



# Essays on International Finance and Asset Pricing

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# Essays on International Finance and Asset Pricing

A dissertation presented

by

Thomas Yang Powers

to

The Committee for the PhD in Business Economics

in partial fulfillment of the requirements

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in the subject of

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## **Essays on International Finance and Asset Pricing**

### **Abstract**

My first essay investigates the relationship between risk and return for investment projects within the firm. I focus on the film industry and find that more volatile movies have higher rates of return, even though this risk is entirely idiosyncratic. My second essay explains the high rates of return on commodity currencies in terms of the procyclicality of commodity prices. Commodity prices are procyclical because commodities are inputs, and thus demand for them is driven by the global business cycle. I also use labor market data to show that increases in labor costs during commodity booms contribute to the higher real exchange rates observed in commodity exporting countries. My final essay, co-authored with Jeffrey Frankel, studies optimal monetary policy in commodity-exporting economies facing a terms-of-trade shock. We build on the previous literature by introducing borrowing constraints, and find that currency depreciation during such a shock leads to higher welfare than either a fixed exchange rate or inflation targeting.

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# Chapter 1

## Expected Returns on Real Investments: Evidence from the Film Industry

### 1.1 Introduction

Businesses expect different returns on different projects, and these differences determine which innovations are developed, which jobs are created, and which investments are available to savers. Nevertheless, despite its relevance, we know relatively little about which factors drive the cross-sectional variation in expected returns for real projects.

In studying this question, our key challenge is that we cannot observe costs. When a firm undertakes a project, it does not have to notify anyone. The cost of financing the project is often unreported, or may be reported only as part of a firmwide aggregate. Investments are not even necessarily priced by outsiders: a price may simply be the shadow price of internal funds faced by corporate decision-makers, which is inherently unobservable.

In this paper, I use a proprietary, project-level dataset on the film industry to study one cross-section of expected returns on real investments. Using this dataset allows me to overcome several issues. First, we have comprehensive data on costs and revenues for a

large cross-section of projects. Second, large studio movies are almost never produced then not released; so selection bias is minimal. Third, movie returns have very low correlations with the market, so we do not need to control for market risk premia when explaining their returns. Finally, movies differ cross-sectionally in observable ways; in particular, certain characteristics might make movies more or less risky, and thus price the cross-section.

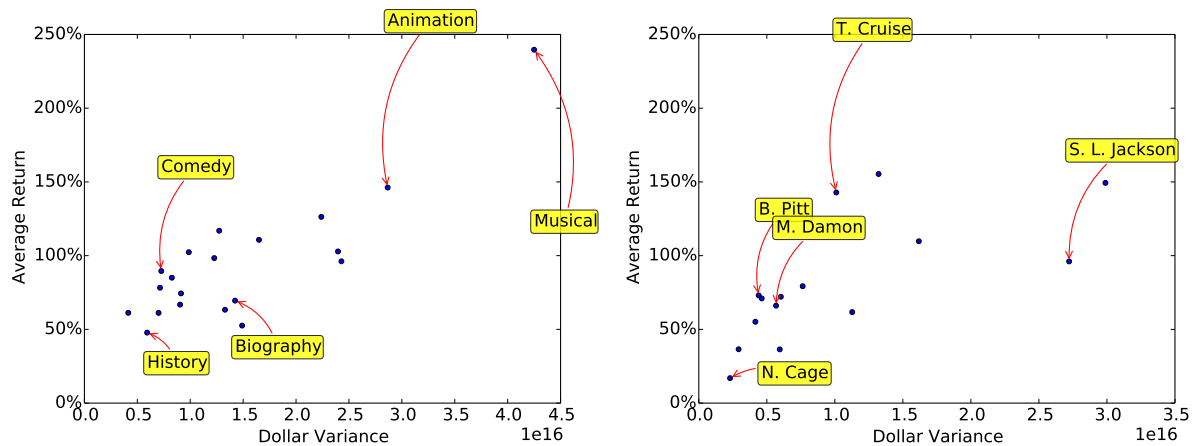
One area in which the pricing of real projects might differ from, say, stocks, is in the pricing of idiosyncratic risk. In liquid secondary markets, we usually think of certain risks as frictionlessly diversifiable, and thus unable to earn any compensation. But in primary markets, frictions may create a spread in expected returns between projects with high and low idiosyncratic risk. For example, a high idiosyncratic risk project might lead to a higher probability of bankruptcy, and if bankruptcy has deadweight costs, firms should demand higher expected returns to produce such a project.

### **Empirical Findings**

The simplest way to measure the risk of a movie is by estimating the dollar variance of its payoff. “Dollar variance” means the variance of the dollar return. Under this metric, the risk of a portfolio of movies is just the sum of the risks of its components (assuming statistical independence). This property captures the intuition that the risk contribution of each movie should be film-specific, rather than a function of the other movies produced by the same studio.

Figure 1.1 plots expected returns on films against these dollar variances. Each point represents movies grouped by a characteristic; for example, in the left panel, I group movies by genre, and then plot the average return and sample dollar variance within each genre as a point. The right panel applies the same process for actors with more than thirty films, a bound I impose to ensure that small sample effects do not drive the results.

Figure 1.1 shows that expected returns are increasing and concave in dollar variance. One way to quantify this statement is to assume parametrically a linear relationship between expected return and dollar *volatility* (the standard deviation of dollar returns), and then



**Figure 1.1:** Returns are increasing and concave in dollar variance. Movies clustered by left: genres; right: actors.

estimate that relationship using GMM. This linear relationship is plotted in figures 10, 13, and 16. In this estimation procedure, both the “right-hand variable” and the slope are estimated, since the risk of each movie must be fitted from its associated characteristics. When I run this procedure, I find that a \$1 million increase in the dollar volatility of payoffs raises expected returns for a film by at least 43 basis points, with the effect being as large as 116 basis points depending on the specification.

When computing these estimates, I take the heavy-tailed nature of the data seriously. Heavy tails can lead to spuriously large rejection regions. Additionally, heavy tails combined with skewness can lead to a small-sample bias. The bias arises if characteristics, say, film genres, have too few movies associated with them. Then, those characteristics that are lucky enough to be associated with positive tail outcomes will exhibit both high realized risk and return. Consequently, we will observe a spurious correlation between the two variables. To address these concerns, I use a bootstrap procedure to correct the initial GMM estimates for bias and compute the appropriate confidence regions. As an alternative, I also compute risk using interquartile ranges, which may be less sensitive to heavy tails, and still observe a positive relationship between risk and return.

A key assumption behind my estimates is that all movie risk is idiosyncratic, so I do not

need to control for market (or other) risk premia. To support this idea, I review previous literature showing that as much as 80% of the variation in box office revenues can be explained by film-specific characteristics, such as critics' ratings, the inclusion of a star actor, the initial budget, or the MPAA rating (G, PG, etc.). Moreover, I study aggregate data and find that the industry's domestic profits are not correlated with the U.S. business cycle. These findings are consistent with previous studies, which I also review. Finally, I argue that even if movies had different amounts of systematic risk, they would have to have implausibly large differences in betas in order to generate the large spreads in expected returns I find.

## **Theory**

Much of the literature assumes project risk can be perfectly passed through to public markets, so they are priced by that market's stochastic discount factor. This assumption is mostly a convenient simplification, however; Holmstrom (1989) argues that in reality, "firms do care about idiosyncratic risk" when deciding the "amount of money to be allocated among proposed projects," even if secondary markets do not.

In fact, corporate finance theory provides us with several possible frictions explaining why firms might price idiosyncratic risk in real investments:

- Bankruptcy might cause deadweight losses; for example, legal fees, or the early liquidation of otherwise good projects. Risk that could lead to bankruptcy therefore carries a cost.
- Projects with a lot of idiosyncratic risk may need to be monitored heavily by financiers, since they won't automatically do well or poorly with the market. The financiers will charge more to finance these projects.
- Managers don't care about shareholders, but rather about their own careers. Taking on risky projects threatens their jobs. Consequently, even if shareholders could perfectly diversify project risk across firms, managers avoid it.

To gain insight, I develop a reduced-form model of the industry incorporating some of the above features. Although my dataset is not rich enough to differentiate sharply between the possible mechanisms, the model is relatively simple and explains the facts.

In the model, firms with more equity in the form of internal cash face smaller frictions. I make the reduced-form assumption that financiers allow these firms to borrow more dollars, and charge them less for taking on risky investments. I model these risk charges as deadweight losses from the frictions, so they are not a risk premium collected by financiers. On the supply side, projects are created by screenwriters and are matched to studios endogenously. Expected returns vary across projects because firms pay different prices for the projects they acquire.

Projects are indivisible, which captures the intuition that there are limits to diversification at the studio level. This assumption poses a technical challenge: in Arrow-Debreu equilibrium, prices can be found using marginal arguments; but it makes no sense to talk about a “marginal amount” of an all-or-nothing investment. Thus, I remove the Walrasian auctioneer and assume that matching takes place in a more general setting, a combinatorial Vickrey-Clarke-Groves auction. The resulting equilibrium maintains similarities to Arrow-Debreu equilibrium (existence, efficiency, incentive compatibility, etc.), while providing a mechanism for pricing indivisible goods.

I prove two key results about prices that are borne out in the data. First, idiosyncratic risk is priced: if the dollar variance of a project’s return is increased (infinitesimally), its equilibrium expected return increases. Second, expected returns are “concave” in the sense that, if two projects have sufficiently different levels of dollar variance, and the riskier project is assigned to a larger firm, the riskier project’s expected return will be less affected by additional risk. The intuition is that the riskier project will be priced by a firm that is larger and, *ipso facto*, less averse to risk.

The rest of the paper is structured as follows. I first review the literature on this topic. Next, I write down my model to explain my findings. I then introduce my dataset. Finally, I describe my empirical methodology and the findings themselves.

## 1.2 Related Literature

### Real investment opportunities

To my knowledge, this is the first paper that uses a large panel of project-level data to test a cross-sectional theory of expected returns. Nevertheless, there have been a number of papers, both theoretical and empirical, that model the real investment opportunities available to firms.

Belo (2010) has some similarities to this paper, and is a good representative for the growing literature on production-based asset pricing. In Belo's model, firms have a technology that allows them to invest in different "projects" (I use quotes because these projects are not discrete). Firms invest until, on the margin, the expected return from further investment, less the marginal adjustment cost, equals the marginal cost of capital, which is set according to a stochastic discount factor in the market for firm-issued liabilities. The fact that the secondary market stochastic discount factor "passes through" to projects undertaken by the firm implies that there are no substantive differences, aside from adjustment costs, in pricing between real projects and secondary market investments.

Greenwood and Hanson (2013) study the time series of returns to owning a dry bulk cargo ship. They find that behavioral factors (extrapolative expected returns) are necessary to explain the joint time variation in investment and returns. This study is similar to the present work in two ways: first, in the use of project-level data, and second, in the development of a theory for the formation of expected returns on the part of managers. I focus more on cross-sectional differences in expected returns between projects, however, rather than how expected returns deviate from rational expectations over time.

### The pricing of idiosyncratic risk

The idea that idiosyncratic risk should not be priced goes back to the original theory of the CAPM (Treyner 1961, Sharpe 1964, Lintner 1965, Mossin 1966). In these two-period models, any risk that can be completely diversified away should not command an expected return,



since investors can frictionlessly divide and share these risks.

Merton (1987) qualified these results by imagining a world in which diversification was necessarily incomplete, because investors either do not know about the whole universe of risks, or cannot access them through standard channels like the stock market. In Merton's world, if a risk has to be less widely held (more concentrated) due to these frictions, its equilibrium expected return should rise, since the holders of that risk have no way to share it.

Ang, Hodrick, Xing, and Zhang (2006) tested these theories. They found that stocks with higher levels of idiosyncratic risk tended to perform worse, not better. This poor performance could not be explained by "exposures to size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness, or dispersion in analysts' forecasts". Fu (2009) provided one explanation of these findings. He argued that Ang, Hodrick, Xing, and Zhang's result is driven by stale (1-month lagged) estimates of volatility; according to Fu, idiosyncratic risk changes very quickly, making lagged estimates of volatility inappropriate. When Fu addresses this issue by using an EGARCH model for conditional volatility, he finds a positive contemporaneous relationship between expected returns and idiosyncratic risk.

### **Corporate finance theories of risk management**

The corporate finance literature has developed several models explain why firms "manage risk" (avoid risk that is not priced by external markets). In general, when firms face costs of external finance, they will manage their risk. The reason is that firms want to preserve their internal cash so as to avoid these costs. This point is made Froot, Scharfstein, and Stein (1993). My paper builds on that literature by thinking of investing in low-risk projects as a form of risk management.

It is also possible that external financing becomes more costly when firms take on riskier projects. This would generate an effect on top of the one just mentioned. For example, bankruptcy might have deadweight costs, so risk that increases the probability of bankruptcy

would be avoided by the firm (Smith and Stulz 1985). There may also be higher monitoring costs associated with liabilities issued by riskier firms; a firm that issues perfectly riskless liabilities need not be monitored at all.

Alternately, career concerns could explain managerial risk aversion (Hirshleifer and Thakor 1992). If a \$100 million movie bombs at the box office, it will hurt the careers of the associated producers. This “limits to diversification” story is particularly plausible because producers need to be exposed to exactly this sort of idiosyncratic risk to have the proper incentives for success.

### **1.3 A matching model of the film industry**

In this model, projects and firms exist exogenously, with firms subject to financial constraints (implemented in a reduced form). Projects match to firms endogenously, leading to price formation. The variation in the prices leads to variation in expected returns.

In particular, because of the financial constraints, firms are more averse to risk than the secondary market is. Thus, when they buy projects, the price they are willing to pay depends on the project’s risk, and riskier projects must pay higher expected returns in equilibrium. Moreover, since riskier projects are held by firms that are better cost avoiders for risk, marginal risk added to a very risky project has a less of an effect on expected return than it would for a low-risk project. This claim is similar to the concavity in expected returns we observe in the data.

After explaining the model, I prove the above points, along with a Modigliani-Miller-like result showing that expected returns should be constant across all projects if we remove financial constraints.

#### **Supply Side**

There are  $N$  screenplays ready for production, numbered 1 to  $N$ , comprising the set  $S$ . Each screenplay has two characteristics. First, it has an investment size  $I_i$  (measured in dollars).

Second, the payoff of the project is the random variable  $I_i (1 + \tilde{Q}_i)$ , and every project has a cumulative distribution function  $\mathcal{F}_i$  for the gross return  $1 + \tilde{Q}_i$ .  $\mathcal{F}_i$  has bounded first and second moments. The project returns are statistically independent and independent of the market return.

Each screenplay will either match to a production studio or expire worthless. If produced, a screenplay will be purchased by a studio for a price  $p_i$ , on top of the investment cost  $I_i$ . Thus, the return to the studio will be

$$1 + \tilde{R}_i = 1 + \tilde{Q}_i - \frac{p_i}{I_i}$$

For simplicity, we assume that variation in expected studio returns comes from variation in the acquisition prices of the projects, which are driven by screenplay characteristics. Thus, we set  $E(\tilde{Q}_i)$  to be identical across screenplays and greater than the risk-free rate  $R_f$ . Intuitively, this restriction implies movies are heterogenous only in size and “risk” (non-first-moments), and we focus solely on how these two factors affect expected returns.

### **Demand Side**

There are  $J$  firms, numbered 1 to  $J$ . The firms raise financing using debt contracts, which can be risky. I adopt a reduced-form specification for financial frictions. Every firm  $j$  has a level of internal cash  $w^j$ , and a borrowing limit  $e^j \geq 0$ , which is a function of  $w^j$ . We can think of this limit as a leverage constraint that prevents the equityholders from having an incentive to take too much risk at the expense of the debtholders. Second, firms face a deadweight cost of risk (variance)  $\gamma^j > 0$ . This can be thought of as a present value of deadweight bankruptcy costs that might be incurred in the future. Both parameters,  $e^j$  and  $\gamma^j$ , are increasing in internal cash  $w^j$ .

We think of each firm having a utility function defined over the bundle  $b$  of projects it ends up developing. A bundle is simply a subset of  $S$ , the set of all projects. The utility

function is quadratic:

$$U^j(b) = \underbrace{E \left[ \sum_{i \in b} I_i (\tilde{Q}_i - R_f) \right] - \gamma^j \text{Var} \left[ \sum_{i \in b} I_i \tilde{Q}_i \right]}_{V^j(b)} - P_{b_k}$$

where  $V^j(b)$  is the value the firm  $j$  assigns to bundle  $b$ , and  $P_b$  is the acquisition price for bundle  $b$ .

The riskless rate term  $R_f$  in the expectation represents the fact that the firm has the outside option of investing its cash in a riskless account. The leverage constraint is imposed on the aggregate size of the firm:

$$\sum_{i \in b_k} I_i \leq e^j$$

which must be satisfied for any bundle  $b_k$  for which a firm wishes to place a positive bid. The idea is that a firm will not be willing to bid anything for a bundle of projects that it cannot afford to produce.

## Matching

Screenplays are sold to firms in a combinatorial Vickrey-Clarke-Groves auction. This setup allows us to price “lumpy” projects rather than the infinitely divisible assets of the Arrow-Debreu model. Moreover, firms can bid on sets of projects rather than individual projects. This feature is necessary to capture (anti-)complementarities: for example, a firm may bid \$1 for project A, \$1 for project B, but \$0 for both A and B because it cannot afford to produce both.

The VCG auction retains many of the useful properties of Arrow-Debreu equilibrium: first, we have a notion of asset prices, which is not necessarily the case in matching algorithms. An “equilibrium” always exists, prices are unique, and the allocation maximizes the welfare of the participants (although not necessarily that of the auctioneer). Furthermore, under this mechanism, it always a dominant strategy for a firm to bid its true valuations, so we need not assume an omniscient central planner.

Economically, a VCG auction differs from other matching algorithms in that it makes particular implicit assumptions about the bargaining power of screenwriters. Namely, screenwriters do not collude to revenue-maximize. Instead, their bargaining power is limited to their ability to match with a different firm. For example, if only one studio were interested in a particular bundle of screenplays, that studio would be able to bargain the price of that bundle down to zero.

The auction works as follows:

1. Studios place bids for each bundle  $b_k$ , where  $k$  is an index of a bundle, equal to their valuations  $V^j(b_k)$ . Given the rest of the mechanism, it is well-known that bidding truthfully is a dominant strategy.
2. The projects are allocated to the studios by a function  $M(j)$ : the index of the bundle to be developed by firm  $j$ . This function is defined so that the match maximizes total welfare subject to the constraint that the same project cannot be developed twice. Welfare is defined as

$$V = \sum_{j=1}^J V^j(b_{M(j)})$$

3. Bundle prices are determined as follows. First, let  $V_{-j}^*(x^*)$  mean the welfare of all agents *except*  $j$  at the welfare-maximizing allocation  $x^*$  that we solved in step 2:

$$V_{-j}^*(x^*) = \sum_{j' \neq j} V^{j'}(b_{M(j')})$$

Second, let  $V_{-j}^*(x_{-j}^*)$  mean the welfare of all agents *except*  $j$  at the allocation  $x_{-j}^*$  that would maximize *their* welfare:

$$V_{-j}^*(x_{-j}^*) = \sum_{j' \neq j} V^{j'}(b_{M'(j')})$$

where  $M'(j')$  maximizes  $\sum_{j' \neq j} V^{j'}(b_{M'(j')})$  subject to the constraint that the same project cannot be developed twice.

The price of the bundle assigned to  $j$  is the second value minus the first: it is what the other players have to give up in order to assign the bundle  $M(j)$  to  $j$  instead of keeping it among themselves.

$$P_{b_{M(j)}} = V_{-j}^*(x_{-j}^*) - V_{-j}^*(x^*)$$

4. The price  $p_i$  imputed to screenplay  $i$  is its pro-rata share of the price of the bundle to which the project belongs. For example, if project  $i$  is part of bundle  $b_k$ ,

$$p_i = \frac{I_i}{\sum_{j \in b_k} I_j} P_{b_k}$$

### Characterizing matches

A *match* is an allocation of bundles and prices resulting from the above auction. This section characterizes these matches.

**Theorem 1.** *A match always exists, and prices are unique.*

*Proof.* Because the welfare function is bounded, it has a (finite) supremum. Because the set of allocations is finite, the supremum is attained. Thus, at least one welfare-maximizing allocation can always be constructed. Prices are computed by taking differences of these welfare-maximizing allocations for different sets of agents.

Any welfare-maximizing allocation is not necessarily unique; for example, all firms could be identical, in which case the assignment of bundles to firms would be arbitrary. However, the maximized value of the welfare function is by definition unique, since if it were not, one of the multiple values would not be a maximum. Thus, the price expression is unique as it is the difference between two uniquely defined terms.  $\square$

**Definition.** A match is *locally stable with respect to a level of risk*  $s_i^2$  (the dollar variance  $I_i^2 \sigma_i^2$ ) if there exists a  $\varepsilon$ -neighborhood  $B$  around  $\sigma_i^2$  such that, if project  $i$ 's risk  $\sigma_i^2$  were changed to any other  $\sigma_i'^2 \in B$ , the matching function  $M(j)$  would remain constant. (The purpose of this definition is to prevent the matching from changing discontinuously when we take derivatives around a level of  $s_i^2$ .)

**Theorem 2.** [Idiosyncratic risk premium.] Let the risk of project  $i$  be defined as its dollar variance:  $s_i^2 = I_i^2 \sigma_i^2$ . A small, exogenous increase in risk for a screenplay raises its expected return:

$$\frac{\partial E(\tilde{R}_i)}{\partial s_i^2} \geq 0$$

where the derivative is taken holding  $I_i$  constant, and evaluated for any level of risk  $s_i^2$ , for any project  $i$ , such that the matching is locally stable with respect to that level of risk.

*Proof.* Suppose that project  $i$  is part of bundle  $b_k$ . Then, by substituting the expression for  $p_i$  into the definition of expected return and taking expectations, we get:

$$E(\tilde{R}_i) = E(\tilde{Q}) - \frac{P_{b_k}}{\sum_{j \in b_k} I_j}$$

where  $E(\tilde{Q})$  is the common value of all  $E(\tilde{Q}_i)$ . Thus, the comparative static of interest has the same sign as

$$-\frac{\partial P_{b_k}}{\partial s_i^2}$$

To use this fact, note that the formula for  $P_{b_k}$  is

$$P_{b_k} = V_{-j}^*(x_{-j}^*) - V_{-j}^*(x^*)$$

where  $j$  is the index of the firm producing bundle  $b_k$ . This formula can be expanded as

$$P_{b_k} = \sum_{j' \neq j} \left[ \sum_{i' \in b_{M(j')}} E(\tilde{Q}_{i'} - R_f) I_{i'} - \gamma^{j'} \text{Var} \left( \sum_{i' \in b_{M(j')}} I_{i'} \tilde{Q}_{i'} \right) \right] \\ - \sum_{j' \neq j} \left[ \sum_{i' \in b_{M'(j')}} E(\tilde{Q}_{i'} - R_f) I_{i'} - \gamma^{j'} \text{Var} \left( \sum_{i' \in b_{M'(j')}} I_{i'} \tilde{Q}_{i'} \right) \right]$$

where  $M(j')$  is the true match and  $M'(j')$  is the match that would have been created if firm  $j$  did not exist.

In this expanded expression, note that the risk of project  $i$  only affects at most one term in the many summations. The reason is that, in the first set of summations, project  $i$  does not appear at all because it is not assigned to any firm  $j' \neq j$ . In the second set of summations,

it appears at most once because project  $i$  has been assigned to some other firm now (or to no firm), and thus contributes to either that firm's risk, or to no firm's risk. Using this fact, along with the independence of the project returns, we can write the partial derivative as

$$\frac{\partial P_{b_k}}{\partial s_i^2} = \begin{cases} \gamma^{j'} & \text{if assigned to a new firm } j' \text{ in the absence of } j \\ 0 & \text{if not assigned to any firm in the absence of } j \end{cases}$$

Thus  $-\partial P_{b_k} / \partial s_i^2 \geq 0$ , and  $\partial E(\tilde{R}_i) / \partial s_i^2 \geq 0$  as required.  $\square$

**Theorem 3.** [Riskier projects are less affected by marginal risk.] For any project  $i$  produced by studio  $j$  and a distinct, riskier project  $i'$  produced by studio  $j'$ , if:

1.  $i$  would be produced in the absence of  $j$ , and  $i'$  would be produced in the absence of  $j'$ ,
2. The risk gap  $s_{i'}^2 / s_i^2$  is sufficiently large:

$$\frac{s_{i'}^2}{s_i^2} > \underbrace{\frac{I_i E(\tilde{Q}_i - R_f)}{\gamma^{N_{-j}(i)} s_i^2}}_{\text{benefit/cost for } i} \cdot \frac{I_{i'}}{I_i}$$

where  $N_{-j}(i)$  is the index of the firm that produces  $i$  in the absence of  $j$ ,

3. The studio producing the riskier project has an equal or larger budget than the other studio:

$$\sum_{w \in b_{M(j)}} I_w \leq \sum_{w \in b_{M(j')}} I_w$$

Then, the riskier project's expected return will be less sensitive to risk:

$$\frac{\partial E(\tilde{R}_i)}{\partial s_i^2} \geq \frac{\partial E(\tilde{R}_{i'})}{\partial s_{i'}^2}$$

where again the  $I$ s are treated as constants and the derivatives are evaluated at points  $s_i^2, s_{i'}^2$  at which the matching is locally stable.

*Proof.* Intuitively, riskier projects are produced by studios with lower risk aversion, so marginal risk is less costly. The key is to formalize that intuition: it is not exactly right



because projects are priced based on the “second price” bidder rather than the actual firm that produces the project. Thus, we need the riskier project to be “sufficiently riskier” that we can strictly compare the second price bidders.

Now for the proof. Parametric condition 2 can be cross-multiplied and simplified to:

$$\gamma^{N-j(i)} s_i^2 > I_i E(\tilde{Q}_i - R_f)$$

Because  $i'$  is produced without  $j'$ , the risk cost is less than the return benefit:

$$\gamma^{N-j'(i')} s_{i'}^2 \leq I_{i'} E(\tilde{Q}_{i'} - R_f)$$

Using the fact that  $E(\tilde{Q}_i) = E(\tilde{Q}_{i'})$ , we can combine the two inequalities to get the useful result:

$$\gamma^{N-j(i)} > \frac{I_{i'} E(\tilde{Q}_{i'} - R_f)}{s_{i'}^2} \geq \gamma^{N-j'(i')}$$

Now we directly compute expected returns. The expected return on film  $i$  is

$$E(\tilde{R}_i) = E(\tilde{Q}) - \frac{P_{b_{M(j)}}}{\sum_{w \in b_{M(j)}} I_w}$$

Take a derivative to get:

$$\frac{\partial E(\tilde{R}_i)}{\partial s_i^2} = \frac{\gamma^{N-j(i)}}{\sum_{w \in b_{M(j)}} I_w}$$

where I have used the result from the previous proof: the slope of the bundle price with respect to the risk of a project is equal to the risk aversion of firm  $N-j(i)$ , assuming  $i$  is produced without  $j$ .

Similarly, the expected return on film  $i'$  satisfies

$$\frac{\partial E(\tilde{R}_{i'})}{\partial s_{i'}^2} = \frac{\gamma^{N-j'(i')}}{\sum_{w \in b_{M(j')}} I_w}$$

Since  $\gamma^{N-j'(i')} < \gamma^{N-j(i)}$  and  $\sum_{w \in b_{M(j')}} I_w \geq \sum_{w \in b_{M(j)}} I_w$  (by the third condition in the theorem

statement), it follows that

$$\frac{\partial E(\tilde{R}_i)}{\partial s_i^2} \geq \frac{\partial E(\tilde{R}_{i'})}{\partial s_{i'}^2}$$

as required.  $\square$

**Theorem 4.** [Modigliani-Miller.] *Expected returns are the same for all screenplays if there is at least one studio that is not borrowing constrained or risk-penalized.*

*Proof.* The social welfare function is

$$W = \sum_{j=1}^J \left[ \sum_{i \in b_{M(j)}} E(\tilde{Q}_i - R_f) I_i - \gamma^j \text{Var} \left( \sum_{i \in b_{M(j)}} I_i \tilde{Q}_i \right) \right]$$

This expression will be maximized by the VCG mechanism. The maximum occurs when all screenplays are assigned to a firm that is not borrowing constrained or risk-penalized. The reason is that we completely avoid any variance penalties, while not giving up anything on expected returns.

Because one firm is producing all projects, the price for each screenplay will be its pro-rated fraction of the bundle corresponding to the entire set of screenplays  $S$ :

$$p_i = \frac{I_i}{\sum_j I_j} P_S$$

Regardless of the value of  $P_S$ , this implies that the expected return to the studio is

$$E(\tilde{R}_i) = E(\tilde{Q}_i) - \frac{P_S}{\sum_j I_j}$$

which is constant for all projects  $i$ .  $\square$

## 1.4 Data

I use data from Nash Information Services' proprietary OpusData database. This dataset is primarily marketed to the film industry and contains detailed information on movie characteristics and financials. A public slice of the dataset can be viewed at <http://www.>

the-numbers.com/, and the full dataset can be purchased at <http://www.opusdata.com/>.

The data are collected from industry sources. For example, domestic and international theatrical distributors report daily box office takes, which can be aggregated to get total box office grosses for each film. For some films, the database relies on publicly available numbers published in trade press, such as *Variety*, *Screen International*, *Hollywood Reporter*, etc. Additional information comes from domestic and international trade groups, such as the Digital Entertainment Group, European Audiovisual Observatory, National Association of Theater Owners, MPAA, BFI, etc. DVD tracking information come from retail sales surveys.

Because OpusData has a relatively small number of genres, I also use genre classifications from the Internet Movie Database (IMDB), a property now fully owned by Amazon.com. Most people interact with this database through the website <http://www.imdb.com/>, which is the 45th most visited site in the world as of January 2014 (Alexa 2014). The full dataset is available as text files from <http://www.imdb.com/interfaces>; although some of these files require substantial processing before they can be inserted into a database. Matching films by title and release year, I can merge the genre classifications from IMDB with financials from OpusData.

As of April 2014, the OpusData dataset contains information on 19,973 movies. The earliest movie is from 1915, but the majority of films are recent: 83.3% of the films were released during or after 1990. The dataset also contains 3,988 directors, 45,258 actors, and 11,489 producers, all linked to their associated movies.

I prune the sample to address biases. First, I use only films with release year before 2013 to minimize under-reporting returns for films that are either still in theaters, or have yet to sell an appreciable number of DVDs. Second, to address selection bias within the remaining sample, I focus on the population of films that report spending over \$10 million 2012 dollars. Within this population, nearly all films are released (since the budget is a sunk

cost), so there is no censorship problem.<sup>1</sup> After removing these films, I am left with 2,853 films, 87.0% of which were released during or after 1990.

I compute the return on film  $i$  using the following formula:

$$1 + \tilde{R}(i) = \frac{70\% \times \text{US box office} + 35\% \times \text{Int'l box office} + 50\% \times \text{US DVD \& Blu-Ray Sales}}{\text{Negative cost}}$$

The percentages used in the numerator are typical values for the fraction of sales the studio itself is able to capture, based on Vogel (2011) and discussions with professionals in the industry. For example, domestic exhibitors (theaters) generally take about 30% of box office revenues, even though exhibition contracts are generally more complex than a simple percentage. Additionally, since distributing films outside the U.S. is often outsourced to foreign distributors, in that case the studio collects about half as much income per box office dollar as in the U.S.

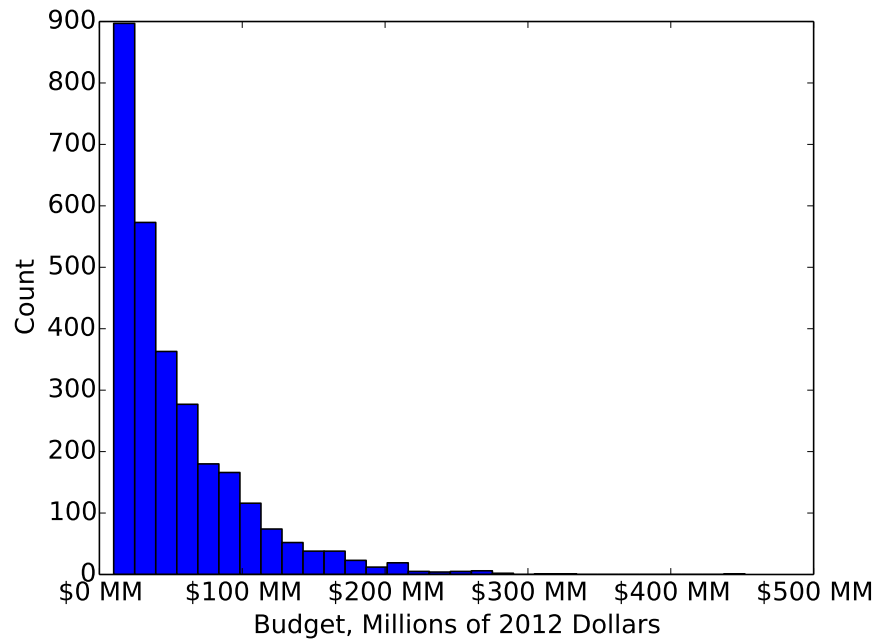
The “negative cost” of a film is the cost of producing the “negative,” or physical film. It does not include distribution or advertising costs. The reason for excluding these costs is that studios may inflate the latter figures to reduce taxes and payments to people owed a share of the profits. *Forrest Gump* is a classic example of this phenomenon; the screenwriter threatened legal action against the studio for declaring that the film was unprofitable after its hugely successful release (Weinraub 1995). The use of negative costs is standard in the literature; for example, see Ravid (1999).

One might reasonably ask whether the returns should be annualized in some way, to account for the fact that different movies take different amounts of time to complete. Specifically, movies that take longer to produce (in general, more expensive movies) might also be riskier; then, the higher measured returns on riskier movies would simply be due to a failure to annualize. Although it seems clear from Figures 6, 7, and 8 that size is not related to risk or return, it is worth addressing the annualization concern directly.

To annualize returns properly, I acquired production duration data from the Internet

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<sup>1</sup>On this point, see the relatively short list of “high profile films that remain unreleased” here: <http://styleblazer.com/176218/hollywood-on-hold-14-high-profile-films-that-remain-unreleased/>

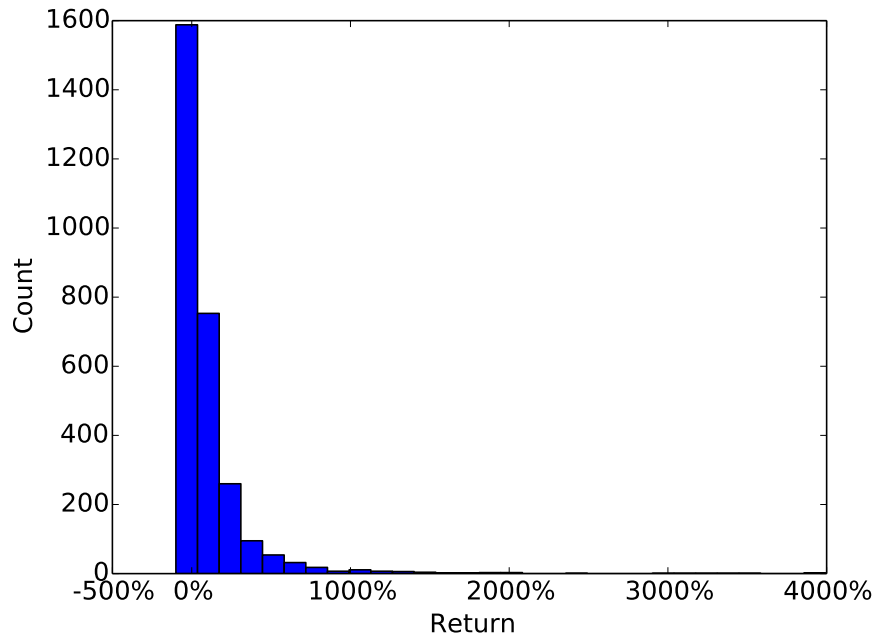


**Figure 1.2:** *The distribution of budgets is skewed*

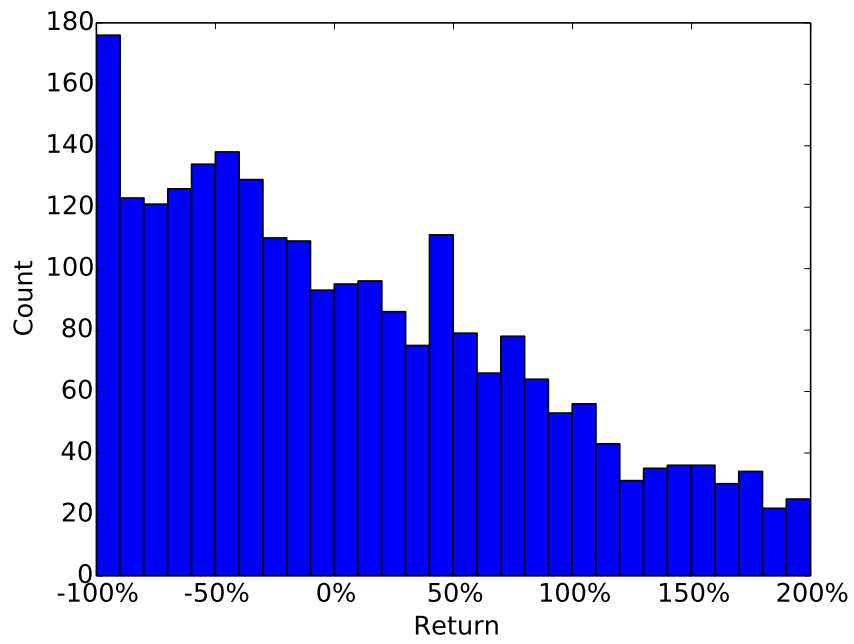
Movie Database. These numbers are available only for a 220-film subset, so the subset is too small to replicate my analysis when grouping movies by actors and directors (without introducing significant small-sample bias). Nevertheless, I find that the results with genre groupings are just as strong if we annualize properly; this is shown in Figure 12. My main results, however, do not include this annualization since I would like to use the full dataset.

The following plots illustrate key features of the dataset. Figure 1.2 shows that the distribution of film budgets has an extremely long tail. Most movies above my \$10 million threshold cost less than \$50 million to make; yet there are some movies that cost more than \$200 million.

Figures 1.3 and 1.4 illustrate that the distribution of film returns has an extreme positive skew. As Ferrari and Rudd (2006) put it, “while the average ROI for all films in recent years has been positive, the majority of films yielded negative returns.” Thus, film finance is very similar to entrepreneurial finance; financiers generally lose money, but do so in order to capture rare, extreme gains.



**Figure 1.3:** *The distribution of returns is extremely skewed*



**Figure 1.4:** *The distribution of returns, left end of the distribution*

**Table 1.1:** *Highest budget movies*

	Budget (2012\$)	Return	Year
Avatar	\$451 MM	233%	2009
Pirates of the Caribbean 3	\$328 MM	99%	2007
Cleopatra	\$314 MM	8%	1963
Titanic	\$286 MM	398%	1997
Spider-Man 3	\$282 MM	91%	2007
John Carter	\$275 MM	-48%	2012
The Dark Knight Rises	\$275 MM	122%	2012
Tangled	\$273 MM	38%	2010
Harry Potter 6	\$266 MM	105%	2009
Superman Returns	\$264 MM	7%	2006

Table 1.1 presents financial data on the biggest budget films in my dataset, and Table 1.2 presents summary statistics for the entire dataset. These are provided to show that data are reasonably accurate and also that there is a huge variation in returns.

## 1.5 Empirical Strategy

My ultimate goal is to demonstrate an idiosyncratic risk premium in the cross-section of expected returns for films. The empirical strategy has five distinct steps:

1. Systematic risk is negligible. I review the evidence on the determinants of realized returns, and argue that nearly all of the variation in realized returns is explained by movie-specific factors. Individual movie returns do not correlate with the market, so we can treat them as idiosyncratic.
2. GMM restrictions for estimating risk. I use the dollar variance of each film as a proxy for idiosyncratic risk, as suggested by my matching model. Because we do not directly observe the ex-ante variance of returns for each film, I develop moment restrictions to estimate these parameters based on the characteristics of each film; namely, the genre, actors, and directors involved with the film.

**Table 1.2:** *Summary statistics*

Number of films	2,853
Median Budget	\$38.0 MM
Mean Budget	\$53.6 MM
Median \$ Return	\$6.0 MM
Mean \$ Return	\$36.5 MM
Median % Return	17.0%
Mean % Return	87.0%
Std. Dev. of \$ Returns	\$111.7 MM
Interquartile Range \$ Returns	\$62.2 MM
Std. Dev. of % Returns	281.8%
Interquartile Range % Returns	161.6%

3. GMM restrictions for estimating the risk premium. After estimating the risk of each movie, I develop a simple model in which there is a concave, square-root relationship between dollar variance and expected returns. This parameterization describes the data well (based on plots) and leads to a linear moment condition relating dollar volatility and expected returns.
4. Econometric issues. I discuss some of the econometric issues related to estimating GMM models with heavy-tailed data, and I propose a methodology based on bootstrapping to solve these issues.
5. I discuss an alternative risk measure for heavy-tailed data (the interquartile range), and show that it also predicts higher returns.

