# Variability in Blazars: Clues from PKS 2155–304

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**ABSTRACT**

Rapid variability on a time-scale much faster than the light-crossing time of the central supermassive black hole has been seen in TeV emission from the blazar PKS 2155−304. The most plausible explanation for this puzzling observation is that the radiating fluid in the relativistic jet is divided into a large number of subregions which move in random directions with relative Lorentz factors $\approx \gamma_j'$. The random motions introduce new relativistic effects, over and above those due to the overall mean bulk Lorentz factor $\Gamma_b$ of the jet. We consider two versions of this 'jets in a jet' model. In the first, the 'subjets' model, stationary regions in the mean jet frame emit relativistic subjets that produce the observed radiation. The variability time-scale is determined by the size of the subregions in the mean jet frame. This model, which is loosely based on magnetic reconnection, has great difficulty explaining the observations in PKS 2155−304. In the alternate 'turbulence' model, various subregions move relativistically in random directions and the variability time-scale is determined by the size of these regions in their own comoving frames. This model fits the data much more comfortably. Details such as what generates the turbulent motion, how particles are heated, and what the radiation process is, remain to be worked out. We consider collisions between TeV photons and soft photons and find that, in both the subjets and turbulence models, the mean bulk Lorentz factor $\Gamma_b$ of the jet needs to be $>25$ to avoid the pair catastrophe.

**Key words:** black hole physics – magnetic reconnection – relativistic processes – turbulence – BL Lacertae objects: individual: PKS 2155−304 – galaxies: jets.

## 1 INTRODUCTION

On 2006 July 28, the high-frequency peaked BL Lac source PKS 2155−304 had a strong flare in the TeV band, with an average flux that was more than 10 times larger than the typical flux seen at other times (Aharonian et al. 2007). During this flare, which lasted for about an hour, rapid variability on a time-scale $\approx 300\,\mathrm{s}$ was observed. With a galaxy bulge luminosity $M_R = -24.4$ (Kotilainen, Falomo & Scarpa 1998), the expected mass of the black hole (BH) in the nucleus of the galaxy is $M_{BH} \approx 1$–$2 \times 10^9\,\mathrm{M_\odot}$ (Bettoni et al. 2003). This corresponds to a gravitational radius $R_g \equiv 2GM/c^2 \approx$ 3–$6 \times 10^{14}$ cm, and a corresponding time-scale $T_g = R_g/c \approx 1$–$2 \times 10^4$ s. Amazingly, $T_g$ is nearly 2 orders of magnitude longer than the variability time.

It is generally accepted that TeV emission in blazars comes from relativistic jets that are pointed towards the observer (Hinton & Hofmann 2009). Opacity arguments suggest that the bulk Lorentz factor of the jet in PKS 2155−304 must be very large, $\Gamma_b > 50$ (Begelman, Fabian & Rees 2008). At first sight, it might appear that this large Lorentz factor will also explain the rapid variability

observed in the source. However, while relativity can certainly cause the observed variability time to be shorter than $R/c$, where $R$ is the size of the emitting region, there is no simple way for the variability time to be shorter than the gravitational time-scale $T_g$ of the central engine. In PKS 2155−304, the variability is faster by a factor of several tens.

The only way to understand the rapid variability in PKS 2155−304 is if one of the following conditions holds: (i) the entire source (engine and emitting region) moves towards the observer with a Lorentz factor $\approx 50$, or (ii) the emitting region alone moves rapidly towards the observer, and the variability is caused by some local instability in the radiating gas which is insensitive to any time-scale associated with the central BH engine, or (iii) the supermassive BH is $\approx 50$ times less massive than we estimate. The first option is obviously impossible. We do not expect a $10^9\,\mathrm{M_\odot}$ BH to move with a Lorentz factor of 50. The third option is also unlikely since the BH mass estimate is obtained via the well-known (BH mass)–(bulge luminosity) relation (Magorrian et al. 1998), which has an uncertainty of no more than a factor of a few.[1] We are thus

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[1] An exception would arise, of course, if the system consists of a binary BH and the smaller BH produces the flare (Volpe & Rieger 2011).

led to the second option, but even this is highly non-trivial. The size of the emitting region is surely at least as large as $R_g$, which means, given the short variability time-scale, that only a small fraction of the emitting volume can be involved in the variability. Yet, the large amplitude of the observed fluctuations suggests that the whole emitting region must contribute to the flare. There is an inherent paradox in these conflicting indications.

Building on ideas developed earlier to explain variability in gamma-ray bursts (GRBs; see Lyutikov & Blandford 2004; Lyutikov 2006; Narayan & Kumar 2009; Lazar, Nakar & Piran 2009), Giannios et al. (2009; see also Giannios, Uzdensky & Begelman 2010; Nalewajko et al. 2011) proposed a 'jets in a jet' model as a possible explanation of the TeV flare observed in PKS 2155−304. According to this model, a relativistic jet moves with an overall bulk Lorentz factor $\Gamma_b$ towards the observer, but the TeV emission is produced in emitting regions that themselves move relativistically with respect to the mean frame of the jet. As shown in the context of GRBs (Narayan & Kumar 2009; Lazar et al. 2009), such a model naturally produces large amplitude variability on time-scales much shorter than the light-crossing time of the emitting volume.

In this work we explore the 'jets in a jet' model and obtain several constraints that must be satisfied for this model to operate. Our analysis follows the work of Narayan & Kumar (2009) and Lazar et al. (2009) on the equivalent problem for GRBs. In Section 2 we summarize the relevant observations of PKS 2155−304. In Section 3 we discuss the overall issue of time-scales in relativistic jets. We then describe the 'jets in a jet' model and its two basic variants – 'subjets' (Section 3.1) and 'turbulence' (Section 3.2) – and analyse the conditions under which each can fit the observations. Both variants of the model involve a large number of emitting regions moving in different directions. Photons emitted in one region can interact with those emitted in another, and if the optical depth is sufficiently large, colliding high-energy photons will pair produce and make the source opaque. We examine the optical depth to this process in Section 5 and consider how the constraint $\Gamma_b > 50$ obtained by Begelman et al. (2008) is modified in the 'jets in a jet' scenario. In Section 6 we summarize our results and consider the implications. Additional details are given in Appendices A and B.

## 2 OBSERVATIONS

The TeV flare under consideration in PKS 2155−304 was already in progress when observations began on 2006 July 28 (Aharonian et al. 2007). It lasted for about $T_{obs} \approx 4000$ s, after which the flux fell to a much lower value. For the analysis in this paper, we need to know the total duration $T$ of the flare. In Appendix A, we use a Bayesian analysis to estimate the probability distribution of $T$. We estimate the median duration of the flare to be $T_{median} \approx 2T_{obs} \approx 8000$ s, and the mean duration to be $T_{mean} = \ln(T_{max}/T_{obs}) T_{obs} \approx (3–10) T_{obs} \approx 12\,000–40\,000$ s. For the present analysis, we choose $T \approx 20\,000$ s.

