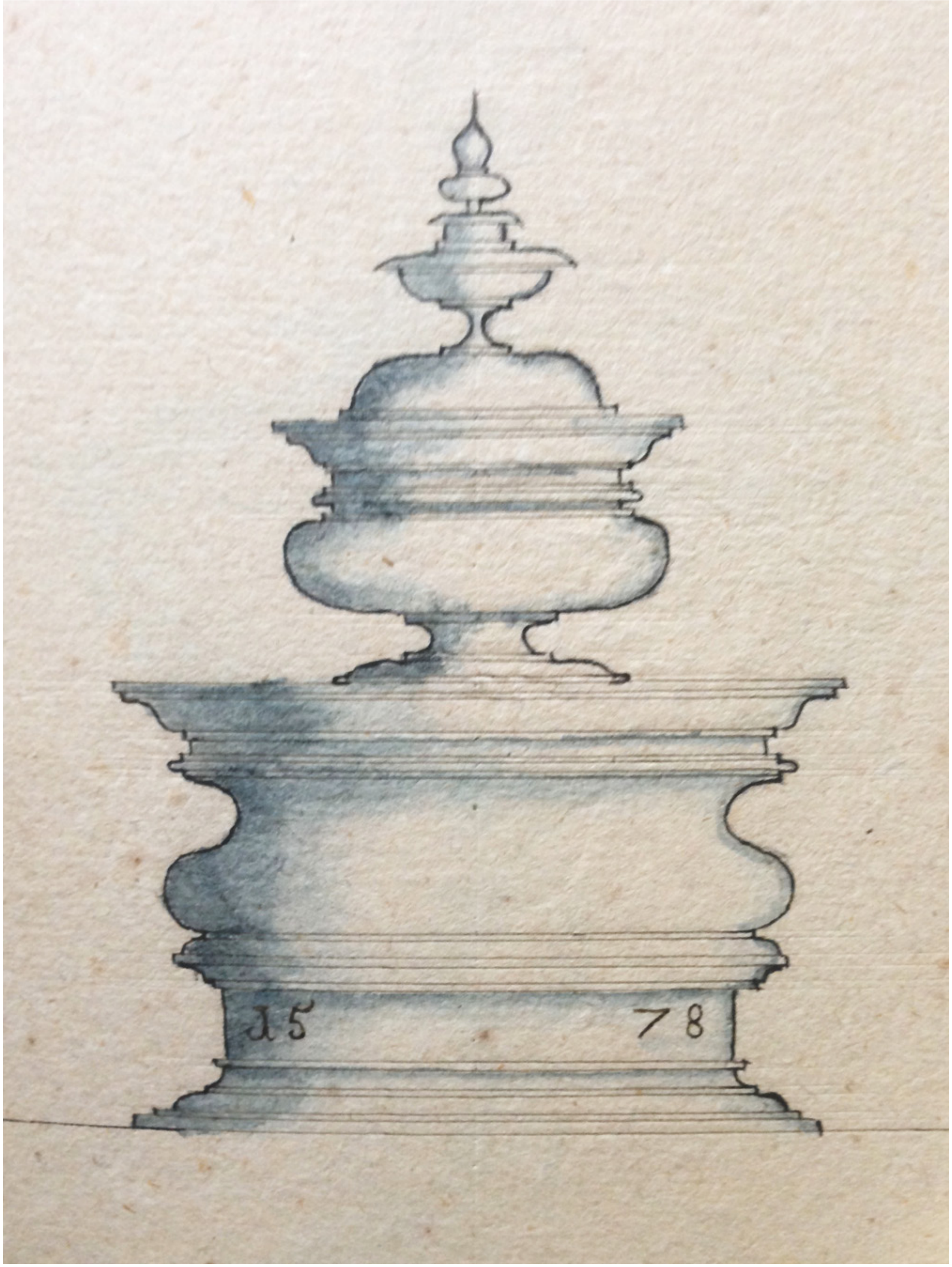


Figure 6.12: A drawing of a turned ivory vessel from 1578, enclosed as a loose sheet with the Inventar Drechselkammer, Dresden 1684.