### **Negligible systematic risk in films**

A key assumption in this paper is that films have market betas that are close to zero (more technically, betas to the stochastic discount factor). Thus, if I can find characteristics that create spreads in risk, and those spreads in risk correlate with returns, we are picking up an idiosyncratic risk premium rather than a systematic risk premium. But is the assumption of



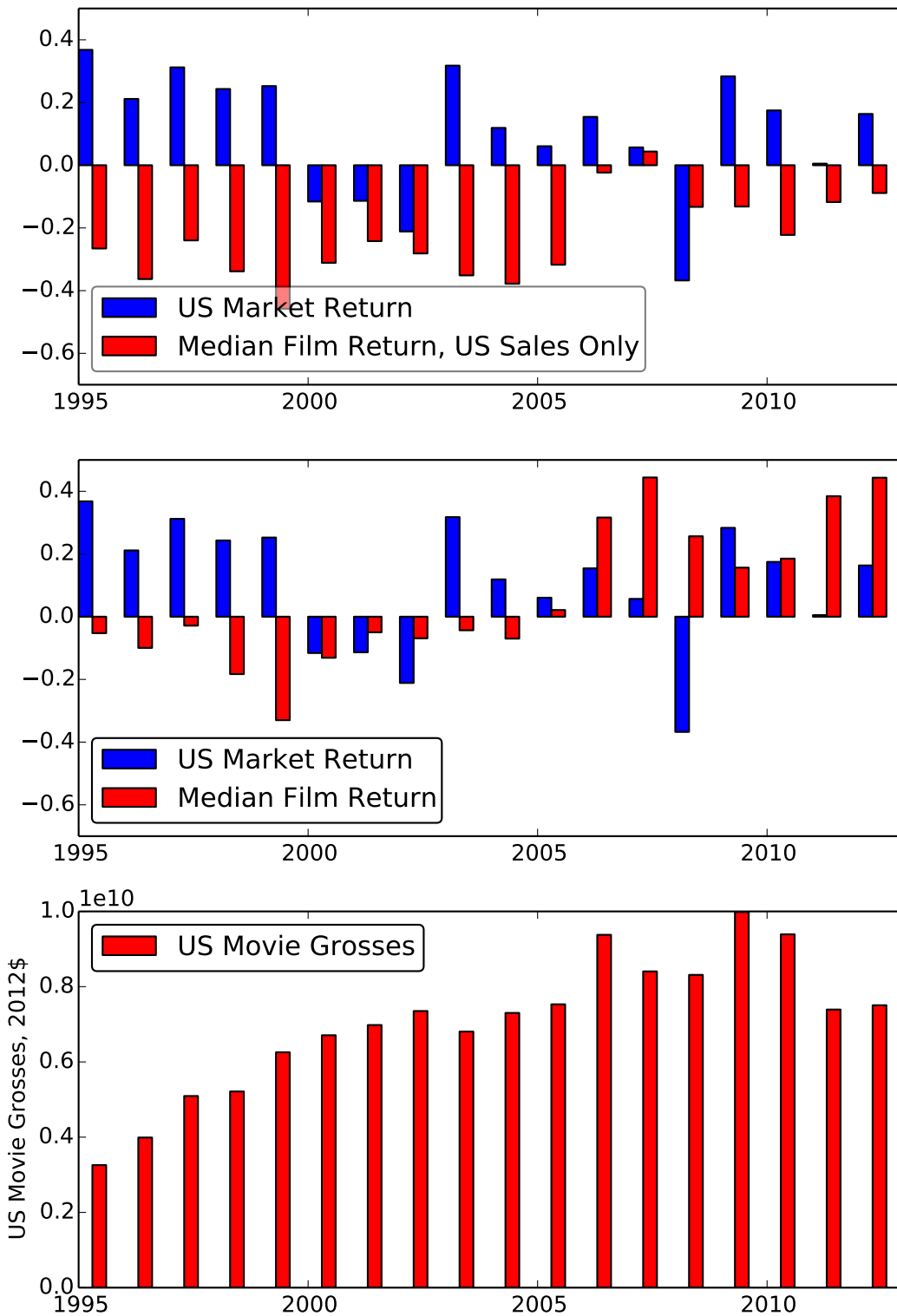
zero betas reasonable?

Regressions suggest that film returns are mostly driven by whether the individual film is good or not, rather than by macroeconomic conditions. Basuroy, Chatterjee, and Ravid (2003) report that 79% of the cross-sectional variation in first-week box office revenue can be explained by reviews, whether the movie features a star, MPAA ratings (G, PG, etc.), and other non-business-cycle characteristics. Nor do macro factors kick in later: Moon, Bergey, and Iacobucci (2009) find that up to 90% of the variation in later-week box office revenue is explained by first-week revenue, in addition to standard movie-specific factors.

On the aggregate level, the film industry is either acyclical or mildly countercyclical. Nardone (1982) documents a study conducted by A. Kapusinski who “matched 42 economic measures of the motion-picture industry... against similar variables used to assess the performance of the whole economy” and found that ticket sales were actually slightly countercyclical (cited in Vogel 2011). Summarizing the findings on the cyclicity of ticket sales, Vogel (2011) states that “ticket sales often remain steady or rise during early to middle stages of a recession, faltering only near the recession’s end.”

Figure 1.5 shows the acyclicity of film returns in my sample. In the top panel, I plot returns on the CRSP value-weight U.S. market portfolio alongside the median film return in my sample, excluding international box office grosses (so that U.S. data are being compared to U.S. data). The years along the  $x$ -axis correspond to the release years for movies. We can see that bad times for the market, such as 2000 and 2008, are not necessarily bad times at all for the film industry. The middle panel shows the same plot, just with the international data added back in for robustness. Finally, the bottom panel plots U.S. movie grosses, which includes box office take along with DVD and Blu-Ray sales. Thus number grows over time but does not seem to be correlated with either the equity market or output. Overall, these results are consistent with the findings just reviewed.

Finally, given the double-digit percentage point spreads in expected returns that I find, betas would have to be implausibly large for a systematic risk story to “work.” For example, when grouping movies by both genre and director (see Figure 1), I find that some genres



**Figure 1.5:** Studio profits are acyclical. Top: CRSP value-weighted U.S. market return and the median return on all films, excluding the international box office. Middle: same as Top, but including the international box office. Bottom: total U.S. film grosses, including DVD and Blu-Ray sales.

and directors differ by over 100 percentage points in expected return. Even if this return were earned over two years, given the size of the equity risk premium (perhaps 5%), one would need different types of movies to have beta differences of at least ten to explain my results using systematic risk. It is hard to believe we have movies that go up in value by 10% whenever the market appreciates by 1%, and it is just as hard to understand how two movies could differ by 10 units in beta. Even if the returns were earned over five years, we would need beta differences as large as four to explain the results.

Although one can find news articles suggesting that recessions tighten financing conditions for movies, this is different from saying that *realized* movie returns covary with *realized* market returns. At best, it says that the level of expected returns on films rises and falls with the expected return on the market.

### **Modeling idiosyncratic risk**

I use the dollar variance of a movie as my measure of risk. This measure, which is simply the expected squared deviation of the dollar return from its mean, is convenient for the following reason. If firms are averse to “risk,” by using dollar variance as our risk measure, we ensure that the quantity of risk a film adds to any studio’s portfolio is exactly the same. The reason for this is that variances of independent dollar returns are additive, which other measures like standard deviation are not (standard deviation is subadditive). Consequently, if we had used one of these other measures, a key factor in determining a movie’s risk would be the identity of the other films produced by the same studio, which would be tricky to deal with parametrically. Later, we will see that two other measures of risk, volatility and interquartile range, generate similar spreads in expected returns, so this choice is fairly innocuous.

We don’t observe dollar variances, but we can estimate them based on characteristics of movies. For example, movies with certain actors or directors, or belonging to certain genres, tend to have different levels of risk. Interestingly, movies with different budget sizes do not necessarily have different levels of risk: the bigger-budget movies which one might imagine

to be riskier also have less uncertainty about their percentage returns, which tends to work against the first effect.

To formalize this, let a “group” be a set of movies that share a characteristic; for example, “movies featuring Tom Cruise” would be a group. Let  $1_{ig}$  be the indicator variable equal to one when movie  $i$  is in group  $g$ . Then, let us suppose (parametrically) that the idiosyncratic dollar cash-flow variance on movie  $i$  is the average of weights  $w_g$  corresponding to the groups  $g$  to which  $i$  belongs:

$$\sigma_{\$,i}^2 \equiv \frac{\sum_{g \in G} w_g 1_{ig}}{\sum_{g \in G} 1_{ig}}$$

For robustness, I also consider a geometric average specification for  $\sigma_{\$,i}^2$ , although the results are quite similar:

$$\sigma_{\$,i}^2 \equiv \exp \left( \frac{1}{\sum_{g \in G} 1_{ig}} \sum_{g \in G} \ln(w_g) 1_{ig} \right)$$

In either case, the denominator term normalizes for the number of groups in which movie  $i$  is a member.

Then, we can write down a moment restriction for each group  $g$  that forces the values of this parameter  $\sigma_{\$,i}^2$  to actually be close to the realized dollar variance in that group:

$$E \left( \left( (R_i - E(R_i))^2 I_i^2 - \sigma_{\$,i}^2 \right) 1_{ig} \right) = 0 \quad G \text{ restrictions}$$

where  $I_i$  is the budget for movie  $i$ .

These  $G$  conditions are sufficient to identify the  $G$  parameters  $\{w_g\}$ . However, we still have to develop a parametric form for  $E(R_i)$  that takes into account the idiosyncratic risk premium. This will add two additional parameters and two additional moment restrictions.

### **Modeling the idiosyncratic risk premium**

Theory does not necessarily predict a particular parametric relationship between idiosyncratic risk (as measured by dollar variance) and expected return. The model I developed above suggests the relationship should be piecewise linear, but there may be many pieces,

and the pieces will generally have decreasing slopes. Graphs of idiosyncratic risk versus returns suggest that a well-fitting parametric model would be

$$E(R_i) = \lambda \sqrt{\sigma_{\$,i}^2} + R_f$$

where  $\lambda$  is a measure of the idiosyncratic risk premium and  $R_f$  is the real interest rate. Figures 10, 14, and 17 plot these variables together.

This model immediately leads to the following two moment conditions. First, pricing errors must be zero in expectation:

$$E(R_i - \lambda \sigma_{\$,i} - R_f) = 0$$

Second, the pricing errors must be uncorrelated with the explanatory variable, so we aren't getting a well-fit model by just having it overestimate for low risk movies and underestimate for high risk movies:

$$E((R_i - \lambda \sigma_{\$,i} - R_f) \sigma_{\$,i}) = 0$$

These two moment conditions are standard conditions for regressions. We now have  $G + 2$  parameters:  $w_g$ ,  $\lambda$ , and  $R_f$ , and we have  $G + 2$  moment conditions as well. The model is just-identified. I estimate the model using the Nelder-Mead simplex algorithm and confirm the algorithm output by testing whether the objective value equals zero at the parameter estimate.

### Econometric issues

Standard GMM estimates have a few econometric issues:

- The explanatory variable  $\sigma_{\$,i}^2$  is estimated, not observed precisely. So we will have the standard attenuation bias where the coefficient estimate for  $\lambda$  will be biased towards zero. This is not a big deal, since if we corrected for it, it would go in our favor (make our estimate for  $\lambda$  even higher than it already is).
- The error in estimating  $\sigma_{\$,i}^2$  is correlated with the error in  $R_i$ . This is a small-sample

bias resulting from the fact that the distribution of movie returns is heavily skewed. Specifically, a small group with high average returns may simply be a group that happened to get an extreme return from the right tail; in this case, it would seem like high variance of returns is correlated with high average returns, even when it is not.

- When data are heavy-tailed, asymptotic standard errors can be quite inaccurate for GMM estimates, especially in small samples. Also, the true (finite-sample distribution) rejection regions may not be symmetric like the asymptotic distributions.

I use bootstrapping to address these issues. Following Horowitz (2001), bootstrap estimates benefit from “asymptotic refinements,” or differences from the asymptotic distribution that make its approximation error to the true finite-sample distribution of smaller order in the number of data points.

Bootstrapping can also be useful for bias reduction. We can write bias as a function of the true population distribution:

$$E(\hat{\lambda}) = \lambda_0 + b(F)$$

where  $\lambda_0$  is the true value of the parameter and  $F$  is the population distribution. The bootstrap analog is to substitute  $\hat{\lambda}$  for  $\lambda_0$ , and the empirical distribution  $F_N$  for  $F$ :

$$E^*(\hat{\lambda}) = \hat{\lambda} + b(F_N)$$

Thus, the expression  $E^*(\hat{\lambda}) - \hat{\lambda}$  will consistently estimate the bias  $b(F)$  as  $N \rightarrow \infty$ .

I find that the bias reductions are not too large, although the estimates are consistently reduced for every grouping characteristic.

### **The interquartile range: a more robust estimator?**

When distributions are sufficiently heavy-tailed, we might be concerned that measures like variance simply become undefined. Even if the second moment is defined, our estimates of it might still be highly sensitive to tail events. A general approach to such problems is to use empirical quantiles of the data; for example, using the median instead of the mean, as

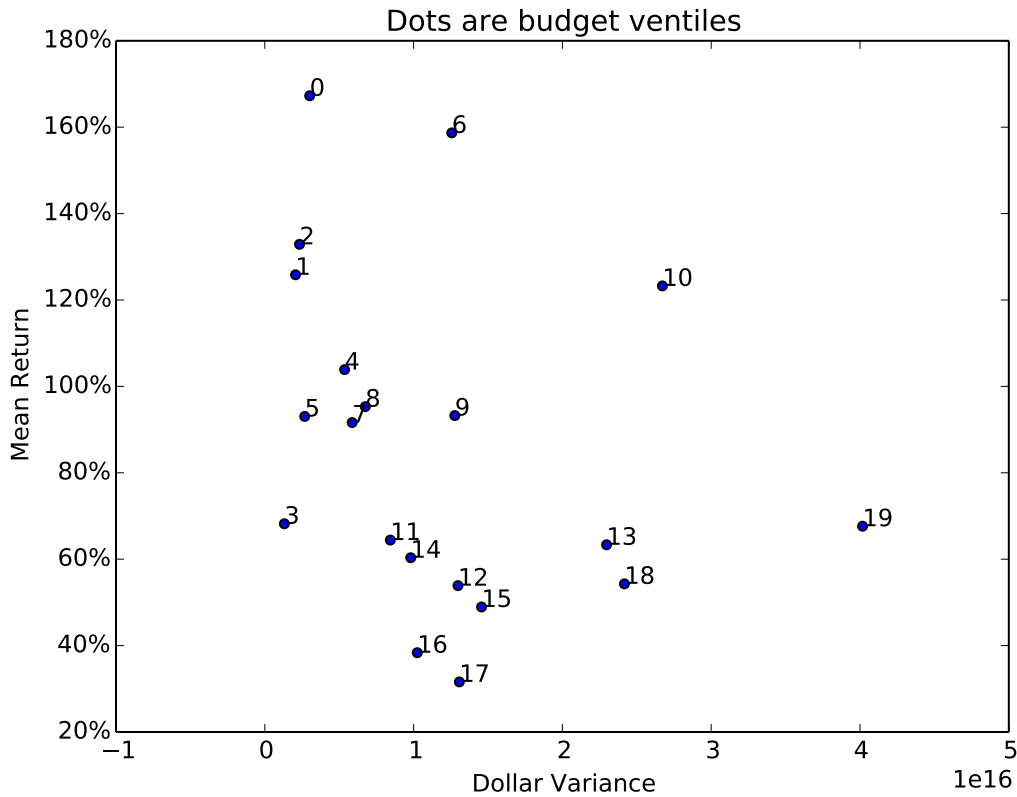
we did in Figure 5. For measuring the spread of a distribution, we can use the interquartile range, or the difference between the 3rd and 1st quartiles of the distribution. Figures 8, 11, 15, and 18 plot expected returns against interquartile ranges for different groupings of movies (size, genre, director, and actor). The resulting patterns are consistent with the plots using dollar variance as the  $x$ -variable.

## 1.6 Results

Graphs of returns against risk and tables with GMM estimates can all be found at the end of the document. This section discusses these graphs and tables.

### Graphs

- Figures 1.6, 1.9, 1.13, and 1.16: these figures plot expected returns against dollar variance for different projects. Each point represents a set of movies that share the same characteristic; for example, they might all be in the smallest 5% of movies by budget, or they might have the same genre. The first plot, figure 1.6, shows that budget size does not sort movies well for the purpose of creating spreads in expected returns; the other characteristics do.
- Figures 1.7, 1.10, 1.14, and 1.17: these figures are the same as the above, but put dollar volatility (standard deviation of dollar returns) instead of dollar variance on the  $x$ -axis. The results are fairly similar, and show that the relationship between dollar volatility and expected return is approximately linear.
- Figures 1.8, 1.11, 1.15, and 1.18: these figures are the same as above, but put interquartile range of dollar returns on the  $x$ -axis. They show that even when we use a measure of risk that is more robust to heavy tails, the qualitative results still go through.
- Figure 1.12: this figure is the same as Figure 1.11, except with returns properly annualized (this applies only to a subset of the data for which I have production start and end dates).



**Figure 1.6:** No clear relationship when movies are bucketed by budget ventile (0 = smallest ventile)

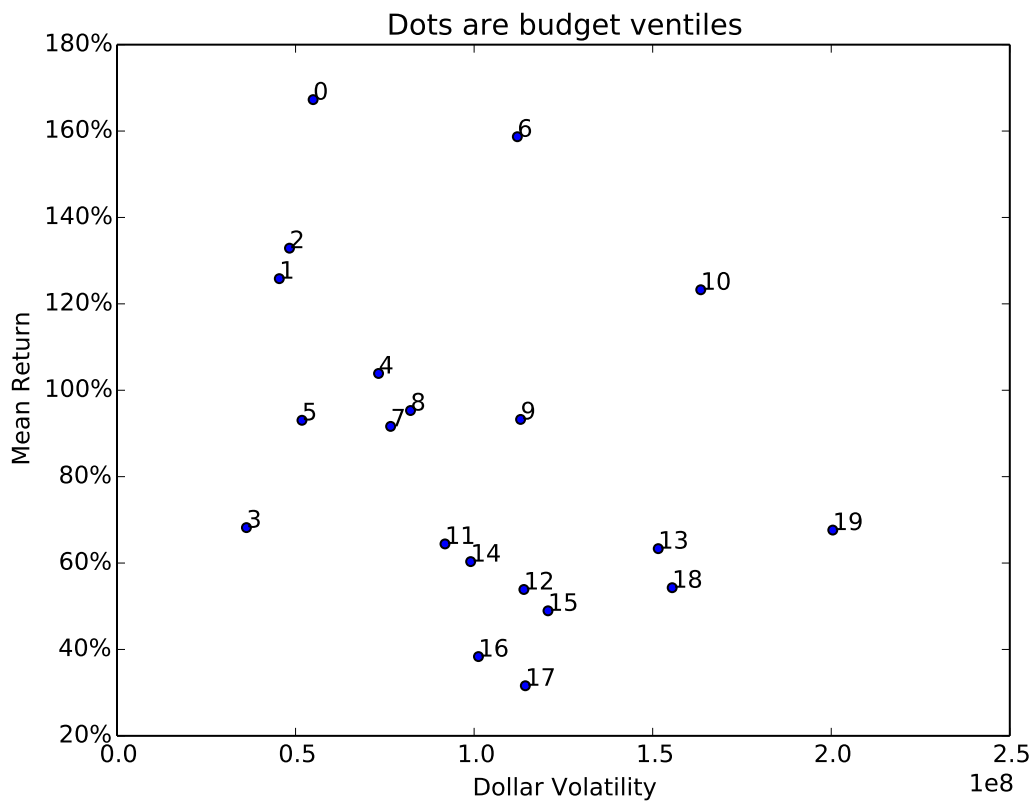
### GMM Estimates

Table 1.3 shows the main GMM results. The column  $\hat{\lambda}$  shows the estimates of the idiosyncratic risk premium; for the three rows corresponding to grouping by genre, director, and actor, these range from 45 to 100 basis points per \$1 MM of dollar volatility. (I do not find statistically significant results when grouping by budget ventile, as shown in figure 6.) The  $\hat{\lambda}_{bc}$  column shows bias-corrected estimates; these range from 44 to 119 basis points per million dollars of volatility.

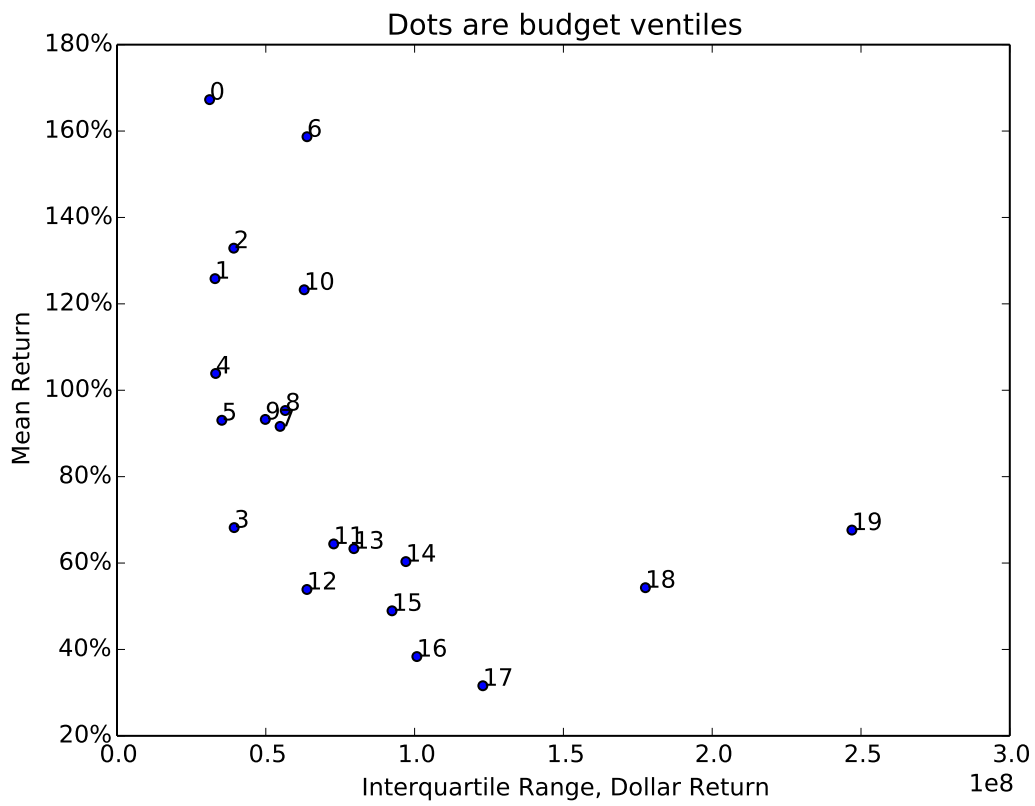
The 95% confidence sets are generated using bootstrapping. The final three rows are statistically significant. The  $R^2$  values are low because the differences in expected returns across films explains very little of the (extremely high) uncertainty about film returns.

To assess economic significance, I calculate the cross-sectional standard deviation of

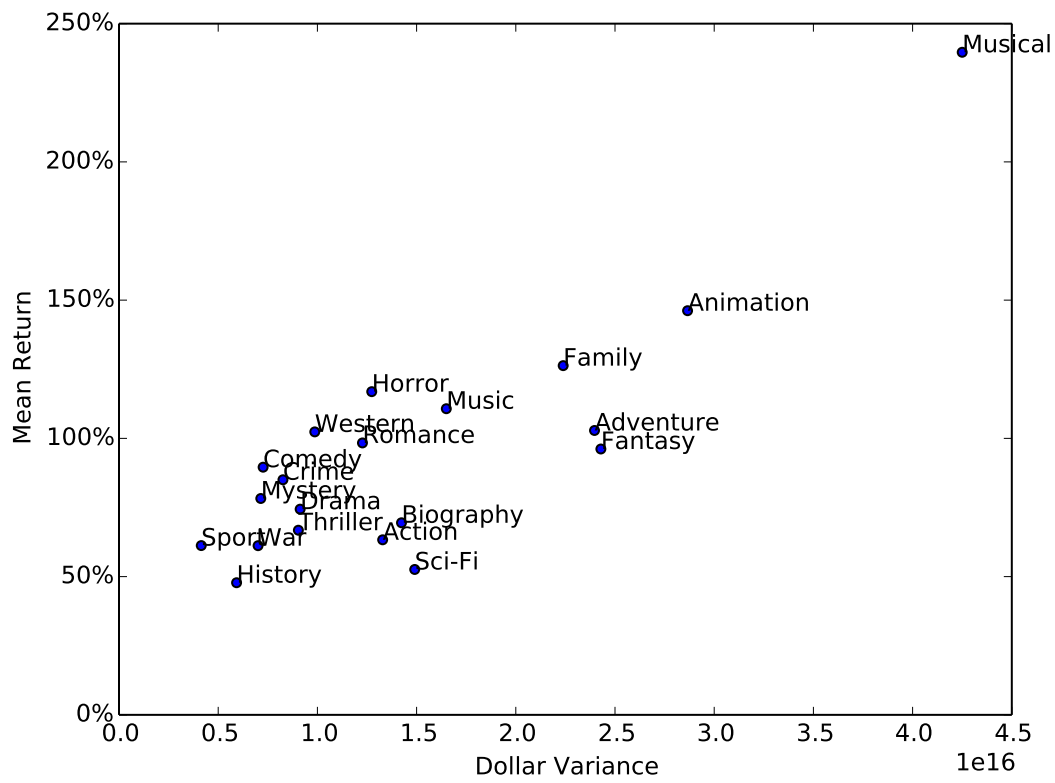




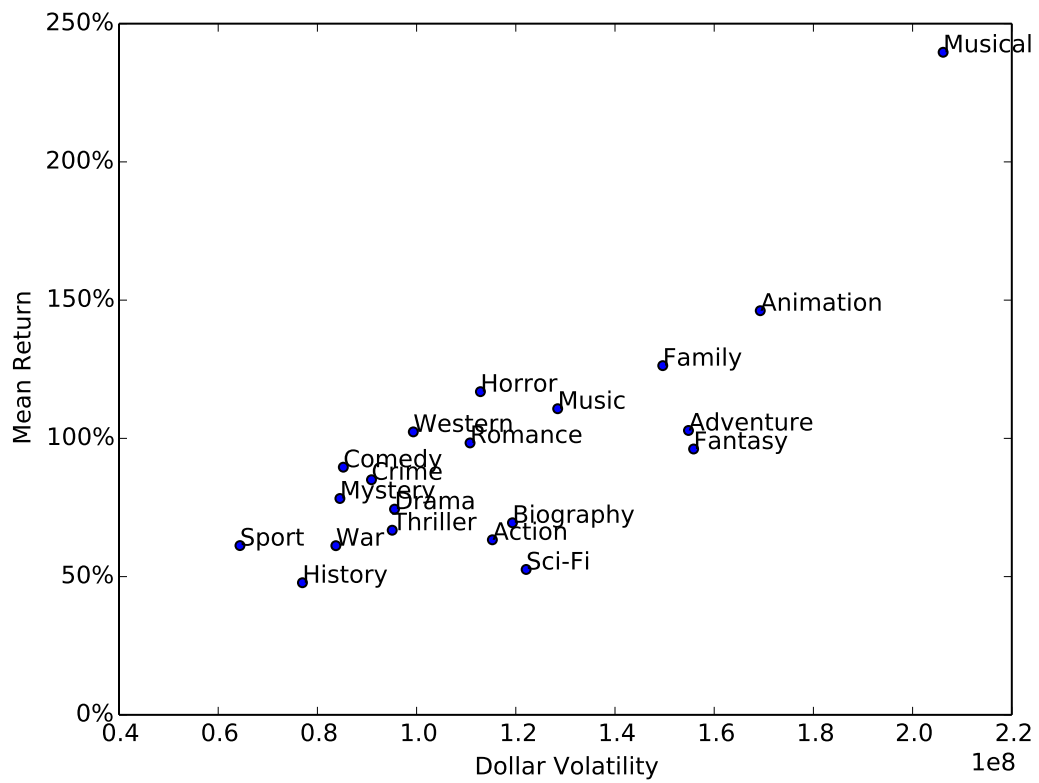
**Figure 1.7:** Budget Ventiles, dollar volatility on the x-axis (0 = smallest ventile)



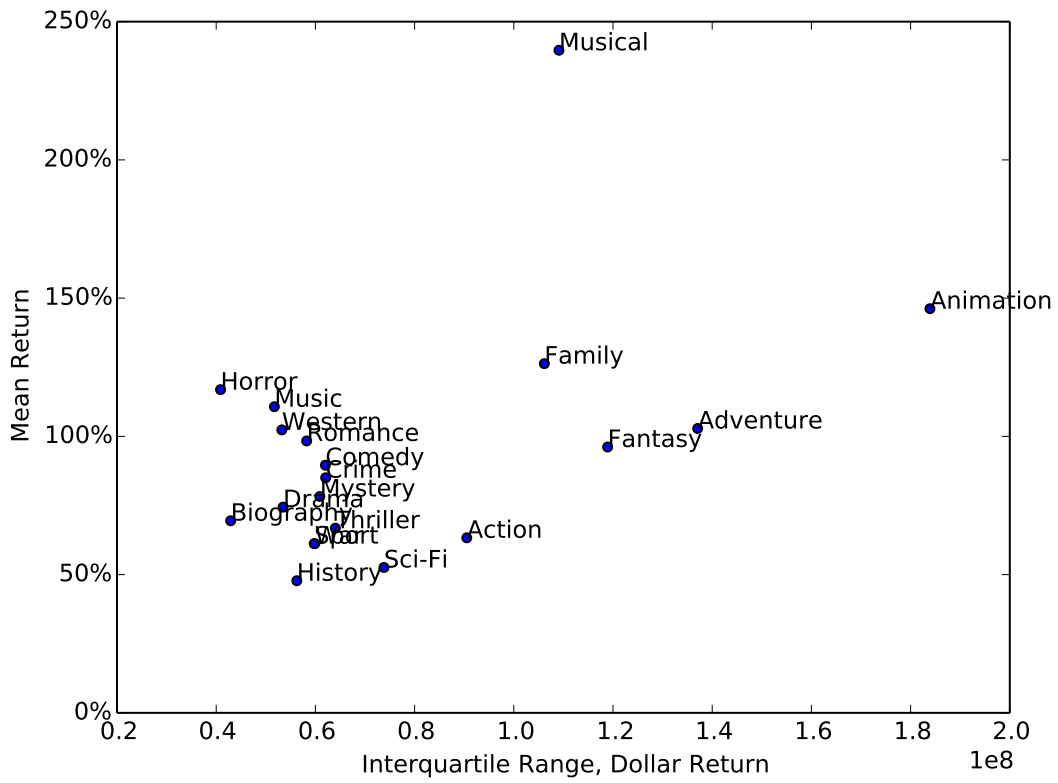
**Figure 1.8:** Budget Ventiles, interquartile range on the x-axis (0 = smallest ventile)



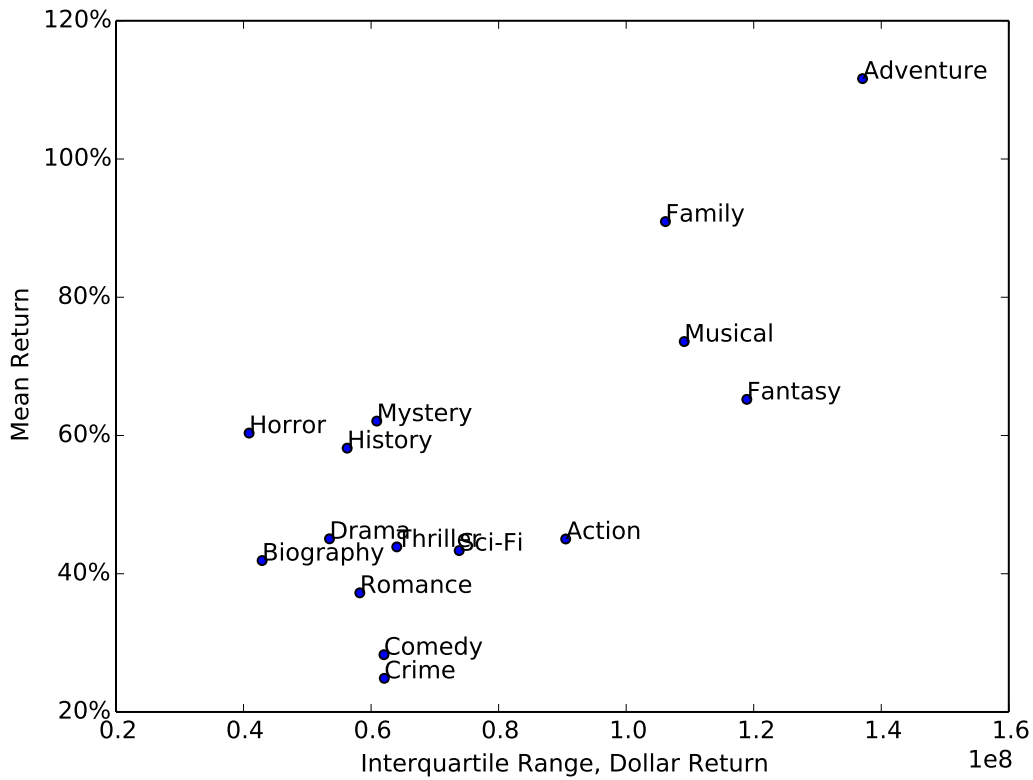
**Figure 1.9:** *Idiosyncratic risk premium, grouping movies by genre*



**Figure 1.10:** Idiosyncratic risk premium, grouping movies by genre, dollar volatility on the x-axis



**Figure 1.11:** *Idiosyncratic risk premium, grouping movies by genre, interquartile range on the x-axis*



**Figure 1.12:** Idiosyncratic risk premium, grouping movies by genre, interquartile range on the x-axis. The returns are annualized using IMDB data and winsorized at 300%. Some genres do not appear since they have fewer than ten movies; the cutoff used in Figures 1.9, 1.10, and 1.11 is thirty movies.

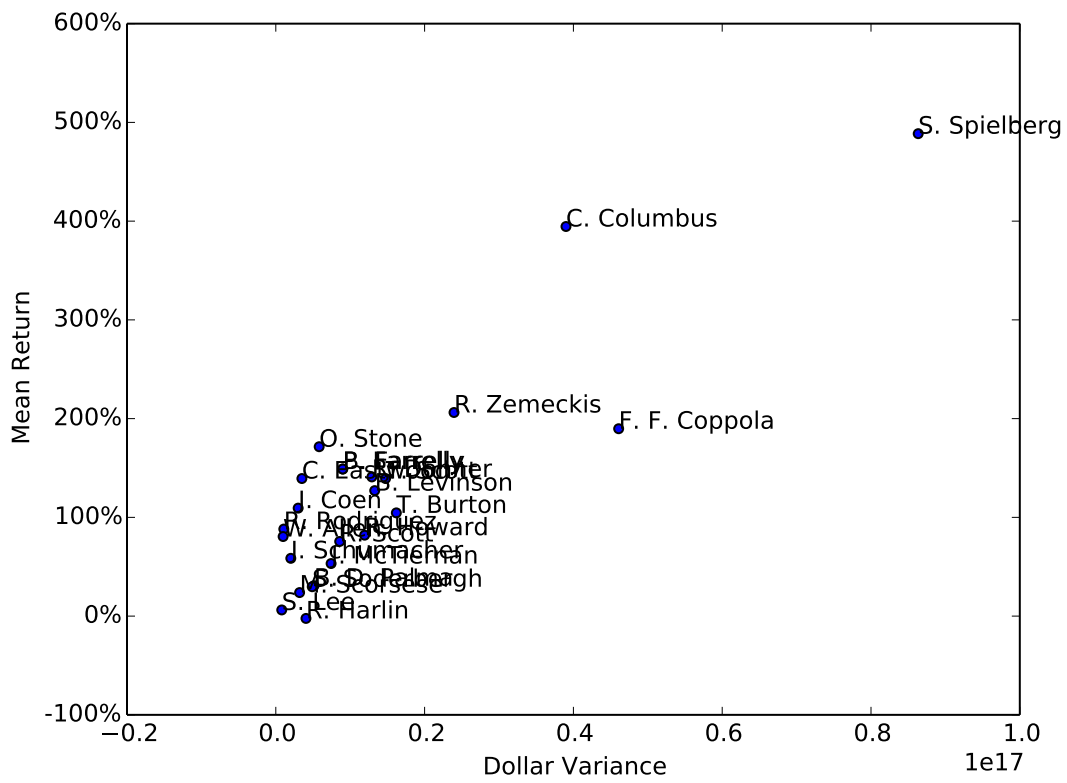


Figure 1.13: Idiosyncratic risk premium, grouping movies by director

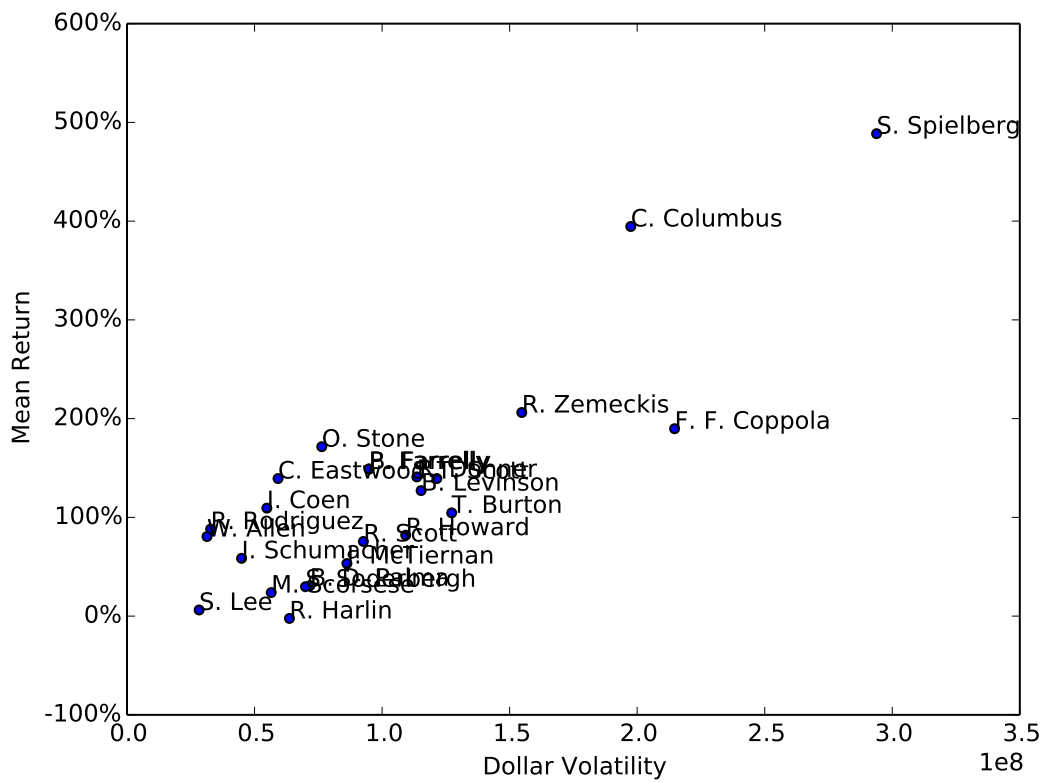


Figure 1.14: Idiosyncratic risk premium, grouping movies by director, dollar volatility on the x-axis