The second quantity we need is the variability time-scale. Aharonian et al. (2007) found very rapid rise times $\tau_r \sim 100–200$ s and longer decay times $\tau_d \sim 200–600$ s (see their table 1). They also defined a doubling time $T_2$, and state that the fastest $T_2 = 224 \pm 60$ s and the average $T_2 = 330 \pm 40$ s. Based on this information, we choose the characteristic variability time to be $\delta t \approx 300$ s (as in Begelman et al. 2008). The ratio of the total duration of the flare to the variability time is an important quantity.[2] For the above choices,

we find

$$\frac{T}{\delta t} \approx 70, \qquad T \approx 20\,000\,\text{s}, \qquad \delta t \approx 300\,\text{s}. \qquad (1)$$

During the 4000 s of observations of the flare in PKS 2155−304, there were five major pulses in the TeV emission. Scaling this number to the total duration $T$, we estimate the total number of pulses in the flare to be $N_p \approx 25$. From this we estimate the duty cycle $\xi$ of the pulsed emission:

$$\xi \equiv \frac{N_p \delta t}{T} \approx 0.4, \qquad N_p \approx 25. \qquad (2)$$

The numerical values given in equations (1) and (2) are for our fiducial estimates of $T$ and $\delta t$. As a very extreme case, we also sometimes consider $T = T_{obs} \approx 4000$ s, $\delta t \approx 600$ s, which give $T/\delta t \approx 7$, $\xi \approx 0.8$.

As mentioned earlier, the mass of the BH in PKS 2155−304 is estimated to be $M_{BH} \approx 1–2 \times 10^9\,M_\odot$, so its gravitational time-scale is

$$T_g \approx 10\,000\,M_9\,\text{s}, \qquad M_9 \equiv \frac{M_{BH}}{10^9\,M_\odot} \approx 1–2. \qquad (3)$$

In the following we use a canonical value of $M_9 = 2$, such that $T_g$ matches our estimate of the flare duration $T$. However, there is no direct measurement of the BH mass, so we also occasionally consider a lower value: $M_9 = 0.5$.

## 3 THE MODEL

Naively, one might think that relativistic motion can cause the observed variability time of a source to be arbitrarily small and so rapid variability can always be explained. However, the situation is more complicated.

Consider a smooth homogeneous jet moving towards the observer with a Lorentz factor $\Gamma_b$. Let the jet be at a distance $R$ from the central engine and let us model it as a piece of a spherical shell with radial width $\Delta R$ as measured in the 'lab' frame, i.e. the frame of the host galaxy. The emission is beamed into an angle $1/\Gamma_b$, so a distant observer receives radiation from only a region of lateral size $\sim R/\Gamma_b$. The angular time-scale $t_{ang}$, the time delay between the centre and edge of the visible patch, is $\sim R/2c\Gamma_b^2$. Since the jet is assumed to be smooth, any observed variability can be no faster than this.[3]

In the comoving frame of the jet, the size of a causally connected region is $R' \sim R/\Gamma_b$, where (and throughout the paper) we use a prime to distinguish length and time-scales in the comoving frame from those in the lab frame. The radial width $\Delta R'$ of the shell of material in the jet cannot be smaller than this size. (If it starts out smaller, it will quickly expand to this size as a result of internal pressure gradients.) Thus, two photons emitted simultaneously in the fluid frame from the front and back of the shell will reach the observer with a time difference $\Delta R'/2c\Gamma_b \gtrsim R/2c\Gamma_b^2$. This radial time-scale $t_{rad}$ is of the same order as, or longer than, the angular time-scale $t_{ang}$.

Remarkably, $R/2c\Gamma_b^2$ is also the observed time difference between two photons emitted by a single fluid element, one emitted when the fluid is at a distance $R$ from the engine and the other emitted at a distance $2R$. This dynamical time-scale $t_{dyn}$ is relevant,

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[2] This quantity is sometimes called the source variability $V$ in the GRB literature (Sari & Piran 1997; Narayan & Kumar 2009).

[3] Throughout this paper, for simplicity, we ignore the cosmological time dilation factor $(1 + z) = 1.116$ between the host galaxy of PKS 2155−304 and the observer. This factor will be more important for a source at a higher redshift. In that case, time-scales in the observer frame will need to be multiplied by $(1 + z)$.

for instance, if the jet slows down as a result of interaction with a surrounding medium. It is also relevant for a freely streaming jet, since it is the time-scale on which the energetic particles in the jet are cooled by adiabatic expansion.

For large values of $\Gamma_b$, all three time-scales described above are much shorter than the Newtonian time-scale $R/c$ that one would compute based on the size of the source as measured in the lab frame. Thus, relativistic motion is indeed capable of producing very short time-scales compared to the dimensions of the emitting region.

However, there is a fourth time-scale. From a simple light-crossing argument applied to the power source of the jet, the radial width of the emitting shell of material cannot be smaller than the size $R_g$ of the central engine. Thus, the shell width in the lab frame must satisfy $\Delta R \geq R_g$, or equivalently, in the jet frame $\Delta R' \geq \Gamma_b R_g$. This additional constraint means that the observed variability time-scale cannot be shorter than $T_g$. However, in PKS 2155−304, we have $T_g \approx 20\,000$ s, whereas the variability time is much shorter, $\delta t \approx 300$ s. This is the problem we are faced with.

In order to solve the above problem, we have to give up the assumption of a smooth jet and must allow the radiation observed at any given time to be emitted from a tiny region of the source with a size $l' \ll R'$. However, $R'$ is the size of a causally connected region. How can a significant amount of energy, as needed to produce the observed large amplitude pulses of TeV emission, be squeezed into a region much smaller than $R'$? The answer is (Narayan & Kumar 2009; Lazar et al. 2009; Giannios et al. 2009; see Appendix B for a table comparing the notations used in these papers) that the radiating region under consideration must move relativistically with respect to the mean comoving frame of the jet. This causes the radiation from a tiny source region to be beamed into a narrow cone, thereby amplifying the observed luminosity without enhancing the energy requirement of the emitting region. This is the key idea of the 'jets in a jet' model. Note that Ghisellini et al. (2009) suggest an alternative way of reducing the size of the emitting region, which we do not consider in this paper.