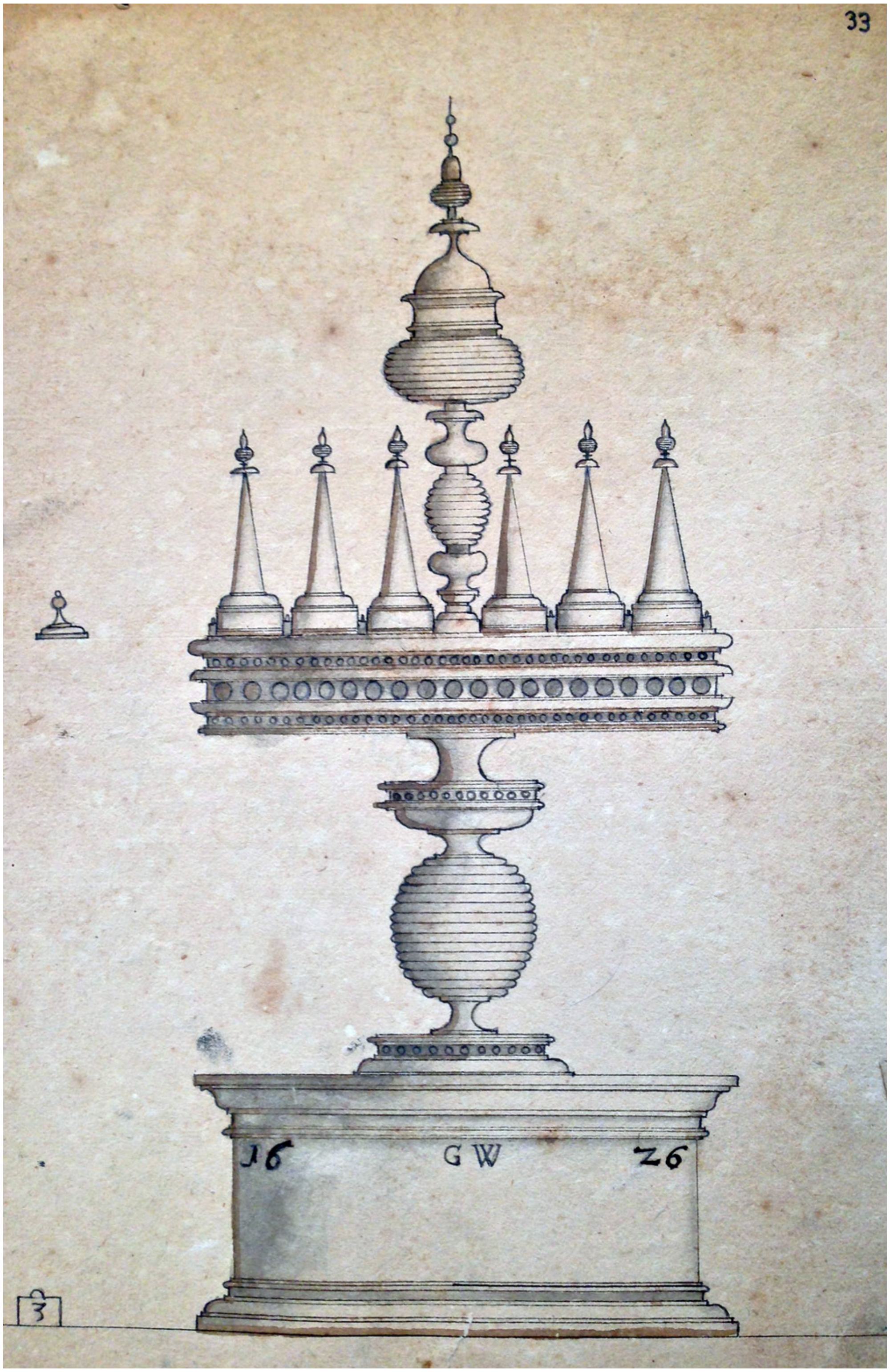


Figure 6.13: A drawing of a turned ivory centerpiece, possibly unrealized, by Georg Wecker and dated 1626. It was similarly included in the Inventar Drechselkammer 1684.