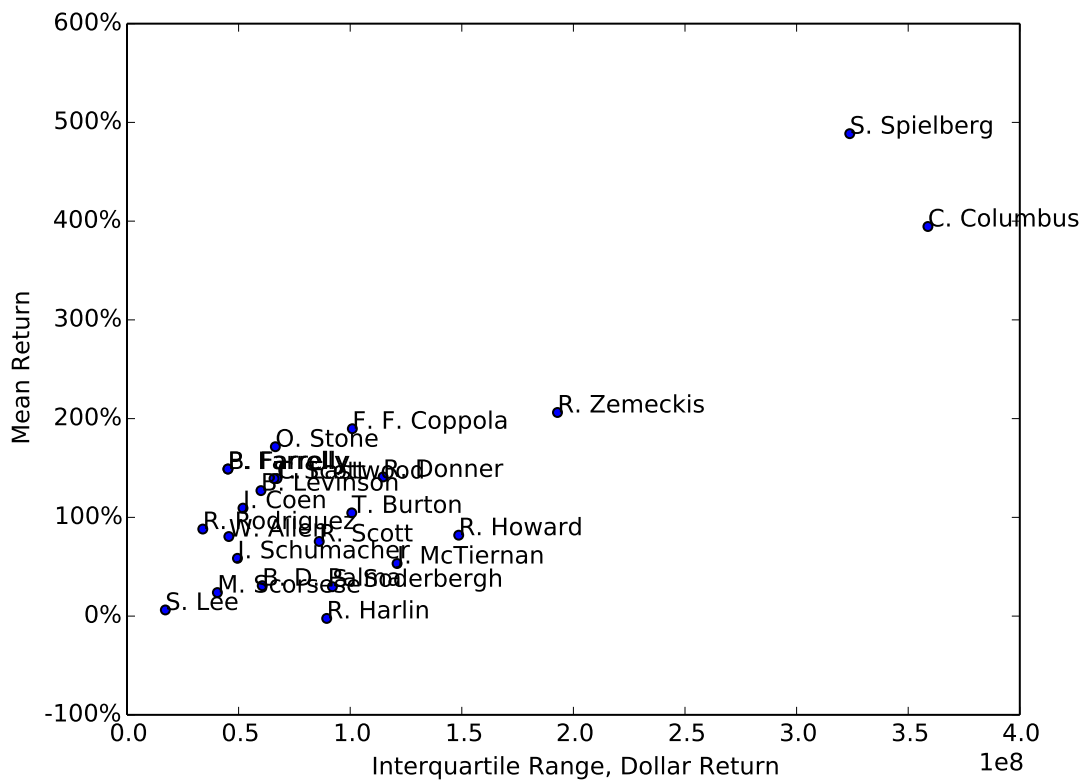
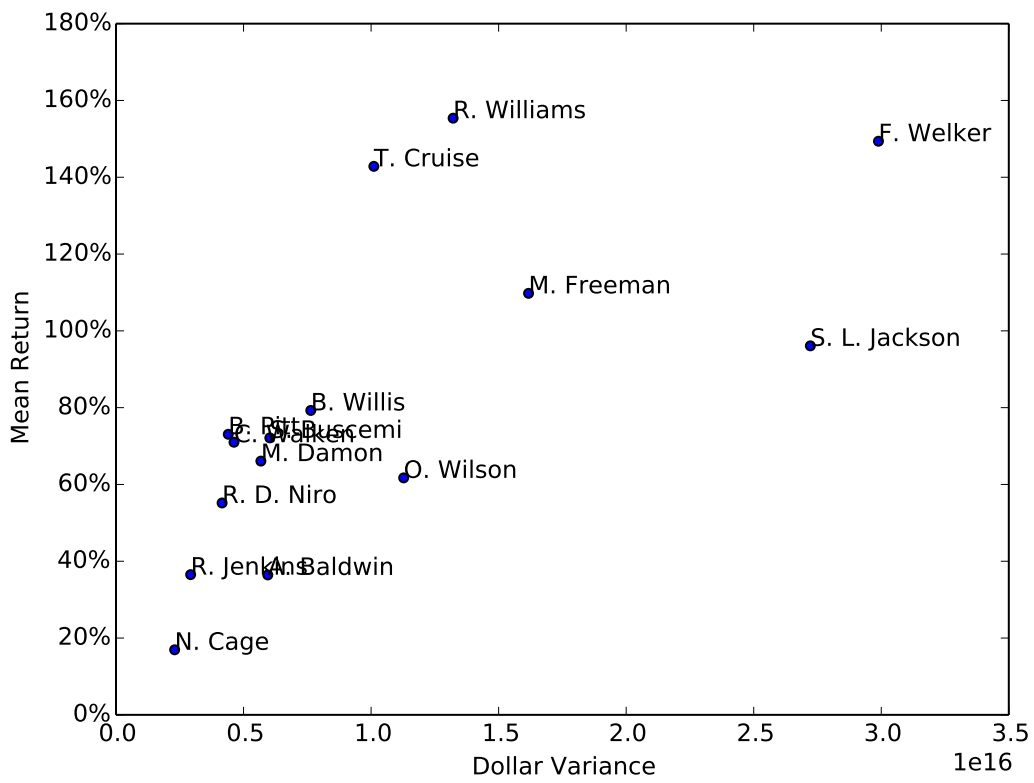


Figure 1.15: Idiosyncratic risk premium, grouping movies by director, interquartile range on the x-axis



**Figure 1.16:** *Idiosyncratic risk premium, grouping movies by actor*

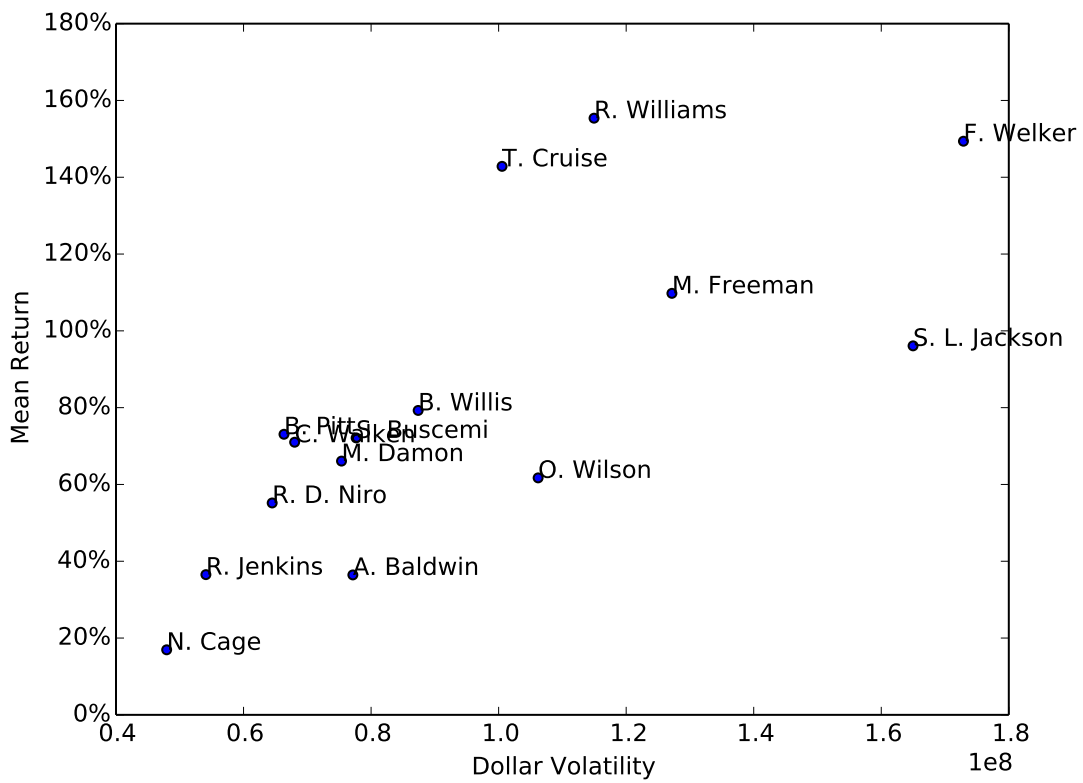
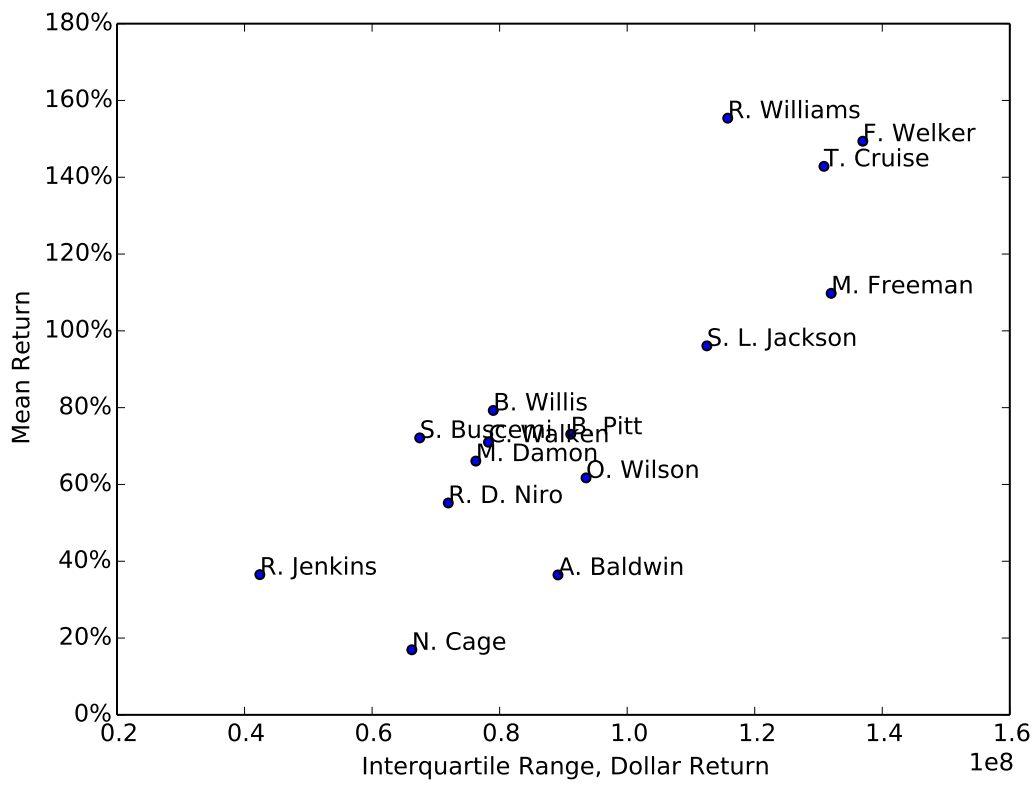


Figure 1.17: Idiosyncratic risk premium, grouping movies by actor, dollar volatility on x-axis



**Figure 1.18:** *Idiosyncratic risk premium, grouping movies by actor, interquartile range on x-axis*

**Table 1.3:** GMM estimates of the price of idiosyncratic risk, with arithmetic averaging for  $\sigma_{\$i}^2$ .

$$E(R_i) = \lambda \sqrt{\sigma_{\$i}^2} + R_f$$

$$\sigma_{\$i}^2 = \sum_g w_g 1_{ig} / \sum_g 1_{ig}$$

Estimated:  $\{w_g\}, \lambda, R_f$

Grouping Characteristic	$\hat{\lambda}$	$\hat{\lambda}_{bc}$	Bootstrap 95%			
			Confidence Set	$R^2$ with $\hat{\lambda}_{bc}$	$\hat{R}_f$	$\sigma(\hat{E}(R_i))$
Budget Ventile	-34 bps/MM\$	-50 bps/MM\$	[-49, 17] bps/MM\$	0.0%	121%	14.5%
<b>Genre</b>	<b>45</b>	<b>44</b>	<b>[39, 52]</b>	<b>0.3%</b>	<b>45%</b>	<b>8.3%</b>
<b>Director (<math>\geq 10</math> films)<sup>1</sup></b>	<b>117</b>	<b>116</b>	<b>[73, 151]</b>	<b>10.2%</b>	<b>10%</b>	<b>118%</b>
<b>Actor (<math>\geq 30</math> films)<sup>1</sup></b>	<b>100</b>	<b>119</b>	<b>[41, 135]</b>	<b>3.3%</b>	<b>-15%</b>	<b>35.3%</b>

expected returns computed according to the model fit. I find that my estimates of the idiosyncratic risk premium  $\lambda$  are large enough to induce sizeable variation in expected returns: for example, the fit using genre groupings implies a standard deviation in expected returns of 8.3 percentage points (compared to a median return of 17% in my dataset); and this is the lowest estimate.

In the rows corresponding to grouping movies by directors and actors, I use reduced samples. In the “Director” row, I keep only those films directed by directors with at least ten films; for the “Actor” row, I keep only those films that feature actors with at least thirty films. This helps mitigate the small-sample bias discussed above that might bias my results upward. Since these samples are smaller, they feature higher  $R^2$  values, and the  $\sigma(\hat{E}(R_i))$  values are also calculated with respect to these smaller samples.

Table 1.4 recalculates the results using geometric averaging in the specification for  $\sigma_{\$i}^2$  rather than arithmetic averaging. One might argue that geometric averaging is more appropriate in some cases; for example, we might imagine that having Brad Pitt in a movie reduces its risk to zero (hypothetically) since he is a big star. In that case, we would want a multiplicative factor representing Brad Pitt that just takes risk down to zero the moment he is in a film, even if there are other less-well-known stars in the movie. In any case, the

results are not too different using this alternative measure.

**Table 1.4:** GMM estimates of the price of idiosyncratic risk, with geometric averaging for  $\sigma_{\$,i}^2$ .

$$E(R_i) = \lambda \sqrt{\sigma_{\$,i}^2} + R_f$$

$$\sigma_{\$,i}^2 = \exp\left(\frac{\sum_g \ln(w_g) 1_{ig}}{\sum_g 1_{ig}}\right)$$

Estimated:  $\{w_g\}, \lambda, R_f$

Grouping Characteristic	$\hat{\lambda}$	$\hat{\lambda}_{bc}$	Bootstrap 95%			
			Confidence Set	$R^2$ with $\hat{\lambda}_{bc}$	$\hat{R}_f$	$\sigma(\hat{E}(R_i))$
Budget Ventile	-34 bps/MM\$	-50 bps/MM\$	[-49, 17] bps/MM\$	0.0%	121%	14.5%
<b>Genre</b>	<b>47</b>	<b>43</b>	<b>[17, 89]</b>	<b>0.7%</b>	<b>43%</b>	<b>22.3%</b>
<b>Director (<math>\geq 10</math> films)</b>	<b>117</b>	<b>116</b>	<b>[73, 151]</b>	<b>10.2%</b>	<b>10%</b>	<b>118%</b>
<b>Actor (<math>\geq 30</math> films)</b>	<b>82</b>	<b>91</b>	<b>[39, 119]</b>	<b>2.6%</b>	<b>-3%</b>	<b>31.1%</b>

## 1.7 Conclusions

This paper has studied a cross-section of expected returns on real investments. In particular, I have focused on how film studios price risk. The evidence supports the idea that projects with higher levels of idiosyncratic risk will be developed only if they have higher expected returns. This finding contradicts the typical idea that diversifiable risk should not be priced.

Although the data do not clearly elucidate a mechanism for this phenomenon, we have seen a model in which agency frictions can explain this behavior. Specifically, there are deadweight costs imposed by financiers (perhaps monitoring costs, or expected bankruptcy costs) whenever firms take on risky investments. This causes the cost of capital to be higher for funding such investments, and makes corporate managers avoid them.

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## Chapter 2

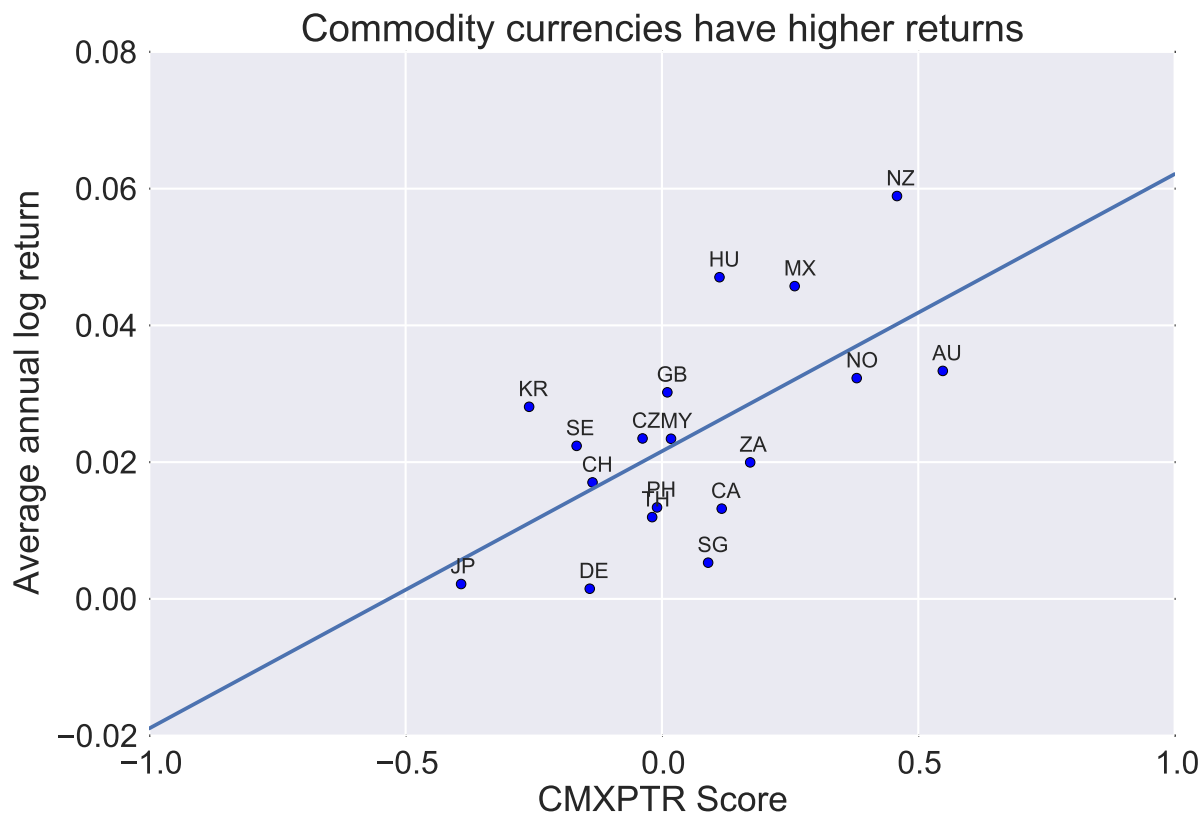
# The Commodity Currency Puzzle

### 2.1 Introduction

Suppose the Australian dollar interest rate is 5%, and the U.S. dollar interest rate is 2%. How much of an excess return can one earn from borrowing in the U.S., converting currencies, and lending in Australia?

The classic theory of uncovered interest parity (UIP) says that currency expected excess returns must be 0%. The 3% interest spread should coincide with an expected 3% depreciation of the Australian dollar, which would net out to a 0% return for the investor.

A challenge to UIP can be found in commodity exporters, whose currencies have excess returns well above zero. Figure 2.1 shows the fact documented by Ready, Roussanov, and Ward (2013). Commodity importers like Japan and Switzerland have mean currency excess returns of zero, while commodity exporters such as Australia and Norway have mean currency excess returns as high as 3% per year. The authors show that a trading strategy that goes long commodity currencies and short non-commodity currencies earns a 4.5% excess return per year, with a Sharpe ratio of 0.5, both of which are comparable to the equity premium. Moreover, as shown in Figure 2.1, as much as a third of the differences in long-run average returns across major, floating-rate currencies can be explained by differences in commodity exposure.



**Figure 2.1: Commodity currencies have higher real excess returns.** *CMXPTR* measures the extent to which a country exports commodities relative to finished goods. Each country's average *CMXPTR* score from 1984 to 2000 (the final date in the NBER-Comtrade dataset) is plotted. The y-axis is the average real log excess return on that country's currency, annualized. The labels are ISO 2-letter country codes. The sample is based on Hassan and Mano's (2015) selection of 36 currencies traded between 1989 and 2007, and I filter currencies that left the sample early due to Eurozone accession (most of them in 1999) and countries that never floated their exchange rate during 1985-2015. For countries that floated for only part of the sample, the averages are computed over the floating period only. I allow the Deutsche Mark (DE) to continue as the Euro after accession.

This paper develops a theory to explain why commodity currencies outperform non-commodity currencies. My theory shows that exchange rates in commodity countries rise in good times and crash in bad times, so investments in these currencies earn a risk premium. Moreover, the economic mechanism behind this cyclicalities is *labor cost disease*.

Commodity booms raise wages in exporter countries, and thus make local, nontradable goods and services less affordable, raising the cost of living (real exchange rate). Because the real exchange rate moves procyclically with commodity prices, the currency inherits a commodity risk premium that resolves the puzzle.

This explanation is appealing because it falls out from a classic theory for the real exchange rate; namely, the Balassa-Samuelson theory that labor costs determine the exchange rate (Balassa 1964, Samuelson 1964). While the original Balassa-Samuelson papers focused on long-run productivity growth in the export sector as the main driver of labor costs, a rapid boom in commodity prices can raise “productivity” just as much, since export productivity is measured as what you can buy on the world market, per unit of labor. My idea is simply to build a stochastic Balassa-Samuelson model, apply it to the case of a commodity exporter, and then compute asset prices.

An additional reason to favor this type of model is that models of cost disease have already been widely used to describe commodity exporters in the international macro literature (Bruno and Sachs 1982; Corden 1984). The literature on commodity currencies, in particular, uses stochastic models of this type (Chen and Rogoff 2005, Cashin, Céspedes, and Sahay 2004). I build on this literature by extending it to solve a puzzle in financial markets.

The paper’s central contribution is a model of the labor cost disease mechanism that makes asset pricing predictions for commodity exporters. This model is a dynamic, quantitative, two-country real business cycle model with production, a distinction between tradable and non-tradable goods, and a distinction between commodities and finished goods. I use a standard rare disaster parameterization to get a realistic amount of risk in the model.<sup>1</sup>

After laying out this model, I split my empirical analysis into two parts. First, I estimate key economic relationships that the model should fit. For example, in commodity exporters, real exchange rates and real commodity prices should be correlated; otherwise, there would be no reason for currencies to inherit commodity risk. Currency risk premia should be about

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<sup>1</sup>I follow Barro (2006), Barro and Ursúa (2008), Barro and Ursúa (2012), Barro, Nakamura, Steinsson, and Ursúa (2013), and use their consumption data where applicable.

as large as implied by their commodity risk exposure and commodity risk premia. Finally, real unit labor costs should co-move with commodity prices. All of these relationships need to be estimated, and this section covers the datasets, methods, and results.

Second, to check quantitative predictions of the model, I calibrate the model and solve it numerically. Simulations show that the model can match the high returns on commodity currencies and the fact that most of this return comes from high real interest rates. The model is also consistent with low short-run forecastability of real exchange rates, the volatility of the real exchange rate, and the relatively high elasticity of exchange rates to commodity price movements in commodity countries. The model does not rely on counterfactually large movements in labor costs.

Still, because the model uses certain standard assumptions, it inherits some well-known limitations. For example, I assume financial markets are complete, so households share risk perfectly. While complete markets is a common assumption, it leads to what is known as the Backus-Smith (1993) puzzle. Assuming a standard consumer utility function, a country should be assigned more consumption during times when its consumption is relatively cheap, but we do not observe such a correlation. A recent literature suggests that this puzzle can be resolved with more sophisticated forms for household preferences that allow the marginal utility of consumption to depend additional factors beyond the level of contemporaneous consumption.<sup>2</sup>

In the final section of the paper, I build a New Keynesian extension of the model in which nominal wages are sticky and labor costs may not adjust frictionlessly as a result.<sup>3</sup>

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<sup>2</sup>Colacito and Croce (2011, 2013) suggest a solution for the Backus-Smith puzzle based on the idea of Epstein-Zin preferences and long-run consumption risks. This paper fits into a broader literature on international long-run risks models that includes, among others, Bansal and Shaliastovich (2012), Backus, Telmer, Gavazzoni, and Zin (2013), and Colacito, Croce, Gavazzoni, and Ready (2015). Stathopoulos (2012) takes a habit-formation approach to the Backus-Smith puzzle, in the spirit of Campbell and Cochrane (1999) and Verdelhan (2010). Brandt, Cochrane, and Santa-Clara (2006) develop a related risk-sharing puzzle, Lewis and Liu (2015) distinguish between sharing persistent and transient consumption risks, and Buraschi, Sener, and Menguturk (2012) point out limits to arbitrage in currency markets.

<sup>3</sup>Obstfeld and Rogoff (1995) and Gali and Monacelli (2005) develop textbook sticky-price models of the open economy, and Erceg, Henderson, and Levin (2000) develop the approach to wage stickiness I adopt in this paper.

Instead, households have infrequent, randomly arriving opportunities to reset the wage they demand. I find that, assuming the central bank adopts a floating exchange rate and targets a zero output gap, nominal wage stickiness does not hinder the adjustment of real relative labor costs. Even if labor costs are fixed in their own currencies, the nominal exchange rate can move flexibly to adjust relative costs. A central bank defending a currency peg, however, will not be able to adjust labor costs so quickly, and will experience output booms when commodity prices are high, and busts when commodity prices are low. The risk premium on the real exchange rate is, naturally, lower in the case of the peg, since the real exchange rate simply moves much less.

My explanation for high commodity currency returns complements the one provided by Ready, Roussanov, and Ward (2013). Like me, they argue that real exchange rates in commodity countries are cyclical, and thus get a risk premium. In their model, however, the cyclicality comes from shipping costs. Commodity exporters need to import consumer goods from abroad, and their cost of living rises during good times because shipping costs tend to be higher. My paper supplies an alternative reason why the cost of living would rise in booming commodity exporters: higher labor costs.

The present paper is also related to Farhi and Gabaix (2015) in that both papers express real exchange rates as a function of export productivity, and both papers take a rare disasters approach to asset pricing.<sup>4</sup> In that paper, many countries all export the same tradable good, potentially with different levels of productivity. Since my paper focuses on commodities, I explicitly make commodities and finished goods different, by making commodities a non-consumable input good. This setup provides a simple mechanism for commodity prices to be cyclical, as input prices rise during times of high world productivity. Additionally, I focus on labor costs specifically, while Farhi and Gabaix take a more general approach of having tradable and non-tradable industries split a stochastic endowment.

This paper also links with the extensive literature on carry trades and the forward

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<sup>4</sup>Gabaix (2012), Daniel, Hodrick, and Lu (2015), and Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2015) also explore crash risk in currencies.

premium puzzle. Carry trades are trading strategies that go long currencies with high interest rates and short currencies with low interest rates. Carry trades make money in two ways: *fixed bets* on currencies that have permanently (or at least, persistently) high interest rates, and *time-varying bets* on currencies based on their current interest rate being higher or lower than average. Commodity currencies fall in the former category of currencies with persistently high interest rates, while the “forward premium puzzle” refers to the latter way of making money.<sup>5</sup> Ready, Roussanov, and Ward (2013) find that their “commodity trade” explains about half the variation in monthly returns on an unconditional carry trade that only makes fixed bets on high-interest-rate countries.

In terms of other literature, I build on the existing work on commodity currencies (Chen and Rogoff 2003; Cashin, Céspedes, and Sahay 2004; Clements and Fry 2008; Kearns 2007; Tokarick 2008) and I also contribute to the growing literature on country-specific currency risk premia. Several other variables have been identified that seem correlated with risk premia, including country size (Hassan 2013), country “systemic”-ness (Hassan, Mertens, and Zhang 2015), trade network centrality (Richmond 2015), monetary policy (Backus, Telmer, Gavazzoni, and Zin 2013; Chinn and Zhang 2015), financial imbalances (Della Corte, Riddiough, and Sarno 2015); and status as a world reserve asset (Rey 2013; Maggiori 2012, 2013; Mueller, Porchia, and Vedolin 2014; He, Krishnamurthy, and Milbradt 2015).

I also connect with recent work measuring commodity risk premia. A number of papers find significant excess returns to commodity portfolios (Gorton and Rouwenhorst 2006; Gorton, Hayashi, and Rouwenhorst 2013; Yang 2013; Bhardwaj, Gorton, and Rouwenhorst 2015). Dhume (2010) suggests a consumption-based explanation for these commodity premia, and Lettau, Maggiori, and Weber (2014) price both commodities and currencies using a downside-risk CAPM.

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<sup>5</sup>Hassan and Mano (2015) provide details on the differences between, and returns on, the two types of strategies. See Fama (1984) for early empirical findings on the forward premium puzzle. A number of explanations have been suggested for the forward premium puzzle, including expectational errors (Froot and Frankel 1989), habit formation preferences (Verdelhan 2010), variable rare disasters (Gabaix 2012, Farhi and Gabaix 2015), long-run risks (Colacito and Croce 2011, 2013), partially segmented financial markets (Gabaix and Maggiori 2015), and infrequent portfolio decisions (Bachetta and van Wincoop, 2010).

## 2.2 Model

### 2.2.1 Setup

Time is discrete and indexed by integers  $t$ , where  $t \geq 0$ . At each time  $t$ , there are different possible states; the state at time 0 is  $s_0$ , at time 1  $s_1$ , etc. A history  $s^t = (s_0, s_1, \dots, s_t) \in S^t$  is a tuple of states from the beginning of the model to date  $t$ .  $\mathcal{P}(s^t)$  is the probability of reaching history  $s^t$  conditional on information at time zero.

There are two countries: Australia and the United States. At history  $s^t$ , Australia produces  $Y_C^*(s^t)$  units of a non-consumable commodity and  $Y_{NT}^*(s^t)$  units of a non-tradable good. The US produces  $Y_F(s^t)$  units of a finished good, which uses the commodity as an input, and  $Y_{NT}(s^t)$  units of its own non-tradable good.

Tradable goods are traded frictionlessly, while non-tradable goods can only be consumed locally.

#### Australia

Markets are complete, so we can think of the Australian household as simply performing a time-0, “static” optimization in which consumption at each history  $s^t$  (for all  $t$ ) is a different good.

Assuming time-separable power utility, the household maximizes:

$$U^* \equiv \sum_t \sum_{s^t \in S^t} \mathcal{P}(s^t) \beta^t \left( \frac{C^*(s^t)^{1-\gamma}}{1-\gamma} \right)$$

where  $\beta$  is the household’s subjective discount factor and  $C^*(s^t)$  is a Cobb-Douglas consumption index. This index aggregates finished good consumption,  $C_F^*(s^t)$  and non-tradable consumption  $C_{NT}^*(s^t)$ :

$$C^*(s^t) \equiv \kappa C_F^*(s^t)^\chi C_{NT}^*(s^t)^{1-\chi}$$

In these equations,  $\gamma$  is the coefficient of relative risk aversion,  $\chi$  is the finished good share of nominal consumption, and  $\kappa$  is a normalizing factor equal to  $\chi^{-\chi} (1-\chi)^{-(1-\chi)}$ .



The household is endowed with a single unit of labor, implying that

$$L_C^* (s^t) + L_{NT}^* (s^t) = 1$$

for all  $s^t$ . The two variables on the left-hand side are labor used producing commodities and labor used producing the non-tradable good.

The household's budget constraint, measured in units of the time-0 consumption basket, is:

$$A^* + \sum_t \sum_{s^t \in S^t} \psi^* (s^t) W^* (s^t) = \sum_t \sum_{s^t \in S^t} \psi^* (s^t) C^* (s^t)$$

In this equation,  $A^*$  is net initial claims that Australia has on the US;  $W^* (s^t)$  is the wage at history  $s^t$ , measured in units of consumption at that history; and  $\psi^* (s^t)$  is the price, in terms of time-0 consumption, of an Arrow-Debreu security paying 1 unit of consumption at history  $s^t$ .

In this constrained maximization, the household's control variables are the consumption and labor functions:  $C_F^* (s^t)$ ,  $C_{NT}^* (s^t)$ ,  $L_C^* (s^t)$ , and  $L_{NT}^* (s^t)$ .

In Australia, representative firms, acting in competitive markets, produce both goods using labor as the only input. Firms maximize profits. For simplicity, output is linear in labor:

$$Y_C^* (s^t) = Z^* (s^t) L_C^* (s^t)$$

$$Y_{NT}^* (s^t) = Z^* (s^t) L_{NT}^* (s^t)$$

where  $Z^* (s^t)$  is an exogenous productivity process for Australia.

### **The labor cost mechanism**

We can already see how labor cost disease links commodity prices and real exchange rates. Profit maximization implies that, for each firm, the marginal benefit of producing one more unit of output equals the marginal cost of hiring someone for  $1/Z^*$  hours. In particular, for

the commodity sector and the non-tradable sector:

$$P_C^* (s^t) = W^* (s^t) / Z^* (s^t) \quad (2.1)$$

$$P_{NT}^* (s^t) = W^* (s^t) / Z^* (s^t) \quad (2.2)$$

where  $P_C^* (s^t)$  and  $P_{NT}^* (s^t)$  are the prices of the commodity and the Australian non-tradable good, measured in units of the contemporaneous consumption basket.

Anything that increases commodity prices must increase non-tradable prices, because they both depend on the same labor input. The rise in non-tradable prices will raise the real exchange rate, since tradable goods have the same real price in both countries.

Given that commodity prices and real exchange rates are correlated, we still need commodities to have a risk premium in order for commodity currencies to get one, too. I build a simple microfoundation for the risk premium on commodity prices, based on demand for commodities being cyclical. However, any stochastic discount factor that gives a risk premium to commodities (which is, in fact, observed; see Figure 2.3) should also give a similar risk premium to commodity currencies.

## United States

The United States household faces a similar problem to the Australian household. It maximizes

$$U \equiv \sum_t \sum_{s^t \in S^t} \mathcal{P} (s^t) \beta^t \left( \frac{C (s^t)^{1-\gamma}}{1-\gamma} \right)$$

where the aggregate consumption index is now:

$$C (s^t) \equiv \kappa C_F (s^t)^\chi C_{NT} (s^t)^{1-\chi}$$

The US household is also endowed with a single unit of labor, implying that

$$L_F (s^t) + L_{NT} (s^t) = 1$$

for all  $s^t$ . The two variables on the left-hand side are labor used producing the finished good and labor used producing the non-tradable good.

The household's budget constraint, measured in units of its time-0 consumption basket, is:

$$A + \sum_t \sum_{s^t \in S^t} \psi(s^t) W(s^t) = \sum_t \sum_{s^t \in S^t} \psi(s^t) C(s^t)$$

where  $A$  is net initial claims on Australia,  $W(s^t)$  is the wage in terms of consumption at history  $s^t$ , and  $\psi(s^t)$  is the price for a consumption claim paid at history  $s^t$ .

In the US, production of the finished good uses a Cobb-Douglas technology with two inputs: labor, with share  $\alpha$ , and the commodity, with share  $1 - \alpha$ . Commodity input equals total commodity output, since there is only one use of the commodity. Non-tradables are produced with a simple linear production function.

$$Y_F(s^t) = Z(s^t) L_F(s^t)^\alpha Y_C^*(s^t)^{1-\alpha}$$

$$Y_{NT}(s^t) = Z(s^t) L_{NT}(s^t)$$

To see directly why commodity prices are cyclical, take the finished good producer's first-order condition with respect to commodity input:

$$\underbrace{P_C(s^t)}_{\text{marginal cost}} = \underbrace{P_F(s^t) (1 - \alpha) Z(s^t) L_F(s^t)^\alpha Y_C^*(s^t)^{-\alpha}}_{\text{marginal benefit}} \quad (2.3)$$

where  $P_C(s^t)$  is the commodity price, and  $P_F(s^t)$  is the finished good price, both relative to the contemporaneous US consumption basket. All else equal, an increase in US productivity  $Z$  raises commodity prices  $P_C$ , making commodity prices cyclical so long as these demand shocks have a larger impact on commodity prices than any Australian supply shocks. Of course, commodity prices and productivity will not be exactly proportional, since, in equilibrium,  $L_F$  and  $Y_C^*$  will be determined by the value of  $Z$  as well. It can be shown that exact proportionality holds in the case of log utility ( $\gamma \rightarrow 1$ ).

## Specifying the shocks

There are two exogenous processes for productivity,  $Z(s^t)$  and  $Z^*(s^t)$ . Since we are interested in risk premia rather than absolute rates of return, I abstract from trend growth, so productivity is stationary around a steady state in both countries.<sup>6</sup>

Productivity in Australia follows an AR(1) in logs. I use lowercase letters to denote logs of uppercase letters:

$$z_t^* = az_{t-1}^* + u_t$$

where  $u_t \sim \mathcal{N}(0, \sigma_u^2)$ .

Productivity in the US also has an AR(1) structure, but with rare disasters:

$$z_t = az_{t-1} + v_t$$

The shock  $v_t$  has distribution:

$$v_t = \begin{cases} h_t & \text{w.p. } 1 - d_p \\ h_t + b & \text{w.p. } d_p \end{cases}$$

where  $h_t \sim \mathcal{N}(0, \sigma_h^2)$ ,  $B < 1$  is the factor by which productivity is multiplied when there is a disaster, and  $d_p$  is the probability of a disaster, which is constant. I assume that  $h_t$  and  $u_t$  are drawn from a multivariate normal distribution with correlation  $\rho_{uh}$ .

## The real exchange rate

The real exchange rate  $Q(s^t)$  is the price of one unit of consumption in Australia, measured in units of US consumption. The real exchange rate is determined by the relative price of non-tradable goods, because if consumption was entirely of tradable goods, any differences in the cost of living could be arbitrated away.

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<sup>6</sup>The nominal exchange rate can still be nonstationary. For example, productivity, and by extension, every real variable in the model, could be constant, but price levels in each country could be random walks that are not cointegrated. Lustig, Stathopoulos, and Verdelhan (2015) discuss the stationarity of nominal exchange rates.

To see the real exchange rate in terms of nontradables, we need to do a bit of algebra. First, we make use of the fact that, with a homothetic consumption aggregator, if prices are measured in terms of the consumption good, then the total cost of consumption equals the quantity consumed:

$$P_F (s^t) C_F (s^t) + P_{NT} (s^t) C_{NT} (s^t) = C (s^t)$$

Since we have appropriately normalized the Cobb-Douglas consumption index, both sides can be divided by  $C (s^t)$  to yield:

$$P_F (s^t)^\chi P_{NT} (s^t)^{1-\chi} = 1$$

Similarly, in Australia,

$$P_F^* (s^t)^\chi P_{NT}^* (s^t)^{1-\chi} = 1$$

A single unit of Australian consumption can be sold for  $1/P_F^* (s^t)$  units of the finished good (a higher price for the finished good means a lower quantity of units purchased). These units can then be exported to the US and sold at a price  $P_F (s^t)$  per unit, measured in the US consumption good. Thus, the real exchange rate is

$$Q (s^t) = \frac{P_F (s^t)}{P_F^* (s^t)}$$

Using this fact, and plugging in our Cobb-Douglas identities, the equation for the real exchange rate simplifies to:

$$q (s^t) = \left( \frac{1-\chi}{\chi} \right) \left( p_{NT}^* (s^t) - p_{NT} (s^t) \right)$$

where lowercase letters denote natural logs.

This formula simply reflects the intuition that higher non-tradable prices in Australia require a higher equilibrium real exchange rate.

## 2.2.2 Solving the model

### Equilibrium

The model uses a standard definition of competitive equilibrium.

1. Households maximize utility subject to their budget constraints. Formally, the control variables  $C_F(s^t)$ ,  $C_{NT}(s^t)$ ,  $C_F^*(s^t)$ ,  $C_{NT}^*(s^t)$ ,  $L_F(s^t)$ ,  $L_{NT}(s^t)$ ,  $L_C^*(s^t)$ , and  $L_{NT}^*(s^t)$  must solve the household maximization problems.
2. Second, firms maximize profits. The firm's choice variables are  $L_F(s^t)$ ,  $L_{NT}(s^t)$ ,  $L_C^*(s^t)$ , and  $L_{NT}^*(s^t)$ .
3. Finally, goods markets clear. Since we are using the Arrow-Debreu formulation, this is equivalent to financial market clearing, as the financial assets are equivalent to goods at different histories. Formally, this means

$$\begin{aligned} C_F(s^t) + C_F^*(s^t) &= Y_F(s^t) \\ C_{NT}(s^t) &= Y_{NT}(s^t) \\ C_{NT}^*(s^t) &= Y_{NT}^*(s^t) \end{aligned}$$

for all histories  $s^t$ .

### Social planning formulation

We simplify the problem by taking a “social planning” approach. Because markets are complete, the competitive equilibrium is Pareto optimal. Consequently, the equilibrium quantities can be written as the solutions to a social planner's dynamic control problem:

$$\max \sum_t \sum_{s^t \in S^t} \beta^t \left[ \nu \left( \frac{C(s^t)^{1-\gamma}}{1-\gamma} \right) + (1-\nu) \left( \frac{C^*(s^t)^{1-\gamma}}{1-\gamma} \right) \right]$$

where the control variables are  $C_F(s^t)$ ,  $C_{NT}(s^t)$ ,  $C_F^*(s^t)$ ,  $C_{NT}^*(s^t)$ ,  $L_F(s^t)$ ,  $L_{NT}(s^t)$ ,  $L_C^*(s^t)$ , and  $L_{NT}^*(s^t)$ . The constraints include both identities and resource constraints. The identities

are:

$$C(s^t) = \kappa C_F(s^t)^\chi C_{NT}(s^t)^{1-\chi}$$

$$C^*(s^t) = \kappa C_F^*(s^t)^\chi C_{NT}^*(s^t)^{1-\chi}$$

The resource constraints are:

$$C_F(s^t) + C_F^*(s^t) = Z(s^t) L_F(s^t)^\alpha (Z^*(s^t) L_C^*(s^t))^{1-\alpha}$$

$$C_{NT}(s^t) = Z(s^t) L_{NT}(s^t)$$

$$C_{NT}^*(s^t) = Z^*(s^t) L_{NT}^*(s^t)$$

$$L_F(s^t) + L_{NT}(s^t) = 1$$

$$L_C^*(s^t) + L_{NT}^*(s^t) = 1$$

Because there are no endogenous state variables and the objective function is linear by state, this problem can be solved independently for each  $s^t$ . That simplified static, deterministic problem is as follows (after substituting in constraints):

$$\max_{C_F, L_F, L_C^*} \nu \left( \frac{(C_F^\chi (Z(1-L_F))^{1-\chi})^{1-\gamma}}{1-\gamma} \right)$$

$$+ (1-\nu) \left( \frac{((ZL_F^\alpha (Z^*L_C^*)^{1-\alpha} - C_F)^\chi (Z^*(1-L_C^*))^{1-\chi})^{1-\gamma}}{1-\gamma} \right)$$

where the variables are now scalars, solved for the state indexed by  $(Z, Z^*)$ .