In models of magnetically accelerated jets, during the acceleration phase the radius and Lorentz factor of the jet generally scale as $R \sim \Gamma_b^2 R_g$ (e.g. Tchekhovskoy, McKinney & Narayan 2008). This gives a causal scale $R' = R/\Gamma_b \sim \Gamma_b R_g$. If acceleration ceases and the radiation is produced when the jet is in a coasting phase, then $R' > \Gamma_b R_g$. To allow for both possibilities, we write

$$R = f_c \Gamma_b^2 R_g, \qquad R' = \frac{R}{\Gamma_b} = f_c \Gamma_b R_g, \tag{4}$$

where the numerical factor $f_c \geq 1$ allows for coasting. In the fireball model of GRBs, during acceleration we have $R \sim \Gamma_b R_g$, and it would appear that we could have $f_c < 1$. However, energy dissipation in a baryon-loaded fireball typically occurs only at a distance $R \approx \Gamma_b^2 c\, t_{engine}$, or equivalently, $t_{dyn} \approx t_{engine}$, where $t_{engine}$ is the characteristic time-scale of the central engine (Sari & Piran 1997). Clearly $t_{engine} \geq T_g$, and thus once again we obtain equation (4) with $f_c \geq 1$.

Events such as the bright flare observed in PKS 2155−304 by Aharonian et al. (2007) are clearly rare. Most of the time the source is much fainter. To model this behaviour we assume that the source has more or less steady low-level jet activity, but occasionally goes through short periods of time when the engine power becomes very much larger. We are concerned with the properties of these bursts of more energetic activity; one of these bursts presumably produced the bright flare seen in PKS 2155−304. Let $t_{engine}$ be the duration that the source spends in an energetic state. Clearly $t_{engine} \geq T_g$. In addition, spreading will cause a shell that starts out radially very thin to expand to a width $\sim R/\Gamma_b^2$ ( in the lab frame), as discussed earlier. We thus write the radial width of the emitting region as

$$\Delta R = \max\left(ct_{engine}, \frac{R}{\Gamma_b^2}\right) \equiv f_d \frac{R}{\Gamma_b^2} = f_d \frac{R'}{\Gamma_b} = f_d f_c R_g, \tag{5}$$

$$\Delta R' = \Gamma_b \Delta R = f_d R' = f_d f_c \Gamma_b R_g, \tag{6}$$

where the engine duration factor $f_d \geq 1$. The overall duration of the flaring activity is determined by $\Delta R$, since the angular spreading time and dynamical time are always shorter than or equal to the radial time. Hence,

$$T \approx \frac{\Delta R}{c} = f_d \frac{R'}{\Gamma_b c} = f_d f_c T_g. \tag{7}$$

In PKS 2155−304, $T$ is roughly equal to $T_g$ for our fiducial choice of the BH mass ($M_9 = 2$). Thus the product $f_d f_c$ must be of order unity. Since $f_d$ and $f_c$ are individually larger than or equal to unity, this implies that each is of order unity. If we assume a lower mass for the BH, e.g. $M_9 = 0.5$, then $f_d f_c \approx 4$ and we have some freedom in choosing the values of $f_d$ and $f_c$.

As explained above, the key idea of the 'jets in a jet' model is that the radiating fluid in the jet is subdivided into a number of independent volumes, each moving relativistically with respect to the mean frame of the jet. In order to explain the rapid variability, it is necessary for each subvolume to be much smaller than the causality scale $R'$, or the engine-related scale $\Delta R'$. The breaking up of the jet into kinematically distinct subvolumes cannot be related to the engine since the length-scale involved is too small. Rather, it must result from a local instability of some sort. Such a situation can arise naturally in a highly magnetized outflow, for instance through magnetic reconnection or magnetohydrodynamic (MHD) turbulence. However, in the following, we do not assume anything specific about the nature of the outflow or what causes the instability, though we do assume some features of the variability time-scale.

There are two main variants of the 'jets in a jet' model (Lazar et al. 2009). The distinction is whether the emission duration of individual pulses in the flare is determined by physics in the comoving frame of the jet or in a frame moving relative to the jet.

### 3.1 Subjets and reconnection

In the model described by Giannios et al. (2009; see also Lyutikov 2006; Lazar et al. 2009, in the context of GRBs), which we denote hereafter as the 'subjets' model, magnetic field reconnection cells arise sporadically within the strongly magnetized jet fluid. Each reconnection event leads to the ejection of twin subjets of relativistic plasma with a typical Lorentz factor $\gamma_j'$ as measured in the mean frame of the jet. The subjets emit the observed TeV emission. A single pulse in the observed TeV light curve corresponds to a single subjet. Hence the observed duration of a pulse is equal to the time taken for the completion of a reconnection event as measured in the jet frame, divided by $\Gamma_b$ (to transform to the observer frame). While this model is strongly motivated by magnetic reconnection, there is nothing in the following discussion that depends on the specific details of this mechanism. The model is valid for any process in which local processes in the jet frame produce relativistic subjets.

Each reconnection event dissipates the magnetic energy in a certain characteristic volume in the jet frame. Let $l'$ be the typical length-scale of this volume, and let $\beta' = v'/c$ be the typical speed with which the magnetic energy flows into the central reconnection

zone. The duration of the reconnection event, as measured in the jet frame, is $\delta t' \approx l'/\beta' c$, and so the observed variability time is

$$\delta t \approx \frac{l'}{\Gamma_b \beta' c}. \tag{8}$$

We remind the reader that $l'$ is measured in the jet frame, i.e. the volume of interest is at rest in this frame. If the reconnection volume itself moves relativistically with respect to the jet frame, then the relevant analysis is that given in Section 3.2. As explained there, the time-scale $\delta t$ becomes shorter by one power of the Lorentz factor.

Clearly, if a subjet changes its direction by more than $1/\gamma_j'$ before consuming all the energy within the reconnection volume, then the observed pulse will be shorter than the estimate given in equation (8).[4] In this case, we simply redefine $l'$ such that the subjet direction is constant to within $1/\gamma_j'$ during the time $\delta t' = l'/\beta' c$. Similarly, if the reconnection site operates intermittently, emitting blobs instead of a continuous jet, then the scale $l'$ is the distance from which energy flows in during the time that a single blob is emitted. Making use of equation (7), the ratio of the flare duration to the variability time is

$$\frac{T}{\delta t} \approx f_d \beta' \frac{R'}{l'}. \tag{9}$$

For a given observer, the region from which radiation can be seen has a size $\sim R'$ perpendicular to the line of sight and a size $\sim \Delta R' = f_d R'$ (measured in the jet frame) along the line of sight. Each independent reconnection region has a volume[5] $\sim l'^3$. The total number of subjets within the observed volume is

$$n_{tot} \approx 2 f_d \left( \frac{R'}{l'} \right)^3 = \frac{2}{f_d^2 \beta'^3} \left( \frac{T}{\delta t} \right)^3. \tag{10}$$