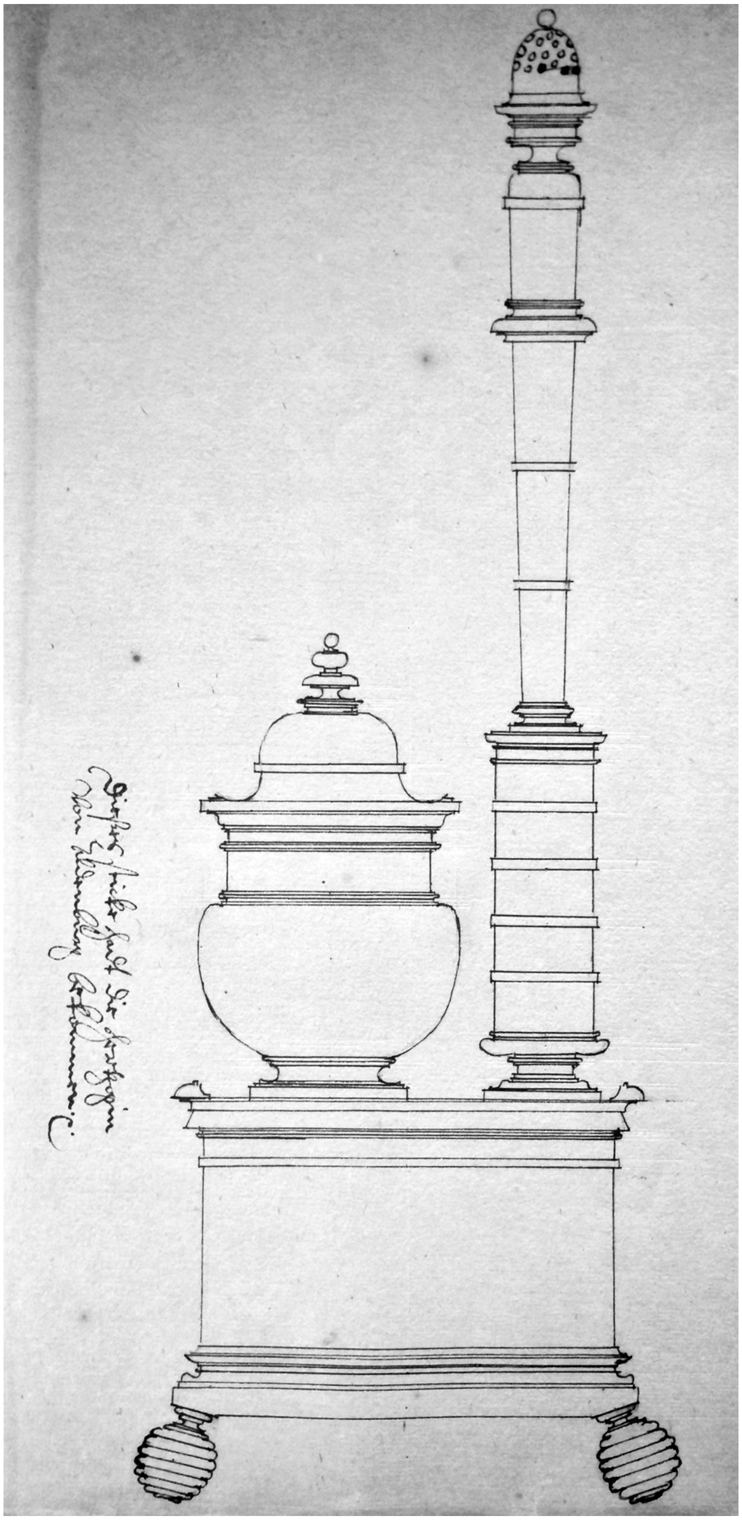


Figure 6.14: Measured drawing of a turned ivory column enclosed as a loose sheet with the Inventar Drechselkammer, Dresden 1684. The handwriting reads “The archduchess of Wurdenberg received this piece.”