Once this simplified problem has been solved for a particular history  $s^t$ , the other quantities can be backed out using the constraints on the dynamic social planning problem. For consumption goods, prices can be computed as ratios of marginal utilities. For the commodity, the price is computed using the firm first-order condition

$$P_C = (1-\alpha) P_F Z L_F^\alpha Y_C^{*-\alpha}$$

where the other terms have already been solved for.

## Numerical solution method

We know that the competitive equilibrium is Pareto optimal, but we do not know the Pareto weights  $\nu$  and  $1 - \nu$ . I compute these numerically using the approach of Negishi (1960), as described in Ljungqvist and Sargent (2000). One makes an initial guess,  $\nu_0$ , and then checks the budget constraints. One then iterates this process, guessing  $\nu_{k+1} < \nu_k$  if the present value of US consumption exceeds the present value of US income. I assume  $A = A^* = 0$  in these solutions. The state prices in the budget constraint are computed as the ratio of marginal utilities at history  $s^t$  and history  $s^0$ :

$$\psi(s^t) = \mathcal{P}(s^t) \beta^t \left( \frac{C(s^t)}{C(s^0)} \right)^{-\gamma}$$

### 2.2.3 Accounting for non-commodity exports

As written, the model forces the commodity country to be a 100% commodity exporter. In reality, even for a country like Australia, commodities make up only about half of exports (although this figure varies over time). Accounting for this difference is not important for the main conclusions of the model, but it will be important quantitatively when we turn to calibration, since otherwise we will predict effects that are too strong.

To capture alternative exports, I take a reduced-form approach. I introduce an alternative export good  $A$  in Australia. Output of this good, like the commodity, is linear in labor:

$$Y_A^*(s^t) = Z^*(s^t) L_A^*(s^t)$$

To make this sector bigger or smaller, we simply embed a bigger or smaller preference  $\phi$  for consuming this good in the US:

$$C(s^t) = \kappa_{US} C_F(s^t)^\chi C_A(s^t)^\phi C_{NT}(s^t)^{1-\chi-\phi}$$

where  $\kappa_{US} \equiv \chi^{-\chi} \phi^{-\phi} (1 - \chi - \phi)^{-(1-\chi-\phi)}$ .



Since the good is not consumed in Australia, market clearing implies:

$$C_A(s^t) = Z^*(s^t) L_A^*(s^t)$$

so the only new endogenous variable is  $L_A^*(s^t)$ .

The final, state-specific social planning problem simply becomes:

$$\begin{aligned} \max_{C_F, L_F, L_C^*, L_A^*} \quad & v \left( \frac{(C_F^\chi (Z(1-L_F))^{1-\chi})^{1-\gamma}}{1-\gamma} \right) \\ & + (1-v) \left( \frac{((ZL_F^\alpha (Z^*L_C^*)^{1-\alpha} - C_F)^\chi (Z^*L_A^*)^\phi (Z^*(1-L_C^* - L_A^*))^{1-\chi})^{1-\gamma}}{1-\gamma} \right) \end{aligned}$$

The taste parameters  $\chi$  and  $\phi$  can then be calibrated to match data on the commodity sector share of exports and the export share of GDP.

## 2.3 Empirics

This section describes how I empirically evaluate the model. First, I make estimates of the key economic mechanisms. This task includes using panel data, merged with currency and commodity data, to measure how much exchange rates, commodity prices, and real wages are moving. Second, I do a quantitative calibration exercise to see if the model can match economic variables for the Australia-US country pair.

The appendix, I fully describe the data sources and construction of individual time series. For the body of the paper, I simply provide short descriptions as I introduce each new series.

### 2.3.1 Key Estimates

#### Currency-commodity elasticities and correlations

In my model, commodities are cyclical, and their risk premium is inherited by commodity currencies through the labor cost mechanism.<sup>7</sup> Thus two things need to be true in the data: first, commodity currencies are highly correlated with commodity prices; second,

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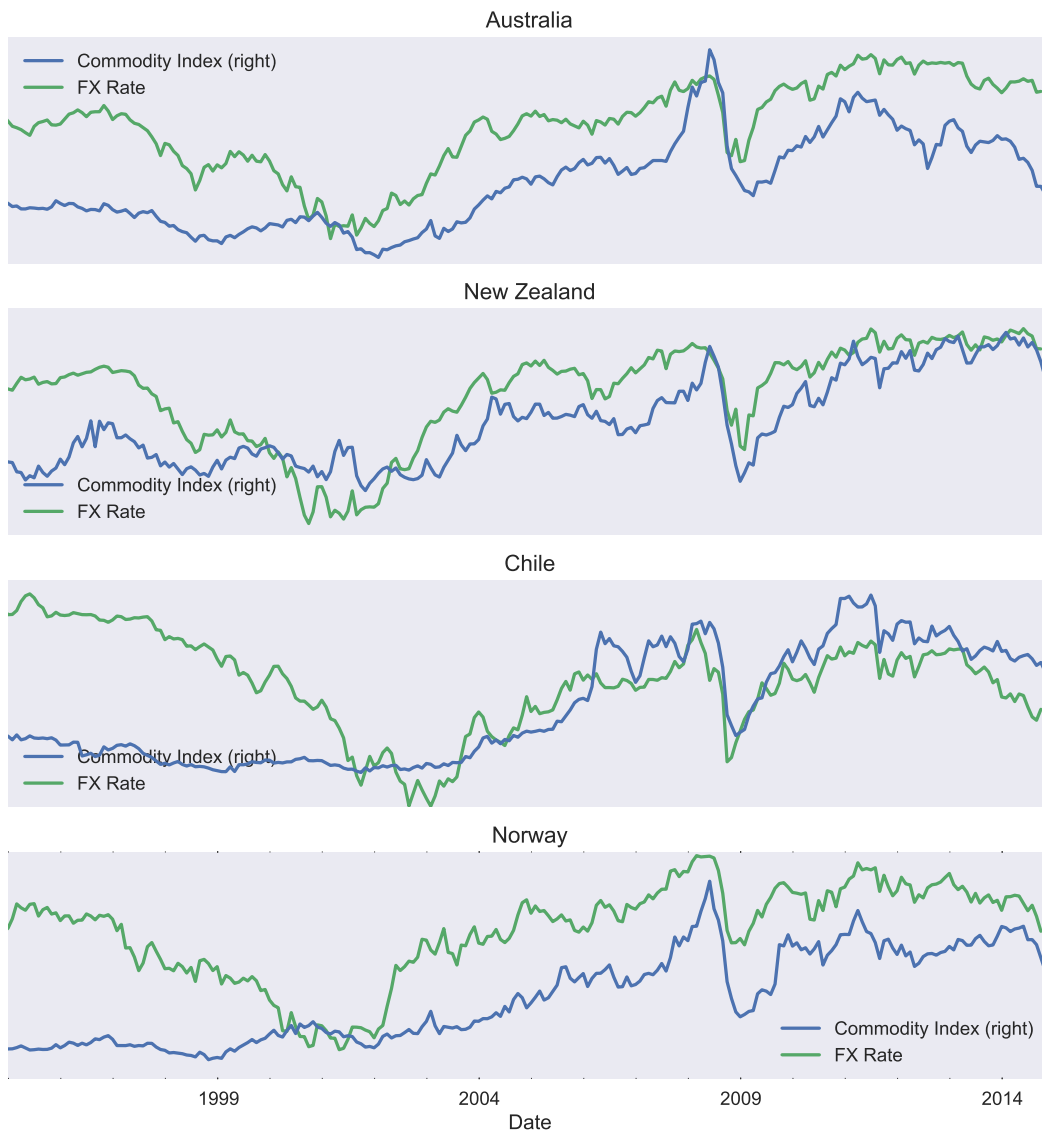
<sup>7</sup>See equations (1)-(3) for details and discussion.

commodity currencies are highly elastic to commodity prices. (To expand on the latter point, it is difficult to get a big risk premium on something that doesn't move very much, even if it is cyclical.)

Figure 2.2 plots exchange rates alongside commodity export prices for a variety of commodity exporters. One can immediately see that they are quite correlated. The commodity price index is export-weighted and specific to each country.<sup>8</sup>

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<sup>8</sup>Figure 2.2 uses nominal variables.



**Figure 2.2: Commodity currencies load strongly on commodity risks.** *The above four plots compare nominal exchange rates to matched, export-weighted commodity price indexes, also measured in dollars. The exchange rates are oriented so that “up” means an appreciation of the named currency. Export weights come from the UN-Comtrade dataset. Log scale is used.*

To compute this commodity price index, I use data from the Commodity Research

Bureau (CRB); one can purchase a CD with daily data on the entire futures curve for a comprehensive set of futures traded on many exchanges. The details of how spot prices, returns, and commodity basis are calculated from these data are provided in the appendix.

I estimate both short-run and long-run elasticities. Short-run elasticities measure the quarterly change in the real exchange rate associated with a contemporaneous quarterly change in commodity prices. The baseline time-series regression to estimate a short-run elasticity is therefore:

$$\Delta q_t^j = \beta^j \Delta \text{cmpi}_t^j + \alpha^j + \varepsilon_t^j$$

where  $q_t^j$  is the log real exchange rate for country  $j$  against the U.S. dollar, and  $\text{cmpi}_t^j$  is the real export-weighted commodity price index. The commodity price index is “real” in the sense that it is a U.S. dollar nominal quantity deflated by the U.S. CPI. Short-run regressions can be interpreted as measuring the substitutability between currency and commodity investment positions from the perspective of a short-horizon investor. The regression results are reported in Table 2.1.

**Table 2.1: Short-run elasticity of the exchange rate to commodity prices (baseline specification).**

I run quarterly time series regressions of the change in the log bilateral real exchange rate  $q_t^j$  between country  $j$  and the U.S. against the change in the log of a matched, export-weighted real commodity price index. The cross-sectional correlation between the estimated elasticities and CMXPTR, a measure of commodity exports, is 0.47.

$$\Delta q_t^j = \beta \Delta \text{cmppi}_t^j + \alpha + \varepsilon_t^j$$

Country $j$	Estimated $\beta$	Standard Error	$R^2$
Australia	0.30**	0.10	11.1%
Canada	0.21***	0.03	34.1%
Switzerland	0.09	0.09	1.3%
Chile	0.29*	0.12	23.1%
Czech Republic	0.14	0.10	2.5%
Germany/Euro	0.12	0.07	3.5%
Denmark	0.04	0.06	0.5%
United Kingdom	0.18***	0.04	19.8%
Hungary	0.28**	0.09	11.0%
India	0.16*	0.08	5.8%
Japan	0.03	0.09	0.1%
South Korea	0.16	0.10	3.4%
Mexico	0.13*	0.05	7.5%
Malaysia	0.07	0.06	1.8%
Norway	0.14***	0.04	14.1%
New Zealand	0.31**	0.10	10.8%
Philippines	0.01	0.06	0.0%
Sweden	0.26**	0.08	11.9%
Singapore	0.09*	0.03	8.0%
Thailand	-0.02	0.08	0.1%
South Africa	0.24*	0.11	5.8%

Because changes in commodity prices sometimes have a delayed effect on real exchange rates, to avoid bias in the contemporaneous coefficient, we can include a lag of the right-hand variable in the regression. These alternative estimates are given in Table 2.2, and I find that they are slightly better correlated with a measure of commodity exports (CMXPTR) than the baseline estimates (correlations are 0.55 versus 0.47).

**Table 2.2: Short-run elasticity of the exchange rate to commodity prices (alternative specification).**

I run quarterly time series regressions of the change in the log bilateral real exchange rate  $q_t^j$  between country  $j$  and the U.S. against the change in the log of a matched, export-weighted real commodity price index. Unlike in Table 2.1, I include one lag of the right-hand-side variable to reduce any bias due to non-instantaneous adjustment of the real exchange rate. The cross-sectional correlation between the estimated elasticities and CMXPTR, a measure of commodity exports, is 0.56.

$$\Delta q_t^j = \beta \Delta \text{cmppi}_t^j + \gamma \Delta \text{cmppi}_{t-1}^j + \alpha + \varepsilon_t^j$$

Country $j$	Estimated $\beta$	Standard Error	$R^2$
Australia	0.42***	0.11	17.1%
Canada	0.22***	0.04	34.4%
Switzerland	0.06	0.09	2.9%
Chile	0.40**	0.12	36.2%
Czech Republic	0.17	0.12	2.8%
Germany/Euro	0.13	0.08	4.1%
Denmark	0.04	0.06	0.6%
United Kingdom	0.19***	0.04	22.2%
Hungary	0.26**	0.10	11.7%
India	0.17*	0.08	5.7%
Japan	0.02	0.10	0.2%
South Korea	0.16	0.10	3.4%
Mexico	0.14**	0.05	10.0%
Malaysia	0.05	0.06	4.3%
Norway	0.15***	0.04	15.2%
New Zealand	0.33**	0.11	11.4%
Philippines	0.00	0.06	0.2%
Sweden	0.26**	0.09	11.9%
Singapore	0.08*	0.04	8.7%
Thailand	-0.01	0.09	0.1%
South Africa	0.25	0.13	5.8%

Long-run elasticities measure the long-run effect of a commodity price change on the real exchange rate. A long-run elasticity can be interpreted as measuring the degree of substitutability between currency and commodity investments for a long-term investor. The long-run effect may be larger than the short-run effect because commodity price changes could have a delayed effect on the real exchange rate. Additionally, at short (quarterly) horizons, much of the variation in real exchange rates is not related to commodity

prices, even for commodity currencies; at longer horizons, this transient noise becomes less important.

The previous literature on commodity currencies has mostly focused on measuring long-run elasticities (see, for example, Chen and Rogoff 2003, and Céspedes, Cashin, and Sahay 2004). Following the literature, I estimate long-run elasticities by regressing the log real exchange rate on the log commodity price index:

$$q_t^j = \beta^j \text{cmpi}_t^j + \alpha^j + \varepsilon_t^j$$

The specification of the regression in levels, rather than differences, is consistent with the assumption of cointegration between the two variables, which is usually maintained in the literature. The idea is that a levels specification picks up delayed effects of commodity prices on exchange rates. This approach would be problematic if both processes did have a unit root but were not cointegrated, but that does not seem consistent with Figure 2.2. The results of these regressions are displayed in Table 2.3.

**Table 2.3: Long-run elasticity of the exchange rate to commodity prices.** *I run time series regressions of the log bilateral real exchange rate  $q_t^j$  between country  $j$  and the U.S. against the log of a matched, export-weighted real commodity price index. Specifying the regression in levels (as opposed to differences, as in Tables 2.1 and 2.2) is consistent with a long-run cointegrating relationship. This approach is also more comparable with the previous literature (e.g., Chen and Rogoff 2003; Cashin, Céspedes, and Sahay 2004). The cross-sectional correlation between the estimated elasticities and CMXPTR, a measure of commodity exports, is 0.55.*

$$q_t^j = \beta \text{cmpi}_t^j + \alpha + \varepsilon_t^j$$

Country $j$	Estimated $\beta$	Standard Error	$R^2$
Australia	0.96***	0.10	56.4%
Canada	0.36***	0.02	87.8%
Switzerland	0.20***	0.03	42.2%
Chile	0.24***	0.05	50.5%
Czech Republic	0.29	0.21	2.5%
Germany/Euro	0.38***	0.07	26.7%
Denmark	0.26***	0.05	26.8%
United Kingdom	0.02	0.02	0.7%
Hungary	0.59***	0.13	20.3%
India	0.13	0.09	2.9%
Japan	-0.02	0.04	0.4%
South Korea	0.13**	0.04	12.6%
Mexico	0.07*	0.03	8.0%
Malaysia	0.06	0.04	3.1%
Norway	0.16***	0.02	38.8%
New Zealand	1.30***	0.17	43.8%
Philippines	0.23***	0.04	25.6%
Sweden	0.24***	0.06	17.6%
Singapore	0.12***	0.03	20.0%
Thailand	0.27***	0.04	35.6%
South Africa	0.32**	0.10	12.7%

Although using long-run elasticities is consistent with previous work, significant caution is required when interpreting them. As Céspedes, Cashin, and Sahay (2004) show, there are a number of cases in which the relationship between the real exchange rate and commodity prices “regime shifts” to a new level. Failing to control for that shift, which is not always in the form of an discrete jump, may bias the estimate of  $\beta^j$  relative to a reasonable value for  $\beta^j$  assuming that such a shift will not occur in the future. Rather than try to identify



these shifts — which involves some subjective judgment — I will only use these long-run elasticities for cases such as Australia in which there are no obvious jumps and there have been no major changes in monetary policy in recent history.

The general conclusion of all three tables of regressions is that commodity prices do have significant effects on commodity currencies, both in the short and long run.

### **Currency risk premia and commodity risk premia**

If commodities themselves are not particularly cyclical, or not have much of a risk premium, then it is impossible for commodity currencies to inherit that risk premium. Fortunately, a recent literature (Gorton and Rouwenhorst 2006; Gorton, Hayashi, and Rouwenhorst 2013; Yang 2013; Bhardwaj, Gorton, and Rouwenhorst 2015) finds significant excess returns to commodities, and higher returns on more cyclical commodities (Dhume 2010). I confirm high excess returns in Figure 2.3 of this paper, which is based on data from CRB.<sup>9</sup>

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<sup>9</sup>Lettau, Maggiori, and Weber (2014) use a downside-risk CAPM to price both commodities and currencies.



**Figure 2.3: A commodity's basis is a good proxy for its expected excess return.** A commodity's excess return is the percentage return on a strategy of going long the nearest contract and rolling into the next-nearest contract on the month-end before the original contract is scheduled to expire. The contract is assumed to be fully collateralized by T-bills. Basis is the component of the return earned if the commodity spot price does not move; it is knowable in advance. The text and appendix detail the construction of these variables, based on Gorton, Hayashi, and Rouwenhorst (2006).

The set of commodities is the same as in Dhume (2010), with the addition of Iron Ore 62% Fe (TSI) for delivery to Tianjin, traded on Globex. This metal has only two years of history, and is the outlier on the right of the chart; returns have been bad recently. Annualized returns are plotted.

Indeed, not only should commodities have a risk premium, but it is important that currency risk premia be consistent with commodity risk premia. For example, a currency should not have a expected excess return of 10%, if the commodities it exports have an expected excess return of 1%, and the currency-commodity elasticity is only 0.5. (That said, we need not expect a perfect relationship, because, as we saw in Tables 2.1 and 2.2, even

countries like Australia have most of their short-run currency risk coming from factors besides commodities.)

Before we can estimate the relationship between currency and commodity risk premia, however, we need a measure of commodity expected returns; I use the commodity *basis*. Many commodities are traded on futures exchanges, and the futures price being higher or lower than the current spot market price creates a natural return that would be earned in the absence of any changes in commodity prices. This “carry” on a long commodity position is called the commodity basis; Figure 2.3 shows that it is highly correlated with realized commodity returns. The appendix gives details on how basis is computed for each commodity; it is expressed as an annualized percentage.

Although basis can predict commodity returns, it is a somewhat negatively biased predictor. This bias may be due to commodity storage costs, which imply returns have to be slightly higher for the short party (the promising to deliver the commodity). I unbiased the basis values by simply adding the intercept term from a cross-sectional regression of commodity returns on basis values.

We also need estimates of commodity expected returns at the *national* level, not at the *commodity* level. I construct export-weighted basis values for each country. These are not time series indexes; they are just one value of each country, based on time-series-average export weights and time-series-average basis values for each commodity. The reason for not creating time series is that some (quite important) commodities, such as iron and coal, do not start being traded on futures exchanges until relatively recently; thus, any countries exporting these commodities would have inaccurate measures of commodity risk before those dates.<sup>10</sup>

To estimate the relationship between currency risk premia and commodity risk premia, I

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<sup>10</sup>This point is related to the interesting result of Kearns (2007). He finds that the risk premium on the Australian dollar is much higher than the risk premium on the commodities exported by Australia. While we use different data sources, one possible reason for the difference in results is that, because his paper was written originally around 2001, his commodity dataset ends in 2000, before the introduction of futures contracts on coal and iron ore (in late 2001 and 2013, respectively). These are Australia’s two biggest commodity exports, and they both have large basis values in my dataset. I get results similar to his if I exclude these commodities.

estimate the four cross-sectional regressions. In each case, I regress a proxy for expected returns on currencies on a proxy for expected returns on commodities, scaled by the currency's exposure to commodity risk. The differences between the cases lie in the different proxies used for each term.

Specifically, I estimate the following four models, with results shown in Table 2.4:

$$\begin{aligned}
 (1) \quad rxh^j &= \beta \left( \hat{\eta}^j \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j \\
 (2) \quad r^j - r &= \beta \left( \hat{\eta}^j \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j \\
 (3) \quad rxh^j &= \beta \left( CMXPTR^j \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j \\
 (4) \quad r^j - r &= \beta \left( CMXPTR^j \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j
 \end{aligned}$$

For the left-hand side variable, in regressions (1) and (3), the expected real excess return on each currency is proxied by  $rxh^j$ , the time series average. In regressions (2) and (4), the average real interest rate differential  $r^j - r$  (real carry) is used instead. For the right-hand side variable, in all regressions (1)-(4), the export-weighted average basis is used as a proxy for commodity expected returns. In equations (1) and (2), the average basis is scaled by the estimated currency-commodity elasticity  $\hat{\eta}^j$  from Table 2.2. As a robustness check, equations (3) and (4) simply use  $CMXPTR^j$ , a measure of commodity exports, in place of the estimated currency-commodity elasticity.

Ideally, we would observe a  $\beta$  of near one in regressions (1) and (2), implying that currency and commodity returns are roughly consistent. Indeed, we find estimates of 0.60 and 0.78, both of which are not significantly different from 1, but are significantly different from zero. (The coefficients estimated in regressions 3 and 4 are not particularly interpretable, because  $CMXPTR$  is normalized to be between -1 and 1.) Moreover, across the four models, about a quarter of the cross-sectional variation in average currency returns can be explained by these commodity factors.

**Table 2.4: Consistency of currency returns and commodity risk premia.** *Under this paper's theory, the average return on a commodity currency should be consistent with the average returns on the commodities it exports, adjusted for the elasticity of the currency to commodity prices.*

To check this prediction, I run four cross-sectional regressions. For the left-hand side variable, in regressions (1) and (3), the expected real excess return on each currency is proxied by  $rxh^j$ , the time series average. In regressions (2) and (4), the average real interest rate differential (carry) is used instead. For the right-hand side variable, in all regressions (1)-(4), the export-weighted average basis is used as a proxy for commodity expected returns, following from Figure 2.3. In equations (1) and (2), this commodity return measure is scaled by the estimated currency-commodity elasticity from Table 2.2. As a robustness check, equations (3) and (4) simply use  $CMXPTR^j$ , a measure of commodity exports, in place of the estimated currency-commodity elasticity, although in this case, there is no reason for the coefficient to be near one.

$$\begin{aligned}
 (1) \quad rxh^j &= \beta \left( \hat{\eta} \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j \\
 (2) \quad r^j - r &= \beta \left( \hat{\eta} \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j \\
 (3) \quad rxh^j &= \beta \left( CMXPTR^j \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j \\
 (4) \quad r^j - r &= \beta \left( CMXPTR^j \cdot cmbasis^j \right) + \alpha^j + \varepsilon^j
 \end{aligned}$$

	(1) $rxh^j$	(2) $r^j - r$	(3) $rxh^j$	(4) $r^j - r$
$\hat{\eta}^j \cdot cmbasis^j$ S.E.	0.60* (0.28)	0.78* (0.34)		
$CMXPTR^j \cdot cmbasis^j$ S.E.			0.38* (0.15)	0.40** (0.13)
$R^2$	19.9%	22.1%	26.0%	33.7%
$N$	21	21	21	21

## Labor costs

The main mechanism of my model is labor cost disease. Formally, this is an increase in *relative unit labor costs*. Unit labor costs are labor compensation divided by labor productivity; in my model, compensation rises in the non-tradable sector to make it competitive with the commodity sector, but productivity does not rise, leading to an increase in unit labor costs (compared to the commodity country).

Quarterly unit labor cost data are available from the OECD, and these can be adjusted for inflation and purchasing power parity so that they are comparable across time and countries. A key limitation of this data is that not all OECD countries are included, and, ultimately, only four of the remainder are truly commodity exporters: Australia, New Zealand, Canada, and Norway. Nevertheless, time series regressions in each of these countries supports the idea that labor costs move contemporaneously with commodity prices.

I regress 4-quarter growth in log real unit labor costs, relative to the United States, from four OECD commodity exporters (Australia, New Zealand, Canada, and Norway) on 4-quarter growth in log real commodity prices. The equation to be estimated is for each country  $j$  is:

$$\Delta \ln \left( \text{relative unit labor costs}_t^j \right) = \beta \Delta \ln \left( \text{export cmdty prices}_t^j \right) + \alpha^j + \varepsilon_t^j$$

Commodity prices are country-specific, export-weighted indices computed from CRB futures data. I use Newey-West standard errors to adjust for overlapping observations. The results are shown in Table 2.5.

Broadly speaking, we see unit labor cost elasticities of between 0.1% to 0.5% for every 1% increase in commodity prices, and these estimates are statistically significant in all four countries.

Since these changes in labor costs could simply reflect short-run factors, I also do long-run regressions in levels, in the spirit of the regressions run in Table 2.3. These regressions

**Table 2.5: In commodity countries, real unit labor costs rise with commodity prices.** *I regress 4-quarter growth in log real unit labor costs, relative to the United States, from four OECD commodity exporters (Australia, New Zealand, Canada, and Norway) on contemporaneous 4-quarter growth in log real commodity prices. Commodity prices are country-specific, export-weighted indices computed from CRB futures data. Observations are quarterly. Because the observations overlap, I use Newey-West standard errors with 3 lags. The OECD data are inflation- and PPP- adjusted for comparability across countries and time. The commodity price indices are inflation-adjusted using U.S. CPI. Observation start dates are: Australia (1991-Q1); Norway (1996-Q1); New Zealand (1991-Q1); Canada (1991-Q1). Asterisks denote the following Newey-West (3-lag) significance levels: \*:  $p \leq 0.05$ , \*\*:  $p \leq 0.01$ , \*\*\*:  $p \leq 0.001$ . Data are quarterly.*

$$\Delta \ln \left( \text{relative unit labor costs}_t^j \right) = \beta \Delta \ln \left( \text{export cmdty prices}_t^j \right) + \alpha^j + \varepsilon_t^j$$

	Time-Series			
	AU	NZ	NO	CA
$\Delta \ln \left( \text{export cmdty prices}_t^j \right)$ Newey-West S.E.	0.40*** (0.08)	0.56*** (0.12)	0.14* (0.07)	0.23*** (0.03)
$R^2$ , within-country	21.8%	27.8%	14.9%	41.9%
Num. Countries	1	1	1	1
Min $T$	95	95	75	95
Avg $T$	95	95	75	95
Max $T$	95	95	75	95

**Table 2.6: Long-run estimates of the effect of commodity prices on unit labor costs.** *I regress log real unit labor costs, relative to the United States, from four OECD commodity exporters (Australia, New Zealand, Canada, and Norway) on log real commodity prices. Commodity prices are country-specific, export-weighted indices computed from CRB futures data. Observations are quarterly. The OECD data are inflation- and PPP- adjusted for comparability across countries and time. The commodity price indices are inflation-adjusted using U.S. CPI. Observation start dates are: Australia (1990-Q1); Norway (1995-Q1); New Zealand (1990-Q1); Canada (1990-Q1). All time series go through 2014-Q3. Asterisks denote the following significance levels: \*:  $p \leq 0.05$ , \*\*:  $p \leq 0.01$ , \*\*\*:  $p \leq 0.001$ .*

$$\ln \left( \text{relative unit labor costs}_t^j \right) = \beta \ln \left( \text{export cmdty prices}_t^j \right) + \alpha^j + \varepsilon_t^j$$

	Time-Series			
	AU	NZ	NO	CA
$\ln \left( \text{export cmdty prices}_t^j \right)$	0.95***	0.89***	0.45***	0.47***
S.E.	(0.11)	(0.17)	(0.03)	(0.025)
$R^2$ , within-country	43.0%	22.2%	73.2%	77.7%
Num. Countries	1	1	1	1
Min $T$	99	99	79	99
Avg $T$	99	99	79	99
Max $T$	99	99	79	99

follow the specification

$$\ln \left( \text{relative unit labor costs}_t^j \right) = \beta \ln \left( \text{export cmdty prices}_t^j \right) + \alpha^j + \varepsilon_t^j$$

Table 2.6 displays the results, which are stronger than those run in differences, with the exception that, for New Zealand, the  $R^2$  value is lower for the levels regression than for the differences regression.

Since the OECD data are limited, I augment this analysis with a comparison of oil-exporting US states and non-oil exporting US states. An advantage of this approach is that labor cost data are collected using uniform procedures. We will only be able to study



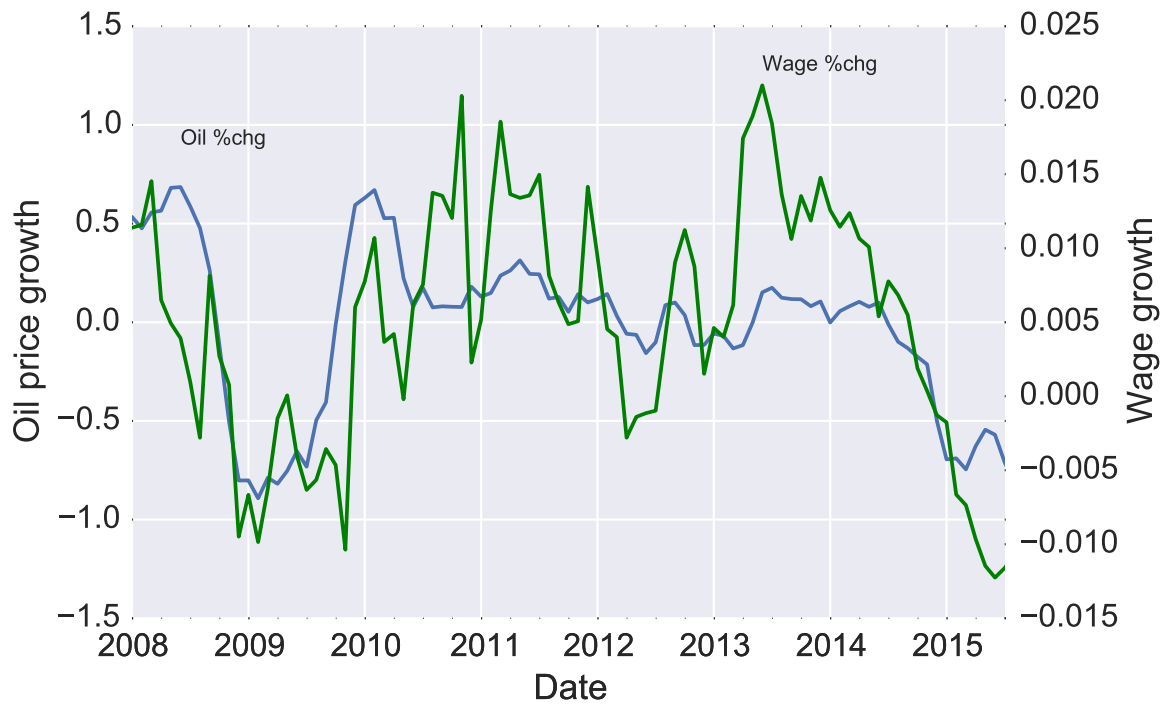
nominal wage data because CPI data are not computed on a state-by-state basis.

In recent years, hydraulic fracturing technology has transformed the U.S. into a significant oil producer. During this period, there were two instances of sharp drops in oil prices: once during the financial crisis, and once toward the end of 2014. If labor costs can be driven by commodity prices, we should observe simultaneous falls in labor costs in oil-producing states, relative to non-oil-producing states.

Assuming non-oil-producing states to be a valid control group is a conservative assumption, because people can move across state borders — particularly, to help a growing sector expand — and thus interstate labor cost differences will be less pronounced than international labor cost differences.

I take my set of oil-producing states from a report by Deutsche Bank on shale oil (Ferro 2015). The set of oil-producing states is: Texas, North Dakota, Oklahoma, Louisiana, Pennsylvania, Wyoming, New Mexico, Colorado, Arkansas, Utah, Kansas, and West Virginia.

Figure 2.4 shows that growth in nominal crude oil prices are highly correlated with growth in oil-state nominal wages.



**Figure 2.4: US wage growth in oil states is driven by oil prices.** *The two lines compare growth in oil prices over the trailing twelve months (left axis) to relative growth in nominal wages (right axis) between oil-producing and non-oil-producing states. This chart shows how commodity price changes can feed into labor costs. The oil price is the spot price of West Texas Intermediate crude.*

I regress the trailing-12-month change in log nominal wages (in oil states relative to non-oil states) on the 12-month change in log dollar oil prices. The specification is:

$$\Delta \ln \left( \frac{\text{oil-state wage}_t}{\text{non-oil state wage}_t} \right) = \beta \Delta \ln (\text{WTI Crude Price}_t) + \alpha + \varepsilon_t$$

This estimate yields an  $R^2$  of 0.32 and a Newey-West  $t$ -statistic of 4.5; details are reported in Table 2.7.

**Table 2.7: In oil-exporting U.S. states, nominal wages rise with U.S. oil prices.** *I regress the trailing 12-month change in log relative nominal wages on the 12-month change in log dollar oil prices (for WTI crude). Relative nominal wages are nominal wages in oil states divided by wages in non-oil states. Asterisks denote the following Newey-West (11 lag) significance levels: \*:  $p \leq 0.05$ , \*\*:  $p \leq 0.01$ , \*\*\*:  $p \leq 0.001$ . Data are monthly.*

$$\Delta \ln \left( \frac{\text{oil-state wage}_t}{\text{non-oil state wage}_t} \right) = \beta \Delta \ln (\text{WTI Crude Price}_t) + \alpha + \varepsilon_t$$

	(1)
$\ln (\Delta \text{ WTI Crude Price}_t)$	0.01***
Newey-West S.E.	(0.002)
$R^2$	32%
$T$	79

Finally, one might worry that labor markets are segmented; an increase in labor compensation in the tradable sector might not have much of an effect in the non-tradable sector, because labor is not perfectly, or rapidly, substitutable between sectors. While I do not explore this possibility in depth, I conduct a very simple consistency check with quarterly OECD nominal wage data. (Nominal wages are the appropriate measure, because they need to be the same across sectors to make the marginal worker indifferent.) I regress changes in log non-tradable wages on log tradable wages, using a panel of 30 OECD countries:

$$\Delta \ln \left( \text{nontradable\_wage}_t^j \right) = \beta \ln \left( \Delta \text{tradable\_wage}_t^j \right) + \nu^j + \mu_t + \varepsilon_t^j$$

A 4-quarter increase in tradable wages of 1% is associated with with a 0.8% increase during the same period in non-tradable wages. This finding suggests that the two markets have wage movements that are, at least, fairly correlated. The regression results are reported in Table 2.8.

**Table 2.8: Real wages in non-tradable professions grow with real wages in tradable professions.** Using a 30-country panel of annual OECD data, I regress annual changes in log real wages in the non-tradable sector on annual changes in log real wages in the tradable sector. I use country and time fixed effects (either separately or together) and cluster standard errors by country.

$$\Delta \ln (\text{nontradable\_wage}_t^j) = \beta \ln (\Delta \text{tradable\_wage}_t^j) + \nu^j + \mu_t + \varepsilon_t^j$$

	(1)	(2)
$\Delta \ln (\text{tradable\_wage}_t^j)$	0.85***	0.74***
Robust S.E.	(0.04)	(0.04)
Country F.E.	Y	Y
Time F.E.	N	Y
$R^2$ , within-country	58.1%	63.8%
Num. Countries	30	30
Min $T$	2	2
Avg $T$	18.9	18.9
Max $T$	44	44

### 2.3.2 Calibration

In this section, I show the results from calibrating the model's parameters to standard values for the Australia-US pair. I run simulations based on these values and compare the

theoretical moments generated by the simulation to actual means, variances, and covariances in the data. Overall, the model provides a decent fit to the data, although it cannot account for certain features, such as the Backus-Smith (1993) puzzle.

### Parameter choices

Table 2.9 shows the calibrated parameter choices, which I explain in the following.

The first set of parameters is calibrated to basic facts about the economy: I take a risk aversion of  $\gamma = 4$  and annual discount factor  $\beta = 0.97$  from Barro (2006). I use a labor share  $\alpha$  of 70% to match US data, and I calibrate  $\chi = 0.14$  and  $\phi = 0.06$  to match two key pieces of data: Australian commodity exports as a share of total exports (52%) and Australian exports as a fraction of GDP (20%); these data points are averages between 1995 and 2014.

The second set of parameters governs the stochastic processes in the model, namely, productivity in the US and in Australia. These are calibrated to match the consumption data from Barro (2006) and Barro and Ursúa (2013). Specifically, I take the probability of a disaster ( $p = 1.7\%$ ) and the average loss in consumption conditional on disaster (29%, implying  $B = 0.71$ ) from Barro (2006). I then compute consumption growth in both countries using the Barro and Ursúa dataset on an annual basis from 1995 - 2009, the end of that dataset. This is a period without disasters for both countries, according to their definition, which puts a lower bound on a disaster of a 10% fall in aggregate consumption. I use these data to calibrate  $\sigma_u$ ,  $\sigma_h$ , and  $\rho_{uh}$ , which govern productivity shocks in the absence of disasters. I set the productivity persistence parameter  $a$  to 0.87 to provide a 5-year half-life on the real exchange rate, as suggested by the literature on purchasing power parity.

Table 2.10 shows that the chosen parameters replicate the microeconomic data to which the model is calibrated.

**Table 2.9:** Calibration parameters. Most of the choices are directly from Barro (2006), or in the case of the shocks, matched to the consumption data in Barro and Ursúa (2013). The taste parameters are chosen to match Australian export data.

*Economy*

Risk aversion $\gamma$	4
Labor share $\alpha$	0.7
Discount factor $\beta$	0.97
Taste for finished goods $\chi$	0.14
Taste for the alternative good $\phi$	0.06

*Shocks*

Productivity AR(1) $a$	0.87
Disaster probability $d_p$	1.7%
Shock volatility $\sigma_u$	0.18
Shock volatility $\sigma_h$	0.21
Shock correlation $\rho_{uh}$	-0.2
Disaster factor $B$	0.70

**Table 2.10:** *Economic statistics used for calibration. The model parameters are determined by certain microeconomic facts, like the volatility of consumption growth and the fall in consumption upon disaster measured in Barro (2006) and Barro and Ursúa (2013). The model replicates these statistics by construction.*

Statistic	Model	Data
AU Commodity exports / Gross exports	0.51	0.52
AU Gross exports / GDP	0.22	0.20
US consumption growth volatility	1.6%	1.6%
AU consumption growth volatility	1.3%	1.3%
US-AU consumption growth correlation	0.74	0.73
Disaster probability	1.7%	1.7%
Consumption drop on disaster	29%	29%

### Key predictions

First, I discuss the unconditional real risk premium on the commodity currency. The risk premium is the sum of two parts: the average real interest rate differential  $r_t^* - r_t$ , and the average real appreciation of the Australian dollar,  $\Delta q_{t+1}$ . The data are averages taken over the past 20 years (that is, starting in 1995-Q1), using data sources described in the appendix. Since the data moments are estimated, 95% confidence intervals around the estimates are also displayed. These means are unconditional means.

**Table 2.11:** Means of variables

Mean of...	Model	Data (95% CI)
$r_t^* - r_t$	1.6%	1.8% $\pm$ 0.4%
$\Delta q_{t+1}$	0.0%	1.2% $\pm$ 2.8%
$r_t^* - r_t + \Delta q_{t+1}$	1.6%	2.8% $\pm$ 2.8%

The model succeeds in matching a large real interest rate spread. The fact that average currency appreciation is zero in the model is actually a strong point: while the Australian dollar has appreciated over the last two decades, the appreciation is not statistically significant, and indeed exchange rate movements seem to be effectively unforecastable in the short run (Rogoff and Stavrakeva 2008).