The factor of 2 is because each reconnection site produces two subjets moving in opposite directions. Purely from relativistic beaming, each subjet would illuminate a solid angle $\sim \pi/\gamma_j'^2$ in the jet frame. Since there might be additional beam broadening due to intrinsic velocity fluctuations within the subjet, we write the solid angle illuminated by one subjet as $\Omega_j' \equiv \pi f_j/\gamma_j'^2$ with $f_j \geq 1$. Assuming that subjets from different reconnection regions are uncorrelated and are oriented randomly, a given observer receives radiation from a fraction $\sim \Omega_j'/4\pi$ of the subjets. Each visible subjet produces one pulse in the observed light curve. We thus estimate the duty cycle in this model to be:[6]

$$\xi \approx \frac{f_j}{2\gamma_j'^2 f_d^2 \beta'^3} \left( \frac{T}{\delta t} \right)^2 \approx \frac{f_j}{2\beta'} \left( \frac{R'}{\gamma_j' l'} \right)^2. \tag{11}$$

[4] Correspondingly, the shape of the pulse in the light curve will no longer be determined by the onset and decline of the reconnection event, but will be determined by the motion of the jet and will generally be symmetric.

[5] Here we assume that the volume feeding a given reconnection site is roughly spherical in shape of size $l'$, and that the energy in the whole volume is consumed during the reconnection event. If the reconnection site operates intermittently or if the direction of the jet changes with time, a spherical shell rather than the whole sphere provides energy for a given observed event. In this case the perpendicular size could be somewhat larger than $l'$ and the volume is equal to $f_v^2 l'^3$, where $f_v l'$ is the typical dimension in the perpendicular direction. For simplicity, we do not carry through the factor $f_v$ in all the equations, but merely mention its effect in footnotes following equations (11) and (13).

[6] An additional factor of $f_v^{-2}$ appears on the right-hand side of this equation if $f_v \neq 1$.

Using our fiducial numbers for PKS 2155−304, namely $T/\delta t \approx 70$ and $\xi \approx 0.4$, equation (11) gives

$$\frac{\gamma_j'^2 f_d^2 \beta'^3}{f_j} \approx 6000. \tag{12}$$

We showed earlier that $f_d \approx 1$, and by definition we have $f_j \geq 1$. In addition, current understanding of relativistic reconnection suggests that $\beta' \sim 0.1$ (Lyubarsky 2005). Writing $\beta' = 0.1\beta_{-1}'$, we thus find

$$\gamma_j' \approx 2500 f_j^{1/2} f_d^{-1} \beta_{-1}'^{-3/2}. \tag{13}$$

This is an extremely large value,[7] particularly when we recall that $\gamma_j' \sim \sqrt{\sigma}$ (Lyubarsky 2005), where $\sigma$ is the magnetization parameter (Kennel & Coroniti 1984). We require $\sigma$ to be truly enormous, which leads to other problems (see below). Even if we set $\beta' = 1$ ($\beta_{-1}' = 10$), which is unlikely for magnetic reconnection, we obtain $\gamma_j' \approx 80$. The subjets thus need to move highly relativistically with respect to one other and with respect to the mean jet frame.

Extreme values of the parameters, e.g. $T \approx 4000\,s$ (which requires a lower BH mass $M_9 < 0.4$ to maintain $T \geq T_g$), $\delta t \approx 600\,s$ (the longest time-scale consistent with the observations), improve the situation somewhat. With $\beta' = 0.1$, this gives $\gamma_j' \approx 170$, still rather extreme, while with $\beta' = 1$, we obtain $\gamma_j' \approx 5$. Alternatively, we could keep $T$ fixed at $20\,000\,s$ but assume that the BH has a smaller mass than our fiducial value $M_9 = 2$. For instance, if we take $M_9 = 0.5$ and assume $f_c = 1$, we obtain $f_d = 4$. If we further set $f_j = 1$, $\beta' = 1$, then $\gamma_j' \sim 20$.[8]

Note that, in order for relativistic reconnection to efficiently convert magnetic energy into particle thermal energy, we need the inflowing magnetic field lines on the two sides of the reconnection region to be aligned to within an angle $\sim 1/2\sqrt{\sigma}$ (Lyubarsky 2005). Recalling that $\gamma_j' \sim \sqrt{\sigma}$ and that the model requires a large $\gamma_j'$, it appears that any 'jets in a jet' model that is based on magnetic reconnection is more likely to convert magnetic energy into random bulk kinetic energy than into relativistic particles. Thus, the model may be closer in spirit to the relativistic turbulence model discussed in the next subsection.

For our fiducial parameters, namely $T/\delta t \approx 70$ and $\xi \approx 0.4$, equation (10) indicates that there must be $n_{tot} \approx 7 \times 10^8 \beta_{-1}'^{-3}$ independent subjets. Although this number appears to be somewhat large, it is quite likely that whatever causes the subjets phenomenon will operate over a range of scales, and the smallest scales, which presumably determine the observed variability time, could be quite small. In the context of magnetic reconnection, we note that the reconnecting layer is likely to be unstable and to form islands and plasmoids so that the volume associated with a single subjet could be quite small compared to the size of a coherent reconnection zone. This would make the effective value of $l'$ in our analysis very small and correspondingly make $n_{tot}$ large. Note, however, that making $l'$ small does not eliminate the need for a large $\gamma_j'$. Whatever may be the effective size of $l'$, we still expect the variability time-scale to be determined by $l'/\beta' c$. Therefore, the uncomfortably large values of $\gamma_j'$ estimated earlier will survive.

As already mentioned, one key assumption is that the time-scale is determined by physics in the mean frame of the jet. This is what enables us to write the time in terms of the quantity $l'/\beta' c$. In the

[7] If $f_v \neq 1$, this estimate should be divided by $f_v$, and its value could thus be a little smaller.

[8] With these values of the parameters and an additional factor of $f_v \sim$ a few, it is possible to obtain a reasonable solution for the reconnection subjet model. But it requires pushing all parameters to somewhat extreme limits.

magnetic reconnection model with plasmoids described in the previous paragraph, if the plasmoids move relativistically with respect to the mean jet frame and if reconnection on the smallest scales occurs within this moving frame, then it is possible to relax the strong constraint on $\gamma_j'$ described above. Such a model is, however, virtually indistinguishable from the turbulence model discussed next.