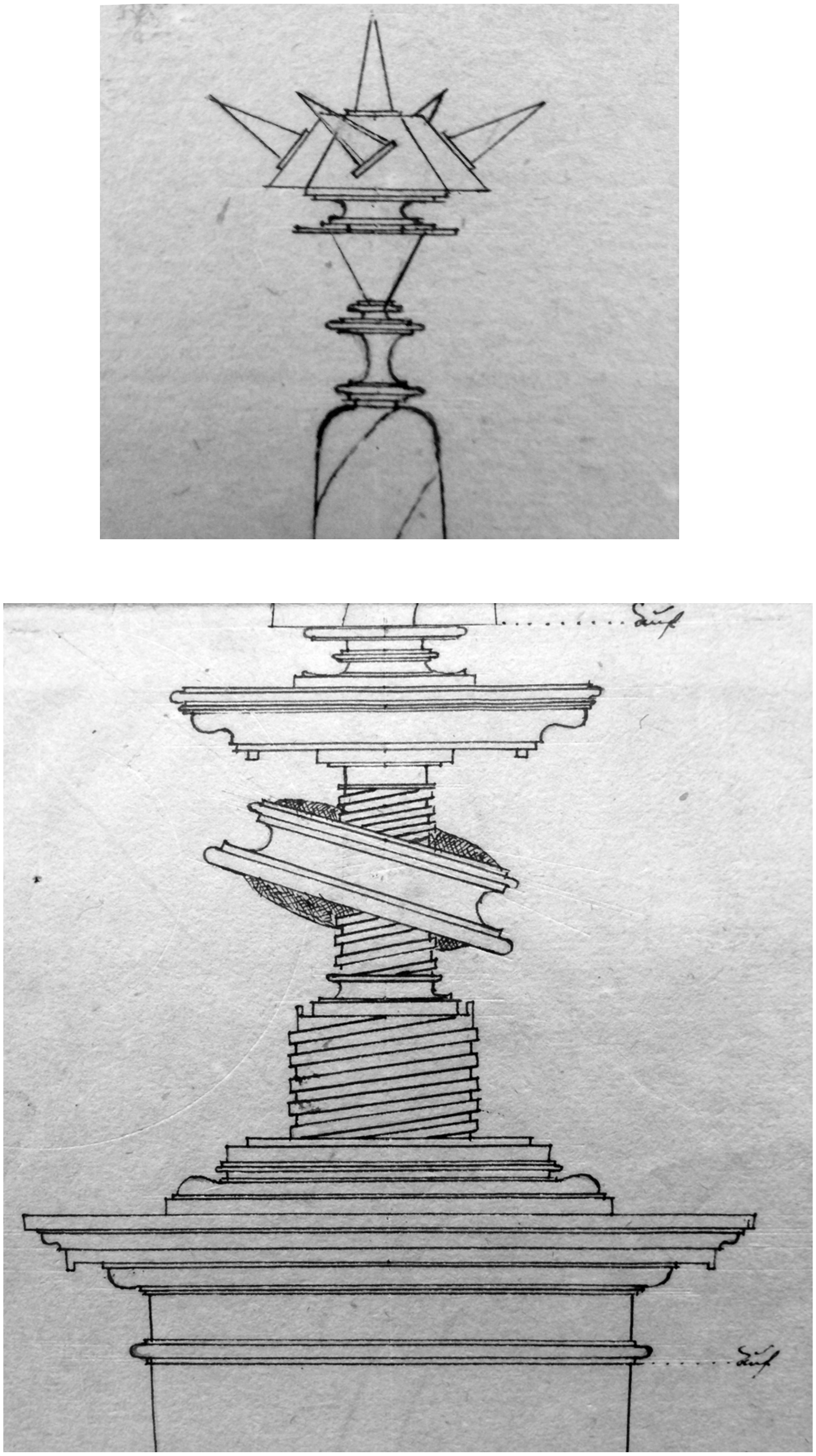


Figure 6.15: Details of a drawing of an ivory column enclosed as a loose sheet with the Inventar Drechselkammer, Dresden 1684. A handwritten note, not pictured, reads "Archduke Leopold Wilhelm received this piece."

Figure 6.16: A page from *Historische Nachricht von den nürnbergischen Mathematicis und Künstlern...* (1730) depicting the work of the turner Lorentz Zick. The turned ivory piece on the right of the page (labelled Fig. 4) shows a nested polyhedra with a star piercing the outermost layer.

Figure E.1: Portrait of Oswald von Eck with lesser rhombicosidodecahedron (1553), Hanns Lautensack (ca. 1520-1564/66), Acc. Num. 41.1.143, Metropolitan Museum of Art.