Turning to the dynamics of the real exchange rate, we find that the model-implied volatility of the real exchange rate during a period of no disasters, 4.6%, is comparable to the observed volatility of the component of the real exchange rate explained by commodity prices, 5.0%. Isolating this component is important because there is significant, non-commodity-related variation in the real exchange rate in the short run. It would be strange if the model were able to explain this other variation, too, since the model only has the commodity channel for moving the exchange rate.



**Table 2.12:** *Variances and correlations*

Moment	Model	Data (95% CI)
$\sigma(\Delta q_t)$	4.6%	5.0% (4.2%, 5.8%)
$\rho(\Delta q_t, \Delta c_t - \Delta c_t^*)$	1.00	0.34 (-0.23, 0.73)
$\rho(\Delta q_t, \Delta c_t)$	0.64	0.24 (-0.32, 0.68)
$\rho(\Delta q_t, \Delta c_t^*)$	-0.04	-0.03 (-0.50, 0.54)

The second row in the above table, also computed for a no-disaster period, shows the Backus-Smith (1993) puzzle. In models with complete markets and standard, time-separable CRRA preferences, an increase in the real exchange rate is only possible through faster growth in US consumption than Australian consumption. Consequently, simulated movements in the real exchange rate are perfectly correlated with the difference in consumption growth rates, while the correlation is much less in the data.

Ideas for fixing the Backus-Smith puzzle in the context of complete markets generally revolve around modifying the household utility function so that the marginal utility of a dollar — which determines the relative value of currency — is determined not only by current consumption, but by additional variables. Colacito and Croce (2011, 2013) suggest a solution for the Backus-Smith puzzle based on the idea of Epstein-Zin preferences and long-run consumption risks. This paper fits into a broader literature on international long-run risks models that includes, among others, Bansal and Shaliastovich (2012), Backus, Telmer, Gavazzoni, and Zin (2013), and Colacito, Croce, Gavazzoni, and Ready (2015). Stathopoulos (2012) takes a habit-formation approach to the Backus-Smith puzzle, in the spirit of Campbell and Cochrane (1999) and Verdelhan (2010).

The third and fourth rows in the above table show correlations between changes in the real exchange rate and consumption growth in each country. The data are not significantly different from either model prediction, although the first correlation is admittedly on the

**Table 2.13:** *Elasticities*

Elasticity of...	Model	Data (95% CI)
$\Delta q_{t+1}$ to $\Delta p_{C,t+1}$	0.88	0.96 $\pm$ 0.20 (Long-run) 0.42 $\pm$ 0.22 (Short-run)
$\Delta r_{ulc,t+1}$ to $\Delta p_{C,t+1}$	0.82	0.95 $\pm$ 0.22 (Long-run) 0.40 $\pm$ 0.16 (Short-run)

high end. The main surprise here is simply that the model's exchange rate is not extremely correlated with consumption growth in both countries, since that would help get a high risk premium on the real exchange rate. It turns out that the correlation between the exchange rate and consumption growth need be high only during disasters, during which everything crashes; the rest of the time, supply shocks can be frequent enough that commodity prices often fall during global growth, which is important for matching the data.

Previously, we estimated two key elasticities: the elasticity of exchange rates to commodity prices, and the elasticity of labor costs to commodity prices. When it comes to the exchange rate elasticity, it is appropriate to compare the model results to the long-run elasticity, since, as discussed previously, the short-run figure does not account for delayed effects of changes in commodity prices that occur instantaneously and frictionlessly in the model. By this standard, the model does fairly well. The unit labor cost elasticity to commodity prices is also close to that observed in the data.

95% confidence intervals are given around the regression estimates. The finding here suggests that we do not need to assume too large movements in labor costs in order to explain the commodity currency puzzle.

## 2.4 Monetary model

This section develops a sticky-wage New Keynesian extension for the model we have seen so far, in the style of Erceg, Henderson, and Levin (2000). A monetary model helps us answer three separate questions:

- Real-life currency trades are nominal; they involve nominal interest rates and exchange rates. How does the nominal exchange rate move compared to the real exchange rate?
- Labor costs cannot adjust rapidly through nominal wages if nominal wages are sticky. Instead, the nominal exchange rate (which may be flexible) needs to adjust. Can nominal exchange rates adjust “enough” to replicate the flexible-price allocation?
- Central banks often respond to terms-of-trade shocks by adjusting their nominal exchange rate. With nominal rigidities, the size of this response will affect the covariance between the real exchange rate and commodity prices. What effect does this have on the real currency risk premium?

I study two regimes: floating and fixed nominal exchange rates, the latter of which I refer to as a currency peg. With a floating rate, the central bank tries to replicate the flexible-price allocation.

The main results are as follows. First, under a peg, the nominal exchange rate does not move, so the flexible-price allocation cannot be replicated. The real exchange rate moves less than in the flexible-price allocation, leading to a smaller real exchange rate risk premium. Output gaps are common.

Second, under floating rates, the flexible-price allocation can be replicated, and the results are identical to those in the purely real model considered previously, up to a small distortion from monopolistic competition. The nominal exchange rate moves more than the real exchange rate in order to achieve this outcome.

This model focuses on wage stickiness because labor costs are the main mechanism studied in this paper. The model could be extended, as in Erceg, Henderson, and Levin (2000), to include price stickiness as well, by allowing monopolistic competition between firms. Since I do not use price stickiness, the model also does not directly take a stance on producer- versus consumer-currency pricing, as described in, for example, Betts and Devereux (2000), Devereux and Engel (2003), Gopinath, Itskhoki, and Rigobon (2007), or Gopinath, Gourinchas, Hsieh, and Li (2011).

### 2.4.1 Key notational differences

I make the following substitutions relative to the non-monetary model:

- $S_t$  means the nominal exchange rate (defined below).
- $\phi$  means the Frisch elasticity of labor supply.

### 2.4.2 Numeraires

In the non-monetary model, all prices were expressed in terms of the consumption bundle; according to this numeraire, the CPI was 1 by definition. Now, all prices are nominal, relative to some notional unit of account (the US or Australian dollar). The consumer price indexes in the US and Australia are then  $P_t$  and  $P_t^*$ , respectively.

Let  $S_t$  be the nominal exchange rate between the USD and the AUD (up implies appreciation of the AUD); this is the rate of substitution between the two different units of account.

Let  $Q_t$  be the real exchange rate, or the value of the Australian consumption basket in terms of the US basket:

$$Q_t \equiv \frac{S_t P_t^*}{P_t} \quad (2.4)$$

The terms of trade  $\Theta_t$  is the price of exports in terms of imports (from the point of view of Australia, so it moves in the same direction as the exchange rate):

$$\Theta_t \equiv \frac{P_{C,t}}{P_{F,t}} \quad (2.5)$$

This relative price was used implicitly in the non-monetary model, but we now give it a symbol to simplify our notation.

### 2.4.3 Risk Sharing

We continue with the assumption of complete markets. For simplicity, we solve the log utility case, which, given the rest of the setup and no net initial financial claims, implies

balanced trade in each period (Cole and Obstfeld 1991). Consequently, neither country accumulates financial claims against the other.

#### 2.4.4 Australia

##### Australian households

If firms could use the labor of each household interchangeably, it would be difficult for wages to be sticky. Households would have to constantly revise their wages, or else face complete unemployment when their wage happened to be higher than the lowest wage offered on the market. Consequently, we introduce a continuum of households in Australia, indexed by  $j$ , and the labor of these households will be imperfectly substitutable from the perspective of firms.

Household  $j$  has log utility over consumption and power utility in labor hours:

$$U_t^*(j) = E_t \sum_{k=0}^{\infty} \beta^k \left( \ln C_{t+k}^*(j) - \frac{L_{t+k}^*(j)^{1+\phi}}{1+\phi} \right) \quad (2.6)$$

Because markets are complete domestically, and every household faces the same prices, consumption will be the same for all households. This statement is true even though households may offer different wages and, by extension, face different levels of labor demanded. We denote this common level of consumption  $C_{t+k}^*$ :

$$C_{t+k}^*(j) = C_{t+k}^* \quad \forall j \quad (2.7)$$

As before, for household  $j$ , consumption is a Cobb-Douglas aggregate of tradable and non-tradable goods:

$$C_t^*(j) \equiv \kappa C_{F,t}^*(j)^\chi C_{NT,t}^*(j)^{1-\chi} \quad (2.8)$$

where  $0 < \chi < 1$  and  $C_{F,t}^*(j)$  and  $C_{NT,t}^*(j)$  are the consumption levels of the finished and non-tradable goods, and  $\kappa \equiv \chi^{-\chi} (1 - \chi)^{-(1-\chi)}$  is a normalizing factor. Again, because of complete markets and the homotheticity of the consumption aggregator, the household-level

consumption of each good equals aggregate national consumption of that good:

$$C_{F,t}^*(j) = C_{F,t}^* \quad \forall j \quad (2.9)$$

$$C_{NT,t}^*(j) = C_{NT,t}^* \quad \forall j \quad (2.10)$$

The consumer price index (CPI) based on the optimal consumption bundle is:

$$P_t^* \equiv P_{F,t}^{*\chi} P_{NT,t}^{*(1-\chi)} \quad (2.11)$$

where  $P_{F,t}^*$  and  $P_{NT,t}^*$  are the prices of the finished and non-tradable goods in Australian dollars.

Household  $j$ 's budget constraint is:

$$C_t^*(j) P_t^* = W_t(j)^* L_t(j)^* + D_t(j)^* \quad (2.12)$$

where  $W_t(j)^*$  is the nominal wage and  $L_t(j)^*$  is labor supply, and  $D_t(j)^*$  is the (stochastic) income assigned to household  $j$  under the complete-markets allocation.

### Australian production

In Australia, there are two sectors of production; in the commodity-exporting sector,  $Y_{C,t}^*$  units are produced, and in the non-tradable sector,  $Y_{NT,t}^*$  units are produced. To follow the non-monetary model as closely as possible, I make total output linear in the "aggregate labor" hired by each firm, but those aggregates are computed with an aggregator function that makes labor from different households imperfect substitutes:

$$Y_{C,t}^* = \underbrace{\left( \int L_{C,t}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}}_{L_{C,t}^*} \quad (2.13)$$

$$Y_{NT,t}^* = \underbrace{\left( \int L_{NT,t}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}}_{L_{NT,t}^*} \quad (2.14)$$

where  $L_{C,t}^*(j)$  and  $L_{NT,t}^*(j)$  are the labor supplies from each household  $j$  to the two industries, substituted into the production function. For simplicity, I omit the local supply shock,  $Z^*$ ,

which was in the non-monetary model.

We introduce a nominal wage aggregate:

$$W_t^* \equiv \left( \int W_t^*(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (2.15)$$

Dixit-Stiglitz demand functions give us labor demand for each household in terms of that household's relative wage. The higher the relative wage of household  $j$ , the lower the labor demanded as a fraction of total labor hired by the firm:

$$L_{C,t}^*(j) = \left( \frac{W_t^*(j)}{W_t^*} \right)^{-\varepsilon} L_{C,t}^* \quad (2.16)$$

$$L_{NT,t}^*(j) = \left( \frac{W_t^*(j)}{W_t^*} \right)^{-\varepsilon} L_{NT,t}^* \quad (2.17)$$

Define "aggregate output" or real GDP as:

$$Y_t^* \equiv \kappa Y_{C,t}^{*\chi} Y_{NT,t}^{*1-\chi} \quad (2.18)$$

Real GDP is the result of deflating nominal GDP by the GDP deflator, which is defined as:

$$G_t^* \equiv P_{C,t}^{*\chi} P_{NT,t}^{*1-\chi} \quad (2.19)$$

The distinction between the CPI (consumer prices) and the GDP deflator (producer prices) will be much more important in this model than in the previous one. The GDP deflator measures the extent to which Australian households are resetting their wages and making locally produced goods more expensive, while the CPI also measures fluctuations in import prices.

### Optimal wage setting

In each period, a randomly selected fraction  $1 - \omega$  of households can reset its wage. The wage-resetting problem is, in principle, the same as the household optimization problem stated above with an extra control variable, the reset wage  $W_r^*$ . However, the problem can be simplified greatly by noting that any terms in the household utility function not affected

by the choice of the reset wage may be dropped. We are left with the objective function:

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\omega)^k u(C_{t+k}^*(j), L_{t+k}^*(j)) \right\} \quad (2.20)$$

where the function  $u(\cdot)$  is the period utility function, which is now maximized with respect to the budget constraint (2.12) with the reset wage substituted in:

$$W_r^* L_{t+k}^*(j) + D_{t+k}^*(j) = P_{t+k}^* C_{t+k}^*(j) \quad (2.21)$$

A second simplification in this problem is to write the household's labor supply  $L_{t+k}^*(j) \equiv L_{C,t+k}^*(j) + L_{NT,t+k}^*(j)$  as a function of the reset wage  $W_r^*$ . Because it is not optimal to set the reset wage so low that there is excess labor demand, the household engineers labor supply to exactly equal labor demand under the selected reset wage. The appendix shows that this condition can be written as:

$$L_{t+k}^*(j) = \left( \frac{W_r^*}{W_{t+k}^*} \right)^{-\varepsilon} L_{t+k}^*$$

The full problem is solved in the appendix. Intuitively, households set their wage to a markup over a weighted average of projected marginal costs during the period over which they are not able to reset their wage.

### The natural rate of output

In this type of model, it is useful to compare the true rate of output with its natural rate, which would prevail under flexible prices. The "output gap" between the two may be a target of monetary policy. The appendix shows that the natural rate of log output in Australia is:

$$y_{n,t}^* = -\frac{1}{1+\phi}\mu \quad (2.22)$$

where  $\mu$  is the log of the steady-state wage markup over marginal cost. Intuitively, the household would normally work for 1 hour, as in the non-monetary model, but it is trying to earn a monopolistic rent by working slightly less. The bigger the markup it can charge,



the less it works.

### AS curve

Intuitively, a positive output gap occurs when households are working too much; they would like to reset their wages upward. Those who are able to raise their wages in period  $t$  also raise the costs of Australian firms, which respond by raising prices. These higher prices are measured as an increase in the GDP deflator,  $\pi_t^{g*}$ .

To make this notion precise, we combine the solution to the wage-resetting problem with the definition of the natural rate of output. We can show (see appendix) that time- $t$  GDP deflator inflation depends on future expected inflation and the current output gap:

$$\pi_t^{g*} = \beta E_t (\pi_{t+1}^{g*}) + \zeta (y_t^* - y_n^*) \quad (2.23)$$

in which:

$$\zeta \equiv (1 + \phi) \lambda \quad (2.24)$$

$$= (1 + \phi) \frac{(1 - \omega)(1 - \beta\omega)}{\omega(1 + \phi\varepsilon)} \quad (2.25)$$

The coefficient  $\zeta$  measures the pass-through from the output gap to current inflation, assuming future expected inflation is held constant.

### AD curve

The model's AD curve comes from the regular Euler (IS) condition:

$$c_t^* = E_t (c_{t+1}^*) - (i_t^* - E_t \pi_t^* - \rho - f^*(\mathbf{z}_t)) \quad (2.26)$$

where  $\pi_t^*$  is CPI inflation. The unusual term  $f^*(\mathbf{z}_t)$  is simply the precautionary saving term that is generally dropped when log-linearizing the Euler condition. We need to keep this term since it is the source of the currency risk premium. The argument of the  $f^*(\cdot)$  function,  $\mathbf{z}_t$ , is a vector representing the current state of the economy; the state includes both the current level of productivity  $z_t$  and the wage distribution at time  $t$ .

Assuming either a peg or a flexible exchange rate, is possible to show that  $f^*(\mathbf{z}_t)$  is constant, so the inclusion of this term does not create an obstacle to solving the model as a system of log-linear equations. This analytical tractability, however, is simply a convenient result of assuming log utility over consumption; in general, one would have to run simulations or use higher-order approximations. Moreover, we are still retaining the standard log-linearizations in deriving the Phillips curve, so although we have removed the approximation error in computing the currency risk premium from consumption processes, the consumption processes themselves are subject to the standard approximation error in this type of model.

By using relationships between consumption and output and between the CPI and the GDP deflator, we can transform this equation into:

$$y_t^* - y_n^* = E_t(y_{t+1}^* - y_n^*) - (i_t^* - E_t\pi_{t+1}^{\delta^*} - \rho - f^*(\mathbf{z}_t)) \quad (2.27)$$

which is our AD curve. The details are shown in the appendix. Note that  $\rho + f^*(\mathbf{z}_t)$  is the “natural rate of interest” here.

#### 2.4.5 America

America is different from Australia in two major ways:

- Wages are flexible. This assumption just simplifies things without taking away any of the intuition.
- As in the real model, the production technology is different; commodity imports are used to produce the finished good, and both non-tradable and finished good output may be subject to TFP shocks.

### American households

As in Australia, the American household's utility is Cobb-Douglas with finished good share  $\chi$ :

$$C_t \equiv \kappa C_{F,t}^\chi C_{NT,t}^{(1-\chi)} \quad (2.28)$$

the CPI  $P_t$  is defined analogously.

The representative household's utility function is:

$$U_t = E_t \sum_{k=0}^{\infty} \beta^k \left( \ln C_{t+k}^* - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right) \quad (2.29)$$

The American budget constraint is

$$C_t P_t = W_t N_t \quad (2.30)$$

### American production

The U.S. has two sectors, a non-tradable sector and a finished-goods-producing sector:

$$Y_{F,t} = Z_t L_{F,t}^\alpha Y_{C,t}^{*1-\alpha} \quad (2.31)$$

$$Y_{NT,t} = Z_t L_{NT,t} \quad (2.32)$$

where  $Z_t$  is total factor productivity,  $L_{F,t}$  the labor used in finished goods production, and  $L_{NT,t}$  is the labor used in the non-tradable sector.

Output is defined *using real value added*:

$$Y_t \equiv \gamma (Y_{F,t} - \Theta_t Y_{C,t}^*)^\chi Y_{NT,t}^{1-\chi} \quad (2.33)$$

And the GDP deflator is:

$$G_t \equiv P_{F,t}^\chi P_{NT,t}^{1-\chi} \quad (2.34)$$

## Terms of trade

As shown in the appendix, under the production functions defined above, the log terms of trade follows:

$$\theta_t = z_t - \alpha (y_t^* - y_n^*) + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu \quad (2.35)$$

Intuitively, commodity prices rise with productivity  $z_t$  as in the non-monetary model. If there is excess output in Australia (positive output gap), this increase in commodity prices will be muted, because there will be a greater supply of commodities on the market.

### 2.4.6 Summary of model

Putting the AS, AD, and terms-of-trade equations together, we get the following system, reminiscent of the baseline New Keynesian model (e.g., among others, Yun 1996; Clarida, Galí, and Gertler 1999):

$$y_t^* - y_n^* = E_t(y_{t+1}^* - y_n^*) - (i_t^* - E_t\pi_{t+1}^{\delta*} - \rho - f^*(z_t)) \quad (2.36)$$

$$\pi_t^{\delta*} = \beta E_t\pi_{t+1}^{\delta*} + \zeta (y_t^* - y_n^*) \quad (2.37)$$

$$\theta_t = z_t - \alpha (y_t^* - y_n^*) + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu \quad (2.38)$$

### 2.4.7 Monetary policy in the US

In the US, the central bank targets GDP deflator (equivalently, CPI) inflation to zero. It uses the well-known Taylor rule

$$i_t = \rho + E_t\Delta c_{t+1} + f(z_t) + \psi^{US} \pi_t^{\delta} \quad (2.39)$$

with  $\psi^{US} > 1$  and  $f(z_t)$  being the US analogue of the term  $f^*(z_t)$ .

This specification of US monetary policy rules out coordination with Australia, which seems realistic. Monetary policy in the major, non-commodity-producing nations does not seem to be run with an eye to stability in commodity exporters.

### 2.4.8 Results under a floating exchange rate

If Australia wants to float its exchange rate, we will assume that the central bank tries to replicate the flexible-price allocation. In this type of model, such an outcome is usually achieved by targeting (setting to zero) the rate of output price inflation,  $\pi_t^{\delta*}$ , with a Taylor rule. Intuitively, output price inflation would occur only if wage costs are rising, and that would only happen if labor demand were above its flexible-price level. Consequently, the central bank adopts the Taylor rule:

$$i_t^* = \rho + f^*(z_t) + \psi \pi_t^{\delta*} \quad (2.40)$$

For a more complete discussion of optimal policy rules in the open economy, and, in particular, a discussion of the right inflation metric to target, see Frankel and Chinn (1995), Huang and Liu (2005), or Anand, Prasad, and Zhang (2015).

Substituting this rule into the system (2.36), (2.37), (2.38) immediately shows us that this policy rule achieves an output gap of zero, with zero GDP deflator inflation. The appendix shows that the ‘‘Taylor principle,’’ or  $\psi > 1$ , is required to rule out other equilibria.

This policy results in a simple, unit-elastic relationship for the nominal exchange rate:

$$s_t = z_t + \text{const.} \quad (2.41)$$

As before, the real exchange rate has an elasticity to productivity less than 1:

$$q_t = (1 - \chi) z_t + \text{const.} \quad (2.42)$$

### 2.4.9 Results under a peg

By definition, the nominal exchange rate is fixed at a level  $\bar{S}$ . The appendix shows that the real exchange rate is, in logs:

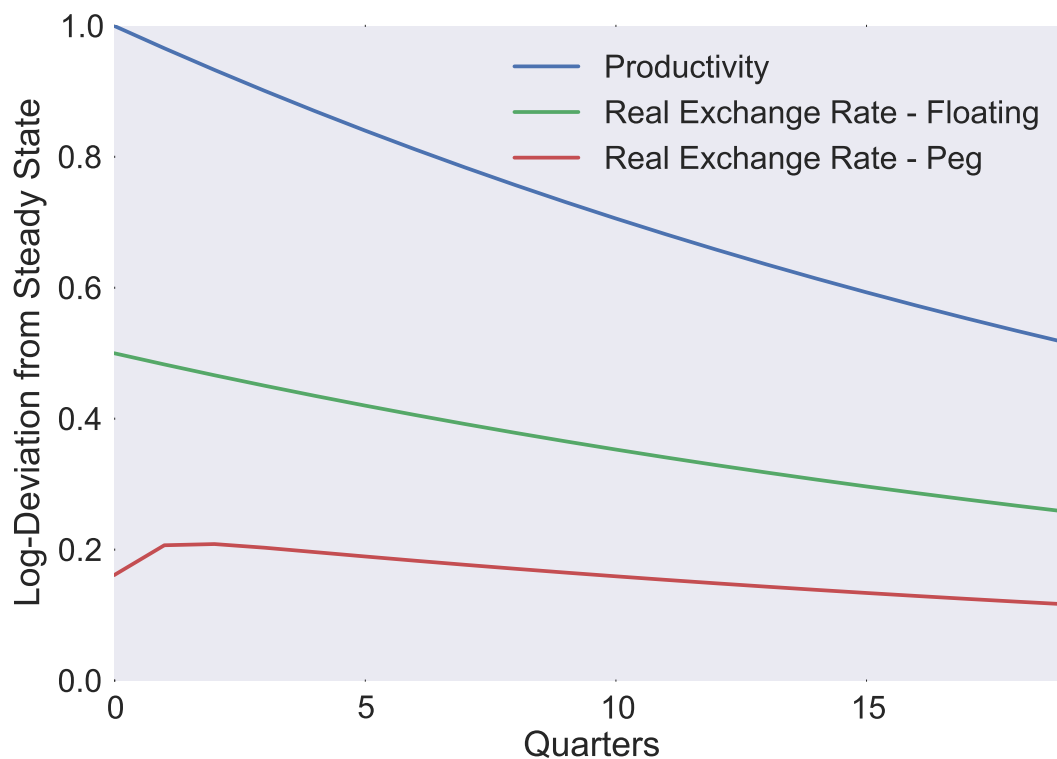
$$q_t = \underbrace{(1 - \chi) z_t}_{\text{flexible-price solution}} - \underbrace{(1 - \chi) (\alpha + (1 - \chi)) \left( (y_t^* - y_n^*) - \frac{\mu}{1 + \phi} \right)}_{\text{sticky-wage effect}} \quad (2.43)$$

There are two components to the real exchange rate: the  $1 - \chi$  elasticity derived in the floating-rate solution, and an additional term increasing in the output gap. Intuitively, an increase in  $z_t$  under floating rates leads to an appreciation of the real exchange rate. But under a currency peg, the real exchange rate can't rise quickly, since neither price levels nor the nominal exchange rate are flexible. Instead, labor costs remain temporarily too low, causing excess demand (positive output gap  $y_t^* - y_n^*$ ). The size of this effect on the real exchange rate is then proxied by a negative multiple of the output gap.

The real exchange rate can be fully solved out in terms of  $z_t$ , and the derivation is shown in the appendix. The resulting elasticity of the real exchange rate with respect to productivity is:

$$\frac{dq_t}{dz_t} = \underbrace{(1 - \chi)}_{\text{flexible-price elasticity}} - \underbrace{(1 - \chi) (\alpha + (1 - \chi)) \left( \frac{\eta (1 - a\beta)}{1 - a\beta + \zeta\eta} \right)}_{\text{sticky-wage effect}}$$

A full, numerical computation of the impulse response functions (again, derived in the appendix) shows that for a typical value of  $\chi = 0.5$ , the flexible-price elasticity of the exchange rate to the productivity shock is 50% on impact, while the sticky-price elasticity is only 16%. The full impulse response functions are plotted in Figure 2.5.



**Figure 2.5: Fixed vs. floating: the elasticity of the exchange rate to productivity shocks.** *This graph shows the elasticity of the real exchange rate to productivity shocks under a calibrated sticky wage model. Under a currency peg, the real exchange rate moves much less with productivity shocks (and, by extension, commodity prices).*

## 2.5 Conclusions

The tight covariance between commodity prices and commodity currencies leads to a risk premium for commodity currencies. This paper has presented a simple model of the currency-commodity covariance. Labor cost disease implies that booms in commodity prices raise the cost of non-tradables, and thus raise real exchange rates at the same time. When calibrated to reasonable parameter values, this model can match currency and commodity asset price data with realistic movements in labor costs.

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## Chapter 3

# Monetary Policy Under Terms-of-Trade Shocks<sup>1</sup>

### 3.1 Introduction

Inflation targeting is the preferred policy of many central banks today. Nevertheless, economists recognize the need for a certain amount of flexibility in meeting these targets on a strict annual basis. This paper investigates a common situation in which such flexibility is useful: a small open economy facing a terms-of-trade shock.

We argue that a deterioration in a country's dollar export prices should be met by above-target consumer price inflation. Such a policy appropriately reduces the *real* value of home-produced goods without the need to adjust local-currency output prices or wages, either of which may be sticky. Moreover, this policy softens the sharp reduction in imports required under a realistic constraint on external financing.

While the idea that a country should depreciate on a terms-of-trade shock is often discussed, our goal is to demonstrate this result with a fully microfounded model that is nevertheless simple to solve and illustrate graphically. We particularly differ from most international DSGE models with nominal rigidities in that we impose external financing

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<sup>1</sup>Co-authored with Jeffrey Frankel

constraints on households, so their ability to smooth terms-of-trade shocks is limited.

Although the standard literature on international DSGE models with nominal rigidities does indeed show that countries should depreciate upon a terms-of-trade shock, they do so for “internal balance” reasons. In other words, while it is necessary to inflate to reduce real output prices and wages, and thus support demand, the effects of the shock on a country’s consumption are small. The reason is either that markets are complete (as in Gali and Monacelli 2005), so the country is insured against the shock, or that borrowing is possible (as in Obstfeld and Rogoff 1995) so the shock can be smoothed adequately. In our model with financial constraints, neither of these options is possible.

While there is also a literature that studies external financing constraints (sometimes in the context of terms-of-trade shocks), these constraints are usually imposed on firms, as in Cespedes, Chang, and Velasco (2004), Devereux, Lane, and Xu (2006), Gertler, Gilchrist, and Natalucci (2007), and Cespedes and Velasco (2012). In this paper, we apply borrowing constraints directly to households, both for simplicity and also because we think household borrowing constraints are also important for welfare. In this approach, we follow Lorenzoni (2014).

This external financing literature also tends to focus on different issues than the one considered in this paper. Some of these papers analyze the balance sheet effects of depreciation, because liabilities and assets may not be denominated in the same currency. Others calibrate fairly complex, but more quantitatively reasonable models of the economy for empirical purposes. Here, we try to focus on a relatively simple mechanism and derive results using analytic approximations.

While external borrowing constraints provide a stronger motive for currency depreciation in response to an adverse terms-of-trade shock, this is not necessarily true for *internal* borrowing constraints (constraints on some households but not others within the same country). Anand, Prasad, and Zhang (2015) show that *not* depreciating can preserve high real wages for borrowing-constrained workers just when their borrowing constraint might otherwise bind.



This paper also builds on the literature on nominal GDP targeting, export price targeting, and product price targeting in countries subject to terms-of-trade shocks. Frankel (2011) provides a summary, and Frankel and Bhandari (2014) provide an analysis similar to the one in this paper, but under an IS-LM-type of setting.

## 3.2 Model

### 3.2.1 Setup

We model a small open economy whose currency is the *peso*. This country hosts a unit measure of households, indexed by  $j \in [0, 1]$ . There are two periods: the “short run” (period  $t$ ) in which a randomly selected fraction  $\omega$  of households may reset their wages, and a “long run” (period  $t + 1$ ) in which all households may reset their wages. Wages can be different for different households because each household’s labor is an imperfect substitute for the labor of another household.

Consumption in this economy consists of finished goods imported from abroad and locally produced nontradables, combined in a Cobb-Douglas consumption aggregate:

$$C_t = \gamma C_{F,t}^\chi C_{NT,t}^{1-\chi}$$

The outside world uses the *dollar* and finished goods always cost one dollar. Thus, the nominal exchange rate  $S$ , expressed as the value of the dollar in terms of pesos, also gives the peso price of the finished good. The price index is therefore

$$P_t = S_t^\chi P_{NT,t}^{1-\chi}$$

where  $P_{NT,t}$  is the peso price of the nontradable good. These Cobb-Douglas relationships hold for period  $t + 1$  as well.

The country exports a commodity to pay for its imports. The ratio of the dollar commodity price to the dollar finished good price is simply  $P_C^*$ , and is the terms of trade.

The only asset available for external financing is dollar bonds, traded at an exogenous

gross nominal interest rate of  $I^*$ .

### 3.2.2 Household's Problem

Each household  $j$  has a standard power utility function that is both time-separable and separable in consumption and hours worked:

$$U = \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t(j)^{1+\phi}}{1+\phi} \right) + \beta \left( \frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \frac{L_{t+1}^{1+\phi}}{1+\phi} \right)$$

where  $L_t$  is hours worked. We will be studying the effects of an unexpected terms-of-trade shock occurring right at the beginning of the short run; after that shock and after the household finds out whether it may reset its wage, all endogenous variables feeding into the utility function are deterministic. Consequently, we do not need to use an expectation operator in this expression.

The household's budget constraint in the long run equates peso expenditure with peso earnings:

$$P_{t+1}C_{t+1} + S_{t+1}D_t I_t^* = W_{t+1}(1 + \tau)L_{t+1} + S_{t+1}A^s I_t^* + T_{t+1}$$

$D_t$  is *dollar* debt issued at time  $t$ .  $\tau$  is a wage subsidy paid by the government in order to induce the efficient level of labor, equal to the long-run equilibrium wage markup that occurs given the structure of competition in the labor market.  $T_{t+1}$  is the lump-sum tax used to pay for this subsidy.  $A^s$  is the level of dollar assets held by the government (say reserves) in steady state (before the beginning of the model); we will assume these assets are liquidated at the end of the model.

In the short run, the household's budget constraint is:

$$P_t C_t + S_t D^s I^{*s} = W_t(j)(1 + \tau)L_t(j) + S_t D_t + T_t(j) + S_t A^s (I_t^* - 1)$$

Here,  $D^s$  represents the steady-state level of dollar debt assumed to be equal to the steady-state level of dollar assets (so that there is no need to run an external deficit or surplus in the steady state).  $I^{*s}$  is the exogenous dollar interest rate in steady state that is inherited on

the existing steady state debt. The final term is the interest earned on the reserves.

Consumption is the same for all households because markets are assumed to be complete internally, while they are constrained externally. The external constraint is that the household may not take on any more debt at time  $t$  than it held in steady state:

$$D_t \leq D^s$$

While it may seem more realistic for this constraint to apply only in the case of a negative terms-of-trade shock, we will see that the end result is exactly the same, since the constraint will not bind under a positive terms-of-trade shock.

For those households who may reset their wage in the short run, the reset wage  $W_r$  is an additional choice variable in the household's problem. (This value is the same for all households who reset, so there is no need to index by  $j$ ). The reset wage enters the problem in two ways: first, it enters the budget constraint directly, and second, it enters the labor demanded by firms for that household's variety of labor.

### 3.3 Production

The economy produces two types of goods: commodities, which it exports for finished goods and does not consume, and non-tradables, which it does consume. The production functions for these goods are standard CES aggregators of all the different types of labor:

$$Y_{C,t} = \underbrace{\left( \int_0^1 L_{C,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}}_{L_{C,t}}$$

$$Y_{NT,t} = \underbrace{\left( \int_0^1 L_{NT,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}}_{L_{NT,t}}$$

With these production functions, it can be shown that demand for household  $j$ 's labor will be:

$$L_t(j) = \left( \frac{W_r}{W_t} \right)^{-\epsilon} L_t$$

where

$$\begin{aligned}
 L_t(j) &= L_{C,t}(j) + L_{NT,t}(j) \\
 L_t &= \left( \int L_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
 W_t &= \left( \int W_t(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}
 \end{aligned}$$

It can also be shown that the two goods will have the same price, since they have the same marginal cost of  $W_t$ . Thus we have that  $P_C = P_{NT} = W_t$ .

We will also define the notion of nominal GDP:

$$G_t Y_t = P_{C,t} Y_{C,t} + P_{NT,t} Y_{NT,t}$$

We decompose this into  $G_t$ , the GDP deflator, equal to  $P_{C,t} = P_{NT,t} = W_t$ , and real GDP  $Y_t$ .

### 3.3.1 Interest rates

As noted, the dollar interest rate  $i^{*s} = \rho$ , where lowercase letters denote logs, and  $\rho = -\ln \beta$ . For our purposes, we will assume the dollar rate stays at this value during both the short run and long run, although one could use our model to derive the effects of an external interest rate shock as well.

Due to interest parity, the peso interest rate is:

$$i_t = \rho + (s_{t+1} - s_t)$$

Since the nominal exchange rate equals the peso finished good price,  $s_{t+1} - s_t$  equals finished good price inflation, measured in pesos. Since the terms of trade don't change after the initial shock, finished good inflation is the same as overall consumer price inflation, since the ratio of the nontradable price and import prices also equals the (constant) terms of trade.

Consequently, the peso nominal interest rate is:

$$i_t = \rho + \pi_{t+1}$$

### 3.3.2 Steady state allocation

We will imagine the economy as existing in a sustainable steady state until it is hit by a terms of trade shock at the beginning of period  $t$  (the short-run period).

The key assumptions for a steady state are:

- The external dollar interest rate  $I^{*s} = 1/\beta$ . (If not, then consumers would have either increasing or decreasing consumption paths.)
- Without loss of generality, we will measure finished good in units so they have the same price as the commodity in the steady state. Thus the terms of trade in steady state is  $P_C^{*s} = 1$ .

Any strictly positive values for the steady state debt and reserves values, which we will denote  $D^s$  and  $A^s$ , can lead to a steady state, so long as those two values are equal (otherwise the country would have to run a persistent trade surplus or deficit to make up the difference).

Under these assumptions, it is possible to verify that steady-state consumption, output, and labor supply all equal 1 in the steady state. Moreover, if there is no terms of trade shock, all these variables will continue at that value in both the short run and long run.

### 3.3.3 Efficient allocation

It will be useful to study the efficient allocation, in other words, the allocation that occurs when wages are flexible and there is no borrowing constraint (which is efficient due to the First Welfare Theorem).

It can be shown that the log of the short-run efficient level of output depends positively on the terms of trade and negatively on new debt issuance. Intuitively, a higher terms of trade acts like a positive productivity shock, while a current account deficit implies people

can work less (less output) to get the same consumption.

$$y_t^e = \frac{\chi(1-\sigma)}{\phi+\sigma} p_C^* - \frac{\alpha\sigma}{\phi+\sigma} (d_t^e - d^s)$$

In this expression,  $\alpha$  is a log-linearization coefficient equal to the steady-state nominal debt to nominal GDP ratio.

In the long run, a similar relationship holds, except debt issued at time  $t$  has a positive impact on output, since households have to work more in the long run to repay the debt they took on in the short run.

$$y_{t+1}^e = \frac{\chi(1-\sigma)}{\phi+\sigma} p_C^* + \frac{\alpha\sigma}{\beta(\phi+\sigma)} (d_t^e - d^s)$$

We can conjecture and verify that the debt constraint does not bind in the efficient equilibrium. Intuitively, since the terms of trade is shocked permanently, there is no reason to “smooth” the shock by borrowing or lending. (This will not be true when we introduce nominal rigidities, since those rigidities may cause short-run booms or recessions that do need to be smoothed.)

Plugging this assumption into the above equations, we obtain the efficient levels of output to be:

$$y_t^e = y_{t+1}^e = \frac{\chi(1-\sigma)}{\phi+\sigma} p_C^*$$

Solving for consumption using log-linear approximations to the budget constraints, we obtain:

$$c_t^e = c_{t+1}^e = \frac{\chi(1+\phi)}{\phi+\sigma} p_C^*$$

### 3.3.4 Natural allocation

It will be useful to introduce  $Y_t^n$ , the natural rate of output in the short run. This is the level of output in the short run that would occur if all households were able to reset their wages freely in the short run, but still with the assumption of a borrowing constraint.

As we have shown, because the efficient allocation involves no new debt issuance, the

borrowing constraint won't bind under a ToT shock so long as wages are flexible. Thus, the natural allocation is identical to the efficient allocation. This result relies heavily on two specific assumptions we have made: first, that reserves are sufficient to balance trade in the efficient allocation, and second, that there is a labor subsidy ensuring that labor supply is efficient in the natural allocation, despite the existence of monopolistic competition.

### 3.3.5 Actual allocation

#### Supply side

With sticky wages, there is a Phillips curve relationship where inflation is determined by real variables. In particular, it is possible to derive that:

$$p_t - p^s = \kappa (y_t - y_t^n) + \theta (d_t - d^s) - \chi (p_C^* - p_C^s)$$

where:

$$\kappa = \frac{(1 - \omega)(\phi + \sigma)}{\omega(1 + \varepsilon\phi)}$$

$$\theta = \frac{(1 - \omega)\sigma\alpha}{\omega(1 + \varepsilon\phi)}$$

The first term,  $\kappa (y_t - y_t^n)$ , is the effect of output above its natural level creating upward pressure on wages. Those who can reset their wages will choose to set them higher when demand for their services is high, causing inflation in domestic good prices. Since those prices are part of the CPI, CPI inflation rises too.

The second term,  $\theta (d_t - d^s)$ , captures the labor supply effect of saving or borrowing. For example, if times are sufficiently good, the household may choose not to spend their extra income on leisure, but work slightly harder, save externally, and work less in the future. Thus, a more positive current account (which is a negative value of  $d_t - d^s$ ) coincides with higher labor supply, and thus lower wage inflation for the same amount of output demand.

Finally, the terms of trade enter into the Phillips curve, since an increase in the relative price of exports to imports implies a decrease in consumer prices (which include imports)

relative to wages (which are what are actually resetting under a demand shock).

### **Demand side**

If the debt constraint is binding, then

$$d_t = d^s$$

Otherwise, it can be shown that a standard IS curve obtains:

$$y_t - y_t^n = -\eta (d_t - d^s) - \frac{1}{\sigma} (i_t - \pi_{t+1} - \rho)$$

where

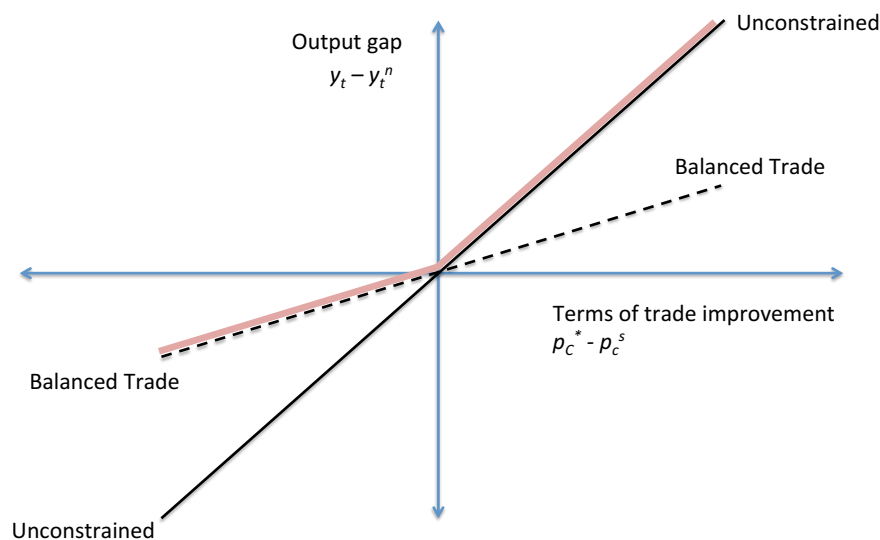
$$\eta = \alpha \left( 1 + \frac{1}{\beta(\sigma + \phi)} \right) > 0$$

The second term is the usual closed-economy relationship between consumption and the real interest rate (a higher real rate discourages consumption today). The second term adjusts this relationship to put it in terms of output: a higher current account surplus implies higher output for the same level of consumption.