## 3.2 Relativistic turbulence

The scenario advocated by Narayan & Kumar (2009; discussed further by Lazar et al. 2009), and which we denote the 'turbulence' model, is one in which relativistic turbulence is generated, possibly because of some MHD instability in the jet fluid. As a result, blobs of fluid move in random directions with a typical Lorentz factor $\gamma_j'$ as measured in the mean jet frame. Each blob is roughly spherical in its own frame, which means it has a transverse size $l'$ and longitudinal size $l'/\gamma_j'$ in the jet frame. At any instant, the radiation from a given blob is focused into a solid angle $\sim \pi/\gamma_j'^2$ in the jet frame. However, over time the beam orientation might wander, so we write the solid angle illuminated by a blob as $\Omega_j' = \pi f_j/\gamma_j'^2$. Narayan & Kumar (2009) suggested that the velocity vector of a blob might wander by about a radian during the time a blob radiates, which corresponds to $f_j \sim \gamma_j'$. However, more general situations, including $f_j \approx 1$, can also be considered (Lazar et al. 2009).

As a turbulent blob changes its direction of motion with time, the observer receives radiation only for the short period that the beam points in the right direction. The fraction of the dynamical time of a blob that is visible to the observer is $\approx 1/f_j$. If this fraction is small, one generically expects to see pulses with a symmetric time profile, as observed in PKS 2155−304.

The duration of a pulse as measured in the jet frame[9] is given by the longitudinal size of a blob, i.e. $\delta t' \approx l'/\gamma_j' c$. Transforming to the observer frame, the observed pulse duration is

$$\delta t \approx \frac{l'}{\Gamma_b \gamma_j' c}. \tag{14}$$

The ratio of the duration of the flaring event to the duration of a single pulse is then

$$\frac{T}{\delta t} \approx f_d \gamma_j' \frac{R'}{l'}. \tag{15}$$

The total number of emitting regions within the observed volume is (there is no additional factor of 2 here since each blob radiates into a single beam)

$$n_{tot} \approx f_d \left(\frac{R'}{l'}\right)^3 = \frac{1}{f_d^2 \gamma_j'^3} \left(\frac{T}{\delta t}\right)^3, \tag{16}$$

and the duty cycle is

$$\xi \approx \frac{f_j}{4\gamma_j'^5 f_d^2} \left(\frac{T}{\delta t}\right)^2 \approx \frac{f_j}{4\gamma_j'} \left(\frac{R'}{\gamma_j' l'}\right)^2. \tag{17}$$

Compared to the subjets model, we see that $\beta' \to \gamma_j'$, with an additional factor of 2 because of the different number of beams per blob in the two models.

Generally, we find that the turbulence model has a much easier time satisfying the observational constraints. For example, with $T/\delta t \approx 70$, $\xi \approx 0.4$, $f_j = 1$, $f_d = 1$, we obtain $\gamma_j' \approx 5$ and $n_{tot} \approx 3000$, which are quite reasonable. If $f_j \approx \gamma_j'$ (as suggested by

Narayan & Kumar 2009), then $\gamma_j' \approx 7$ and $n_{tot} = 800$, which is again acceptable.

The huge difference in the predictions of the subjets and turbulence models can be traced to differences in how the variability time-scale is related to the blob size $l'$ in the two models. In the subjets model, we have $\delta t' \approx l'/\beta' c$, i.e. it is the crossing time of a region of size $l'$, as measured in the mean jet frame, at speed $\beta' c$. In the turbulence model, on the other hand, we have $\delta t' \approx l'/\gamma_j' c$, i.e. it is the crossing time of a region of size $l'$, as measured in the frame of the moving blob, at speed $c$. The factor $\beta' \sim 0.1$ arises in the subjets model because we believe reconnection is limited to a speed substantially below $c$ (Lyubarsky 2005). There is no equivalent factor in the turbulence model, mostly by assumption, since we do not have as complete a physical picture of this model as we do for the subjets model. The second factor $\gamma_j'$ is because of another key difference between the two models. In the subjets model, the time-scale for a reconnection event is determined by physics in the mean frame of the jet, since we assume that the reconnection cell – or a reconnecting plasmoid or island within the reconnection layer – is at rest in this frame. In the turbulence model, on the other hand, everything is determined by physics in the comoving frame of the blob. The net factor of $\gamma_j'/\beta'$ between the two models is quite a large number and this leads to drastic differences in their predictions.

## 4 OPACITY LIMITS

As discussed by Cavallo & Rees (1978), if a source of non-thermal high-energy radiation is spatially more compact than a certain limit, then photon–photon collisions will be very frequent and there will be copious electron–positron pair production. The source will then be opaque to its own high-energy radiation and will be unable to produce the observed non-thermal spectrum. Assuming a spatially homogeneous jet, and assuming that both the observed TeV photons and the soft $\approx 1$ eV photons that provide most of the cross-section for pair production are produced within the jet, Begelman et al. (2008) estimated that the pair production optical depth in PKS 2155−304 is

$$\tau_{\gamma\gamma} \approx 2 \times 10^{10} L_{46} t_{300}^{-1} \Gamma_b^{-6}, \tag{18}$$

where $L_{46}$ is the (isotropic equivalent) luminosity of the jet in units of $10^{46}$ erg s$^{-1}$ and $t_{300}$ is the variability time in units of 300 s.[10] Setting $L_{46} \approx t_{300} \approx 1$, the requirement $\tau_{\gamma\gamma} < 1$ gives $\Gamma_b > 50$.

In the 'jets in a jet' model, the radiation received by the observer during any given pulse in the light curve comes from a single subjet or blob in the source. Since the emitting region moves with respect to the observer with a net Lorentz factor $\approx \Gamma_b \gamma_j'$, one might be tempted to replace $\Gamma_b$ by $\Gamma_b \gamma_j'$ in equation (18) to thereby obtain $\Gamma_b \gamma_j' > 50$. This would loosen the constraint on $\Gamma_b$ by a large factor (at least 5, and potentially several tens). However, the argument is incorrect.[11]

The result given in (18) corresponds to a single radiating region moving relativistically towards the observer. It describes the optical depth for the radiation to escape from its own local emission region. Since there are many emission regions in the 'jets in a jet' model,

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[9] See Lazar et al. (2009) for a discussion of additional relevant time-scales.

[10] The exponent on $\Gamma_b$ in equation (18) depends on the spectrum of the source. Piran (1999) gives $4 + 2\alpha$ instead of 6, where $\alpha$ is the spectral index ($F_\nu \propto \nu^{-\alpha}$). The exponent 6 thus corresponds to $\alpha = 1$, which is a reasonable value for blazars and GRBs.

[11] Giannios, Uzdensky & Begelman (2009) derived a bound $\Gamma_b > 9$ for PKS 2155−304 using what appears to be, apart from a number of details, essentially this incorrect argument.

even after a beam escapes from its original blob, it is likely to encounter other beams of radiation on its way towards the observer. In order for the radiation to reach the observer, we require the net optical depth due to all these encounters to be small. To calculate the corresponding optical depth, it is most convenient to work in the mean jet frame (the primed frame in our notation).