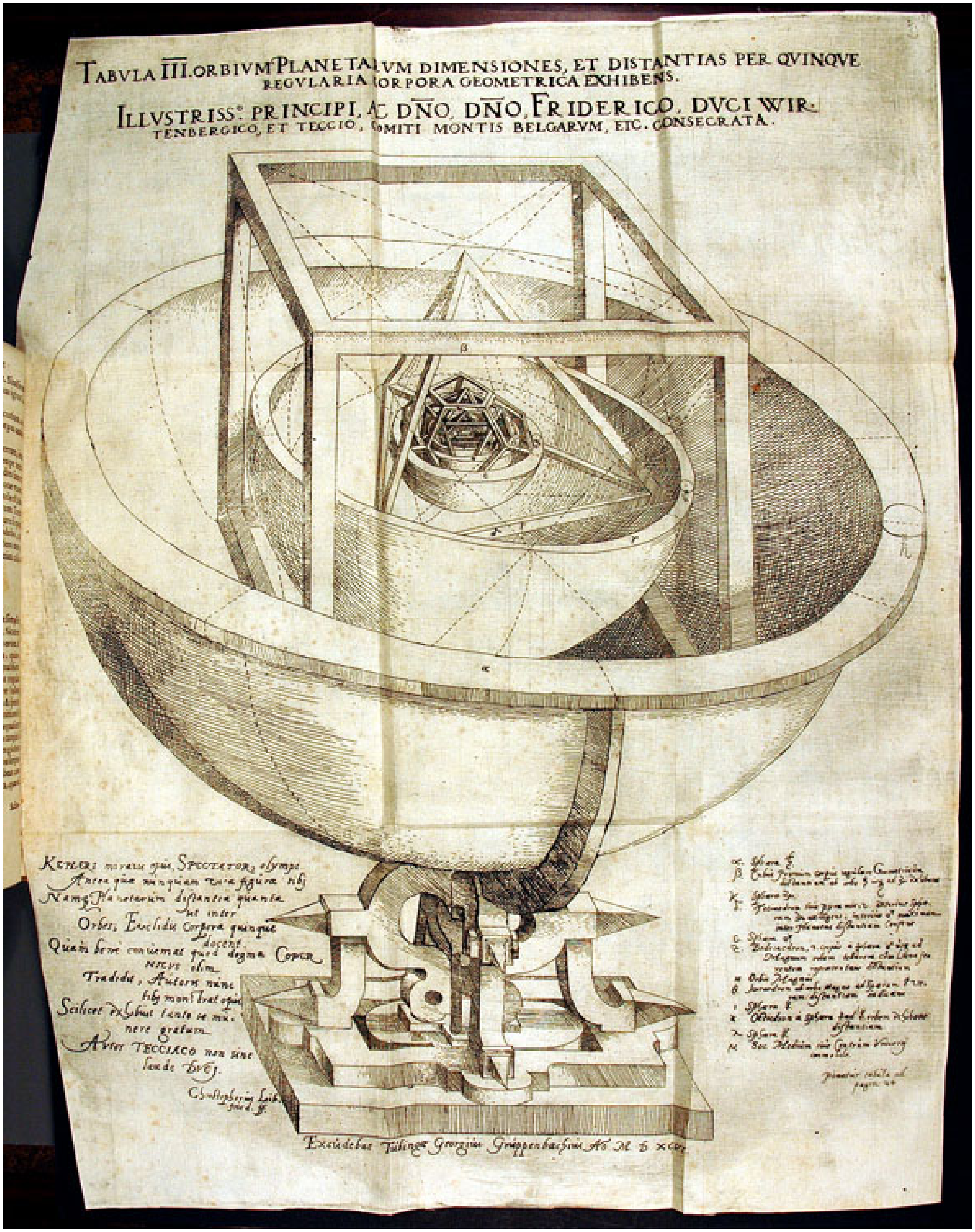


Figure E.2: Tabula III showing the planetary intervals as Platonic Solids from Johannes Kepler's *Mysterium Cosmographicum* (1596).