## Example

### Output Gap Under CPI Inflation Targeting



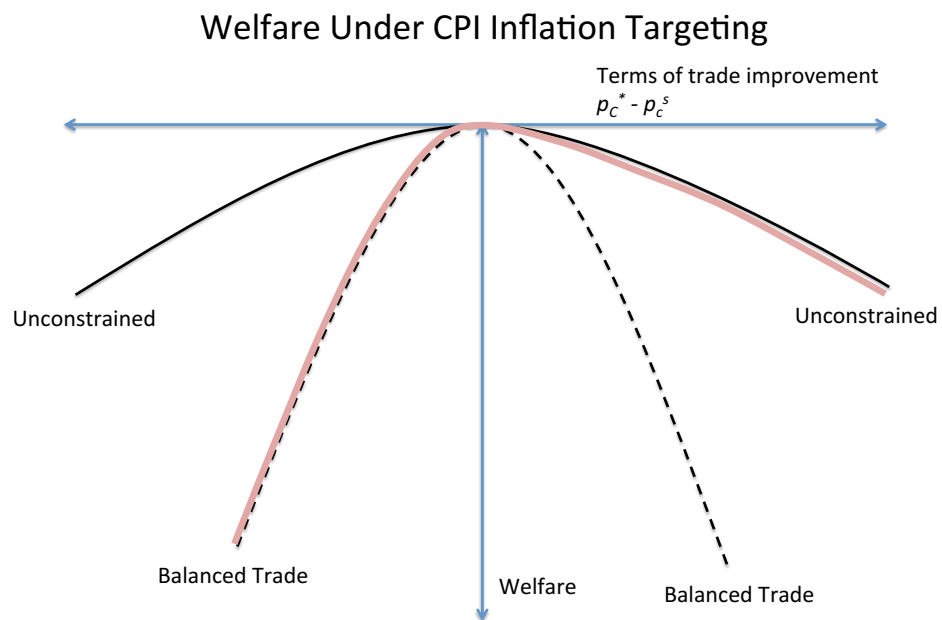
**Figure 3.1:** The output gap under a CPI target. The line marked “balanced trade” shows the typical result when the country cannot run a current account surplus or deficit. In that case, an improvement in the terms of trade forces the central bank to engineer a boom to keep inflation from falling.

Under the “unconstrained” equilibrium, the boom induced by a stronger terms of trade makes people want to smooth their earnings over time by saving abroad. They work a little harder today and aim to work less in the future. Thus output is higher today than in the balanced trade equilibrium.

When the terms of trade improve, the unconstrained equilibrium holds (since lending is not constrained) but the balanced trade equilibrium holds on the left side of the graph, since borrowing is constrained. The solid line shows the actual equilibrium that results from joining these two lines together at the steady state (the origin).

The workings of the equilibrium described above are illustrated in Figures 3.1 and 3.2. Both diagrams plot the outcome of under a CPI inflation target; in other words, the central bank sets  $p_t - p^s = 0$ .

In Figure 3.1, the line marked “balanced trade” shows the typical result when the country cannot run a current account surplus or deficit. In that case, an improvement in the terms of trade forces the central bank to engineer a boom to keep inflation from falling.



**Figure 3.2:** Welfare under a CPI target. The line marked “balanced trade” shows the result when the country cannot run a current account surplus or deficit. The solid line depicts the unconstrained equilibrium. The effect of the constraint, which is binding only for negative terms-of-trade shocks, is to make welfare lower, and faster-decreasing, for negative terms-of-trade shocks.

Under the “unconstrained” equilibrium, the boom induced by a stronger terms of trade makes people want to smooth their earnings over time by saving abroad. They work a little harder today and aim to work less in the future. Thus output is higher today than in the balanced trade equilibrium.

When the terms of trade improve, the unconstrained equilibrium holds (since lending is not constrained) but the balanced trade equilibrium holds on the left side of the graph, since borrowing *is* constrained. The solid line shows the actual equilibrium that results from joining these two lines together at the steady state (the origin).

While Figure 3.1 illustrates the effects of a terms-of-trade shock on the output gap, Figure 3.2 illustrates the effects of the same shock on welfare. Again, the solid line depicts the unconstrained equilibrium, and the dashed line depicts the balanced trade equilibrium. The effect of the constraint is to make welfare lower, and faster-decreasing, for negative terms-of-trade shocks.

## 3.4 Monetary Policy Design

### 3.4.1 Welfare

Let superscript “e” denote variables in the efficient equilibrium, which is identical to the natural allocation in a world with no borrowing constraint. Let tildes denote deviations between the actual equilibrium and the efficient equilibrium; for example,  $\tilde{c}_t = c_t - c_t^e$ .

It can then be shown that the welfare loss relative to the efficient equilibrium is, to second order, given by:

$$\tilde{W} = -\frac{1}{2} \left( y_t - y_t^n \right)^2 - \frac{\beta}{2} \left( y_{t+1} - y_{t+1}^n \right)^2 - \frac{\zeta}{2} \left( (p_t - p^s) + \chi (p_C^* - p_C^{*s}) \right)^2$$

where

$$\zeta = \left( \frac{\varepsilon(1 + \varepsilon\phi)}{\phi + \sigma} \right) \left( \frac{\omega}{1 - \omega} \right)$$

**Table 3.1:** Comparison of several monetary policies. Fixing the exchange rate to commodity prices works the best because any fall in the terms of trade is met by offsetting inflation. This inflation ensures that real wages are able to fall to their equilibrium level without any short-run period of wage distortions (some households having reset their wages and others not having had the chance to do so).

The inflation target achieves zero CPI inflation, but this means that non-tradable prices have to fall as finished good prices rise. This fall in non-tradable prices can only be generated by a fall in the cost of production; in other words, the central bank needs to generate a recession to reduce wages. That has an output gap cost.

The FX peg is the worst policy. Under this policy, the entire burden of adjustment falls on nominal non-tradable prices, as import prices remain fixed in pesos. The same logic as under the CPI inflation target holds, except now the recession needs to be larger in order to generate the necessary (larger) fall in wages. The policy is actually deflationary in CPI terms.

Policy	$p_t - p^s$	$y_t - y_t^n$	Welfare
FX Peg $s_t - s^s = 0$	$(1 - \chi) \tilde{p}_C^*$	$\frac{1}{\kappa - \frac{\theta}{\eta}} \tilde{p}_C^*$	$-\frac{1}{2} \left( \frac{1}{\kappa^2} + \zeta \right) \tilde{p}_C^{*2}$
Inflation Target $p_t - p^s = 0$	0	$\frac{\chi}{\kappa - \frac{\theta}{\eta}} \tilde{p}_C^*$	$-\frac{1}{2} \left( \frac{1}{\kappa^2} + \zeta \right) \chi^2 \tilde{p}_C^{*2}$
Commodity Basket $s_t - s^s = -(p_C^* - p_C^{*s})$	$-\chi \tilde{p}_C^*$	0	0

### 3.4.2 Comparing policies

Table 3.1 compares three policy options assuming a negative terms of trade shock of size  $\hat{p}_C^* = p_C^* - p_C^{*s}$ . We posit that the shock is negative since we wish to consider the equilibrium under the financial constraint.

First, we consider a fixed nominal exchange rate peg to the dollar:  $s_t - s^s = 0$ . Second, we consider an inflation target:  $p_t - p^s = 0$ . Finally, we consider a policy of targeting the exchange rate to the dollar price of the commodity, so that it appreciates whenever commodity prices rise:  $s_t - s^s = \hat{p}_C^*$ .

The three policies can be welfare ranked: the commodity peg is the best, the fixed FX peg is the worst, and the inflation target is in the middle.

Fixing the exchange rate to commodity prices works the best because any fall in the terms of trade is met by offsetting inflation. This inflation ensures that real wages are able to fall to their equilibrium level without any short-run period of wage distortions (some households having reset their wages and others not having had the chance to do so).

The inflation target achieves zero CPI inflation, but this means that non-tradable prices have to fall as finished good prices rise. This fall in non-tradable prices can only be generated by a fall in the cost of production; in other words, the central bank needs to generate a recession to reduce wages. That has an output gap cost.

The FX peg is the worst policy. Under this policy, the entire burden of adjustment falls on nominal non-tradable prices, as import prices remain fixed in pesos. The same logic as under the CPI inflation target holds, except now the recession needs to be larger in order to generate the necessary (larger) fall in wages. The policy is actually deflationary in CPI terms.

## 3.5 Conclusions

We have developed a tractable yet microfounded model in which we can explore optimal monetary policy under a terms-of-trade shock. This model allows for nominal rigidities

in wages and an external borrowing constraint that binds on a negative shock, but allows dollar lending when there is a positive shock. Our main contribution is to confirm the idea, often found in international IS-LM-type models, that a country ought to depreciate (or allow consumer price inflation to rise above target) upon a negative terms-of-trade shock. Our contribution relative to the New Open Economy Macroeconomics literature is a simple, graphical approach to imposing a more realistic borrowing constraint; we find that this borrowing constraint may significantly increase the cost of sticking with a fixed consumer price inflation target in the face of an adverse terms-of-trade shock.

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# Appendix A

## Derivations for Chapter 2

### A.1 Introduction

This Appendix provides the full derivation of the sticky wage model results used in my job market paper, “The Commodity Currency Puzzle.”

### A.2 Model Setup

#### A.2.1 Key notational differences

To conform to notational standards, I make the following substitutions relative to the non-monetary model given in the paper.

- $S_t$  means the nominal exchange rate (discussed below). I will no longer index variables by history  $s^t$ .
- $\phi$  means the Frisch elasticity of labor supply. It is no longer the a taste parameter for non-commodity exports in the non-monetary model.

#### A.2.2 Numeraires

Let  $S_t$  be the nominal exchange rate between the USD and the AUD (up implies appreciation of the AUD). Let  $P_t$  and  $P_t^*$  be the CPI values in the U.S. and Australia; we no longer



normalize them to 1.

Let  $Q_t$  be the real exchange rate

$$Q_t \equiv \frac{S_t P_t^*}{P_t} \quad (\text{A.1})$$

The terms of trade  $\Theta_t$  is the price of exports in terms of imports (from the point of view of Australia, so it moves in the same direction as the exchange rate):

$$\Theta_t \equiv \frac{P_{C,t}}{P_{F,t}} \quad (\text{A.2})$$

### A.2.3 Risk Sharing

We continue with the assumption of complete markets. For simplicity, we solve the log utility case, which, given the rest of the setup and no net initial financial claims, implies balanced trade in each period (Cole and Obstfeld 1991). Consequently, neither country accumulates financial claims against the other.

### A.2.4 Australia

#### Australian households

In Australia, there is a continuum of households indexed by  $j$ . Household  $j$  has log utility over consumption and power utility in labor hours:

$$U_t^*(j) = E_t \sum_{k=0}^{\infty} \beta^k \left( \ln C_{t+k}^*(j) - \frac{L(j)_{t+k}^{*1+\phi}}{1+\phi} \right) \quad (\text{A.3})$$

Because markets are complete domestically, and every household faces the same prices, consumption will be the same for all households. We denote this level of consumption  $C_{t+k}^*$ :

$$C_{t+k}^*(j) = C_{t+k}^* \quad \forall j \quad (\text{A.4})$$

For household  $j$ , consumption is a Cobb-Douglas aggregate of tradable and non-tradable goods:

$$C_t^*(j) \equiv \kappa C_{F,t}^*(j)^\chi C_{NT,t}^*(j)^{1-\chi} \quad (\text{A.5})$$

where  $0 < \chi < 1$  and  $C_{F,t}^*(j)$  and  $C_{NT,t}^*(j)$  are the consumption levels of the finished and non-tradable goods, and  $\kappa \equiv \chi^{-\chi} (1 - \chi)^{-(1-\chi)}$  is a normalizing factor. Again, because of complete markets and the homotheticity of the consumption aggregator, the household-level consumption of each good equals aggregate national consumption of that good:

$$C_{F,t}^*(j) = C_{F,t}^* \quad \forall j \quad (\text{A.6})$$

$$C_{NT,t}^*(j) = C_{NT,t}^* \quad \forall j \quad (\text{A.7})$$

The consumer price index (CPI) based on the optimal consumption bundle is:

$$P_t^* \equiv P_{F,t}^{*\chi} P_{NT,t}^{*(1-\chi)} \quad (\text{A.8})$$

where  $P_{F,t}^*$  and  $P_{NT,t}^*$  are the prices of the finished and non-tradable goods in Australian dollars.

Household  $j$ 's budget constraint given balanced trade is:

$$C_t^*(j) P_t^* = W_t(j)^* L_t(j)^* + D_t(j)^* \quad (\text{A.9})$$

where  $W_t(j)^*$  is the nominal wage and  $L_t(j)^*$  is labor supply, and  $D_t(j)^*$  is the (stochastic) income assigned to household  $j$  under the complete-markets allocation.

### Australian production

In Australia, there are two sectors of production that both depend on labor only. In the commodity-exporting sector,  $Y_{C,t}^*$  units are produced, and in the non-tradable sector,  $Y_{NT,t}^*$

units are produced, as follows:

$$Y_{C,t}^* = \underbrace{\left( \int L(j)_{C,t}^* \frac{\varepsilon-1}{\varepsilon} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}}_{L_{C,t}^*} \quad (\text{A.10})$$

$$Y_{NT,t}^* = \underbrace{\left( \int L(j)_{NT,t}^* \frac{\varepsilon-1}{\varepsilon} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}}_{L_{NT,t}^*} \quad (\text{A.11})$$

where  $L_{C,t}^*(j)$  and  $L_{NT,t}^*(j)$  are the labor supplies from each household  $j$  to the two industries, substituted into the production function. These production technologies are linear in the labor hour aggregators  $L_{C,t}^*$  and  $L_{NT,t}^*$ . For simplicity, I omit the local supply shock,  $Z^*$ , which was in the non-monetary model.

We introduce a nominal wage aggregate:

$$W_t^* \equiv \left( \int W_t^*(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (\text{A.12})$$

Dixit-Stiglitz demand functions give us labor demand for each household in terms of that household's relative wage:

$$L_{C,t}^*(j) = \left( \frac{W_t^*(j)}{W_t^*} \right)^{-\varepsilon} L_{C,t}^* \quad (\text{A.13})$$

$$L_{NT,t}^*(j) = \left( \frac{W_t^*(j)}{W_t^*} \right)^{-\varepsilon} L_{NT,t}^* \quad (\text{A.14})$$

Adding these conditions, we can write an expression for household  $j$ 's total labor hours demanded across both sectors:

$$L_t^*(j) = \left( \frac{W_t^*(j)}{W_t^*} \right)^{-\varepsilon} L_t^* \quad (\text{A.15})$$

Given these conditions, nominal labor compensation equals  $W_t^* L_t^*$ :

$$\int W_t^*(j) L_t^*(j) = W_t^* L_t^* \quad (\text{A.16})$$

Define "aggregate output" or real GDP as:

$$Y_t^* \equiv \kappa Y_{C,t}^* \chi Y_{NT,t}^* \quad (\text{A.17})$$

Real GDP is the result of deflating nominal GDP by the GDP deflator, which is defined as:

$$G_t^* \equiv P_{C,t}^* P_{NT,t}^{*1-\chi} \quad (\text{A.18})$$

The GDP deflator is related to CPI as follows:

$$g_t^* = p_t^* + \chi\theta_t \quad (\text{A.19})$$

The standard first-order optimality conditions equate the marginal product of labor with the wage:

$$P_{C,t}^* = W_t^* \quad (\text{A.20})$$

$$P_{NT,t}^* = W_t^* \quad (\text{A.21})$$

The above results imply that the GDP deflator equals the nominal wage, since both sectors have output prices equal to the wage (equations A.18, A.20, and A.21):

$$G_t^* = W_t^* \quad (\text{A.22})$$

### **Relationship between output and consumption**

With balanced trade, nominal consumption equals nominal output:

$$P_t^* C_t^* = Y_t^* G_t^* \quad (\text{A.23})$$

This statement can be proved by combining the definitions of the aggregates with the fact that  $P_{C,t}^* Y_{C,t}^* = P_{F,t}^* C_{F,t}^*$  for trade to be balanced.

Taking logs of both sides, we can relate real consumption to real output as follows:

$$c_t^* = y_t^* + \chi\theta_t \quad (\text{A.24})$$

where  $\theta_t$  is the log terms of trade.

### National budget constraint

The national budget constraint is found by summing the household constraint (A.9) over households  $j$ :

$$\begin{aligned}\int P_t^* C_t^* &= \int W_t^*(j) L_t^*(j) \\ P_t^* C_t^* &= W_t^* L_t^*\end{aligned}\tag{A.25}$$

The Arrow-Debreu security payments drop out here, because they net to zero within the country (there are no international financial claims on net).

### Aggregate production function

Real output is linear in labor. To prove this, simply combine the relationship between nominal consumption and nominal output (A.23) with the national budget constraint (A.25). Using the fact that the GDP deflator equals the nominal wage (A.22), we have that

$$Y_t^* = L_t^*\tag{A.26}$$

### Optimal wage setting

In each period, a randomly selected fraction  $1 - \omega$  of households can reset its wage. The wage-resetting problem is, in principle, the same as the household optimization problem stated above with an extra control variable, the reset wage  $W_r^*$ . However, the problem can be simplified greatly by noting that any terms in the household utility function not affected by the choice of the reset wage may be dropped. We are left with the objective function:

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\omega)^k u(C_{t+k}^*(j)_{t+k}, L_{t+k}^*(j)) \right\}\tag{A.27}$$

where the function  $u(\cdot)$  is the period utility function, which is now maximized with respect to the budget constraint (A.9) with the reset wage substituted in:

$$W_r^* L_{t+k}^*(j) + D_{t+k}^*(j) = P_{t+k}^* C_{t+k}^*(j)\tag{A.28}$$

A second simplification in this problem is to write the household's labor supply  $L(j)_{t+k}^*$  as a function of the reset wage  $W_r^*$ . Because it is not optimal to set the reset wage so low that there is excess labor demand, the household engineers labor supply to exactly equal labor demand under the selected reset wage. Thus, we can simply use equation (A.15), understanding that the notation  $L_{t+k}^*(j)$  is appropriate to mean both the quantity supplied and demanded:

$$L_{t+k}^*(j) = \left( \frac{W_r^*}{W_{t+k}^*} \right)^{-\varepsilon} L_{t+k}^*$$

Solving this problem leads to a relationship between wage inflation and wage markups (full solution details are given in Section 4). Intuitively, the household would like to set its wage at a constant markup above its marginal cost of labor. If labor demand is surprisingly high, then the household is willing to work harder (because its wage is still above marginal cost) but its markup will be compressed. Thus, at an economy-wide level, we expect households to reset their wages upwards when average markups are compressed relative to desired markups.

Specifically, the wage inflation is related to markups as follows:

$$\pi_t^{*w} = \beta E_t(\pi_{t+1}^{*w}) - \lambda(\mu_t^* - \mu) \quad (\text{A.29})$$

$\pi_t^{*w}$  is wage inflation:

$$\pi_t^{*w} \equiv w_t^* - w_{t-1}^* \quad (\text{A.30})$$

$\mu$  is the steady-state log markup that households would like to add on top of their marginal cost of labor:

$$\mu \equiv \ln \left( \frac{\varepsilon}{\varepsilon - 1} \right) \quad (\text{A.31})$$

$\mu_t^*$  is the prevailing average markup in the economy, defined as the wedge between the prevailing real wage and the aggregate marginal rate of substitution between consumption

and leisure:

$$\mu_t^* \equiv (w_t^* - p_t^*) - mrs_t^* \quad (\text{A.32})$$

The aggregate MRS is defined as:

$$mrs_{t+k}^* \equiv \phi l_{t+k}^* + \sigma c_{t+k}^* \quad (\text{A.33})$$

The coefficient  $\lambda$  is given by:

$$\lambda \equiv \frac{(1 - \omega)(1 - \beta\omega)}{\omega(1 + \phi\varepsilon)} \quad (\text{A.34})$$

The formula for  $\lambda$  depends on wage stickiness,  $\omega$ . The stickier wages are, the harder it is to generate wage inflation, so the lower  $\lambda$  will be through the numerator. At the same time, however, very sticky wages will make a household want to adjust its wage significantly when it does get the opportunity to do so, so there will be an offsetting effect in the denominator. Under complete wage stickiness,  $\omega = 1$  and  $\lambda = 0$ ; in the flexible-price limit,  $\omega \rightarrow 0$  and  $\lambda \rightarrow \infty$ .

### The actual rate of output

We now derive the level of real GDP in Australia. Using the previous definitions (A.32, A.33), the average markup can be written as:

$$\mu_t^* = (w_t^* - p_t^*) - \phi l_t^* - c_t^* \quad (\text{A.35})$$

Plugging in the relationship between the CPI and the GDP deflator (A.19), the relationship between real consumption and output (A.24), the equality between output and labor supply (A.26), and the equality between the GDP deflator and the nominal wage (A.22), we can simplify this equation to:

$$y_t^* = -\frac{1}{1 + \phi} \mu_t^* \quad (\text{A.36})$$

## The natural rate of output

In this type of model, it is useful to compare the true rate of output with its natural rate, which would prevail under flexible prices. The “output gap” between the two may be a target of monetary policy.

As shown in the Appendix on optimal wage setting, in a steady state with zero inflation, all households behave as though they were in a flexible-price world. This level satisfies the well-known consumption-leisure first-order condition, that the real wage equals the marginal of substitution between an hour of leisure and a unit of consumption, plus markup:

$$\frac{W^{ss}}{P^{ss}} = MRS^{ss} \mathcal{M}$$

where  $W^{ss}/P^{ss}$  is the real wage in the steady state,  $MRS^{ss}$  is  $-u_L/u_C$  in the steady state, and  $\mathcal{M}$  is the monopolistic markup  $\varepsilon/(\varepsilon - 1)$ .

Under flexible prices, that first-order condition must be obeyed in all states:

$$\frac{W_{n,t}^*}{P_{n,t}^*} = C_{n,t}^* Y_{n,t}^{*\phi} \mathcal{M}$$

where we have used the fact that  $Y_{n,t}^* = L_{n,t}^*$ , proved analogously to its sticky-wage counterpart (A.26).

We can eliminate consumption since we have derived it in terms of output (A.24 is true regardless of whether wages are flexible):

$$y_{n,t}^* + \chi\theta + \phi y_{n,t}^* + \mu = w_{n,t}^* - p_{n,t}^*$$

Substituting in the GDP deflator using its relationship with the CPI (A.19), we can eliminate the term  $\chi\theta$  from both sides:

$$y_{n,t}^* + \phi y_{n,t}^* + \mu = w_{n,t}^* - g_{n,t}^*$$

Finally, use the fact that the wage equals the GDP deflator (A.22) to get

$$y_{n,t}^* = -\frac{1}{1 + \phi} \mu \tag{A.37}$$



Intuitively, the household would normally work for 1 hour, as in the real model, but it is trying to earn a monopolistic rent by working slightly less. The bigger the markup it can charge, the less it works.

### AS curve

The previous two sections have put actual and natural rates of output in terms of markups. We can therefore take our equation for wage inflation, which is expressed in terms of markups, and put it in terms of the output gap instead. This will be our AS or Phillips Curve.

Specifically, we simply combine equations (A.29), (A.36), and (A.37) to get the relationship:

$$\pi_t^{w*} = \beta E_t (\pi_{t+1}^{w*}) + \zeta (y_t^* - y_{n,t}^*)$$

in which:

$$\begin{aligned} \zeta &\equiv (1 + \phi) \lambda \\ &= (1 + \phi) \frac{(1 - \omega)(1 - \beta\omega)}{\omega(1 + \phi\varepsilon)} \end{aligned}$$

We will actually use a version of the AS curve in terms of the GDP deflator, which, in this case, is no different, since wages equal output prices in Australia:

$$\pi_t^{g*} = \beta E_t (\pi_{t+1}^{g*}) + \zeta (y_t^* - y_n^*) \quad (\text{A.38})$$

### AD curve

The AD curve comes from the regular Euler (IS) condition:

$$c_t^* = E_t (c_{t+1}^*) - (i_t^* - E_t \pi_t^* - \rho - f^*(z_t)) \quad (\text{A.39})$$

where  $\pi_t^*$  is CPI inflation. The unusual term  $f(z_t)$  is simply the precautionary saving term that is generally dropped when log-linearizing the Euler condition; here, we need to keep

this term since it is important for the currency risk premium.

We convert consumption to output using (A.24), and CPI to the GDP deflator using (A.19).

Substituting in for consumption first, we get:

$$y_t^* + \chi\theta_t = E_t(y_{t+1}^* + \chi\theta_{t+1}) - (i_t^* - E_t(p_{t+1}^* - p_t^*) - \rho - f^*(z_t))$$

Now we substitute in for prices:

$$y_t^* = E_t(y_{t+1}^* + \chi\theta_{t+1}) - \chi\theta_t - (i_t^* - E_t(g_{t+1}^* - \chi\theta_{t+1} - g_t^* + \chi\theta_t) - \rho - f^*(z_t))$$

The terms of trade terms cancel out to yield:

$$y_t^* = E_t(y_{t+1}^*) - (i_t^* - E_t(g_{t+1}^* - g_t^*) - \rho - f^*(z_t))$$

Subtracting the natural rate of output (which is constant) from both sides, we obtain:

$$y_t^* - y_n^* = E_t(y_{t+1}^* - y_n^*) - (i_t^* - E_t\pi_{t+1}^{g^*} - \rho - f^*(z_t)) \quad (\text{A.40})$$

which is our AD curve. Note that  $\rho + f^*(z_t)$  is the “natural rate of interest” here.

### Labor supply in each sector

As in the real model, labor supply is split in fixed fractions between the two sectors, but now the total amount of labor  $L_t^*$  moves around based on demand.

To show the fixed splitting, start with the consumer demand for non-tradable goods, which follows from the Cobb-Douglas functional form and the fact that nominal income is  $L_t^*W_t^*$ :

$$C_{NT,t}^* = (1 - \chi) \left( \frac{L_t^*W_t^*}{P_{NT,t}^*} \right)$$

Imposing market clearing conditions and the firm first-order condition that  $W_t^*/P_{NT,t}^* = 1$ ,

we have

$$L_t^* - L_{C,t}^* = (1 - \chi) L_t^*$$

This immediately implies our result:

$$L_{C,t}^* = \chi L_t^* \tag{A.41}$$

$$L_{NT,t}^* = (1 - \chi) L_t^* \tag{A.42}$$

### A.2.5 America

America is different from Australia in two major ways:

- Wages are flexible. This just simplifies things without taking away any of the intuition.
- As in the real model, the production technology is different; commodity imports are used to produce the finished good, and both non-tradable and finished good output may be subject to TFP shocks.

#### American households

As in Australia, the American household's utility is Cobb-Douglas with finished good share  $\chi$ :

$$C_t \equiv \kappa C_{F,t}^\chi C_{NT,t}^{(1-\chi)}$$

the CPI  $P_t$  is defined analogously.

The representative household's utility function is:

$$U_t = E_t \sum_{k=0}^{\infty} \beta^k \left( \ln C_{t+k}^* - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right)$$

The American budget constraint is

$$C_t P_t = W_t N_t$$

## American production

The U.S. has two sectors, a non-tradable sector and a finished-goods-producing sector:

$$Y_{F,t} = Z_t L_{F,t}^\alpha Y_{C,t}^{*1-\alpha}$$

$$Y_{NT,t} = Z_t L_{NT,t}$$

where  $Z_t$  is total factor productivity,  $L_{F,t}$  the labor used in finished goods production, and  $L_{NT,t}$  is the labor used in the non-tradable sector.

The first-order conditions set marginal product equal to marginal cost:

$$P_{F,t} \alpha Z_t L_{F,t}^{\alpha-1} Y_{C,t}^{*1-\alpha} = W_t \quad (\text{A.43})$$

$$P_{F,t} (1 - \alpha) Z_t L_{F,t}^\alpha Y_{C,t}^{*-\alpha} = P_{C,t} \quad (\text{A.44})$$

$$P_{NT,t} Z_t = W_t \quad (\text{A.45})$$

Output is defined *using real value added*:

$$Y_t \equiv \gamma (Y_{F,t} - \Theta_t Y_{C,t}^*)^\chi Y_{NT,t}^{1-\chi}$$

And the GDP deflator is:

$$G_t \equiv P_{F,t}^\chi P_{NT,t}^{1-\chi}$$

## Relationship between consumption and output

Again, because there are no net exports, nominal output equals nominal consumption:

$$\begin{aligned} G_t Y_t &= \gamma (P_{F,t} Y_{F,t} - P_{C,t} Y_{C,t})^\chi (P_{NT,t} Y_{NT,t})^{1-\chi} \\ &= \gamma (P_{F,t} Y_{F,t} - P_{F,t} C_{F,t}^*)^\chi (P_{NT,t} C_{NT,t})^{1-\chi} \\ &= \gamma (P_{F,t} C_{F,t})^\chi (P_{NT,t} C_{NT,t})^{1-\chi} \\ &= P_t C_t \end{aligned}$$

In real terms, consumption and output are linked by the same identity as in Australia:

$$c_t = y_t + g_t - p_t$$

But note that the GDP deflator is the same as CPI for the US, so log consumption equals log output:

$$c_t = y_t$$

### **Labor supply and the natural rate of output**

Labor supply in the U.S. works the same way as in the real model: one hour of labor, and fixed shares of labor in each sector.

To prove this, note that the consumption-leisure first-order condition equates the real wage with the marginal rate of substitution between an hour of leisure and a unit of consumption:

$$\frac{W_t}{P_t} = C_t L_t^\phi$$

Using the budget constraint  $P_t C_t = W_t L_t$ , this condition simplifies to:

$$W_t = W_t L_t^{1+\phi}$$

which implies unit labor supply

$$L_t = 1$$

The division of labor between the two sectors can be found as usual. Using the demand function for the non-tradable good, we have

$$C_{NT,t} = (1 - \chi) \frac{W_t L_t}{P_{NT,t}}$$

Market clearing implies that

$$Z_t (L_t - L_{F,t}) = (1 - \chi) \frac{W_t L_t}{P_{NT,t}}$$

$$Z_t \left(1 - \frac{L_{F,t}}{L_t}\right) = (1 - \chi) \frac{W_t}{P_{NT,t}}$$

Since  $L_t = 1$ ,

$$L_F = \chi \tag{A.46}$$

$$L_{NT} = 1 - \chi \tag{A.47}$$

### Terms of trade

The firm first-order condition for imports is:

$$(1 - \alpha) Z_t L_F^\alpha Y_{C,t}^{*-\alpha} = \frac{P_{C,t}}{P_{F,t}}$$

$$(1 - \alpha) Z_t \chi^\alpha \chi^{-\alpha} L_t^{*-\alpha} = \frac{P_{C,t}}{P_{F,t}}$$

where the second line comes from substituting in the labor supplies.

This equation can be put in terms of the terms of trade:

$$(1 - \alpha) Z_t L_t^{*-\alpha} = \Theta_t$$

$$\theta_t = z_t + \ln(1 - \alpha) - \alpha l_t^*$$

In Australia, output is equal to labor (A.26), so labor can be written in terms of the output gap:

$$l_t^* - l_n^* = y_t^* - y_n^*$$

$$l_t^* = -\frac{1}{1 + \phi} \mu + (y_t^* - y_n^*) \tag{A.48}$$

Substituting this expression for Australian labor into the expression for the terms of trade yields:

$$\theta_t = z_t - \alpha (y_t^* - y_n^*) + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu$$

## A.2.6 Summary of model

It's easiest to think of the model as a set of three main equations: the AS and AD curves from Australia, and the ToT (terms-of-trade) equation from America.

$$y_t^* - y_n^* = E_t (y_{t+1}^* - y_n^*) - (i_t^* - E_t \pi_{t+1}^{\delta*} - \rho - f^*(z_t)) \quad (\text{A.49})$$

$$\pi_t^{\delta*} = \beta E_t \pi_{t+1}^{\delta*} + \zeta (y_t^* - y_n^*) \quad (\text{A.50})$$

$$\theta_t = z_t - \alpha (y_t^* - y_n^*) + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu \quad (\text{A.51})$$

## A.3 Monetary Policy Analysis

### A.3.1 Shocks

There is only one exogenous process in the sticky wage model: US productivity. It follows a standard Gaussian AR(1) in logs:

$$z_t = az_{t-1} + \varepsilon_t$$

where  $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ .

### A.3.2 US monetary policy rule

In the US, it is assumed that the central bank targets GDP deflator (equivalently, CPI) inflation to zero. It can do this with the well-known Taylor rule

$$i_t = \underbrace{\rho + E_t \Delta c_{t+1} + f(z_t)}_{\text{"natural rate"}} + \psi^{US} \pi_t^{\delta} \quad (\text{A.52})$$

with  $\psi^{US} > 1$ .

We will consider two monetary policy regimes for Australia: floating and fixed nominal exchange rates.

### A.3.3 Real commodity prices

We are particularly interested in whether the elasticity of the real exchange rate to commodity prices is greater or smaller in countries with a peg. To compute this, we need to compute log real commodity prices:  $p_C - p$ . (In the non-monetary model, this is just written  $p_C$ , since the CPI is normalized to 1.)

Commodity prices are, by definition, finished good prices times the terms of trade:

$$p_{C,t} - p_t = p_{F,t} - p_t + \theta_t \quad (\text{A.53})$$

Finished good prices are given by the Cobb-Douglas consumption first-order condition:

$$p_{F,t} + c_{F,t} = \ln \chi + c_t + p_t \quad (\text{A.54})$$

Plugging (A.54) into (A.53), we obtain

$$p_{C,t} - p_t = \ln \chi + c_t - c_{F,t} + \theta_t \quad (\text{A.55})$$

The definition of the consumption aggregate implies that

$$\begin{aligned} \frac{C_t}{C_{F,t}} &= \frac{\kappa C_{F,t}^\chi C_{NT,t}^{1-\chi}}{C_{F,t}} \\ &= \kappa C_{F,t}^{\chi-1} C_{NT,t}^{1-\chi} \end{aligned}$$

Because of market clearing,  $C_{NT,t} = Y_{NT,t}$ , and because labor effort is constant in the US,  $Y_{NT,t} = Z_t (1 - \chi)$ . Thus we have

$$\frac{C_t}{C_{F,t}} = \kappa C_{F,t}^{\chi-1} Z_t^{1-\chi} (1 - \chi)^{1-\chi}$$

Plugging this into (A.55), we obtain

$$p_{C,t} - p_t = \ln \chi + \ln \kappa + (1 - \chi) \ln (1 - \chi) - (1 - \chi) c_{F,t} + (1 - \chi) z_t + \theta_t$$

Balanced trade and log utility implies that  $c_{F,t} = \ln \alpha + y_{F,t}$ . Moreover,  $y_{F,t}$  is given by



the production function

$$Y_{F,t} = Z_t L_F^\chi Y_{C,t}^{*1-\chi}$$

which, again given fixed labor shares, yields

$$y_{F,t} = z_t + \chi \ln \chi + (1 - \chi) l_t^*$$

Plugging this into our previous expression for the commodity price, we obtain

$$p_{C,t} - p_t = \ln \chi + \ln \kappa + (1 - \chi) \ln (1 - \chi) - (1 - \chi) (\ln \alpha + z_t + (1 - \chi) l_t^* + \chi \ln \chi) + \dots \\ (1 - \chi) z_t + \theta_t$$

Using the definition of  $\kappa$ , this can be simplified to

$$p_{C,t} - p_t = (1 - (1 - \chi) \chi) \ln \chi - (1 - \chi) \ln \alpha - (1 - \chi)^2 l_t^* + \theta_t$$

Substituting in the terms of trade using equation (A.51), we find

$$p_{C,t} - p_t = (1 - (1 - \chi) \chi) \ln \chi - (1 - \chi) \ln \alpha - (1 - \chi)^2 l_t^* \\ + z_t - \alpha (y_t^* - y_n^*) + \ln (1 - \alpha) + \frac{\alpha}{1 + \phi} \mu$$

We can also substitute out  $l_t^*$  using the output gap from equation (A.48):

$$p_{C,t} - p_t = (1 - (1 - \chi) \chi) \ln \chi - (1 - \chi) \ln \alpha - (1 - \chi)^2 \left( -\frac{1}{1 + \phi} \mu + (y_t^* - y_n^*) \right) \\ + z_t - \alpha (y_t^* - y_n^*) + \ln (1 - \alpha) + \frac{\alpha}{1 + \phi} \mu$$

Overall, we find that the elasticity of the real commodity price to productivity shocks is

$$\frac{\partial (p_{C,t} - p_t)}{\partial z_t} = 1 - \frac{1}{\eta} \frac{\partial (y_t^* - y_n^*)}{\partial z_t} \quad (\text{A.56})$$

where  $\eta \equiv 1 / (\alpha + (1 - \chi)^2)$ .

In the flexible-price allocation, the second term is zero, so real commodity prices are unit elastic to productivity shocks.

### A.3.4 Floating exchange rate

#### Monetary policy specification

Under floating exchange rates, the central bank tries to replicate the flexible-price allocation. It follows the Taylor rule

$$i_t^* = \rho + f^*(z_t) + \psi \pi_t^{\delta^*} \quad (\text{A.57})$$

Note that there is no need to include the standard term of the expected growth rate of consumption. This term will automatically appear in the *real* (CPI-adjusted) interest rate. The *nominal* interest rate moves only with the precautionary saving term.

To see how rule (A.57) can replicate the flexible-price allocation, suppose that it is substituted into the system (A.49), (A.50), (A.51). This substitution yields the modified system

$$\begin{aligned} y_t^* - y_n^* &= E_t(y_{t+1}^* - y_n^*) - (E_t \pi_{t+1}^{\delta^*} + \psi \pi_t^{\delta^*}) \\ \pi_t^{\delta^*} &= \beta E_t \pi_{t+1}^{\delta^*} + \zeta (y_t^* - y_n^*) \\ \theta_t &= z_t - \alpha (y_t^* - y_n^*) + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu \end{aligned}$$

Write the first two equations of the system in vector form:

$$\mathbf{x}_t = \Xi E_t \mathbf{x}_{t+1} + \Gamma \mathbf{x}_t$$

where

$$\begin{aligned} \mathbf{x}_t &\equiv \begin{bmatrix} y_t^* - y_n^* \\ \pi_t^{\delta^*} \end{bmatrix} \\ \Xi &\equiv \begin{bmatrix} 1 & -1 \\ 0 & \beta \end{bmatrix} \\ \Gamma &\equiv \begin{bmatrix} 0 & -\psi \\ \zeta & 0 \end{bmatrix} \end{aligned}$$

Solving this system recursively implies that:

$$E_t \mathbf{x}_{t+k} = \left[ \Xi^{-1} (\mathbf{I} - \Gamma) \right]^k \mathbf{x}_t$$

So long as  $\psi$  is chosen so that the eigenvalues of  $\Xi^{-1} (\mathbf{I} - \Gamma)$  are outside the unit circle, only the flexible-price, zero-inflation solution will be stationary (not diverge in expectation as  $k \rightarrow \infty$ ).

### Nominal exchange rate

Under floating rates, the nominal exchange rate can then be derived as follows. By no-arbitrage in goods markets,

$$s_t = p_{F,t} - p_{F,t}^* \tag{A.58}$$

The GDP deflators can be written in log-linear form as:

$$\begin{aligned} g_t &= \chi p_{F,t} + (1 - \chi) p_{NT,t} \\ g_t^* &= \chi \theta_t + \chi p_{F,t}^* + (1 - \chi) p_{NT,t}^* \end{aligned}$$

These can be plugged into the initial equation for the exchange rate (A.58). After collecting terms, we have:

$$s_t = \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1 - \chi}{\chi} (p_{NT,t}^* - p_{NT,t}) \tag{A.59}$$

where  $\bar{g}$  is the targeted level of  $g_t$  in the US.