Let us focus on the volume of the jet from which the observer can receive radiation. This volume has a size $R' \times R'$ in the two transverse directions and $\Delta R' = f_d R'$ in the radial direction. Each fluid element in this volume will radiate roughly for a dynamical time $t_{dyn} \approx R'/c$. Multiplying this time by $c$ to convert it to a length, the total four-volume that is visible to the observer is

$$V_{total}^{(4)} \approx f_d R'^4. \tag{19}$$

Consider first the subjets model. Each subjet illuminates a conical volume with a solid angle $f_j \pi / \gamma_j'^2$, and so the average three-volume of the cone is $(f_j \pi / 3 \gamma_j'^2)\langle r^3 \rangle$, where $r$ is the length of the cone inside the reference volume. For $r$ distributed uniformly between 0 and $R'$, the mean three-volume of a subjet is $(f_j \pi / 12 \gamma_j'^2)R'^3$. The time for which a subjet shines (as measured in the jet frame) is $l'/\beta' c$. Multiplying by this factor, and also by the number of subjets $n_{tot}$ given in equation (10), the total four-volume occupied by all subjets is

$$V_{subjets}^{(4)} \approx \frac{f_j \pi}{6 \gamma_j'^2 f_d^2 \beta'^4} R'^3 l' \left(\frac{T}{\delta t}\right)^3. \tag{20}$$

Dividing by $V_{total}^{(4)}$ and making use of equation (11), the fractional four-volume occupied by subjets is

$$f_{subjets} \equiv \frac{V_{subjets}^{(4)}}{V_{total}^{(4)}} \approx \frac{\pi}{3}\xi, \tag{21}$$

i.e. it is roughly equal to the duty cycle of the observed light curve, $\xi \approx 0.4$. This result is not surprising.

A similar calculation can be done for the turbulence model. As before, the solid angle of each beam is $f_j \pi / \gamma_j'^2$, but the duration of the beam in the jet frame is $\approx l'/\gamma_j' c$. Repeating the same steps as above (using equations 16 and 17), we find

$$V_{turbulence}^{(4)} \approx \frac{f_j \pi}{12 \gamma_j'^6 f_d^2} R'^3 l' \left(\frac{T}{\delta t}\right)^3, \tag{22}$$

$$f_{turbulence} \equiv \frac{V_{turbulence}^{(4)}}{V_{total}^{(4)}} \approx \frac{\pi}{3}\xi. \tag{23}$$

The result is the same as for the subjets model.

Before proceeding, let us define a 'reference jet model' which consists of a homogeneous jet with a comoving volume $R' \times R' \times f_d R'$, moving with a Lorentz factor $\Gamma_b$ towards the observer. By construction, the total duration of the observed high-energy radiation from this hypothetical jet is the same as in PKS 2155−304: $T \approx f_d R'/\Gamma_b c$ (see equation 7). We assume that the jet produces the same mean luminosity as that observed in PKS 2155−304. However, being homogeneous, it cannot produce the observed rapid variability.

When applying equation (18) to the reference jet model, we must set the variability time equal to $R'/\Gamma_b c = T/f_d \approx (20\,000/f_d)$ s. As a result, the optical depth and limiting Lorentz factor become (for $L_{46} \approx 1$)

reference jet model : $\quad \tau_{\gamma\gamma} \approx 3 \times 10^8 f_d \Gamma_b^{-6}, \qquad \Gamma_b > 25 f_d^{1/6}.$
$$\tag{24}$$

The larger size of the emitting volume, compared to the model considered by Begelman et al. (2008) where the size was constrained

by the observed variability time of 300 s, leads to a factor of 2 reduction in the minimum bulk Lorentz factor $\Gamma_b$.

Consider now the 'jets in a jet' model. In both the subjets and the turbulence versions of this model, the radiation of the criss-crossing beams occupies a fraction $\approx \xi$ of the total four-volume $V_{total}^{(4)}$. Since the 'jets in a jet' model produces the same average luminosity as the reference jet model, the mean number density of photons in the jet frame must be the same. The only difference is that, in the 'jets in a jet' model, a fraction $\xi$ of the volume is occupied by radiation, and in these regions the local number density is higher than average by a factor $\xi^{-1}$. Any given beam of radiation will intersect many other beams on its way out.[12] A fraction $\xi$ of its path will be through other beams, each of which will on average have a photon number density a factor $\xi^{-1}$ larger than in the reference jet model, and the remainder of its path will be through radiation-free regions. The net result is that each escaping photon will on average interact with exactly the same number of high energy photons as a photon in the reference jet model. Thus, the optical depth to pair production is the same in both models, i.e.

jets in a jet model : $\quad \tau_{\gamma\gamma} \approx 3 \times 10^8 f_d \Gamma_b^{-6}, \qquad \Gamma_b > 25 f_d^{1/6}.$
$$\tag{25}$$

Note that, while the optical depth is the same in both the reference jet model and the 'jets in a jet' model, the latter has the distinction of being able to produce the rapidly varying light curve observed in PKS 2155−304. Interestingly, the model does this without suffering any penalty in the pair production opacity. In fact, the 'jets in a jet' model has a less restrictive limit on the bulk Lorentz factor of the jet, $\Gamma_b \gtrsim 25$, compared to the original limit, $\Gamma_b \gtrsim 50$, obtained by Begelman et al. (2008).

So far we have assumed that the TeV photons as well as the $\approx 1$ eV photons that dominate the pair production cross-section are created within the jet. In the context of our analysis, it does not matter whether the soft photons are produced within individual subjets or are created in the mean frame of the jet. (The latter is favoured since there is no evidence for any correlated variability between the TeV and UV flux.) The result given in equation (25) is valid in either case. However, if the soft photons are produced in the lab frame, e.g. from an accretion disc around the supermassive BH, then there is no constraint on $\Gamma_b$ (Zou, Fan & Piran 2011). Instead we obtain a constraint on the geometry of the jet which we do not explore further.

## 5 IMPLICATIONS AND CONCLUSIONS

The very rapid variability observed in the TeV flux of PKS 2155−304 (Aharonian et al. 2007), which is faster by a factor of 50 than the gravitational time-scale of the central BH, is a remarkable puzzle. Normally, even assuming a relativistic outflow, one does not expect to see variability faster than the gravitational time of the driving engine. PKS 2155−304 violates this expectation by a large factor. A possible way out – apparently the only way out – is to invoke some version of the 'jets in a jet' model. This model was discussed earlier (though this name was not used) in connection with the variability of GRBs (Lyutikov & Blandford 2004; Lyutikov 2006; Narayan & Kumar 2009; Lazar et al. 2009). The same idea has been shown to explain the variability in PKS 2155−304 (Giannios et al. 2009).