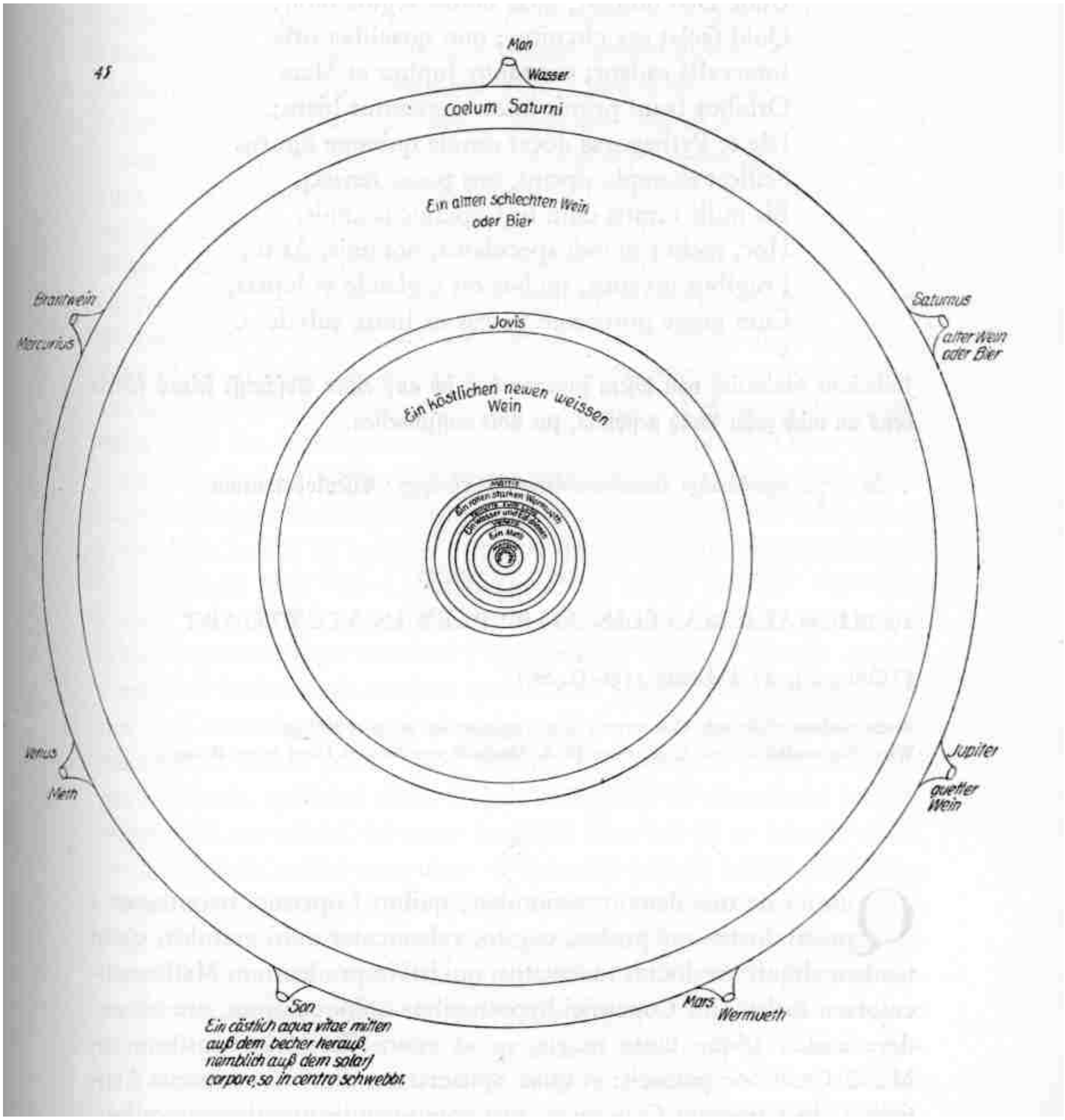


Figure E.3: Schematic diagram (1596) of the first version of the *Mysterium* model by Johannes Kepler. Reprinted in GSW 13, nr. 28. The seven spigots are represented on the outside of the model as are the type of liquid they would secrete. "Vermuth for mars, good wine from Jupiter, old wine or beer from Saturn, water from the moon, Branttwein (vodka) from mercury, Mreth from Venus, and a delicious acqua vita (repeatedly-distilled brandy) emitted out from the chalice, namely out from the solar body which floats in the center."