Because of GDP deflator targeting, the GDP deflator terms can be combined into a constant, and this expression can be simplified to

$$s_t = \text{const} + \frac{1 - \chi}{\chi} (p_{NT,t}^* - p_{NT,t}) + \theta_t \tag{A.60}$$

To derive the term  $p_{NT,t}^* - p_{NT,t}$ , use the first-order conditions for labor in the non-

tradable goods sectors (A.21, A.45) to get:

$$p_{NT,t}^* - p_{NT,t} = w_t^* - w_t + z_t$$

Using equation (A.43), we substitute in the marginal product of labor for the wage in the U.S.:

$$p_{NT,t}^* - p_{NT,t} = w_t^* - \ln \left[ P_{F,t} Z_t L_{F,t}^{\chi-1} L_{C,t}^{*1-\chi} \right] + z_t - w_t^*$$

Because labor shares are constant (A.47, A.42, A.46, A.47) this equation simplifies to:

$$p_{NT,t}^* - p_{NT,t} = w_t^* - p_{F,t} - (1 - \chi) l_t^* \quad (\text{A.61})$$

As shown above in (A.48), if the output gap is stabilized in Australia, labor supply  $l_t^*$  simply equals  $-\frac{1}{1+\phi}\mu$ , a constant. We can fold this into the constant term in (A.60) to get

$$s_t = \text{const} + \frac{1 - \chi}{\chi} (w_t^* - p_{F,t}) + \theta_t$$

Because  $p_{C,t} = p_{F,t} + \theta_t$ , this simplifies to:

$$s_t = \text{const} + \frac{1 - \chi}{\chi} (w_t^* - p_{C,t} + \theta_t) + \theta_t$$

Since  $w_t^* = p_{C,t}^*$  from (A.20), our expression for the exchange rate becomes:

$$s_t = \text{const} + \frac{1 - \chi}{\chi} (\theta_t - s_t) + \theta_t$$

where the additional  $s_t$  on the right-hand side comes from the conversion of  $p_{C,t}$  to  $p_{C,t}^*$ .

This expression can be simplified to

$$s_t \left[ 1 + \frac{1 - \chi}{\chi} \right] = \left[ 1 + \frac{1 - \chi}{\chi} \right] \theta_t + \text{const}$$

Finally, we use the terms-of-trade equation (A.51) to substitute out  $\theta_t$ :

$$\theta_t = z_t + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu$$

This substitution results in the simple, unit-elastic relationship for the nominal exchange

rate:

$$s_t = z_t + \text{const.}$$

### Real exchange rate

The real exchange rate can be derived from the nominal exchange rate as follows. First, using the definition of CPIs, the real exchange rate can be written in terms of individual good prices:

$$\begin{aligned} q_t &= s_t + p_t^* - p_t \\ &= s_t + (\chi p_F^* + (1 - \chi) p_{NT}^*) - (\chi p_F + (1 - \chi) p_{NT}) \end{aligned}$$

Using no-arbitrage in goods markets, the terms with finished goods can be eliminated:

$$q_t = (1 - \chi) s_t + (1 - \chi) (p_{NT,t}^* - p_{NT,t})$$

Re-arranging equation (A.60), we have

$$(1 - \chi) (p_{NT,t}^* - p_{NT,t}) = \chi (s_t - \theta_t) + \text{const}$$

Plugging this ratio into the right-hand side of the equation for  $q_t$ , we get

$$\begin{aligned} q_t &= (1 - \chi) s_t + \chi s_t - \chi \theta_t + \text{const} \\ &= s_t - \chi \theta_t + \text{const} \end{aligned}$$

Finally, because both  $s_t$  and  $\theta_t$  have unit elasticities with respect to  $z_t$ , the real exchange rate has an elasticity of  $1 - \chi$  with respect to  $z_t$ :

$$q_t = (1 - \chi) z_t + \text{const} \tag{A.62}$$

### Commodity price elasticities

Since commodity prices are unit elastic to productivity shocks at the flexible-price allocation (equation A.56, which is replicated under this monetary policy, the real and nominal

exchange rates have the same elasticity to real commodity prices as they do to productivity.

### Currency risk premium on nominal carry trade

As shown in Backus, Foresi, and Telmer (2001), the currency risk premium is given by

$$E_t (i_t^* - i_t + \Delta s_{t+1}) = f^*(z_t) - f(z_t)$$

The  $f(\cdot)$  terms are defined as the nonlinear terms from the Euler equation. Given that consumption growth is conditionally lognormal, in the US, we have

$$f(z_t) = - \left( \frac{1}{2} V_t \Delta c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) \right)$$

Since, in the US, inflation is zero, we simply find:

$$f(z_t) = -\frac{1}{2} V_t \Delta c_{t+1}$$

As shown in Appendix B, this equals

$$f(z_t) = -\frac{1}{2} \sigma_\varepsilon^2$$

In Australia, the nominal interest rate follows a similar lognormal Euler equation, leading to:

$$f^*(z_t) = - \left( \frac{1}{2} V_t \Delta c_{t+1}^* + \frac{1}{2} V_t \pi_{t+1}^* + \text{cov}_t(\Delta c_{t+1}^*, \pi_{t+1}^*) \right)$$

Plugging in the formula for the GDP deflator, we have

$$f^*(z_t) = - \left( \frac{1}{2} V_t \Delta c_{t+1}^* + \frac{1}{2} \chi^2 V_t \Delta \theta_t - \chi \text{cov}_t(\Delta c_{t+1}^*, \Delta \theta_t) \right)$$

As shown in Appendix B, in the flexible-price allocation,  $c_t^*$  has an elasticity of  $\chi$  to  $z_t$ , while  $\theta_t$  has an elasticity of 1 to  $z_t$ . Thus we have

$$\begin{aligned} f^*(z_t) &= - \left( \frac{1}{2} \chi^2 \sigma_\varepsilon^2 + \frac{1}{2} \chi^2 \sigma_\varepsilon^2 - \chi^2 \sigma_\varepsilon^2 \right) \\ &= 0 \end{aligned}$$

So the nominal currency risk premium is:

$$\begin{aligned} E_t (i_t^* - i_t + \Delta s_{t+1}) &= \frac{1}{2} V_t \Delta c_{t+1} \\ &= \frac{1}{2} \sigma_\varepsilon^2 \end{aligned}$$

The real currency risk premium is computed in Appendix B, and it is found to be:

$$E_t (r_t^* - r_t + \Delta q_{t+1}) = \frac{1}{2} (1 - \chi^2) \sigma_\varepsilon^2$$

### A.3.5 Peg

#### Monetary policy specification & nominal exchange rate

By definition, the nominal exchange rate is fixed at a level  $\bar{s}$ . Thus, there is no risk premium on the nominal exchange rate.

#### Real exchange rate

We can use equation (A.59) from the derivations for the floating exchange rate, except substitute in the fixed exchange rate to get:

$$\bar{s} = \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1 - \chi}{\chi} (p_{NT,t}^* - p_{NT,t})$$

Using equation (A.61), we can substitute out the nontradable prices to get:

$$\bar{s} = \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1 - \chi}{\chi} (w_t^* - p_{F,t} - (1 - \chi) l_t^*)$$

Using the fact that  $p_{F,t} = p_{C,t} - \theta_t$ , we get

$$\bar{s} = \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1 - \chi}{\chi} (w_t^* - p_{C,t} + \theta_t - (1 - \chi) l_t^*)$$

Using the fact that  $p_{C,t} = p_{C,t}^* + \bar{s}$ :

$$\bar{s} = \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1 - \chi}{\chi} (w_t^* - p_{C,t}^* - \bar{s} + \theta_t - (1 - \chi) l_t^*)$$

Since  $w^* = p_{C,t}^*$ :

$$\bar{s} = \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1-\chi}{\chi} (-\bar{s} + \theta_t - (1-\chi) l_t^*)$$

Which simplifies to

$$\bar{s} \left(1 + \frac{1-\chi}{\chi}\right) = \left(1 + \frac{1-\chi}{\chi}\right) \theta_t + \frac{1}{\chi} (\bar{g} - g_t^*) + \frac{1-\chi}{\chi} (-(1-\chi) l_t^*)$$

Multiplying both sides by  $\chi$ , we obtain

$$\bar{s} = \theta_t + (\bar{g} - g_t^*) - (1-\chi)^2 l_t^*$$

Using the labor supply equation (A.48), we can substitute out  $l_t^*$  to get

$$\bar{s} = \theta_t + (\bar{g} - g_t^*) - (1-\chi)^2 \left( -\frac{\mu}{1+\phi} + \hat{y}_t^* \right) \quad (\text{A.63})$$

where  $\hat{y}_t^*$  is the output gap.

Now the real exchange rate is defined as

$$q_t = \bar{s} + p_t^* - \bar{g}$$

Using the definition of the GDP deflator, this can be expanded to

$$q_t = \bar{s} - \bar{g} + g_t^* - \chi \theta_t \quad (\text{A.64})$$

The quantity  $g_t^* - \chi \theta_t$  can be derived from equation (A.63):

$$g_t^* - \chi \theta_t = \bar{g} - \bar{s} + (1-\chi) \theta_t - (1-\chi)^2 \left( -\frac{\mu}{1+\phi} + \hat{y}_t^* \right)$$

Plugging this into equation (A.64):

$$q_t = (1-\chi) \theta_t - (1-\chi)^2 \left( -\frac{\mu}{1+\phi} + \hat{y}_t^* \right)$$

Now we use the equilibrium condition for  $\theta_t$ , equation (A.51):

$$q_t = (1-\chi) \left( z_t - \alpha \hat{y}_t^* + \ln(1-\alpha) + \frac{\alpha}{1+\phi} \mu \right) - (1-\chi)^2 \left( -\frac{\mu}{1+\phi} + \hat{y}_t^* \right)$$



This simplifies to

$$q_t = (1 - \chi) z_t - (1 - \chi) (\alpha + (1 - \chi)) \left( \hat{y}_t^* - \frac{\mu}{1 + \phi} \right) \quad (\text{A.65})$$

There are two components to the real exchange rate: the  $1 - \chi$  elasticity derived in the floating-rate solution, and an additional term increasing in the output gap. Intuitively, an increase in  $z_t$  under floating rates leads to an appreciation of the real exchange rate. But under a currency peg, the real exchange rate can't rise so quickly (we derived earlier that the nominal exchange rate would rise even more, with a unit elasticity on  $z_t$ ). Instead, to keep the nominal exchange rate fixed, the real exchange rate either has to rise more slowly, or even fall. Mechanically, the central bank loosens policy relative to the floating-rate, currency-appreciation case, causing a boom (positive  $\hat{y}_t$ ). This boom reduces  $q_t$  relative to the floating-rate solution.

The value of  $\hat{y}_t^*$  is derived by simultaneously solving a differenced version of (A.63) along with the AS curve. First, starting with equation (A.63) and plugging in the terms-of-trade equation (A.51), we get:

$$\bar{s} = z_t - \alpha \hat{y}_t^* + \ln(1 - \alpha) + \frac{\alpha}{1 + \phi} \mu + (\bar{g} - g_t^*) - (1 - \chi)^2 \left( -\frac{\mu}{1 + \phi} + \hat{y}_t^* \right)$$

Without loss of generality, we can assume the peg is at parity ( $\bar{s} = 0$ ). The above equation, then taken together with the AS curve (A.50), gives the system:

$$\begin{aligned} \hat{y}_t^* &= \eta z_t + \eta (\bar{g} - g_t^*) + \eta \tau \\ \pi_t^{g^*} &= \beta E_t \pi_{t+1}^{g^*} + \zeta \hat{y}_t^* \\ g_t^* &= g_{t-1}^* + \pi_{t+1}^{g^*} \end{aligned}$$

in which the new constants are:

$$\begin{aligned} \eta &\equiv \frac{1}{\alpha + (1 - \chi)^2} \\ \tau &\equiv \ln(1 - \alpha) + \frac{\alpha - (1 - \chi)^2}{1 + \phi} \mu \end{aligned}$$

This system can be written in matrix form as:

$$\mathbf{x}_t = \Xi E_t \mathbf{x}_{t+1} + \Gamma \mathbf{x}_t + \Gamma_L \mathbf{x}_{t-1} + \Lambda z_t + \Omega$$

where

$$\mathbf{x}_t \equiv \begin{bmatrix} \hat{y}_t^* \\ \pi_t^{g^*} \\ g_t^* \end{bmatrix}$$

$$\Xi \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma \equiv \begin{bmatrix} 0 & 0 & -\eta \\ \zeta & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Gamma_L \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Lambda \equiv \begin{bmatrix} \eta \\ 0 \\ 0 \end{bmatrix}$$

$$\Omega \equiv \begin{bmatrix} \eta (\bar{g} + \tau) \\ 0 \\ 0 \end{bmatrix}$$

The solution to this recurrence relation depends on both contemporaneous  $z_t$  and lagged  $\mathbf{x}_{t-1}$ . The reason for the dependence on the lagged term is really dependence on  $g_{t-1}^*$ ; the closer the price level is to its “target” value, the less inflation is required at time  $t$ .

Consequently, we guess that the solution has the form:

$$\mathbf{x}_t = \mathbf{A}z_t + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{C} + \boldsymbol{\delta}_t$$

Using the method of undetermined coefficients, we find that:

$$\mathbf{A} = (\mathbf{I} - \boldsymbol{\Gamma} - a\boldsymbol{\Xi})^{-1} \boldsymbol{\Lambda}$$

$$\mathbf{B} = (\mathbf{I} - \boldsymbol{\Gamma} - \boldsymbol{\Xi})^{-1} \boldsymbol{\Gamma}_L$$

$$\mathbf{C} = (\mathbf{I} - \boldsymbol{\Gamma} - \boldsymbol{\Xi})^{-1} \boldsymbol{\Omega}$$

Additionally, assuming that  $\boldsymbol{\Xi}^{-1}(\mathbf{I} - \boldsymbol{\Gamma})$  has eigenvalues outside the unit circle, there is a unique locally bounded solution with  $\boldsymbol{\delta}_t = \mathbf{0}$  for all  $t$ . (This condition holds true for the parameter choices made in the main body of the paper.)

Of particular interest is the elasticity  $dq_t/dz_t$ . In computing this, we will assume that the economy starts out at a  $t = -1$  steady state with

$$g_{-1}^* = \bar{g} + \tau$$

A shock hits at  $t = 0$ . This choice of  $g_{-1}^*$  ensures we are truly at a steady state before the shock: if the shock  $z_0 = 0$ , then the output gap remains zero.

Given this setup, we can compute the response of the real exchange rate to the productivity shock using formula (A.65), to obtain:

$$\frac{dq_t}{dz_t} = \underbrace{(1 - \chi)}_{\text{flexible-price elasticity}} - \underbrace{(1 - \chi)(\alpha + (1 - \chi)) \left( \frac{\eta(1 - a\beta)}{1 - a\beta + \zeta\eta} \right)}_{\text{sticky-wage effect}}$$

### Commodity price elasticity

Under a peg, real commodity prices are no longer unit elastic to productivity. Intuitively, because wages in Australia cannot rise quickly, there is a temporary boom in commodity output when productivity rises, and thus commodity prices do not rise so quickly.

To understand how real exchange rates are related to commodity prices, then, we can

compare equations (A.56) and (A.65):

$$\frac{\partial q_t}{\partial z_t} = (1 - \chi) - (1 - \chi) (\alpha + (1 - \chi)) \frac{\partial \hat{y}_t^*}{\partial z_t}$$

$$\frac{\partial (p_{C,t} - p_t)}{\partial z_t} = 1 - \frac{1}{\eta} \frac{\partial \hat{y}_t^*}{\partial z_t}$$

Combining the two, we find that

$$\frac{\partial q_t}{\partial (p_{C,t} - p_t)} = \frac{(1 - \chi) - (1 - \chi) (\alpha + (1 - \chi)) \frac{\partial \hat{y}_t^*}{\partial z_t}}{1 - \frac{1}{\eta} \frac{\partial \hat{y}_t^*}{\partial z_t}}$$

This expression is easily computed using the impulse response function for  $\hat{y}_t^*$  developed above. As discussed in the main body of the paper, with the parameterization used in the paper, this is about a third as large as  $dq_t/d(p_{C,t} - p_t)$  under floating rates.

### Currency risk premium

As discussed in the section on floating rates, the currency risk premium is given by:

$$E_t (i_t^* - i_t + \Delta s_{t+1}) = f^*(z_t) - f(z_t)$$

The  $f(\cdot)$  terms are defined as the nonlinear terms from the Euler equation.

We need to derive the relationship between consumption and productivity in both countries, first to show that consumption growth is conditionally lognormal, and second to actually compute the values of  $f(z_t)$  and  $f^*(z_t)$ .

Starting with US consumption,

$$C_t = \kappa C_{F,t}^\chi C_{NT,t}^{1-\chi}$$

Using log utility and balanced trade, and market clearing for nontradables, we have:

$$C_t = \kappa (\alpha Y_{F,t})^\chi (Y_{NT,t})^{1-\chi}$$

Using the fact that labor shares are constant,

$$\begin{aligned}
C_t &= \kappa \left( \alpha Z_t L_F^\chi Y_{C,t}^{*1-\chi} \right)^\chi Z_t^{1-\chi} (1-\chi)^{1-\chi} \\
&= \kappa \left( \alpha Z_t L_{C,t}^{*1-\chi} \right)^\chi Z_t^{1-\chi} (1-\chi)^{1-\chi} \\
&= \alpha^\chi Z_t^\chi L_{C,t}^{*\chi(1-\chi)}
\end{aligned}$$

Putting things in logs:

$$c_t = \chi \ln \alpha + z_t + \chi (1-\chi) l_t^*$$

We can substitute out  $l_t^*$  using the output gap from equation (A.48):

$$c_t = \chi \ln \alpha + z_t + \chi (1-\chi) \left( -\frac{1}{1+\phi} \mu + (y_t^* - y_n^*) \right)$$

We showed previously (in the section on the real exchange rate under the peg) that

$$\frac{\partial \hat{y}_t^*}{\partial z_t} = \frac{\eta (1-a\beta)}{1-a\beta + \zeta \eta}$$

Thus, consumption can be written as

$$c_t = \left( 1 + \chi (1-\chi) \frac{\eta (1-a\beta)}{1-a\beta + \zeta \eta} \right) z_t + \text{const.}$$

Now, turning to Australian consumption, we can write it as:

$$C_t^* = \kappa C_{F,t}^{*\chi} C_{NT,t}^{*1-\chi}$$

Using log utility and balanced trade, and market clearing for nontradables, we have:

$$C_t^* = \kappa \left( (1-\alpha) Y_{F,t}^* \right)^\chi \left( Y_{NT,t}^* \right)^{1-\chi}$$

Using the fact that labor shares are constant,

$$\begin{aligned}
C_t^* &= \kappa \left( (1-\alpha) Z_t L_F^\chi Y_{C,t}^{*1-\chi} \right)^\chi (1-\chi)^{1-\chi} \\
&= \kappa \left( (1-\alpha) Z_t L_{C,t}^{*1-\chi} \right)^\chi (1-\chi)^{1-\chi} \\
&= (1-\alpha)^\chi Z_t^\chi L_{C,t}^{*\chi(1-\chi)}
\end{aligned}$$

Putting things in logs:

$$c_t^* = \chi \ln(1 - \alpha) + \chi z_t + \chi(1 - \chi) l_t^*$$

We can substitute out  $l_t^*$  using the output gap from equation (A.48):

$$c_t^* = \chi \ln(1 - \alpha) + \chi z_t + \chi(1 - \chi) \left( -\frac{1}{1 + \phi} \mu + (y_t^* - y_n^*) \right)$$

We showed previously (in the section on the real exchange rate under the peg) that

$$\frac{\partial y_t^*}{\partial z_t} = \frac{\eta(1 - a\beta)}{1 - a\beta + \zeta\eta}$$

Thus, consumption can be written as

$$c_t^* = \left( \chi + \chi(1 - \chi) \frac{\eta(1 - a\beta)}{1 - a\beta + \zeta\eta} \right) z_t + \text{const.}$$

To sum up, under a peg, consumption in both the US and Australia have some extra elasticity to productivity shocks, since they tend to cause a temporary boom in Australia. The extra elasticity is the same in both countries.

Letting  $\aleph \equiv \chi(1 - \chi) \frac{\eta(1 - a\beta)}{1 - a\beta + \zeta\eta}$ , we can start discussing the nominal currency risk premium.

That premium has to equal zero, since there is no nominal exchange rate risk. Thus,  $f(z_t) = f^*(z_t)$  to satisfy the condition of Backus, Foresi, and Telmer (2001). Computing  $f(z_t)$  using the fact that consumption growth is conditionally lognormal:

$$f(z_t) = - \left( \frac{1}{2} V_t \Delta c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) \right)$$

Since, in the US, inflation is zero, we simply find:

$$f(z_t) = -\frac{1}{2} V_t \Delta c_{t+1}$$

Substituting in our expression for US consumption, we have

$$f(z_t) = f^*(z_t) = -\frac{1}{2} (1 + \aleph) \sigma_\varepsilon^2$$

Thus the US and Australian nominal interest rates are (in equilibrium):

$$i_t = \rho + E_t \Delta c_{t+1} + E_t \pi_{t+1} - \frac{1}{2} (1 + \aleph) \sigma_\varepsilon^2$$

$$i_t^* = \rho + E_t \Delta c_{t+1}^* + E_t \pi_{t+1}^* - \frac{1}{2} (1 + \aleph) \sigma_\varepsilon^2$$

The two have to be equal  $i_t = i_t^*$  to rule out arbitrage in currency markets. This equality can be shown using the Backus-Smith (1993) fact that the change in the real exchange rate, which, in this case, is just the change in relative price levels, must equal the change in relative consumption.

The real currency risk premium is computed as:

$$E_t (r_t^* - r_t + \Delta q_{t+1}) = \frac{1}{2} (V_t \Delta c_{t+1} - V_t \Delta c_{t+1}^*)$$

$$= \frac{1}{2} \left( (1 + \aleph)^2 - (\chi - \aleph)^2 \right) \sigma_\varepsilon^2$$

## A.4 Technical Results

### A.4.1 Optimal wage setting

This is the wage reset problem for a household in the monetary model. To avoid excessive complication from solving a special case, we solve the problem with a general utility function, and drop specific notation from the above problem (for example, asterisks for Australia). Also, we use the more standard notation that  $N$  is labor hours and  $M$  is the nominal SDF (these are  $L$  and  $N$ , above).

General household utility function for household type  $j$  resetting their wage at time  $t$ :

$$E_t \sum_{k=0}^{\infty} \beta^k u \left( C_{t+k}^j, N_{t+k}^j \right)$$

The maximization is subject to the budget constraint:

$$P_{t+k} C_{t+k}^j + E_t (M_{t+k+1} A_{t+k+1}) \leq A_t + W_r N_{t+k}^j + D_{t+k}^j$$

where  $W_r$  is the reset wage.

Also subject to the monopolist's demand curve:

$$N_{t+k}^j = \left( \frac{W_r}{W_{t+k}} \right)^{-\varepsilon} N_{t+k}$$

Only some of the future states are affected by the choice of wage today; namely, states in which the wage hasn't been reset since today. We can drop all other terms from the objective function, leaving:

$$E_t \sum_{k=0}^{\infty} (\beta\omega)^k u \left( C_{t+k}^j, N_{t+k}^j \right)$$

where  $\omega$  is the probability of *not* resetting your price in a particular period.

The choice variables are the sequences  $\{C_{t+k}^j, N_{t+k}^j\}$  and  $W_r$ . By using the two constraints, we can substitute out both of the sequences (consumption and labor supply) and reduce the number of choice variables to one, namely the reset wage.

In substituting in for consumption, we use the budget constraint:

$$\begin{aligned} C_{t+k}^j &= \frac{A_t + W_r N_{t+k}^j + D_{t+k}^j - E_t (M_{t+k+1} A_{t+k+1})}{P_{t+k}} \\ &= \frac{A_t + W_r^{1-\varepsilon} W_{t+k}^\varepsilon N_{t+k} + D_{t+k}^j - E_t (M_{t+k+1} A_{t+k+1})}{P_{t+k}} \end{aligned}$$

Then the first-order condition with respect to the reset wage is:

$$\begin{aligned} 0 &= E_t \sum_{k=0}^{\infty} (\beta\omega)^k \left[ U_C \left( C_{t+k}^j, N_{t+k}^j \right) \frac{(1-\varepsilon) W_r^{-\varepsilon} W_{t+k}^\varepsilon N_{t+k}}{P_{t+k}} + \right. \\ &\quad \left. U_N \left( C_{t+k}^j, N_{t+k}^j \right) (-\varepsilon) \left( \frac{W_r}{W_{t+k}} \right)^{-\varepsilon-1} \left( \frac{1}{W_{t+k}} \right) N_{t+k} \right] \\ 0 &= E_t \sum_{k=0}^{\infty} (\beta\omega)^k N_{t+k} \left[ U_C \left( \frac{W_r}{W_{t+k}} \right)^{-\varepsilon} \frac{1}{P_{t+k}} + U_N \mathcal{M} \left( \frac{W_r}{W_{t+k}} \right)^{-\varepsilon-1} \left( \frac{1}{W_{t+k}} \right) \right] \\ 0 &= E_t \sum_{k=0}^{\infty} (\beta\omega)^k N_{t+k}^j \left[ U_C \frac{1}{P_{t+k}} + U_N \mathcal{M} \left( \frac{W_r}{W_{t+k}} \right)^{-1} \left( \frac{1}{W_{t+k}} \right) \right] \\ 0 &= E_t \sum_{k=0}^{\infty} (\beta\omega)^k N_{t+k}^j \left[ U_C \frac{1}{P_{t+k}} + U_N \mathcal{M} \frac{1}{W_r} \right] \\ 0 &= E_t \sum_{k=0}^{\infty} (\beta\omega)^k N_{t+k}^j \left[ U_C \frac{W_r}{P_{t+k}} + U_N \mathcal{M} \right] \end{aligned}$$



Now define

$$MRS_{t+k}^j \equiv -\frac{U_N(C_{t+k}^j, N_{t+k}^j)}{U_C(C_{t+k}^j, N_{t+k}^j)}$$

Then this can be written as

$$0 = E_t \sum_{k=0}^{\infty} (\beta\omega)^k N_{t+k}^j \left[ U_C \frac{W_r}{P_{t+k}} + U_N \mathcal{M} \right]$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta\omega)^k N_{t+k}^j U_C \left[ \frac{W_r}{P_{t+k}} - MRS_{t+k}^j \mathcal{M} \right]$$

Note that  $U_C$  is not a constant (it depends consumption at time  $t+k$ ) so we cannot divide it out.

In a flexible-price steady state with zero inflation, we would have

$$\frac{W^{ss}}{P^{ss}} = MRS^{ss} \mathcal{M}$$

The reason is that we would be solving the same problem as above, except only the first term in the series would exist, since  $\omega = 0$ . Also, everyone would have the same wage,  $W^{ss}$ , rather than there being individual wages and marginal rates of substitution.

Now we use this fact to log-linearize the sticky-price first order condition around the flexible-price condition. The log-linearization is done for the endogenous variables  $N_{t+k}^j$ ,  $U_C$ ,  $W_r$ ,  $P_{t+k}$ , and  $MRS_{t+k}^j$ .

The idea is simply to express each term in the summation as its linear Taylor Series approximation, then replace  $dX$  with  $Xdx$ . This is perhaps the simplest way to log-linearize.

We have

$$\begin{aligned}
0 = E_t \sum_{k=0}^{\infty} (\beta\omega)^k & \left[ U_C^{ss} \underbrace{\left( \frac{W^{ss}}{P^{ss}} - \mathcal{M}MRS^{ss} \right)}_0 N^{ss} dn_{t+k}^j \right. \\
& + N^{ss} \underbrace{\left( \frac{W^{ss}}{P^{ss}} - \mathcal{M}MRS^{ss} \right)}_0 U_C^{ss} du_c \\
& + \frac{N^{ss} U_C^{ss}}{P^{ss}} W^{ss} dw_r - \frac{N^{ss} U_C^{ss} W_r^{ss}}{(P^{ss})^2} P^{ss} dp_{t+k} \\
& \left. + N^{ss} U_C^{ss} \mathcal{M}MRS^{ss} dmrs_{t+k}^j \right]
\end{aligned}$$

The first two terms drop out because they're being multiplied by zero. The last three terms all have the constant coefficient  $W_r^{ss}/P^{ss}$  (or what is equivalent to it). We can divide this out to get:

$$0 = E_t \sum_{k=0}^{\infty} (\beta\omega)^k \left[ dw_r - dp_{t+k} - dmrs_{t+k}^j \right]$$

Expressing this in terms of log-deviations from the steady state:

$$\begin{aligned}
0 &= E_t \sum_{k=0}^{\infty} (\beta\omega)^k \left[ (w_r - w^{ss}) - (p_{t+k} - p^{ss}) - (mrs_{t+k}^j - mrs^{ss}) \right] \\
0 &= -(1 - \beta\omega)^{-1} \mu + E_t \sum_{k=0}^{\infty} (\beta\omega)^k \left[ w_r - p_{t+k} - mrs_{t+k}^j \right] \\
(1 - \beta\omega)^{-1} w_r &= (1 - \beta\omega)^{-1} \mu + E_t \sum_{k=0}^{\infty} (\beta\omega)^k \left[ p_{t+k} + mrs_{t+k}^j \right] \\
w_r &= \mu + (1 - \beta\omega) E_t \sum_{k=0}^{\infty} (\beta\omega)^k \left[ p_{t+k} + mrs_{t+k}^j \right]
\end{aligned}$$

Under the assumption that

$$u \left( C_{t+k}^j, N_{t+k}^j \right) \equiv \frac{C_{t+k}^{j1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{j1+\phi}}{1+\phi}$$

Then

$$\begin{aligned}
 mrs_{t+k}^j &= \ln \left[ -\frac{u_N}{u_C} \right] \\
 &= \ln \left[ \frac{N_{t+k}^{j\phi}}{C_{t+k}^{j-\sigma}} \right] \\
 &= \phi n_{t+k}^j + \sigma c_{t+k}^j
 \end{aligned}$$

We can also define a marginal rate of substitution using aggregate variables:

$$mrs_{t+k} = \phi n_{t+k} + \sigma c_{t+k}$$

Using the fact that there are complete markets domestically,  $c_{t+k}^j = c_{t+k}$ . So then

$$mrs_{t+k}^j = mrs_{t+k} + \phi (n_{t+k}^j - n_{t+k})$$

The difference in labor supply is related to the difference in wages by the demand function:

$$\begin{aligned}
 N_{t+k}^j &= \left( \frac{W_r}{W_{t+k}} \right)^{-\varepsilon} N_{t+k} \\
 n_{t+k}^j - n_{t+k} &= -\varepsilon (w_r - w_{t+k})
 \end{aligned}$$

So we have

$$mrs_{t+k}^j = mrs_{t+k} - \phi \varepsilon (w_r - w_{t+k})$$

Now we can rewrite the optimal wage equation:

$$\begin{aligned}
w_r &= \mu + (1 - \beta\omega) E_t \sum_{k=0}^{\infty} (\beta\omega)^k [p_{t+k} + mrs_{t+k} - \phi\varepsilon(w_r - w_{t+k})] \\
w_r &= \frac{1 - \beta\omega}{1 + \phi\varepsilon} E_t \sum_{k=0}^{\infty} (\beta\omega)^k [\mu + p_{t+k} + mrs_{t+k} + \phi\varepsilon w_{t+k}] \\
w_r &= \frac{1 - \beta\omega}{1 + \phi\varepsilon} E_t \sum_{k=0}^{\infty} (\beta\omega)^k [\mu + p_{t+k} + mrs_{t+k} - w_{t+k} + (1 + \phi\varepsilon) w_{t+k}] \\
w_r &= \frac{1 - \beta\omega}{1 + \phi\varepsilon} E_t \sum_{k=0}^{\infty} (\beta\omega)^k [(1 + \phi\varepsilon) w_{t+k} - [(w_{t+k} - p_{t+k} - mrs_{t+k}) - \mu]] \\
w_r &= \frac{1 - \beta\omega}{1 + \phi\varepsilon} E_t \sum_{k=0}^{\infty} (\beta\omega)^k [(1 + \phi\varepsilon) w_{t+k} - [\mu_{t+k} - \mu]]
\end{aligned}$$

where  $\mu_{t+k} \equiv (w_{t+k} - p_{t+k}) - mrs_{t+k}$  is the a measure of *aggregate* markups, and would be equal to  $\mu$  if prices were flexible.

Now let's introduce the notation  $w_{r,t}$  as the solution to the wage-setting problem at time period  $t$ , and  $w_{r,t+1}$  as the solution at  $t + 1$ , etc. Then we have

$$\begin{aligned}
w_{r,t} - \beta\omega E_t(w_{r,t+1}) &= \frac{1 - \beta\omega}{1 + \phi\varepsilon} E_t \sum_{k=0}^{\infty} (\beta\omega)^k [(1 + \phi\varepsilon) w_{t+k} - [\mu_{t+k} - \mu]] \\
&\quad - \frac{1 - \beta\omega}{1 + \phi\varepsilon} E_t \sum_{k=0}^{\infty} (\beta\omega)^k [(1 + \phi\varepsilon) w_{t+k+1} - [\mu_{t+k+1} - \mu]] \\
&= \frac{1 - \beta\omega}{1 + \phi\varepsilon} [(1 + \phi\varepsilon) w_t - [\mu_t - \mu]] \\
w_{r,t} &= \beta\omega E_t(w_{r,t+1}) + (1 - \beta\omega) w_t - \frac{1 - \beta\omega}{1 + \phi\varepsilon} (\mu_t - \mu)
\end{aligned}$$

Up to a log-linearization, the wage is

$$w_t = \omega w_{t-1} + (1 - \omega) w_{r,t}$$

Thus wage inflation is

$$\begin{aligned}
w_t - w_{t-1} &= (\omega - 1) w_{t-1} + (1 - \omega) w_{r,t} \\
&= (1 - \omega) (w_{r,t} - w_{t-1}) \\
&= (1 - \omega) \left( \beta \omega E_t (w_{r,t+1}) + (1 - \beta \omega) w_t - \frac{1 - \beta \omega}{1 + \phi \varepsilon} (\mu_t - \mu) - w_{t-1} \right) \\
&= (1 - \omega) \left( \beta \omega E_t \left( \frac{w_{t+1} - \omega w_t}{1 - \omega} \right) + (1 - \beta \omega) w_t \right. \\
&\quad \left. - \frac{1 - \beta \omega}{1 + \phi \varepsilon} (\mu_t - \mu) - w_{t-1} \right) \\
w_t - w_{t-1} &= (1 - \omega) \left( \frac{\beta \omega}{1 - \omega} E_t (w_{t+1}) + \left[ -\frac{\beta \omega^2}{1 - \omega} + (1 - \beta \omega) \right] w_t \right. \\
&\quad \left. - w_{t-1} - \frac{1 - \beta \omega}{1 + \phi \varepsilon} (\mu_t - \mu) \right) \\
&= \beta \omega E_t (w_{t+1}) + [(1 - \beta \omega) (1 - \omega) - \beta \omega^2] w_t - (1 - \omega) w_{t-1} \\
&\quad - \frac{(1 - \omega) (1 - \beta \omega)}{1 + \phi \varepsilon} (\mu_t - \mu) \\
&= \beta \omega E_t (w_{t+1}) + [1 - \omega - \beta \omega + \beta \omega^2 - \beta \omega^2] w_t - (1 - \omega) w_{t-1} \\
&\quad - \frac{(1 - \omega) (1 - \beta \omega)}{1 + \phi \varepsilon} (\mu_t - \mu) \\
&= \beta \omega E_t (w_{t+1}) + [1 - \omega - \beta \omega] w_t - (1 - \omega) w_{t-1} \\
&\quad - \frac{(1 - \omega) (1 - \beta \omega)}{1 + \phi \varepsilon} (\mu_t - \mu) \\
&= \beta \omega E_t (w_{t+1} - w_t) - (1 - \omega) (w_t - w_{t-1}) \\
&\quad - \frac{(1 - \omega) (1 - \beta \omega)}{1 + \phi \varepsilon} (\mu_t - \mu)
\end{aligned}$$

$$(w_t - w_{t-1})(1 - (1 - \omega)) = \beta\omega E_t(w_{t+1} - w_t) - (1 - \omega)(w_t - w_{t-1}) - \frac{(1 - \omega)(1 - \beta\omega)}{1 + \phi\varepsilon}(\mu_t - \mu)$$

$$w_t - w_{t-1} = \beta E_t(w_{t+1} - w_t) - \frac{(1 - \omega)(1 - \beta\omega)}{\omega(1 + \phi\varepsilon)}(\mu_t - \mu)$$

$$\pi_t^w = \beta E_t(\pi_{t+1}^w) - \lambda(\mu_t - \mu)$$

## A.5 Data Construction

- **Exports and imports:** To get nominal import and export values by SITC code, I use the annual NBER-Comtrade dataset available on the NBER's website. The classification of SITC codes into "commodity" or "finished good" follows from Ready, Roussanov, and Ward (2015), except I do the classification at the 2-digit level rather than the 4-digit level, since their detailed classification is not posted online. I classify a 2-digit sector as a commodity-producing sector if the majority of the 4-digit sub-sectors are classified as such by Ready, Roussanov, and Ward.

Export-weighted indices of commodity prices and commodity basis are constructed using the NBER-Comtrade dataset, extended to 2011 by the MIT Observatory of Economic Complexity. Exports are mapped to CRB commodity tickers by SITC code. The commodity data are discussed separately.

**Currency data (spot rates and forward rates).** I start with all currencies with spot and 1-month forward prices available on Datastream. Within Datastream, there are several datasets providing these data; when there are conflicts, I use WMR, Thomson Reuters, BBI, and HBSC in that order of preference. I use end-of-month data.

Additionally, I make the following adjustments. First, I exclude the “offshore” Thai baht, which includes data errors, and I use the “onshore” values instead. I also remove zeros from Barclays data (in Datastream) for the Belgian franc spot and forward rate series, replacing them with a “missing observation” code. I also mark as missing the BEF forward rate series from Dec-1989 to Nov-1991, as the given values are incorrect.

I compute forward discounts from forward and spot prices, and use them to measure interest rate differentials.

I filter out currencies during periods in which they did not have a floating exchange rate. “Not floating” is defined as scoring less than 9 on the Ilzetzki, Reinhart and Rogoff (2011) fine classification of exchange rate regimes. This is a monthly classification, so when I take time-series averages, I use only the periods during which the currency was actually floating.

To deal with the formation of the Euro, I allow the Deutsche Mark to continue as the Euro after accession. Whether the other Eurozone currencies are included depends on the application; for long-run cross-sectional comparisons, I exclude them, because many of them became highly correlated with the mark before accession, and thus do not really constitute independent observations.

I use two subsamples. For constructing trading strategies, I simply use the entire set of floating-rate currencies on the dates that are available, that have whatever data are necessary to include or exclude them from a trade. This includes 40 currencies (not necessarily all at the same time).

I also use a “long-history” sample for cross-sectional comparisons, so that various moments can be measured without large standard errors. To construct it, I start with

Hassan and Mano's (2015) selection of 36 currencies traded between 1989 and 2007, and I filter currencies that left the sample early due to Eurozone accession (most of them in 1999). This sample includes following countries: Australia, Canada, Czech Republic, Denmark, Germany, Hungary, India, Japan, Malaysia, Mexico, New Zealand, Norway, the Philippines, Poland, South Africa, Sweden, Switzerland, Taiwan, Thailand, the United Kingdom, South Korea, and Singapore.

- **Commodity data (spot prices and basis).** Spot price and commodity basis data are provided by the Commodity Research Bureau (CRB). CRB provides a CD with historical pricing data on the whole futures curve for many (not just the most-traded) futures.

Since true spot market prices are often observed imperfectly, I follow Gorton and Rouwenhorst (2006) and use as a proxy the price of the first nearby future.

Basis measures the carry earned from holding a long futures position. It is computed as the first nearby futures price divided by the second nearby futures price, annualized by a factor of  $365/(\text{calendar days between the expiry of the two futures})$ , and expressed as an annual arithmetic net return.

For LME-traded metals, the spot price is treated as the first nearby price, the 3-month forward price is used as the second nearby price, and the time between the two contracts is assumed to be 90 days.

Commodity returns are calculated as the excess return on a strategy of going long the nearest contract and rolling into the next-nearest the month-end before the original contract is scheduled to expire. The contract is assumed to be fully collateralized by T-bills. See Gorton, Hayashi, and Rouwenhorst (2006) for details.

The set of commodities is the same as in Dhume (2010), with the addition of Iron Ore



62% Fe (TSI) for delivery to Tianjin, traded on Globex.