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[12] This would not be true if $\xi \ll 1$, but we are working in a different limit where $\xi \sim 1$.

In the 'jets in a jet' model, the fluctuating pulses in the observed radiation are not produced by the entire jet, but by small subregions within the jet. These subregions move relativistically with respect to the mean frame of the jet. As a result, each subregion produces a beam of radiation that is focused tightly in the direction of its motion. Only a few subregions radiate towards the observer, but these appear anomalously bright as a result of relativistic beaming. Thus, the model explains both the large apparent luminosity and the rapid variability, without violating any energy or light-crossing-time constraints.

We have considered in this paper two versions of the 'jets in a jet' model. The first version, the 'subjets' model, assumes that something like magnetic reconnection takes place in many different subregions within the jet fluid and that each reconnection site ejects twin relativistic subjets in opposite directions along the local recombining magnetic field. This model is characteristic of a class of models in which the time-scale of pulses in the observed light curve is determined by a process that is at rest in the frame of the jet. The role of the relativistic subjet is merely to provide a luminosity boost through beaming. The second version, the 'turbulence' model, assumes that some instability in the jet fluid leads to highly relativistic random motions within the medium and as a result the radiation from each subregion is narrowly beamed in the direction of the local motion. This model is characteristic of a class of models in which the observed variability time-scale is determined by a process that takes place in the frame of the relativistically moving subregion. Thus, both the observed variability time-scale and the luminosity are affected by the motion of the subregion (in contrast to the subjets model where only the luminosity is modified). These two models are quite similar in spirit, but they differ in details which become important in the case of extreme objects such as PKS 2155−304.

In the case of the subjets model, using our canonical observational constraints from PKS 2155−304, namely flare duration $T \approx$ 20 000 s, variability time $\delta t \approx 300$ s, pulse duty cycle $\xi \approx 0.4$, reconnection inflow speed $\beta' \approx 0.1$, we find that the model requires the Lorentz factor of the subjets, as measured in the mean jet frame, to be $\gamma_j' \approx 2500$. Given that $\gamma_j' \sim \sqrt{\sigma}$, where $\sigma$ is the magnetization parameter of the jet fluid, this value seems unreasonably large. This requirement becomes less stringent if we assume extreme values for the time-scales, e.g. $T \approx 4000$ s (i.e. setting $T$ equal to the duration of the observations, assuming that the flare started exactly when the observations began) and $\delta t \approx 600$ s (the maximum duration of each pulse; see Aharonian et al. 2007). For this choice, $T/\delta t \approx 7$, $\xi \approx 0.8$, and we find $\gamma_j' \approx 170$.

The crux of the problem in the subjets model is the relatively low fiducial value we assume for $\beta'$ (based on the work of Lyubarsky 2005). The velocity $\beta' c$ is the speed with which magnetized fluid moves in towards the reconnection point. Since this velocity determines the variability time, a low value of $\beta'$ implies a correspondingly small size $l'$ of the reconnection cell. To compensate, $\gamma_j'$ has to be very large so that the small amount of energy available within one reconnection cell is strongly beamed to give the luminosity observed in a single pulse of the light curve.

It is possible that the arguments leading to $\beta' \approx 0.1$ could be circumvented, allowing a reconnection inflow speed close to $c$. It is also possible that some (unknown) mechanism other than reconnection produces the subjets and that this mechanism transports energy to the subjets at the speed of light. However, even if we set $\beta' = 1$, this is still only marginally acceptable. Using fiducial values for the other parameters we find $\gamma_j' \approx 80$. We obtain a reasonable solution only if we choose both a high value of $\beta' \approx 1$ and extreme values of

the time-scales, $T \approx 4000$ s, $\delta t \approx 600$ s. In this case, we find $\gamma_j' \approx 5$. Or, we could choose $\beta' \approx 1$ and instead of changing $T$ and $\delta t$, we could select a smaller BH mass, $M_9 \approx 0.5$, which gives $\gamma_j' \approx 20$.

We thus conclude that the subjets model works only if magnetic reconnection in relativistic plasmas proceeds at the speed of light, much faster than currently thought. This alone may not be sufficient to explain PKS 2155−304. We may also need to push the observational estimates of time-scales or BH mass to extreme values.

The situation in the case of the turbulence model is quite different. In contrast to the subjets model, here, even with fiducial values of parameters, the implied conditions in the jet are quite reasonable. For instance, with $T \approx 20\,000$ s, $\delta t \approx 300$ s, $\xi \approx 0.4$, we find $\gamma_j' \approx 5$–7, $n_{\mathrm{tot}} \approx 10^3$. Such conditions appear quite reasonable for a relativistically moving high-energy source.

The weakness of the turbulence model is that it is short on details. We do not have a physical model of what causes the turbulence and what the limits of this process are. (This is in contrast to the subjets model where we have a very specific process in mind – reconnection – which immediately gives a physical constraint $\beta' \sim 0.1$.) In fact, the word 'turbulence' itself is merely a code for random motions. The actual dynamics may not be truly turbulent in the sense it is normally understood. Certainly, we cannot have hydrodynamic turbulence since motions with $\gamma_j' \approx 5$ are highly supersonic with respect to the maximum allowed sound speed, $c_{s,\mathrm{max}} = c/\sqrt{3}$. In principle, MHD turbulence is compatible with the model since wave speeds can reach up to Lorentz factors $\Gamma_{\mathrm{wave}} \sim \sqrt{\sigma}$, where $\sigma$ is the magnetization parameter. Therefore, with a sufficiently strongly magnetized medium ($\sigma \gg 1$), random motions with $\gamma_j' \approx 5$–7 could be supported.

The heating of particles and the production of radiation is also unexplained in the turbulence model. In the subjets model, reconnection automatically produces relativistic beams of particles and it is not hard to imagine that these particles will radiate in the ambient magnetic field. (Note, however, that the dissipation efficiency in reconnection may be rather low unless the reconnecting fields coming in from the two sides are practically parallel; see Lyubarsky 2005). In the case of the turbulence model, the most obvious candidate for particle heating is a shock, but this is unlikely to work since highly magnetized shocks are very inefficient at particle heating (Kennel & Coroniti 1984; Narayan, Kumar & Tchekhovskoy 2011). Thus, it may be necessary to invoke heating through reconnection inside the turbulent blobs (which would immediately introduce inefficiency through a $\beta'$-like factor, just as in the subjets model). Perhaps the most promising idea is that the particles are heated directly by waves in the turbulent magnetized medium, but the details remain to be worked out.