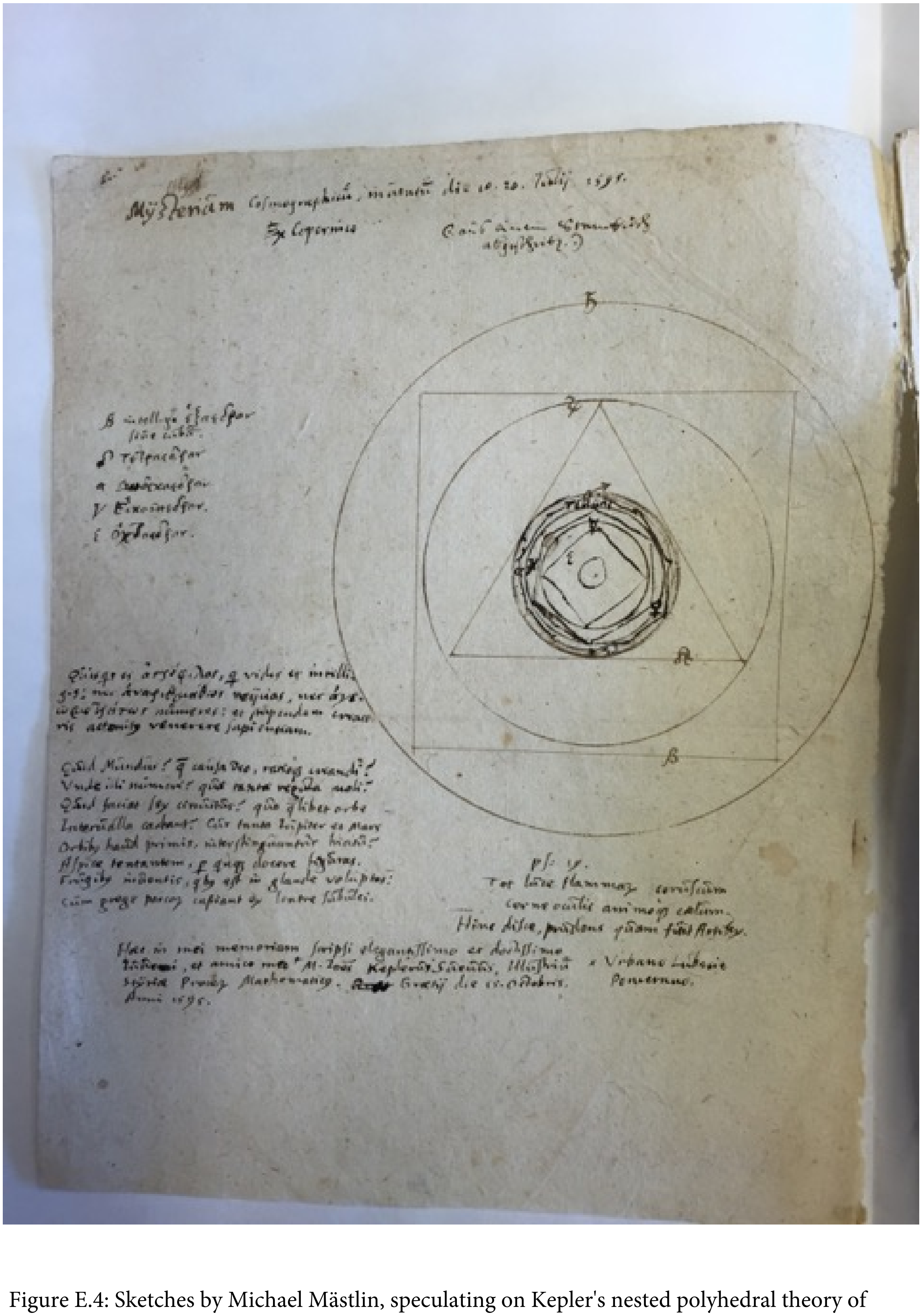


Figure E.4: Sketches by Michael Mästlin, speculating on Kepler's nested polyhedral theory of planetary intervals. Late 16th century, Stuttgart Landesbibliothek.