- **Consumer prices.** I use monthly, quarterly, and annual consumer price index data from the International Monetary Fund's IFS (International Financial Statistics) dataset.
- **Export share of NGDP:** To get nominal exports as a share of nominal GDP, I use annual data from the Penn World Table, version 8.1, available from the University of Groningen's website.
- **U.S. consumption:** I use quarterly real GDP data from the BLS. Consumption is the sum of nondurable and services consumption.
- **Oil prices:** I use end-of-month spot values of West Texas Intermediate (WTI) oil, taken from FRED (series *MCOILWTICO*).
- **U.S. state-level nominal wages:** My wage data are from FRED, titled "Average hourly earnings of all employees: total private in [State]". These series are not seasonally adjusted and come from the Bureau of Labor Statistics. A sample series identifier, for Alabama, is *SMU01000000500000003*. These data are monthly.
- **CMXPTR.** To develop a measure of commodity exports, I follow Ready, Roussanov, and Ward (2014) and introduce the variable CMXPTR, defined for country  $j$  for year  $t$  as

$$\text{CMXPTR}_t^j \equiv \left( \frac{\text{net exports of basic goods} + \text{net imports of complex goods}}{\text{gross trade in all goods}} \right)_t^j$$

When aggregating across time, I use the average CMXPTR value during the period 1985-1994. Figure 6 shows that the time series of CMXPTR seem quite intuitive.

## Appendix B

### Derivations for Chapter 3

We model a small open economy whose currency is the *peso*. Consumption in this economy consists of finished goods imported from abroad and locally produced nontradables, combined in a Cobb-Douglas consumption aggregate:

$$C = \gamma C_F^\chi C_{NT}^{1-\chi}$$

The outside world uses the *dollar* and finished goods always cost one dollar. Thus, the nominal exchange rate  $S$ , expressed as the value of the dollar in terms of pesos, gives the peso price of the finished good. The price index is therefore

$$P = S^\chi P_{NT}^{1-\chi}$$

where  $P_{NT}$  is the peso price of the nontradable good.

The country exports commodities to pay for its imports. The ratio of the dollar commodity price to the dollar finished good price is simply  $P_C^*$ , and is the terms of trade.

The only asset available for external financing is dollar bonds, traded at an exogenous gross nominal interest rate of  $I^*$ . This notation means that one dollar (equivalently, one unit of the finished good) becomes  $I^*$  dollars in one period from now.

The model is completely deterministic and has two periods. Before these periods begin, we will assume the economy is in a “steady state” denoted with superscript  $d$ . We will

consider the impact over two periods of a permanent change in the value of  $P_C^*$ .

## B.1 Consumer's problem

The consumer maximizes

$$U = \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L(j)_t^{1+\phi}}{1+\phi} \right) + \beta \left( \frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \frac{L_{t+1}^{1+\phi}}{1+\phi} \right)$$

Long-run budget constraint:

$$P_{t+1}C_{t+1} + S_{t+1}D_t I_t^* = W_{t+1}(1+\tau)L_{t+1} + S_{t+1}A^s I_t^* + T_{t+1}$$

where  $D_t$  is *dollar* debt issued at time  $t$  and  $A^s$  is the initial level of dollar assets held by the government (say reserves).

Short-run budget constraint:

$$P_t C_t + S_t D^s I^{*s} = W_t(j)(1+\tau)L_t(j) + S_t D_t + T_t(j) + S_t A^s (I_t^* - 1)$$

The term  $\tau$  represents a wage subsidy paid for by lump sum taxes, which are a component of  $T_{t+1}$  and  $T_t(j)$ .  $T_t(j)$  also includes transfers within the country to ensure that everyone has the same consumption (the idea is that there is complete risk sharing within the country, although not across countries). The wage subsidy enables the flexible-price allocation without constraints to be efficient in the presence of monopolistic competition.

The subsidy is set so that  $1 + \tau$  is equal to the wage markup —  $1 + \tau = \varepsilon / (\varepsilon - 1)$ .

The final term is the interest earned on the reserves.

Short-run aggregate budget constraint (integrate over all individual budget constraints):

$$P_t C_t + S_t D^s I^{*s} = W_t L_t + S_t D_t + S_t A^s (I_t^* - 1)$$

## B.2 Production

There are two sectors of production:

$$Y_{C,t} = \underbrace{\left( \int L_{C,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}}_{L_{C,t}}$$
$$Y_{NT,t} = \underbrace{\left( \int L_{NT,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}}_{L_{NT,t}}$$

Output is given by:

$$Y_t = Y_{C,t} + Y_{NT,t}$$

## B.3 Optimal wage setting

Reset wage  $w_r$  is a choice variable in household problem.

Wage enters short-run budget constraint directly, and it also enters indirectly through labor demand:

$$L_t(j) = \left( \frac{W_r}{W_t} \right)^{-\varepsilon} L_t$$

where

$$L_t(j) = L_{C,t}(j) + L_{NT,t}(j)$$
$$L_t = \left( \int L_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$W_t = \left( \int W_t(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$$

## B.4 Steady state allocation

We will imagine the economy as existing in a sustainable steady state until it is hit by a terms of trade shock at the beginning of period  $t$  (the short-run period).

The key assumptions for a steady state are:

- The external dollar interest rate  $I^{*s} = 1/\beta$ . (If not, then consumers would have either increasing or decreasing consumption paths.)
- Monetary policy is conducted *during the steady state* such that the price level and/or exchange rate are constant. (The two are equivalent since there are no terms of trade shocks. The key point is that we want real debt constant in steady state.) Standard assumption, see Gali-Monacelli (2005).

Without loss of generality, we will measure finished good in units so they have the same price as the commodity in the steady state. Thus the terms of trade in steady state is  $P_C^{*s} = 1$ .

Any strictly positive values for the steady state debt and reserves values, which we will denote  $D^s$  and  $A^s$ , can lead to a steady state, so long as those two values are equal (otherwise the country would have to run a persistent trade surplus or deficit to make up the difference).

Because the exchange rate is constant in steady state, the peso nominal rate equals the dollar nominal rate of  $1/\beta$ . Thus the Euler equation tells us that consumption will be constant:

$$C_t^s = C_{t+1}^s$$

Trade is also balanced because the two periods are identical (we can conjecture this and check it):

$$C_t^s = L_t^s$$

$$C_{t+1}^s = L_{t+1}^s$$

Consumption-leisure optimality condition implies

$$1 = C_t^\sigma L_t^\phi$$

Output in both periods is

$$Y_t = 1$$

And this equals consumption and labor supply.

## B.5 Short run BC

First we log-linearize the budget constraints to put consumption in terms of output. In the short run, the budget constraint is:

$$P_t C_t + S_t D^s I^{*s} = W_t L_t + S_t D_t + S_t A^s (I^{*s} - 1)$$

which is log-linearized as:

$$\begin{aligned} (P^s C^s) (p_t - p^s + c_t - c^s) + (S^s D^s I^{*s}) (s_t - s^s) &= (G^s Y^s) (g_t - g^s + y_t - y^s) \\ &+ (S^s D^s) (s_t - s^s + d_t - d^s) \\ &+ (S^s A^s I^{*s}) (s_t - s^s) \\ &- (S^s A^s) (s_t - s^s) \end{aligned}$$

Noting that  $P^s C^s = G^s Y^s$ , we can simplify this to

$$\begin{aligned} (G^s Y^s) (p_t + c_t) + (S^s D^s I^{*s}) (s_t - s^s) &= (G^s Y^s) (g_t + y_t) \\ &+ (S^s D^s) (s_t - s^s + d_t - d^s) \\ &+ (S^s A^s I^{*s}) (s_t - s^s) \\ &- (S^s A^s) (s_t - s^s) \end{aligned}$$

Substituting in steady state values ( $S^s = 1$  and  $I^{*s} = 1/\beta$ ), we have

$$\begin{aligned} (G^s Y^s) (p_t + c_t) + \frac{1}{\beta} D^s s_t &= (G^s Y^s) (g_t + y_t) + D^s (s_t + d_t - d^s) \\ &+ \frac{1}{\beta} A^s (s_t - s^s) - A^s (s_t - s^s) \end{aligned}$$

Letting  $\alpha = S^s D^s / G^s Y^s$ , we have

$$(p_t + c_t) + \frac{\alpha}{\beta} s_t = (g_t + y_t) + \alpha (s_t + d_t - d^s) + \left( \frac{1}{\beta} - 1 \right) \alpha (s_t - s^s)$$

Combining like terms, we have

$$c_t = y_t + \chi (p_C^* - p_C^{*s}) + \alpha (d_t - d^s)$$

The reason that depreciation of the peso has no effect on reducing debt is that it also reduces the assets that can be used to pay debt. These effects exactly offset at the steady state.

## B.6 Long run BC

The long-run budget constraint is

$$P_{t+1} C_{t+1} + S_{t+1} D_t I_t^* = W_{t+1} L_{t+1} + S_{t+1} A^s I_t^*$$

which is log-linearized as:

$$(P^s C^s) (p_{t+1} - p^s + c_{t+1} - c^s) = (G^s Y^s) (g_{t+1} - g^s + y_{t+1} - y^s) + (S^s D^s I^{*s}) (s_{t+1} - s^s + d_t - d^s + i_t^* - i^s) + (S^s A^s I^s) (s_{t+1} - s^s + i_t^* - i^s)$$

Noting that  $P^s C^s = G^s Y^s$ , we can simplify this to

$$(p_{t+1} + c_{t+1}) + \frac{\alpha}{\beta} (s_{t+1} - s^s + d_t - d^s + i_t^* - i^s) = (g_{t+1} + y_{t+1}) + \frac{\alpha}{\beta} (s_{t+1} - s^s + i_t^* - i^s)$$

Collecting like terms, we have

$$c_{t+1} = y_{t+1} + \chi (p_C^* - p_C^{*s}) - \frac{\alpha}{\beta} (d_t - d^s)$$

## B.7 The nominal interest rate

The dollar interest rate  $i^{*s} = -\rho$  and does not change out of steady state. Due to covered interest parity, this implies the peso interest rate is:

$$i_t = \rho + (s_{t+1} - s_t)$$

The change in the exchange rate is related to the change in the terms of trade:

$$\begin{aligned} s_{t+1} - s_t &= p_{F,t+1} - p_{F,t} \\ &= p_{F,t+1} - p_{t+1} - p_{F,t} + p_t + \pi_{t+1} \\ &= -p_{C,t+1}^* - p_{C,t}^* - \pi_{t+1} \end{aligned}$$

Given that the terms of trade does not change from  $t$  to  $t + 1$  (it is assumed to be shocked permanently), this term just equals the inflation rate  $\pi_{t+1}$ . Consequently, the nominal interest rate is:

$$i_t = \pi_{t+1} + \rho$$

## B.8 Efficient allocation

It will be useful to study the efficient allocation, in other words, the allocation that occurs when wages are flexible and there is no borrowing constraint (which is efficient due to the First Welfare Theorem).

First, we begin by working out  $Y_t^e$ , the efficient rate of output in the short run. To compute this, note that the household's consumption-leisure optimality condition equates the real wage (including wage subsidy) to the ratio of the marginal utility of leisure and the marginal utility of consumption, marked up due to monopolistic competition:

$$\frac{W_t^e (1 + \tau)}{P_t^e} = \mathcal{M} \frac{L_t^{e\phi}}{C_t^{e-\sigma}}$$



Noting that  $L_t^e = Y_t^e$ , and  $W_t^e/P_t^e = P_C^{*\chi}$ , we have

$$\chi p_C^* = \phi y_t^e + \sigma c_t^e$$

Substituting in the short-run budget constraint for consumption, we obtain

$$y_t^e = \frac{\chi(1-\sigma)}{\phi+\sigma} p_C^* - \frac{\alpha\sigma}{\phi+\sigma} (d_t^e - d^s)$$

The same consumption-leisure condition holds in the long run, but the budget constraint is different. Substituting in this different budget constraint leads to

$$y_{t+1}^e = \frac{\chi(1-\sigma)}{\phi+\sigma} p_C^* + \frac{\alpha\sigma}{\beta(\phi+\sigma)} (d_t^e - d^s)$$

We will conjecture and then verify that there is an equilibrium in which the debt constraint does not bind. In such an equilibrium, the Euler equation is

$$c_{t+1} - c_t = \frac{1}{\sigma} (i_t - \pi_{t+1} - \rho)$$

Substituting in the budget constraints, we have

$$y_{t+1}^e + \chi p_C^* - \frac{\alpha}{\beta} (d_t^e - d^s) - y_t^e - \chi p_C^* - \alpha (d_t^e - d^s) = 0$$

(it can be shown that  $i_t - \pi_{t+1} - \rho = 0$ ).

This implies that

$$y_{t+1}^e - y_t^e = \alpha \left(1 + \frac{1}{\beta}\right) (d_t^e - d^s)$$

Intuition: higher debt implies you think it is better to consume today than tomorrow (according to the intertemporal rate of substitution given by the real interest rate of 0%) thus you work more tomorrow than today.

Taking the same difference from the consumption-leisure conditions, we have:

$$y_{t+1}^e - y_t^e = \alpha \left(1 + \frac{1}{\beta}\right) \frac{\sigma}{(\phi+\sigma)} (d_t^e - d^s)$$

Intuition: higher debt implies you think it's better to work tomorrow than today (comparing

consumption to leisure). Thus, output is higher tomorrow.

The only way both of these conditions can be true simultaneously is if  $d_t^e = d^s$ , in other words, no new debt is issued on net, thus the debt constraint is not binding.

Plugging this into the equation for output, we have

$$y_t^e = y_{t+1}^e = \frac{\chi(1-\sigma)}{\phi+\sigma} p_C^*$$

And solving for consumption using the budget constraints, we obtain:

$$\begin{aligned} c_t^e &= y_t^e + \chi(p_C^* - p_C^{*s}) \\ &= \chi \left( 1 + \frac{(1-\sigma)}{\phi+\sigma} \right) p_C^* \\ &= \frac{\chi(1+\phi)}{\phi+\sigma} p_C^* \\ c_{t+1}^e &= y_{t+1}^e + \chi(p_C^* - p_C^{*s}) \\ &= \frac{\chi(1+\phi)}{\phi+\sigma} p_C^* \end{aligned}$$

## B.9 Natural allocation

It will be useful to introduce  $Y_t^n$ , the natural rate of output in the short run. This is the level of output in the short run that would occur if all households were able to reset their wages freely in the short run, but still with the assumption of a borrowing constraint.

As we have shown, the efficient allocation involves no new debt issuance, so the borrowing constraint doesn't bind under a ToT shock, so long as wages are flexible. Thus the natural allocation is identical to the efficient allocation.

## B.10 Actual allocation

### B.10.1 Phillips Curve

Optimal wage setting implies that the reset wage (for the fraction  $1 - \omega$  of people who can reset) is

$$\frac{W_r}{P_t} (1 + \tau) = \mathcal{M} \frac{L_r^\phi}{C_t^{-\sigma}}$$

Taking logs of both sides, we obtain:

$$w_r = \phi l_r + \sigma c_t + p_t$$

Substituting in the short run budget constraint and the labor demand curve, we obtain:

$$w_r - p_t = \phi (-\varepsilon (w_r - w_t) + y_t) + \sigma (y_t + \chi (p_C^* - p_C^{*s}) + \alpha (d_t - d^s))$$

Combining like terms, we obtain:

$$(1 + \varepsilon\phi) w_r - (1 + \phi\varepsilon) w_t + (w_t - p_t) = (\phi + \sigma) y_t + \sigma\chi p_C^* + \sigma\alpha (d_t - d^s)$$

Simplifying, we obtain

$$y_t = \frac{1 + \varepsilon\phi}{\phi + \sigma} (w_r - w_t) + \frac{\chi(1 - \sigma)}{\phi + \sigma} p_C^* - \frac{\sigma\alpha}{\phi + \sigma} (d_t - d^s)$$

The latter part of the right-hand side is related to the natural rate of output, so:

$$y_t - y_t^n = \frac{1 + \varepsilon\phi}{\phi + \sigma} (w_r - w_t) - \frac{\sigma\alpha}{\phi + \sigma} (d_t - d_t^n)$$

We can express  $w_r$  in terms of the wage inflation rate:

$$\begin{aligned} w_r - w_t &= w_r - (1 - \omega) w_r - \omega w^s \\ &= \omega (w_r - w^s) \end{aligned}$$

where we have used the log linearization  $w_t = (1 - \omega) w_r + \omega w^s$ . Thus

$$w_r - w^s = \frac{\phi + \sigma}{\omega(1 + \varepsilon\phi)} (y_t - y_n) + \frac{\sigma\alpha}{\omega(1 + \varepsilon\phi)} (d_t - d_t^n)$$

Now we link wage increases to price increases:

$$p_t = w_t - \chi p_C^*$$

This relationship implies:

$$\begin{aligned} p_t - p^s &= w_t - w^s - \chi p_C^* \\ &= (1 - \omega) (w_r - w^s) - \chi p_C^* \end{aligned}$$

We can then write the AS curve as follows, noting that  $d_t^n = d^s$ :

$$p_t - p^s = \frac{(1 - \omega)(\phi + \sigma)}{\omega(1 + \varepsilon\phi)} (y_t - y_t^n) + \frac{(1 - \omega)\sigma\alpha}{\omega(1 + \varepsilon\phi)} (d_t - d^s) - \chi (p_C^* - p_C^s)$$

or

$$p_t - p^s = \kappa (y_t - y_t^n) + \theta (d_t - d^s) - \chi (p_C^* - p_C^s)$$

where

$$\begin{aligned} \kappa &= \frac{(1 - \omega)(\phi + \sigma)}{\omega(1 + \varepsilon\phi)} \\ \theta &= \frac{(1 - \omega)\sigma\alpha}{\omega(1 + \varepsilon\phi)} \end{aligned}$$

Intuition for the terms:

- First term: output above natural level causes people to work harder, and thus demand higher wages
- Second term: debt above steady state implies people can take more leisure for the same amount of output. Thus you have to pay them more (higher wages) if you want them to work.
- Third term: converts wage inflation to price inflation. If the terms of trade improve,

wages can rise without CPI rising.

### B.10.2 AD Curve

The Euler equation is

$$\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} I_t = 0$$

In logs, we have

$$i_t = \rho + \sigma \Delta c_{t+1} + \pi_{t+1}$$

Substituting in our equations for consumption, we have the nominal interest rate (which is a local bond that is not traded externally):

$$i_t = \rho + \sigma \left( y_{t+1} + \chi (p_C^* - p_C^{*s}) - \frac{\alpha}{\beta} (d_t - d^s) - y_t - \alpha (d_t - d^s) - \chi (p_C^* - p_C^{*s}) \right) + \pi_{t+1}$$

Combining like terms, we obtain

$$i_t = \rho + \sigma (y_{t+1} - y_t) - \sigma \alpha \left( 1 + \frac{1}{\beta} \right) (d_t - d^s) + \pi_{t+1}$$

which can be rearranged to the more familiar form

$$y_t = y_{t+1} - \alpha \left( 1 + \frac{1}{\beta} \right) (d_t - d^s) - \frac{1}{\sigma} (i_t - \pi_{t+1} - \rho)$$

Subtracting the natural rate of output at  $t$  from both sides, we obtain:

$$y_t - y_t^n = y_{t+1} - y_{t+1}^n - \alpha \left( 1 + \frac{1}{\beta} \right) (d_t - d^s) - \frac{1}{\sigma} (i_t - \pi_{t+1} - \rho)$$

Period  $t + 1$  output is, based on the labor-leisure condition:

$$\begin{aligned} w_{t+1} - p_{t+1} &= \phi y_{t+1} + \sigma c_{t+1} \\ \chi p_C^* &= \phi y_{t+1} + \sigma \left( y_{t+1} + \chi p_C^* - \frac{\alpha}{\beta} (d_t - d^s) \right) \\ y_{t+1} &= \frac{\chi(1-\sigma)}{\phi + \sigma} p_C^* + \frac{\alpha\sigma}{\beta(\phi + \sigma)} (d_t - d^s) \end{aligned}$$

Thus,

$$y_{t+1} - y_t^n = \frac{\sigma\alpha}{\beta(\phi + \sigma)} (d_t - d^s)$$

Substituting this into the AD curve, we have

$$y_t - y_t^n = -\eta (d_t - d^s) - \frac{1}{\sigma} (i_t - \pi_{t+1} - \rho)$$

where

$$\eta = - \left( \frac{\sigma\alpha}{\beta(\phi + \sigma)} - \alpha - \frac{\alpha}{\beta} \right)$$

This simplifies to

$$\begin{aligned} \eta &= - \frac{\sigma\alpha - \alpha\beta(\phi + \sigma) - \alpha(\phi + \sigma)}{\beta(\phi + \sigma)} \\ &= - \frac{\sigma\alpha - \alpha\beta\phi - \alpha\beta\sigma - \alpha\phi - \alpha\sigma}{\beta(\phi + \sigma)} \\ &= \frac{\alpha\beta\phi + \alpha\beta\sigma + \alpha\phi}{\beta(\sigma + \phi)} \\ &= \alpha \left( \frac{\beta(\sigma + \phi) + \phi}{\beta(\sigma + \phi)} \right) \\ &= \alpha \left( 1 + \frac{1}{\beta(\sigma + \phi)} \right) > 0 \end{aligned}$$

Intuition: higher debt means long run output is above the efficient level, meaning that it must be below the efficient level today and we are running a current account deficit that is paid back in the future.

It is useful to know a relationship between our key constants:

$$\begin{aligned}
\kappa - \frac{\theta}{\eta} &= \frac{(1-\omega)(\phi+\sigma)}{\omega(1+\varepsilon\phi)} - \frac{(1-\omega)\sigma\alpha}{\omega(1+\varepsilon\phi)} \frac{\beta(\sigma+\phi)}{\alpha\beta\phi + \alpha\beta\sigma + \alpha\phi} \\
&= \left( \frac{1-\omega}{\omega(1+\varepsilon\phi)} \right) \left( \phi + \sigma - \frac{\sigma\alpha\beta(\sigma+\phi)}{\alpha\beta\phi + \alpha\beta\sigma + \alpha\phi} \right) \\
&= \left( \frac{1-\omega}{\omega(1+\varepsilon\phi)} \right) (\phi + \sigma) \left( 1 - \frac{\sigma\alpha\beta}{\alpha\beta\phi + \alpha\beta\sigma + \alpha\phi} \right) \\
&= \left( \frac{1-\omega}{\omega(1+\varepsilon\phi)} \right) (\phi + \sigma) \left( 1 - \frac{\sigma\beta}{\sigma\beta + (1+\beta)\phi} \right) > 0
\end{aligned}$$

whose sign is *ambiguous*. In low debt steady states, in particular, it will be negative.

### B.10.3 AD under borrowing constraint

If borrowing is constrained, and the terms of trade move so that the constraint is binding, then  $d_t = d^s$  simply replaces the AD curve.

### B.10.4 Equilibrium Equations

$$\begin{aligned}
y_t - y_t^n &= -\eta(d_t - d^s) - \frac{1}{\sigma}(i_t - \pi_{t+1} - \rho) & \text{if } d_t < d^s \\
d_t &= d^s & \text{otherwise}
\end{aligned}$$

$$p_t - p^s = \kappa(y_t - y_t^n) + \theta(d_t - d^s) - \chi(p_C^* - p_C^s)$$

## B.11 Computing welfare

Let superscript “e” denote variables in the efficient equilibrium, which is identical to the natural allocation in a world with no borrowing constraint. Let tildes denote deviations between the actual equilibrium and the efficient equilibrium; for example,  $\tilde{c}_t = c_t - c_t^e$ .

We expand the utility function around the efficient equilibrium to compute the welfare loss or gain. For simplicity, we will consider only the case in which there is a drop in the terms of trade, in which case the consumer is borrowing-constrained. In that case, the

equilibrium condition  $d_t = d^s$  binds. Moreover, the relationship between consumption and output in each period is given by the budget constraints:

$$\begin{aligned}c_t &= y_t + \chi p_C^* \\c_{t+1} &= y_{t+1} + \chi p_C^* \\c_t^e &= y_t^e + \chi p_C^* \\c_{t+1}^e &= y_{t+1}^e + \chi p_C^*\end{aligned}$$

Thus:

$$\begin{aligned}\tilde{c}_t &= \tilde{y}_t \\ \tilde{c}_{t+1} &= \tilde{y}_{t+1}\end{aligned}$$

The expansion of the aggregate utility function for period  $t$  is:

$$\begin{aligned}\int_0^1 (U(C_t, L_t(j)) - U_t^e) dj &\approx U_C C_t^e \left[ \tilde{c} + \frac{1-\sigma}{2} \tilde{c}^2 \right] \\ &+ U_L L_t^e \left[ \int_0^1 \tilde{l}_t(j) dj + \frac{1+\phi}{2} \int_0^1 \tilde{l}_t(j)^2 dj \right]\end{aligned}$$

where  $U_C$  and  $U_L$  are the partial derivatives of  $U(C_t, L_t(j))$  with respect to  $C_t$  and  $L_t(j)$ , respectively, evaluated at the efficient equilibrium. The terms with  $\sigma$  and  $\phi$  come from dividing the second partial derivatives by the first partial derivatives; the mixed partial term is zero because the utility function is separable in consumption and hours worked.

It can be shown that, up to a second-order log-linearization of the definition of  $L_t$  around the steady state:

$$\tilde{l}_t + \frac{1}{2} \tilde{l}_t^2 = \int_0^1 \tilde{l}_t(j) dj + \frac{1}{2} \int_0^1 \tilde{l}_t(j)^2 dj$$



Additionally, by substituting in the labor demand curve, it can be shown that:

$$\begin{aligned}\int_0^1 \tilde{l}_t(j)^2 dj &= \int_0^1 (\tilde{l}_t(j) - \tilde{l}_t + \tilde{l}_t)^2 dj \\ &= \int_0^1 (-\varepsilon(\tilde{w}_t(j) - \bar{w}_t) + \tilde{l}_t)^2 dj \\ &= \varepsilon^2 \int_0^1 (\tilde{w}_t(j) - \bar{w}_t)^2 dj - 2\varepsilon\tilde{l}_t \int_0^1 (\tilde{w}_t(j) - \bar{w}_t) dj + \tilde{l}_t^2\end{aligned}$$

where we have used the labor demand curve.

Using the lemma below concerning the log-linearization of the nominal wage, the middle term can be shown to be of second order (since the integral is approximately zero), and this expression can be simplified to:

$$\int_0^1 \tilde{l}_t(j)^2 dj = \varepsilon^2 \text{Var}_j(\tilde{w}_t(j)) + \tilde{l}_t^2$$

Substituting these relationships into the aggregate welfare function, we obtain

$$\begin{aligned}\int_0^1 (U(C_t, L_t(j)) - U_t^e) dj &= U_C C_t^e \left[ \tilde{c}_t + \frac{1-\sigma}{2} \tilde{c}_t^2 \right] \\ &\quad + U_L L_t^e \left[ \tilde{l}_t + \frac{1+\phi}{2} \tilde{l}_t^2 + \frac{\varepsilon^2 \phi}{2} \text{Var}_j(w_t(j)) \right]\end{aligned}$$

Substituting in output, we have

$$\begin{aligned}\int_0^1 (U(C_t, L_t(j)) - U_t^e) dj &= U_C C_t^e \left[ \tilde{y}_t + \frac{1-\sigma}{2} \tilde{y}_t^2 \right] \\ &\quad + U_L L_t^e \left[ \tilde{y}_t + d_{w,t} + \frac{1+\phi}{2} \tilde{y}_t^2 + \frac{\varepsilon^2 \phi}{2} \text{Var}_j(w_t(j)) \right]\end{aligned}$$

This substitution is accomplished by noting that, to second order, labor and output are not equal, but differ by a term related to the dispersion of wages:

$$\tilde{l}_t = \tilde{y}_t + d_{w,t}$$

where

$$\begin{aligned}d_{w,t} &= \ln \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon} dj \\ &\approx \frac{\varepsilon}{2} \text{Var}(w_t(j))\end{aligned}$$

Only first-order terms with  $d_{w,t}$  need be included because it is approximately equal to *squared* inflation.

In the efficient allocation,  $U_C C_t^e = -U_L L_t^e$ . Moreover,

$$\begin{aligned} U_L L_t^e &= -Y^{e(1+\phi)} \\ &= -1 \end{aligned}$$

Thus welfare in period  $t$  is:

$$\tilde{W}_t = \left[ \tilde{y}_t + \frac{1-\sigma}{2} \tilde{y}_t^2 \right] - \left[ \tilde{y}_t + \frac{\varepsilon}{2} + \frac{1+\phi}{2} \tilde{y}_t^2 + \frac{\varepsilon^2 \phi}{2} \text{Var}_j (w_t(j)) \right]$$

It can be shown that  $\text{Var}_j (w_t(j)) = \pi_{w,t}^2 \left( \frac{\omega}{1-\omega} \right)$  where  $\pi_{w,t} = w_t - w^s$ , so this can be further simplified to

$$\tilde{W}_t = - \left( \frac{\phi + \sigma}{2} \right) \tilde{y}_t^2 - \left( \frac{\varepsilon(1 + \varepsilon\phi)}{2} \right) \left( \frac{\omega}{1-\omega} \right) \pi_{w,t}^2$$

The same derivation can be done for period  $t + 1$ , although because wages are not sticky, the cross-sectional variance of wages term does not appear. Thus the total welfare function (including both periods) will be:

$$\tilde{W} = \left[ - \left( \frac{\phi + \sigma}{2} \right) \tilde{y}_t^2 - \left( \frac{\varepsilon(1 + \varepsilon\phi)}{2} \right) \left( \frac{\omega}{1-\omega} \right) \pi_{w,t} - \beta \left( \frac{\phi + \sigma}{2} \right) \tilde{y}_{t+1}^2 \right]$$

Note that wage inflation can be put in terms of price inflation:

$$\begin{aligned} \pi_{w,t} &= w_t - w^s \\ &= \chi p_C^* + (p_t - p^s) \end{aligned}$$

So we can substitute in price inflation:

$$\tilde{W} = \left[ - \left( \frac{\phi + \sigma}{2} \right) \tilde{y}_t^2 - \left( \frac{\varepsilon(1 + \varepsilon\phi)}{2} \right) \left( \frac{\omega}{1-\omega} \right) \left( (p_t - p^s) + \chi (p_C^* - p_C^{*s}) \right)^2 - \beta \left( \frac{\phi + \sigma}{2} \right) \tilde{y}_{t+1}^2 \right]$$

Finally, because output is the same in the natural and efficient allocations, we can substitute in the output gap:

$$\tilde{W} = \left[ - \left( \frac{\phi + \sigma}{2} \right) (y_t - y_t^n)^2 - \left( \frac{\varepsilon(1 + \varepsilon\phi)}{2} \right) \left( \frac{\omega}{1-\omega} \right) \left( (p_t - p^s) + \chi (p_C^* - p_C^{*s}) \right)^2 - \beta \left( \frac{\phi + \sigma}{2} \right) (y_{t+1} - y_{t+1}^n)^2 \right]$$

or more succinctly,

$$\tilde{W} = -\frac{1}{2} \left( y_t - y_t^n \right)^2 - \frac{\beta}{2} \left( y_{t+1} - y_{t+1}^n \right)^2 - \frac{\zeta}{2} \left( (p_t - p^s) + \chi (p_C^* - p_C^{*s}) \right)^2$$

where

$$\zeta = \left( \frac{\varepsilon(1 + \varepsilon\phi)}{\phi + \sigma} \right) \left( \frac{\omega}{1 - \omega} \right)$$

### B.11.1 Lemma: Wage log-linearization to second order

The nominal wage is defined as:

$$W_t = \left[ \int W_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$$

This can be simplified to

$$1 = \int \left( \frac{W_t(j)}{W_t} \right)^{1-\varepsilon} dj$$

If we now do a second order expansion of the integrand, we find

$$\left( \frac{W_t(j)}{W_t} \right)^{1-\varepsilon} \approx 1 + (1 - \varepsilon) (w_t(j) - w_t) + \frac{(1 - \varepsilon)^2}{2} (w_t(j) - w_t)^2$$

Substituting this into the integral, we find that

$$E_j (w_t(j) - w_t) = \frac{\varepsilon - 1}{2} E_j \left( (w_t(j) - w_t)^2 \right)$$

Note that under the approximation that  $w_t \approx E_j (w_t(j))$ , we have:

$$E_j \left( (w_t(j) - w_t)^2 \right) = \text{Var}_j (w_t(j))$$

Consequently:

$$E_j (\tilde{w}_t(j) - \tilde{w}_t) = \frac{\varepsilon - 1}{2} \text{Var}_j (w_t(j))$$

$$E_j \left( (\tilde{w}_t(j) - \tilde{w}_t)^2 \right) = \text{Var}_j (w_t(j))$$

## B.12 Outcomes under different monetary policies

Summary: a fixed exchange rate is the worst; inflation target is better; producer price target or commodity basket target is the best.

### B.12.1 Fixed exchange rate

We will make use of the key identity

$$p_t = s_t + (1 - \chi) p_C^*$$

In the first period, we have

$$p_t - p^s = s_t - s^s + (1 - \chi) (p_C^* - p_C^{*s})$$

Thus a fixed exchange rate implies

$$p_t - p^s = (1 - \chi) (p_C^* - p_C^{*s})$$

In the second period, the same condition implies

$$\pi_{t+1} = 0$$

In the first period, the Phillips curve is then

$$(1 - \chi) (p_C^* - p_C^{*s}) = \kappa (y_t - y_t^n) + \theta (d_t - d^s) - \chi (p_C^* - p_C^s)$$

In the constrained equilibrium, we therefore have

$$y_t - y_t^n = \frac{1}{\kappa} (p_C^* - p_C^{*s})$$

So the output gap is more extreme than under the CPI targeting case (because the central bank has to induce a bigger recession to be slightly deflationary rather than just keeping prices constant under a negative TOT shock).

In the unconstrained equilibrium, we have

$$\begin{aligned}(1 - \chi)(p_C^* - p_C^{*S}) &= \left(\kappa - \frac{\theta}{\eta}\right)(y_t - y_t^n) - \chi(p_C^* - p_C^{*S}) \\(p_C^* - p_C^{*S}) &= \left(\kappa - \frac{\theta}{\eta}\right)(y_t - y_t^n) \\y_t - y_t^n &= \frac{1}{\kappa - \frac{\theta}{\eta}}(p_C^* - p_C^{*S})\end{aligned}$$

Again a bigger boom than under CPI inflation targeting. So the fixed FX rate just exaggerates things relative to the CPI target.

Welfare:

$$\begin{aligned}\tilde{W} &= -\frac{1}{2}(y_t - y_t^n)^2 - \frac{\beta}{2}(y_{t+1} - y_{t+1}^n)^2 - \frac{\zeta}{2}\left((p_t - p^s) + \chi(p_C^* - p_C^{*S})\right)^2 \\&= -\frac{1}{2}\left(\frac{1}{\kappa}(p_C^* - p_C^{*S})\right)^2 - \frac{\zeta}{2}\left((1 - \chi)(p_C^* - p_C^{*S}) + \chi(p_C^* - p_C^{*S})\right)^2 \\&= -\frac{1}{2}\left(\frac{1}{\kappa^2} + \zeta\right)(p_C^* - p_C^{*S})^2\end{aligned}$$

The future output gap term is zero because in the constrained region, no new debt is accumulated, so we enter the long run under the same initial conditions as we would have under the efficient allocation. Since wages are also flexible in the long run, the efficient (also natural) allocation occurs in the long run.

### B.12.2 CPI inflation targeting

In the borrowing constrained region, we simply have the equation

$$p_t - p^s = \kappa(y_t - y_t^n) - \chi(p_C^* - p_C^{*S})$$

$p_t$  is determined by monetary policy and  $y_t^n = \frac{\chi(1-\sigma)}{\phi+\sigma}p_C^*$ . Current account deficit is zero, trade is balanced. If we have inflation targeting, the output gap equals

$$y_t - y_t^n = \frac{\chi}{\kappa}(p_C^* - p_C^{*S})$$

In the non-borrowing constrained region, we have

$$\begin{aligned} y_t - y_t^n &= -\eta (d_t - d^s) \\ p_t - p^s &= \kappa (y_t - y_t^n) + \theta (d_t - d^s) - \chi (p_C^* - p_C^{*s}) \end{aligned}$$

Susbsituting the IS curve into the AS curve, we have

$$p_t - p^s = \left( \kappa - \frac{\theta}{\eta} \right) (y_t - y_t^n) - \chi (p_C^* - p_C^{*s})$$

Under inflation targeting, we have

$$y_t - y_t^n = \frac{\chi}{\kappa - \frac{\theta}{\eta}} (p_C^* - p_C^{*s})$$

The boom is “bigger” here, although the effect is not directly comparable since the constrained region applies when the TOT falls and the unconstrained region applies when the TOT rises. The intuition for this bigger boom is that a positive output gap has two effects on inflation when external lending is allowed. First, there is the direct effect that higher demand for outputs makes people work more, and thus they want to raise their wages. Second, there is a contrary effect in that people also increase their *willingness* to work more if they can save the proceeds of the boom abroad, since it will allow them to work less in the long run. The second effect (“supply effect”) means that, a bigger output gap is possible given the same fixed level of inflation.

Welfare computation:

$$\begin{aligned} \tilde{W} &= -\frac{1}{2} \left( y_t - y_t^n \right)^2 - \frac{\beta}{2} \left( y_{t+1} - y_{t+1}^n \right)^2 - \frac{\zeta}{2} \left( (p_t - p^s) + \chi (p_C^* - p_C^{*s}) \right)^2 \\ &= -\frac{1}{2} \left( \frac{\chi}{\kappa} (p_C^* - p_C^{*s}) \right)^2 - \frac{\zeta}{2} (\chi (p_C^* - p_C^{*s}))^2 \\ &= -\frac{1}{2} (p_C^* - p_C^{*s})^2 \chi^2 \left( \frac{1}{\kappa^2} + \zeta \right) \end{aligned}$$

The future output gap term is zero because in the constrained region, no new debt is accumulated, so we enter the long run under the same initial conditions as we would have under the efficient allocation. Since wages are also flexible in the long run, the efficient (also

natural) allocation occurs in the long run.

### B.12.3 Producer price targeting

Producer price targeting implies that  $p_{NT} = p_{NT}^s$ . Thus

$$\begin{aligned} p_t &= \chi p_F + (1 - \chi) p_{NT} \\ p_t - p^s &= \chi (p_{F,t} - p_F^s) + (1 - \chi) (p_{NT,t} - p_{NT}^s) \\ &= \chi (p_{F,t} - p_{NT,t} - (p_F^s - p_{NT}^s)) \\ &= \chi (p_C^* - p_C^{*s}) \end{aligned}$$

In the constrained region, we have the Phillips curve:

$$0 = \kappa (y_t - y_t^n)$$

Thus the output gap is zero.

In the unconstrained region, we have the Phillips curve:

$$\begin{aligned} y_t - y_t^n &= -\eta (d_t - d^s) \\ \chi (p_C^* - p_C^{*s}) &= \kappa (y_t - y_t^n) + \theta (d_t - d^s) - \chi (p_C^* - p_C^{*s}) \end{aligned}$$

which is also solved by the output gap being zero.

Welfare:

$$\begin{aligned} \tilde{W} &= -\frac{1}{2} (y_t - y_t^n)^2 - \frac{\beta}{2} (y_{t+1} - y_{t+1}^n)^2 - \frac{\zeta}{2} \left( (p_t - p^s) + \chi (p_C^* - p_C^{*s}) \right)^2 \\ &= 0 \end{aligned}$$

### B.12.4 Nominal wage targeting

Same as producer price targeting, since  $w_t = p_{NT,t}$ .

### B.12.5 Commodity basket targeting

Here we target  $p_{C,t} = p_C^s$ ; the effect is the same as targeting the wage or non-tradable prices since these are all equal by the firm optimality conditions.

### B.12.6 Nominal GDP targeting

Nominal GDP is

$$G_t Y_t = W_t L_t$$

In logs, the policy is:

$$w_t + y_t = w^s + y^s$$

We know that  $y^s = 0$ , so we have that wage inflation is

$$w_t - w^s = y_t$$

We have that

$$\begin{aligned} w_t - w^s &= y_t^n + (y_t - y_t^n) \\ p_{C,t} - p_C^s &= \frac{\chi(1-\sigma)}{(\phi+\sigma)} p_{C,t}^* + (y_t - y_t^n) \\ p_{C,t}^* - p_C^{*s} &= \frac{\chi(1-\sigma)}{(\phi+\sigma)} p_{C,t}^* + (y_t - y_t^n) \\ \left(1 - \frac{\chi(1-\sigma)}{(\phi+\sigma)}\right) p_C^* &= y_t - y_t^n \end{aligned}$$

The constrained region Phillips curve gives us a similar relationship:

$$p_t - p^s = \kappa(y_t - y_t^n) - \chi(p_C^* - p_C^{*s})$$

Plugging in for the output gap, we get

$$p_t - p^s = \left(\kappa - \frac{\kappa\chi(1-\sigma)}{(\phi+\sigma)} - \chi\right) p_C^*$$

Thus there can be CPI inflation or deflation depending on the size of the constants, and



the output gap can be either positive or negative. Obviously GDP deflator will go in the opposite direction of the output, and output unambiguously moves in the same direction as the terms of trade.