Summarizing, we find the situation far from clear. The subjets model with magnetic reconnection provides a natural way to produce the required 'jets within a jet'. However, given our current understanding of relativistic reconnection (Lyubarsky 2005), namely that the inflow speed towards the reconnection site can be no larger than a tenth the speed of light, this model has great difficulty explaining the observations in PKS 2155−304. The turbulence model (Narayan & Kumar 2009; Lazar et al. 2009) apparently has no difficulty fitting the data. However, all we have is a broad outline of this model, and there is no physical picture of how the 'turbulence' in this model is produced or how this turbulence heats particles and produces the observed radiation. In the end, we suspect that some version of magnetic instability and eventually energy dissipation via reconnection is probably the solution. However, instead of reconnection behaving as in our static subjets model in which the reconnecting zone is at rest with respect to the mean jet frame,

it probably drives relativistic turbulence and thereby acquires the characteristics of our favoured turbulence model. In other words, the reconnecting volume may itself move relativistically with respect to the jet frame, in which case the scalings become closer to those in the turbulence model.

In Section 4 we analyse the pair production opacity for TeV photons in the 'jets in a jet' model. We show that collisions of beams emitted from different subregions are common and hence the opacity due to these beam–beam collisions is more important than the opacity within a single emitting subregion. Allowing for this effect we find that, regardless of which version of the 'jets in a jet' model we consider, the minimum bulk Lorentz factor $\Gamma_b$ of the jet is $\approx 25$, which is lower than the limit $\approx 50$ obtained by Begelman et al. (2008). Although the reduction in the value of $\Gamma_b$ is only a factor of 2, it is nevertheless a significant revision since the new value is closer to the typical bulk Lorentz factors $\approx 10$–20 found in relativistic jets in other blazars. If the soft photons that provide most of the opacity for pair production are not produced within individual subjets or in the mean jet frame, but are created externally (e.g. in the accretion disc), then these constraints on $\Gamma_b$ are no longer valid.

## ACKNOWLEDGMENTS

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## APPENDIX A: BAYESIAN ESTIMATE OF THE DURATION OF THE FLARE IN PKS 2155−304

Let us suppose that an astronomical source is on when we first start observing it. It remains on for a time $T_{obs}$ and then shuts off. We would like to estimate the total on-time $T$ of the source, including the time it was on before we began observing.

Let us suppose that the source was observed previously a time $T_{max}$ before the current turn-off time and that, at that time, the source was off. Therefore, the minimum and maximum possible on-time of the source are $T_{obs}$ and $T_{max}$, respectively. We wish to evaluate the probability distribution of $T$ between these two limits.

In the absence of other information, the probability distribution of $T$ is logarithmically flat, i.e.[13]

$$P(T)\mathrm{d}T \propto \mathrm{d}\ln T = \mathrm{d}T/T. \tag{A1}$$

For a given value of $T$, the chance that the observed on-time will lie between $T_{obs}$ and $T_{obs} + \mathrm{d}T_{obs}$ is clearly uniform for all values of $T_{obs}$ between 0 and $T$. Thus, for a true on-time $T$, the probability of an observed on-time $T_{obs}$ is simply

$$P(T_{obs}|T)\mathrm{d}T_{obs} = \mathrm{d}T_{obs}/T, \quad 0 \leq T_{obs} \leq T. \tag{A2}$$

This distribution is normalized such that the total integrated probability is unity.

Bayes' theorem states that $P(T|T_{obs})$, the probability that the true duration is $T$, given an observed duration $T_{obs}$, is

$$P(T|T_{obs}) = C\,P(T_{obs}|T)P(T), \tag{A3}$$

where $C$ is a normalization constant. Thus

$$P(T|T_{obs})\mathrm{d}T = (T_{obs}/T^2)\mathrm{d}T, \quad T_{obs} \leq T \leq T_{max}, \tag{A4}$$

where the term $T_{obs}$ in the numerator on the right-hand side takes care of the normalization (assuming for simplicity that $T_{max} \gg T_{obs}$). The probability distribution $P(T|T_{obs})$ peaks at $T = T_{obs}$ (which is reasonable), but it has a long tail extending all the way to $T = T_{max}$. The median and mean values of $T$ are easily calculated from this probability distribution:

$$T_{median} \approx 2T_{obs}, \tag{A5}$$

$$T_{mean} = \ln(T_{max}/T_{obs})\,T_{obs}. \tag{A6}$$

For the particular flare of interest in PKS 2155−304, we have $T_{obs} \approx 4000$ s, so $T_{median} \approx 8000$ s. To compute the mean we need to know $T_{max}$. If the source was observed the night previous to the flare and it was off at that time, then $T_{max} \sim 10^5$ s, and we obtain $T_{mean} \approx 3T_{obs} \approx 12\,000$ s. However, if the previous observation in the off state was as long ago as 2003 September (Aharonian et al. 2005), then $T_{mean} \sim 10T_{obs} \approx 40\,000$ s.

## APPENDIX B: COMPARISON OF NOTATIONS IN DIFFERENT WORKS

Since the present paper draws on a number of earlier works, notably Narayan & Kumar (2009), Lazar et al. (2009) and Giannios et al. (2009), we compare the notations in Table B1.

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[13] Abramowski et al. (2010) have shown that the flux distribution in PKS 2155−304 has a lognormal distribution. Although there is no direct connection, their observations provide additional motivation for the form of $P(T)$ assumed here.

**Table B1.** Comparison of notations

| | This work | Narayan & Kumar (2009) | Lazar et al. (2009) | Giannios et al. (2009) |
|---|---|---|---|---|
| Radial distance from the BH | $R$ | $R$ | $R, R_0$ | $R = rR_{\rm g}$ |
| Overall flare duration | $T$ | $t_{\rm burst}$ | $T$ | – |
| Individual pulse time-scale | $\delta t$ | $t_{\rm var}$ | $\delta t$ | $t_f$ |
| Duty cycle | $\xi$ | $\approx 1$ | $n_{\rm p}$ | – |
| Bulk Lorentz factor | $\Gamma_{\rm b}$ | $\Gamma$ | $\Gamma$ | $\Gamma_{\rm j}$ |
| (Flare duration)/$(R/2\Gamma_{\rm b}^2 c)$ | $f_{\rm d}$ | 1 | $d$ | – |
| Random Lorentz factor | $\gamma_{\rm j}'$ | $\gamma_t$ | $\gamma'$ | $\Gamma_{\rm co}$ |
| Reconnection region size | $l'$ | – | $l' \equiv \psi_{\rm SJ} R$ | $\tilde{l}$ |
| Turbulent blob size | $l'$ | $r_e$ | $l' \equiv \psi R$ | – |
| Solid angle of each beam | $\Omega_{\rm j}' \equiv \pi f_{\rm j}/\gamma_{\rm j}'^2$ | $1/\gamma_t$ | $\phi^2$ | $1/4\Gamma_{\rm co}^2$ |

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